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Solving a new mathematical model for cellular manufacturing system: A fuzzy goal programming approach

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A fuzzy goal programming-based approach is used to solve a proposed multi-objective linear programming model and simultaneously handle two important problems in cellular manufacturing systems, viz. cell formation and layout design. Considerations of intra-cell layout, the intra-cell material handling can be calculated exactly. The advantages of the proposed model are considering machining cost, inter-cell, intra-cell (forward and backward) material handling, operation sequence and resource constraints on the capacity of machines. To illustrate applicability of the proposed model, an example is solved and computational results are noted.

Keywords: *Cell formation, Layout design, Fuzzy goal programming.*

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1. Introduction

Group technology (GT) has emerged as a useful scientific principle in improving the productivity of batch-type manufacturing systems in which many different types of products having relatively low volumes are produced in small lot sizes. Cellular manufacturing (CM) is a successful application of GT concepts. The design of a cellular manufacturing system (CMS) usually begins with two fundamental grouping tasks: part-family formation and machine-cell formation. Part-family formation is to group parts with similar geometric characteristics or processing requirements to take advantage of their similarities for design or manufacturing purposes. Machine-cell formation is to bring dissimilar machines together and dedicate them to the manufacture of one or more part families. Comprehensive summaries and taxonomies of studies devoted to part-family formation and machine-cell formation problems were presented by [17, 23, 31].

As seen in the literature, many authors [6, 10, 15, 16, 19, 33] adopt either a sequential or a simultaneous procedure to group the parts and machines. The sequential procedure used in some of these studies determines the part families first, followed by machine assignments. On the other hand, the simultaneous procedure determines the part families and machine groups concurrently. Heragu and Kakuturi [10] attempted to integrate machine grouping and layout problems. The machine cells are first formed by a heuristic and near-optimal intra-cell and inter-cell layouts are constructed by a hybrid simulated annealing algorithm. Chiang and Lee [6] developed a genetic algorithm based method with optimal partition for cell formation in a bi-directional linear flow layout, where the objective is to minimize the actual inter-cell flow cost, instead of the typical measure that optimizes the number of inter-cell movements. Mahdavi et al. [16] developed a heuristic algorithm based on flow matrix for cell formation and layout design. The objective is to make use of the valuable information about the flow patterns of various jobs in a manufacturing system and obtain relevant performance measures for the cell design and layout problem.

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Some developed models are more realistic and appealing to real-world applications [3, 9, 13, 32, 34] because they take more factors such as demands, processing times, space availabilities, material handling costs, operating costs and machine capacities into considerations. Tsai and Lee [28] proposed mathematical programming (MP) models and developed a multifunctional MP model that included most of the critical features of the cell formation problem. Wu et al. [32] developed a hierarchical genetic algorithm to simultaneously form manufacturing cells and determined the group layout of a CMS. The objective functions of this model are to minimize the total cost of movement (both inter-cell and intra-cell) and exceptional elements (EEs), respectively. Eski and Ozkarahan [8] proposed a hybrid analytic-simulation fuzzy goal programming (FGP) model for the cell formation problem considering stochastic production requirements and alternative routes.

To have a better understanding of the research existing work, we present the most important manufacturing attributes in Table 1. A sample of 9 recently published articles and the corresponding attributes considered in the articles are given in Table 2. The model provides a larger coverage of the attributes than any other existing work.

Table 1. List of manufacturing attributes

Number	Attribute
1	Machine capacity
2	The number of machines
3	Machine operating cost
4	Machine investment cost
5	Inter-cell transfer of parts
6	Intra –cell transfer of parts
(a)	Intra –cell forward transfer of parts
(b)	Intra –cell backward transfer of parts
7	Processing time
8	The sequence of processing parts
9	The demands of parts
10	Layout within cells
11	Cell size limits

Table 2. CMS design attributes for our work and in a sample of recently published articles

