A Fuzzy Mixed-integer Goal Programming Model for Determining an Optimal Compromise Mix of Design Requirements in Quality Function Deployment

S. Rahimi¹, M.M. Lotfi²*, M.H. Abooie³

Quality function deployment is a well-known customer-oriented design procedure for translating the voice of customers into a final production. This is a way that higher customer satisfaction is achieved while the other goals of company may also be met. This method, at the first stage, attempts to determine the best fulfillment levels of design requirements which are emanated by customer needs. In real-world applications, product design processes are performed in an uncertain and imprecise environment, more than one objective should be considered to identify the target levels of design requirements, and the values of design requirements are often discrete. Regarding these issues, a fuzzy mixed-integer linear goal programming model with a flexible goal hierarchy is proposed to achieve the optimized compromise solution from a given number of design requirement alternatives. To determine relative importance of customer needs, as an important input data, we apply the well-known fuzzy AHP method. Inspired by a numerical problem, the efficiency of our proposed approach is demonstrated by several experiments. Notably, the approach can easily and efficiently be matched with QFD problems.

Keywords: Quality function deployment, Design requirements, Optimization, Fuzzy sets, Goal programming.

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1. Introduction

Global competitiveness has become a big concern for both manufacturing and service companies demanding a high quality in their products/services (Karsak et al. [19]). They require some techniques such as Quality Function Deployment (QFD) to improve the quality of their products/services and satisfy their customers’ needs at a high level (Cherif et al. [12]). QFD, as a widely used customer-driven method in product development and quality engineering, is a systematic process for translating the voice of customers into a final product in various stages (Chen and Weng [10]). Now, companies are successfully using QFD as a powerful tool for making strategic and operational decisions. QFD starts from marketing research and identification of customers. Then, arriving at the analysis process, it attempts to recognize the customer needs (CN) and involve them in the design and production stages. The concept of QFD was introduced in Japan by Akao [1] and described in detail by Revelle et al. [30]. The Kobe shipyards of Mitsubishi heavy industries was the first company which implemented QFD in 1972 (Kim et al. [21]).

Akao [1] defines QFD as “a method for developing a design quality aimed at satisfying the customer and then translating the customers’ demands into design targets and major quality assurance points to be used throughout the production phase”. The primary functions of QFD include product development, quality management and customer needs analysis. Recently, these functions have been

*Corresponding Author.
¹MSc student, Department of Industrial Engineering, Yazd University, Yazd, Iran, Email: rahimisamira67@yahoo.com.
²Department of Industrial Engineering, Yazd University, Yazd, Iran, Email: lotfi@yazd.ac.ir.
³Department of Industrial Engineering, Yazd University, Yazd, Iran, Email: mhabooie@yazd.ac.ir.
extended to some areas such as design, planning, decision making, engineering, management, team work, timing and costing. Moreover, QFD has increasingly been applied to transportation and communication, electronics and electrical utilities, software systems, manufacturing, services, education and research, and many other industries including aerospace, construction, packaging and textile (Chan and Wu [5]). The main objectives of QFD are to reduce the length of product development cycle to improve the quality and to minimize the total production process costs (Kim et al. [21]). As Tseng and Torng [34] report, QFD, if it is appropriately applied, can decrease the development time by one-half down to one-third. For example, Toyota and its suppliers, by using QFD, were able to reduce the start up production costs by 60% and the development time by one-third.

QFD employs four sets of matrix diagrams that resemble connected houses; the first one that is related to the product design stage is the house of quality (HOQ) for transforming the CNs into the design requirements (DRs), a description of product in the language of engineers, which is our concern here. The HOQ has six sections: a CNs section, a competitive assessment section, a DRs section, a relationship matrix, a trade-off matrix, and a target values section (Park and Kim [29]). The objective of HOQ is to determine target levels of a product’s DRs for maximizing the customer’s satisfaction. The major problem with HOQ is that the CNs which tend to be subjective, qualitative, and nontechnical, have to be translated into DRs that should be expressed in the quantitative and technical terms. But, QFD team members usually have established the relationships between CNs and DRs and among the DRs themselves subjectively based on the past experience. Therefore, the process of quantifying such naturally subjective planning issues in HOQ using various types of mathematical programming and corresponding solution techniques has received ever-increasing attention during the past decade. In this regard, a summary of the relevant and supportive body of literature will be reviewed in Section 2.