Paper/Attributes	1	2	3	4	5	6		7	8	9	10	11	Total attributes
						a	b						
Model proposed in our work	x	x	x	x	x	x	x	x	x	x	x	x	12
Wu et al. [32]	x				x	x	x	x	x	x	x	x	9
Eski and Ozkarahan [8]					x			x	x			x	4
Jayaswal and Adil [12]	x	x	x	x	x			x	x				7
Cao and Chen [5]	x	x			x			x	x			x	6
Solimanpur et al. [25]	x	x	x						x				4
Spiliopoulos and Sofianopoilos [26]		x			x				x				3
Uddin and Shanker [29]	x				x			x	x	x		x	6
Zhao and Wu [36]	x	x			x	x		x	x				6
Vakharia and Chang [30]	x				x			x	x	x		x	6

Here, we propose a multi-objective programming model by assuming inter-cell and intra-cell (forward and backward) material handing with different batch sizes and unit costs. In other words, we assume that the part types are shifted with the different batch sizes and costs between and within cells. Because of the conflicting objectives, i.e., machine costs and material handling costs, we propose a fuzzy goal programming (FGP) approach to solve the extended model.

In the goal programming problem a, the general equilibrium and optimization factors are often conflicting, and we wish to balance. However, determining the goal value of each objective is precisely difficult for the decision maker (DM), since possibly only partial information is known. To incorporate uncertainty and imprecision into the formulation, the FGP is used. FGP works according to the three possible styles of the objective function to generate the results consistent with the DM's expectation. We consider five cost-oriented objectives as minimizing machine's costs (constant and variable), inter-cell material handing cost and intra-cell (forward and backward) material handing cost.

The rest of the paper is organized as follows. In the next section, we describe fuzzy goal programming based approach. We formulate our CMS model as a mathematical model and elaborate on its properties in Section 3. Example and sensitive analysis are given in Section 4 to illustrate the proposed model. Conclusions are given in Section 5.

2. Fuzzy Goal Programming Based Approach

Goal programming (GP) is one of the most powerful multi-objective decision-making approaches in practical decision-making. However, application of GP to the real life problems may face two important difficulties. The first is the mathematical expression of the decision maker's imprecise aspiration levels for the goals and the second is the need to optimize all goals simultaneously. Fuzzy goal programming (FGP) is a mathematical decision-making mechanism to incorporate uncertainty and imprecision into the formulation. In decision-making situations, a high degree of fuzziness and uncertainty are included in the data set.

The FGP problem has been addressed using various methods such as probability distribution, penalty function, fuzzy numbers, preemptive fuzzy goal programming, interpolated membership function, and the weighted additive model. Zimmermann [37] first proposed fuzzy programming for solving the multi-objective linear programming problems. A number of researchers have extended the fuzzy set theory to the field of goal programming proposed by Narasimhan [20]. In fact, the fuzzy goal and multi-objective programming have very extensive applications. Some applications of FGP can be found in [1, 8, 11, 18, 24, 27].

The fuzzy model of a generalized multi-objective multi-constrained optimization problem [35] can be expressed as follows. Consider a problem with the following minimization objectives:

$$Z_l\left(X\right) \le g_1, \qquad l = 1,2,...,b, \tag{1}$$

and the constraints imposed as:

$$d_j(X) \le D_j, \qquad j = 1,2,...,m,$$

where,
l = index of goals
b = number of fuzzy-minimum goal constraints
g_l = goal value or target value for objective l given by DM
X = k- dimensional decision vector
$Z_l(X)$ = goal constraints.

$G = \{X | d_j(X) \le D_j, j = 1,...,m\}$ = system constraints or feasible space.
m = number of system constraints.
Let p_l denote the maximum tolerance limit for g_l as determined by the DM. Thus, using the concept of fuzzy sets, the membership function of the objective functions can be defined as follows [37]:

$$\mu_{Zl}(X) = \begin{cases} 1 & if\ Z_l(X) < g_l \\ 1 - \dfrac{Z_l(X) - g_l}{p_l} & if\ g_l \le Z_l(X) \le g_l + p_l \\ 0 & if\ Z_l(X) > g_l + p_l. \end{cases} \tag{2}$$

The term $\mu_{Zl}(X)$ indicates the desirability of the DM to solution X in terms of the objective l. The corresponding graph of Eq. (2) is shown in Fig. 1.

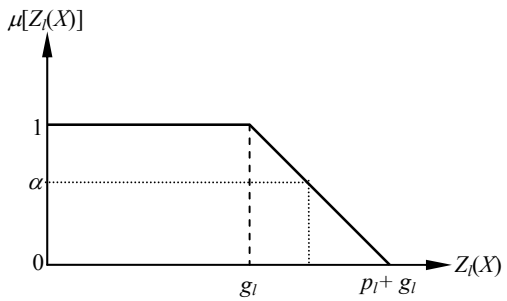

Figure 1. Membership function related to objectives

The α-level sets Z_l^{α} $\forall l$ is defined as:

$$Z_l^{\alpha} = \{X | \mu_{Z_l}(X) \ge \alpha, 0 \le \alpha \le 1\}, \quad l = 1,2,...,b.$$

Then, the decision space is defined as the intersection of the membership functions of objectives (given by Eq. (1)) and system constraints as follows:

$$Z^* = \left\{ \bigcap_{l=1}^{b} Z_l^{a} \right\} \cap G, \qquad 0 \le \alpha \le 1.$$

According to the extension principle, the membership function of Z is defined as follows:

$$\mu_Z(X) = \min_{l=1}^{m} \{\mu_{Zl}(X)\}.$$

Finally, the optimal solution, X^*, must maximize $\mu_Z(X)$ by solving the following mathematical programming model:

$$\begin{aligned} &\max\ \alpha \\ &\text{s.t.} \\ &\quad \alpha \le \mu_{Zl}(X), && l = 1,2,...,b \\ &\quad d_j(X) \le D_j, && j = 1,2,...,m \\ &\quad 0 \le \alpha \le 1. \end{aligned}$$

3. CMS Problem Formulation

Here, a mathematical model for the design of CMS is formulated. The proposed model deals with minimization of the integrated inter and intra-cell (forward and backward) movement costs and the costs of machines.

3.1. Assumptions

The problem is formulated under the following assumptions.
(1) The number of cells is known.
(2) The upper bound and lower bound of the cell size are known.
(3) Each part type has a number of operations to be processed. Operations related to each part type must be processed in the order they have been numbered. Moreover, the sequence of operations is important in the calculation of inter-cell and intra-cell material handling costs since it provides a more accurate calculation of the number of times that a part either has to move between machines of different cells or between machines within the same cell (forward and backward).
(4) The processing times for all operations of part types are known and deterministic.
(5) Parts are moved between and within cells. Inter-cell movement is incurred whenever consecutive operations of the same part type are carried out in different cells. For instance, assume that the operation s of part type i is processed on machine type j in cell k. If the next operation, i.e, $s + 1$, of this part type is processed on any machine but in another cell, then there is an inter-cell movement. The intra-cell movement is incurred whenever consecutive operations of the same part type are processed in the same cell. For instance, suppose the operation s of part type i is processed on machine type j in cell k. If the next operation, i.e., $s + 1$, of this part is processed on any machine within the same cell, then there is an intra-cell movement. To the best of our knowledge, all studies considering this movement have assumed that intra-cell movement occurs between two different machine types [2, 7, 21]. However, in reality, intra-cell movement can occur between same machine types on different locations in one cell. We have considered this situation in our model. Moreover, in the manufacturing systems, a backward movement incurs more expense, and so its cost is assumed greater than a forward movement cost in the proposed model.
(6) All machine types of equal dimension should be located in the locations of cells with straight line type layout.
(7) Parts are moved between cells and within cells in a batch. Inter-cell and intra-cell movements related to each part type have different batch sizes and different costs.
(8) The demand for each part type is given.
(9) Time capacity of each machine type is known.
(10) Machines can have one or more identical duplicates to satisfy capacity requirements and possibly reduce/eliminate inter-cell movements.
(11) The constant cost of each machine type is known and implies maintenance and other overhead costs such as energy cost and general service cost.
(12) Operating cost of each machine type depends on the workload allocated to the machine.