In spite of the favorable quantitative research work till now, QFD still experiences several limits in applications especially in forming a desired HOQ and thus, it can be improved. This is important since a poor HOQ commonly leads to either failure of the product in market or extended product development time and cost. Bouchereau and Rowlands [4] report some problems concerning the QFD technique such as ambiguity in the voice of customer, need to input and analyze large amounts of subjective data, impreciseness in the process of setting target values in HOQ. It necessitates that some kind of fuzziness is taken when resolving such problems in the data preparation, formulation and analysis of QFD.

Furthermore, in the product design process, there are some limitations on the required resources such as time, cost, etc. Because of the multi-dimensional competition, manufacturers should focus not only on the product quality but also on the trade-off between the quality and the other resource constraints. As a result, they need optimization models and techniques to establish a set of DRs for a product maximizing the customer’s satisfaction under given resource constraints (Ting [33] and Lai et al. [22]). Meanwhile, some authors (e.g., Karsak [18]) have recently notified that similar to most real-world applications, the need for establishing a reasonable trade-off between the multiple conflicting objectives in the process of quantifying the HOQ matrix engages us in a multi-objective environment. However, in practice, when realistic multi-objective decision problems are considered, providing crisp definitions of goal priorities/importances is not an easy task. In fact, from decision maker (DM)’s point of view, an inherent uncertainty or vague perception may be latent in priorities of the goals. Moreover, the decision space and correlation between the objectives may also affect the definition of importance relations among the goals.
Briefly, in our work here, a new QFD optimization approach based on fuzzy goal programming concept is suggested. We propose a fuzzy mixed-integer linear goal program to optimize the compromise solution of a given number of DR alternatives. By using the fuzzy analytic hierarchy process (AHP) method, we provide the relative importance of CNs, as a critical input of our model. Moreover, a flexible pre-emptive goal hierarchy is applied to take simultaneously the fuzzy goals and constraints as well as the uncertain hierarchical levels of the fuzzy goals into account. Applying fuzzy set theory into the goal program has the advantage that the DM can express vague aspirations as well as priorities by some types of natural language terms. The remainder of our work is organized as follows. In Section 2, we give a short literature review on QFD optimization studies. In Section 3, the proposed FMILGP model and its equivalent crisp version are provided. Section 4 is devoted to computational and analytical results. Finally, concluding remarks are given in Section 5.

2. Literature Review

Linear programming (LP) is a well-known method which has recently been applied to finding the best set of DRs. Askin and Dawson [3] present an LP model for determining the optimal set of DRs based on customer’s preferences. Fung et al. [15] formulate a linear QFD planning model to maximize the overall customer’s satisfaction in which attainment of DRs is optimized by allocating resources among them. Lai et al. [23] propose a QFD model using a linear physical programming technique to optimize the overall customers’ satisfaction in product design. Moskowitz and Kim [26] develop a decision support system based on an LP formulation to help finding the best set of DRs.

In the above-reviewed studies, it is assumed that the values of DRs can be any point in a continuous range while they are often considered discrete in real-world applications. For example, in reality, there are no light bulbs with the powers of 57 or 133 watts; but, 25, 60 or 100 watts. In such cases, dynamic programming or mixed integer linear programming (MILP) models are suggested (Lai et al. [22]). Wasserman [36] develops a 0-1 integer programming model to optimize the product design problem under certain resource constraints. Park and Kim [29] introduce a quadratic integer programming model in which the correlations between DRs are also incorporated through some cost constraints. Delice and Güngör [13] propose a QFD approach combined with an MILP formulation and the Kano model in order to obtain the optimized solution from a certain set of DRs. Lai et al. [22] develop a dynamic programming approach for the QFD optimization problem. They first suggest an extended HOQ to gather more information. Next, limited resources are allocated to DRs using dynamic programming, and the target level of each DR is optimized.