3.2. Indexing Sets

i index for part types (i =1, 2,…, P)
j index for machine types (j=1,2,...,M)
k index for cells (k =1, 2,…, C)
s index for operations pertaining to part type i (s =1, 2,…, S_i)
l index for location of machines (l =1, 2,…, L).

3.3. Parameters

$\gamma^{\text{inter-cell}}$: Material handling cost between cells.

$\gamma_f^{\text{intra-cell}}$: Forward material handling cost within cells.

$\gamma_b^{\text{intra-cell}}$: Backward material handling cost within cells.

$B_i^{\text{inter-cell}}$: Batch size for inter-cell movements of part type *i*.

$B_{fi}^{\text{intra-cell}}$: Batch size for forward intra-cell movements of part type *i*.

$B_{bi}^{\text{intra-cell}}$: Batch size for backward intra-cell movements of part type *i*.

L_k: Lower bound on the number of machines in cell *k*.

U_k: Upper bound on the number of machines in cell *k*.

N_j : Number of machines of type *j* available for allotment to cells.

t_{isj} : Processing time of operation *s* of part type *i* with machine type *j*.

D_i : Demand quantity of part type *i*.

T_j : The capacity of machine type *j*.

C_j : Constant cost of machine type *j*.

α_j: Operating cost of machine type *j* per time unit.

a_{isj} : 1 if operation *s* of part type *i* can be processed on machine type *j*; 0, otherwise

3.4. Decision Variables

X_{isljk} : 1 if operation *s* of part type *i* is done on machine type *j* assigned to location *l* in cell *k*; 0, otherwise.

Y_{jkl} : 1 if machine type *j* is assigned to location *l* in cell *k*; 0, otherwise.

$Z_{isklj'l'}$: 1 if operation *s* of part type *i* is done on machine type *j* assigned to location *l* and operation *s*+1 of part type *i* is done on machine type j' assigned to location l' in cell *k*; 0, otherwise.

3.5. The Mathematical Model

Based on parameters and variables defined above, we now present the CMS model for optimal cell formation and layout design.

3.5.1. The Objective Functions

The objective function is considered to be minimizing the total sum of inter-cell and intra-cell (forward and backward) movement costs and costs of machines. The first term of Eq. (3) computes the total inter-cell movement costs, where S_i-1 indicates the total number of movements of part *i*. It is the sum of the product of the number of inter-cell transfers resulting from both consecutive operation of each part type $\left\lceil D_i / B_i^{inter-cell} \right\rceil$ and the cost of transferring an inter-cell batch of each part type ($\gamma^{\text{inter-cell}}$). Likewise, the second term of Eq. (3) gives the total intra-cell forward movement cost. They are the sum of the product of the number of intra-cell forward transfers

resulting from both consecutive operation of each part type $\left\lceil D_i / B_{fi}^{\text{int}ra-cell} \right\rceil$ and the cost of transferring an intra-cell batch of each part type ($\gamma_f^{\text{intra-cell}}$). The forward travel distances from machines j to j', which are located in locations l and l', have been shown by $(l'-l)$. The third term of Eq. (3) computes the total intra-cell backward movement cost. It is the sum of the product of the number of intra-cell backward transfers resulting from both consecutive operation of each part type $\left\lceil D_i / B_{bi}^{\text{int}ra-cell} \right\rceil$ and the cost of transferring an intra-cell batch of each part type ($\gamma_b^{\text{intra-cell}}$). The backward travel distances from machines j to j', which are located in locations l and l', have been shown in the third term by $(l-l')$. The first term of Eq. (4) calculates the constant cost of all machines required for cells. Likewise, the second term of Eq. (4) stands for the operating cost of machines. Therefore, the objective functions of the proposed model are written as:

$$\begin{aligned}\text{Min } Z_1 = {} & \gamma^{\text{int}er-cell} \times \sum_{i=1}^{P}\left[\left\lceil \frac{D_i}{B_i^{\text{inter-cell}}}\right\rceil \times \left[(S_i-1)-\left(\sum_{k=1}^{C}\sum_{s=1}^{S_i-1}\sum_{j=1}^{M}\sum_{j'=1}^{M}\sum_{l=1}^{L}\sum_{l'=1}^{L} Z_{isljl'j'}\right)\right]\right] \\ & +\gamma_f^{\text{int}ra-cell}\left(\sum_{i=1}^{P}\sum_{k=1}^{C}\sum_{s=1}^{S_i-1}\sum_{j=1}^{M}\sum_{j'=1}^{M}\sum_{l=1}^{L}\sum_{l'=l+1}^{L}\left\lceil \frac{D_i}{B_{fi}^{\text{intra-cell}}}\right\rceil (l'-l) Z_{isljl'j'}\right) \\ & +\gamma_b^{\text{int}ra-cell}\left(\sum_{i=1}^{P}\sum_{k=1}^{C}\sum_{s=1}^{S_i-1}\sum_{j=1}^{M}\sum_{j'=1}^{M}\sum_{l'=1}^{L}\sum_{l=l'+1}^{L}\left\lceil \frac{D_i}{B_{bi}^{\text{intra-cell}}}\right\rceil (l-l') Z_{iskljkl'j'}\right)\end{aligned} \tag{3}$$

$$\text{Min } Z_2 = \sum_{k=1}^{C}\sum_{j=1}^{M}\sum_{l=1}^{L} C_j Y_{jkl} + \sum_{k=1}^{C}\sum_{i=1}^{P}\sum_{s=1}^{S_i}\sum_{j=1}^{M}\sum_{l=1}^{L} \alpha_j D_i t_{isj} X_{isljk} \,. \tag{4}$$

3.5.2. The Constraints

$$\sum_{j=1}^{M}\sum_{l=1}^{L} Y_{jkl} \geq L_k \qquad \forall k \tag{5}$$

$$\sum_{j=1}^{M}\sum_{l=1}^{L} Y_{jkl} \leq U_k \qquad \forall k \tag{6}$$

$$\sum_{k=1}^{C}\sum_{l=1}^{L} Y_{jkl} \leq N_j \qquad \forall j \tag{7}$$

$$\sum_{j=1}^{M} Y_{jkl} \leq 1 \qquad \forall k, l \tag{8}$$

$$\sum_{j=1}^{M}\sum_{k=1}^{C}\sum_{l=1}^{L} X_{isljk} a_{isj} = 1 \qquad \forall i, s \tag{9}$$

$$\sum_{i=1}^{P}\sum_{s=1}^{S_i} X_{isljk} t_{isj} D_i \leq T_j \qquad \forall j, k, l \tag{10}$$

$$Z_{isljl'j'} \geq X_{isljk} + X_{is+1,l'j'k} - 1 \qquad \forall s = 1 \ldots S_{i-1}, i, j, j', l, l', k \tag{11}$$

$$X_{isljk}, Y_{jkl}, Z_{isklj'l'} \in \{0,1\} \qquad \forall i, j, s, k, l. \tag{12}$$

Inequalities (5) and (6) ensure the lower and upper bounds considerations for the number of machines to be allocated to locations of each cell. Inequalities (7) ensure that the number of machines available for a given type is not bypassed. Inequalities (8) ensure that each machine can be allocated to only one location of each cell at most. Constraints (9) guarantee that each operation will be assigned to a cell which contains the required machine type. Inequalities (10) ensure that the workload of each machine will not exceed its capacity. Constraints (11) ensure that variable $Z_{isklj'l'}$ is 1 if both variables X_{isljk} and $X_{is+1,l'j'k}$ are 1. Constraints (12) dictate decision variables to be binary.