The QFD designers believe that the product design process is actually performed in an uncertain environment. First, customer's preferences are inherently imprecise and more-or-less vague. Second, the relationships between CNs and DRs and also among DRs themselves are qualitatively identified and stated by linguistic terms which should be translated into corresponding numerical scales. This is intensified when developing an entirely new product for which engineers do not have perfect knowledge concerning the impact of engineering characteristics on CNs. In this regard, the use of AHP is favorable because AHP uses a hierarchical structure and enables DM to define high levels strategic objectives and specific metrics for a better assessment of strategic alignment (Kendrick and Saaty [20]). Narasimhan [28] enumerates two advantages of AHP including, (1) facilitating an accurate judgment through systematically formalizing and rendering a subjective decision process and (2) providing information about the implicit weights of evaluation criteria. Another advantage of AHP is that it results in a better communication, leading to a clearer understanding and consensus among members of decision-making groups.
One useful tool for dealing with imprecision as well as vagueness involved in an AHP method is fuzzy set theory. Fuzzy AHP technique was used by researchers to synthesize the opinions of the DM to identify the weight of each criterion. Fuzzy AHP approach has shown to be a convenient method in tackling practical multi-attribute problems due to its capability to capture the vagueness of human thinking and to aid in solving the research problem through a structured manner and by a simple process (Tseng and Lin [35]).

Fung et al. [14] suggest a fuzzy rule-base inference model to facilitate the decisions on the target values of DRs. Based on a fuzzy technical importance rating of DRs, Chen and Weng [9] formulate a fuzzy LP model to find the fulfillment levels of DRs and create a high level of customer’s satisfaction. Kahraman et al. [17] propose an integrated framework based on fuzzy QFD and MILP formulation to determine the DRs to be considered in designing a product. The coefficients of the objective function are obtained from a fuzzy analytic network process approach. Also, fuzzy AHP is used to determine three matrices representing the impact of the CNs on each DR, the inter-dependency of the CNs and the inter-dependency of the DRs, respectively. Chen and Ko [8] employ the same constraints as those in Chen and Weng’s model [9] and present a fuzzy nonlinear model to determine the performance level of DRs to maximize customer’s satisfaction. Unlike the existing research, they apply the Kano model to classify DRs into three categories based on their importance to customer’s satisfaction. Fung et al. [16] develop a fuzzy nonlinear programming formulation of QFD planning under imprecise costs and some technical constraints in which the design budgets are also involved. Chen et al. [11] propose a fuzzy regression-based LP model to determine the optimal set of DRs in which the relationship between CNs and DRs and the correlation among the competitors are simulated in a fuzzy frame. Tang et al. [32] develop fuzzy optimization models including the financial considerations along with a genetic-based interactive solution approach to determine target values of DRs in QFD. Luo et al. [25] propose a methodology involving a market survey, fuzzy clustering, QFD and fuzzy optimization to achieve the optimal target settings of DRs of a new product in a multi-segment market.

In the existing research work, a mixture of DRs was determined considering only a single objective; i.e., maximizing the overall customer’s satisfaction. However, in general, the satisfaction of CNs is not the only goal in product design, but the other criteria such as cost, development time, technical difficulty, and extendibility also need to be involved. In this regard, we need to use a variant of multi-objective programming or multi-attribute decision-making approaches. Kim et al. [21] present a fuzzy multi-attribute LP model for QFD planning in which the DRs of the product are considered as the attributes. In this manner, they formulate a multi-objective optimization model in order to find the target values of DRs (attributes) to maximize the overall customer satisfaction. Among various existing multi-objective optimization approaches, goal programming (GP), originated by Charnes et al. [7], is one of the most powerful and well-applied tools used for modeling, solving and analyzing real-world problems that address multiple conflicting objectives for which the appropriate target values are assigned by a DM. In classical GP models, unwanted deviations from target values defined by the decision maker are minimized in order to reach an acceptable solution.

Karsak et al. [19], by incorporating three goals including cost, extensibility level and manufacturability level, present a 0-1 weighted GP model combined with analytic network process to determine the DRs for the product design. The results show that cost budget goal has the highest weight while extensibility and manufacturability goals are in lower ranks, respectively. Sener and Karsak [31], by considering cost budget, extendibility and technical difficulty in addition to customer’s satisfaction, use a fuzzy regression to estimate the relationships between CNs and DRs, and among DRs themselves. They assign the importance degrees of “very high”, “high”, and
“medium” to the overall customer satisfaction, extendibility, and technical difficulty goals, respectively. Karsak [18] proposes a fuzzy multi-objective programming model to determine the level of fulfillment of DRs that incorporates the inherent imprecise and subjective information in the QFD planning process. In the model, fulfillment of DRs and extendibility are the objectives to be maximized, whereas technical difficulty is to be minimized. Chen and Weng [10] formulate a fuzzy goal programming model to determine a mix of DRs to produce the maximal sum of satisfaction degrees of all the goals (i.e., customer’s satisfaction, cost expenditure and technical difficulty). There, customer’s satisfaction and cost expenditure goals are given a higher priority level than technical difficulty goal.