4. Computational Results

To verify the behavior of the proposed model, an example is presented to illustrate applicability of the proposed model. The example is generated according to the information given in Table 3. It consists of eight part types (P_1, P_2,…, P_8), and six machine types (M_1, M_2,…, M_6), where each part type is assumed to have a number of operations that must be processed respectively as numbered in the order and the processing time as shown in the parentheses. For simplicity, the capacity of all the machines in all the problems is the same (i.e., 1550 hour/period). Table 4 shows the other input parameters for solving the above problem. The computational experiments are carried out with a branch-and-bound (B&B) method in the LINGO 8 software package.

Table 3. The typical data set

M \ P	1	2	3	4	5	6	7	8	N_j	C_j	α_j	T_j
1	0	2 (0.44)	0	3 (0.72)	2 (0.58)	0	3 (0,57)	0	2	600	7	1550
2	1 (0.31)	1 (0.33)	2 (0.63)	0	2 (0.62)	1 (0.22)	4 (0.5)	2 (0.3)	2	900	6	1550
3	1 (0.4)	0	1 (0.52)	3 (0.65)	0	2 (0.52)	0	3 (0.47)	2	750	5	1550
4	0	0	1 (0.41)	1 (0.37)	0	2 (0.45)	1 (0.25)	3 (0.5)	2	700	7	1550
5	2 (0.51)	2 (0.52)	3 (0.4)	0	1 (0.61)	3 (0.35)	0	1 (0.47)	2	600	8	1550
6	0	0	0	2 (0.53)	2 (0.7)	0	2 (0.28)	2 (0.4)	2	800	9	1550
D_i	600	475	250	625	375	700	575	350				
$B_i^{inter-cell}$	20	18	15	23	15	25	18	15				
$B_{fi}^{intera-cell}$	8	6	5	7	5	8	6	5				
$B_{bi}^{intera-cell}$	13	10	9	13	9	14	10	9				

Table 4. Parameter setting model

Parameter	Cell I	Cell II
L_k	2	2
U_k	4	4
Forward intra-cell movement unit cost	4	
Backward intra-cell movement unit cost	10	
Inter-cell movement unit cost	30	

For implementation of the FGP approach, let g_1 and g_2 be the goal values of objectives 1 and 2, respectively. Also, p_1 and p_2 be the maximum tolerance related to objectives 1 and 2, respectively. For obtaining g_1 and g_2, two sub-problems, i.e., Prob.1 and Prob.2 with individual objectives 1 and 2 must be solved as follows:

Prob. 1

$$\text{Min } Z_1$$
$$\text{s.t.}$$
$$\text{Constraints (5)-(12),}$$

Prob. 2

$$\text{Min } Z_2$$
$$\text{s.t.}$$
$$\text{Constraints (5)-(12),}$$

where, after solving, we obtain: Z_1= 5464 and Z_2=50242.
The maximum tolerance values are determined by the DM as p_1= 800 and p_2= 1500. Thus, the membership functions for objectives 1 and 2 are obtained as follows:

$$\mu_{Z_1}(x)=\begin{cases} 1 & if\ \ Z_1(X)<5464 \\ 1-\left((z_1(X)-5464)/800\right) & if\ \ 5464\le Z_1(X)\le 6264 \\ 0 & if\ \ Z_1(X)>6264 \end{cases}$$

$$\mu_{Z_2}(x)=\begin{cases} 1 & if\ \ Z_2(X)<50242 \\ 1-\left((z_2(X)-50242)/1500\right) & if\ \ 50242\le Z_2(X)\le 51742 \\ 0 & if\ \ Z_2(X)>51742. \end{cases}$$

To obtain the optimal solution, the following program must be solved:

$$\max \alpha$$
$$\text{s.t.}$$
$$(5)\,\text{to}\,(14)\,\text{and}$$
$$\alpha \le 1 - \frac{Z_1(X) - 5464}{800}$$
$$\alpha \le 1 - \frac{Z_2(X) - 50242}{1500}$$
$$0 \le \alpha \le 1.$$

The obtained results related to the optimal solution are shown in Table 5. Also, optimal configuration, i.e., formed cells, is shown in Table 6, respectively. Considering the given tolerances, material flow cost is obtained to be \$5464 and machine cost is obtained \$51142 under the aspiration level 0.4.