We believe that, a practical and suitable form of GP in the area of QFD planning is pre-emptive GP, because firms usually have a pre-emptive priority for achieving goals that are not addible even though in the form of a weighted additive function. In fact, a DM may find determining priority levels more straightforward than determining precise weights for the goals. Consequently, a deviation from a higher priority level goal is considered to be infinitely more important than a deviation from a lower priority goal. However, determining precisely target values as well as priorities of the goals, as done in a traditional pre-emptive GP, is also a difficult task along with some errors and shortcomings. In the case of a pre-emptive GP, each goal is set to a certain predefined priority level. A series of mathematical programming problems are solved sequentially, first considering highest priority goals only, and then continuing with lower priority ones, under the constraints imposed by the alternative optimal solutions of the problems including higher priority goals. In fact, a traditional pre-emptive GP model may be unrealistic, because it assumes infinite trade-offs between different levels of goal hierarchy. Moreover, the corresponding sequential techniques may cut-off some interesting parts of the solution space. In order to specify the imprecise target levels in an uncertain environment, fuzzy goal programming (FGP) approach was introduced by Narasimhan [27]. Recently, Aköz and Petrovic [2] have proposed a novel flexible goal hierarchy to implement pre-emptive GP in a fuzzy framework. At first, they formulate the imprecise importance relations between the goals via fuzzy binary relations in order to substitute the existing hard goal hierarchy with a flexible one. Thereafter, a new achievement function is defined as a convex combination of the sum of achievement degrees of the fuzzy goals and satisfaction degrees of the imprecise importance relations between them. In the achievement function, a parameter which is specified by DM adjusts the trade-off between relative priority relations and achievement degrees of fuzzy goals.

Accordingly, here, we propose a new fuzzy mixed-integer linear goal programming (FMILGP) model to optimize a compromise solution of a limited number of DR alternatives. Different from the other studies in the literature, (a) we practically consider different pre-emptive priorities for the three conflicting fuzzy goals using a pre-emptive GP, (b) in the proposed FMILGP model, sum of achievement degrees of fuzzy goals and sum of satisfaction degrees of relationships between them are simultaneously considered through maximization of an appropriate convex combination objective function, (c) fuzzy set theory is employed to resolve vagueness and linguistic characteristics of (1) relative importance of CNs by fuzzy AHP and (2) relationships between CNs and DRs as well as among DRs themselves. Notably, we, as done in other existing researches, apply fuzzy theory to address natural vagueness in providing goals’ target levels as well as inherent ambiguities existing in some critical parameters, say cost budget and development time of DR alternatives.

3. Problem Formulation

In this section, the proposed FMILGP model for QFD optimization problem is formulated. In order to validate our model, we implement it to optimize a washing machine development problem taken from Delice and Güngör [13]. Firstly, HOQ is constructed to represent the information gathered
about the development problem (see Figure 1). Secondly, the proposed FMILILGP model is used to find a combination of optimal values of DRs of the washing machine for maximizing the overall customer’s satisfaction. In the considered problem, the five CNs are “Thorough washing”, “Quiet washing”, “Thorough rinsing”, “No damage to clothes” and “Short washing time” while the five DRs are “washing quality (%)”, “noise level (db)”, “washing time (minutes)”, “rinsing quality (%))” and “clothes damage rate (%))”.

3.1. Preparing HOQ

We first determine the relationships among DRs. As mentioned before, determining precisely the relationships among DRs is usually a difficult task. We use linguistic terms, as defined in Table 1, for this purpose. For each DR, three alternative values are self-made according to the specifications of the washing machine problem. Noteworthy, these alternative values are introduced arbitrarily and a given DM can use his/her own alternative settings. These alternatives together with their cost budgets and development times in the form of triangular fuzzy numbers (TFNs) are presented in Figure 1. TFNs have extensively been used in the related literature due to their various advantages including intuitiveness, simplicity in data acquisition, and computational efficiency (see Figure 2). It is worse to note that most possible values of TFNs for cost budgets are defined based upon the values implemented in Delice and Güngör [13], but those for the development times are set by authors according to the type and values of DR alternatives, since the development time was not considered in [13]. Thereafter, most pessimistic and most optimistic values of TFNs are, as usual, predicted by the following equations:

\[ C_{jr}^P = C_{jr}^m - 0.1 * C_{jr}^m * (1 + Rand(0,1)) \]  
\[ C_{jr}^O = C_{jr}^m + 0.1 * C_{jr}^m * (1 + Rand(0,1)) \]  
\[ t_{jr}^P = t_{jr}^m - 0.1 * t_{jr}^m * (1 + Rand(0,1)) \]  
\[ t_{jr}^O = t_{jr}^m + 0.1 * t_{jr}^m * (1 + Rand(0,1)) \]

where \( C_{jr} = (C_{jr}^P, C_{jr}^m, C_{jr}^O) \) and \( t_{jr} = (t_{jr}^P, t_{jr}^m, t_{jr}^O) \) are the TFNs for cost budget and development time of \( r \)th alternative of \( DR_r \), respectively. The TFNs are then converted to their equivalent crisp values by using the expected value operator proposed by Liu and Liu [24].

To estimate the relative importance (i.e., weight) of CNs in HOQ, we use the well-known fuzzy AHP approach. Since human knowledge on relative importance of CNs is, in essence, imprecise and vague, we apply a fuzzy AHP method to incorporate such uncertainty into AHP formulation. Chang [6] introduces a new approach based on the extent analysis method to handle the pair-wise comparison scale of fuzzy AHP. The first step in this method is to use TFNs for pair-wise comparison by means of the proposed scale. Next, the extent analysis method is employed to obtain priority weights via synthetic extent values. Fuzzy evaluation matrix of all the criteria is constructed through a pair-wise comparison of different attributes relevant to the overall objective using linguistic variables and TFNs. In order to run the fuzzy AHP method, we ask four experts to state relative importance of each CN via linguistic terms presented in Table 2 by appropriate TFNs.
A Fuzzy Mixed-integer Goal Programming Model

Figure 1. HOQ for washing machine development problem

Table 1. Linguistic terms for relationships between CNs and DRs and among DRs

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>TFN</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>(0,0,0)</td>
</tr>
<tr>
<td>Very weak</td>
<td>(0,0.1,0.2)</td>
</tr>
<tr>
<td>Weak</td>
<td>(0.1,0.3,0.5)</td>
</tr>
<tr>
<td>Moderate</td>
<td>(0.4,0.5,0.6)</td>
</tr>
<tr>
<td>Strong</td>
<td>(0.5,0.7,0.9)</td>
</tr>
<tr>
<td>Very strong</td>
<td>(0.8,0.9,1)</td>
</tr>
</tbody>
</table>

Table 2. Linguistic terms for fuzzy AHP

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>TFN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equally significant</td>
<td>(1,1)</td>
</tr>
<tr>
<td>A little significant to significant</td>
<td>(2,3)</td>
</tr>
<tr>
<td>Significant</td>
<td>(3,4.5)</td>
</tr>
<tr>
<td>Very significant</td>
<td>(4,5,6)</td>
</tr>
<tr>
<td>Very significant to completely significant</td>
<td>(5,6,7)</td>
</tr>
<tr>
<td>Completely significant</td>
<td>(7,7)</td>
</tr>
</tbody>
</table>

Let \( X = \{ x_1, x_2, \ldots, x_n \} \) and \( G = \{ g_1, g_2, \ldots, g_m \} \) be an object set and a goal set, respectively. The extent analysis values for each goal \( g_i \) (i.e., \( \tilde{M}_{g_i}^{1}, \tilde{M}_{g_i}^{2}, \ldots, \tilde{M}_{g_i}^{m} \), \( i = 1, 2, \ldots, n \)) are computed where all the \( \tilde{M}_{g_i}^{j} \), \( j = 1, 2, \ldots, m \) are TFNs. Then, the following steps are sequentially performed:
Step 1. The value of fuzzy synthetic extent with respect to the $i$th object is defined:

$$S_i = \sum_{j=1}^{m} \tilde{M}^j_{i} \otimes \left[\sum_{l=1}^{n} \tilde{M}^l_{j}\right]^{-1}.$$  

(5)

Step 2. The degree of possibility of $\tilde{M}_2 \equiv (M^P_2, M^m_2, M^o_2) \geq \tilde{M}_1 \equiv (M^P_1, M^m_1, M^o_1)$ is defined:

$$V(\tilde{M}_2 \geq \tilde{M}_1) = \text{height}(\tilde{M}_1 \cap \tilde{M}_2) = \begin{cases} 1, & \frac{M^m_2 - M^o_1}{(M^P_2 - M^o_2) - (M^P_1 - M^o_1)}, \\ 0, & \frac{M^m_2 - M^o_1}{(M^P_2 - M^o_2) - (M^P_1 - M^o_1)}, \end{cases}$$  