Table 5. Optimal solution

Parameter	Z_1	Z_2	α
Value	5464	51142	0.4

Table 6. The optimal cell configuration

		MACHINES						
		3	2	5	4	6	1	2
PARTS	1	0	1	2	0	0	0	0
	3	1	2	3	0	0	0	0
	5	0	2	1	0	0	0	0
	6	2	1	3	0	0	0	0
	8	3	2	1	0	0	0	0
	2	0	0	0	0	0	2	1
	4	0	0	0	1	2	3	0
	7	0	0	0	1	2	3	4

Table 7 shows a sensitive analysis on the p_1 and p_2 parameters. As shown in Table 7, by decreasing p_2, material handling (Z_1) increases, because limitation of the machine's costs (P_2) must be recovered by material handling cost. In other words, removing a machine is equivalent to adding inter-cell movement with less construct cost. For instance, in configuration related to problem p_1= 800, p_2= 1100 (Table 7), the machine type two is removed in cell 2 and operation 1 of part 2 and operation 4 of part 7 are done by machine 2 in cell one in Table 8. Likewise, comparing tables 6 and 8, an additional unit machine type of 2 is allocated to cell 2 is removed. According to Table 8, the constant cost of machine 2 is less than the one in Table 6. Therefore, the two inter- cell movements occur in Table 8 because operation 1 of part 2 and operation 4 of part 7 are allocated to machine 2 in cell 1. As a result, the decreasing p_2 is recovered by use of the material handling cost.

Table 7. Sensitive analysis on maximum tolerance values

Tolerances		Aspiration Level	Z_1 (\$)			Z_2 (\$)	
p_1 (\$)	p_2(\$)	α	Z_{11}	Z_{12}	Z_{13}	Z_{21}	Z_{22}
800	1500	0.4	0	3408	2230	5250	45892
800	1300	0.31	0	3408	2230	5250	45892

800	1100	0.25	1770	2892	1400	4350	45892

The proposed model is computationally complex as it integrates the cell formation and layout design problems. The cell formation problem (CFP) itself is an NP-hard problem [4, 14]. In addition to CFP, layout of machines within cells has also been considered in the proposed model. The latter problem is also known as an NP-hard problem [22]. Therefore, the proposed model here is NP-hard, since it integrates two NP-hard problems. Due to this fact, it is necessary to develop a heuristic or metaheuristic approach to solve the proposed model for large-sized problems.

Table 8. The optimal configuration for (p_1= 800, p_2= 1100)

		MACHINES					
		3	2	5	4	6	1
PARTS	1	0	1	2	0	0	0
	3	1	2	3	0	0	0
	5	0	2	1	0	0	0
	6	2	1	3	0	0	0
	8	3	2	1	0	0	0
	2	0	1	0	0	0	2
	4	0	0	0	1	2	3
	7	0	4	0	1	2	3

5. Conclusions

We proposed a fuzzy goal programming (FGP) approach to solve a bi-objective cellular manufacturing system by assuming operation sequence, inter/intra cell material handling with different batch sizes and costs, and machine capacities. Because of the existing conflict between the two main objectives in cell formation problem, i.e., machine cost and material flow cost, the fuzzy goal programming can be an efficient approach for achieving a desirable solution from the decision maker point of view. FGP considers a membership function for each objective by a goal value obtained from the single-objective sub-problems and a tolerance value given by the DM. We solve a comprehensive example and show that the proposed approach can determine the optimal cellular configuration with the aim of maximizing aspiration level and with respect to the given tolerance values provided by the DM. Obviously, the cellular configuration may be changed whenever the tolerance values are changed. The advantages of the proposed model are:

- Designing a linear mathematical model which contains two important problems (cell formation and machine layout) in CMS, simultaneously.
- Calculation of forward and backward intra-cell material handling costs by considering the operation sequence and the distance between the locations assigned to machines.
- Calculation of the cost of intra-cell material handling between same machine types in different locations accurately.
- Investigation of influence of material handeling costs and machine costs on each other in the bi-objective mathematical model.

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