(6)

Step 3. The degree of possibility for a convex fuzzy number being greater than $k$ convex fuzzy numbers $\tilde{M}_i, i = 1, 2, ..., k$, is defined:

$$V(\tilde{M} \geq \tilde{M}_1, \tilde{M}_2, ..., \tilde{M}_k) = \text{Min}(V(\tilde{M} \geq \tilde{M}_i)) \quad i = 1, 2, ..., k$$  

(7)

Assuming that $A_i, i = 1, 2, ..., n$, are $n$ elements and $d'(A_i) = \text{Min}(V(\tilde{M}_i \geq \tilde{M}_k)), k = 1, 2, ..., n$, the weight vector is given by $W' = (d'(A_1), d'(A_2), ..., d'(A_n))^T$.

Step 4. The normalized weight vector is as $W = (d(A_1), d(A_2), ..., d(A_n))^T$ where $W$ is a vector of crisp numbers.

According to the above method, the weight of CNs are calculated and reported as in Figure 1. It is confirmed that the most and least important CNs are “No damage to clothes” and “Quiet washing”, respectively.

Another important input required to form the HOQ matrix is the normalized relationships between CNs and DRs (see Figure 1). Since the mentioned relationships are usually vague, in practice, we again employ the linguistic terms presented in Table 1 to estimate the fuzzy relationships among CNs and DRs. Afterwards, linguistic terms are translated into corresponding TFNs and the well-known method of Wasserman [36], which accounts also for the correlations among DRs, is applied to calculate fuzzy normalized relationships between CNs and DRs. Accordingly, the normalized relationships are calculated based upon the following equation:

$$R_{ij}^{\text{norm}} = \frac{\sum_{k=1}^{N} \text{EV}(\tilde{R}_{ik} \cdot \tilde{Y}_{kj})}{\sum_{j=1}^{M} \sum_{k=1}^{N} \text{EV}(\tilde{R}_{ik} \cdot \tilde{Y}_{kj})}, \quad i = 1, ..., M, \quad j = 1, ..., N,$$  

(8)

where $R_{ij}^{\text{norm}}$ denotes the normalized relationship between $i$th CN and $j$th DR, $\tilde{R}_{ik}$ is the TFN for relationship between $i$th CN and $k$th DR, $\tilde{Y}_{kj}$ is the TFN for relationship among $k$th and $j$th DR, $M$ is the number of CNs, and $N$ is the number of DRs. In (8), we use the well-known linear approximation of the multiplication of two TFNs (i.e., $\tilde{R}_{ik} \cdot \tilde{Y}_{kj}$) as given below:

$$\tilde{R}_{ik} \cdot \tilde{Y}_{kj} = \left(R_{ik}^P \cdot \tilde{Y}_{kj}^P, R_{ik}^m \cdot \tilde{Y}_{kj}^m, R_{ik}^o \cdot \tilde{Y}_{kj}^o\right).$$  

(9)
Then, in order to transform the resulting TFN of the multiplication into an equivalent crisp value, the expected value operator (Liu and Liu [24]), named EV in (10) below, is applied. Thus, the expected value of TFN $\tilde{A}$ is defined as follows:

$$EV(\tilde{A}) = \frac{A^p + 2A^m + A^o}{4} \quad (10)$$

Notably, for $i$th CN, we have $\sum_{j=1}^{N_j} R_{ij}^{norm} = 1$. In Subsection 3.2, we develop our FMILGP model according to the prepared HOQ.

**Figure 2. Membership function of TFN $\tilde{p}$**

### 3.2. Proposed FMILGP Model

Hereafter, we define the sets, parameters and decision variables used to formulate the FMILGP model.

#### Sets

- **CN**: Set of CNs (CN index: $i=1,2,...,M$)
- **DR**: Set of DRs (DR index: $j=1,2,...,N$)
- **Alt$_j$**: Set of alternatives for $j$th DR ($j\in$DR, Alt$_j$ index: $r=1,2,...,I_j$)

#### Parameters

- $W_i$: Relative importance (weight) of $i$th CN
- $R_{ij}^{norm}$: Normalized relationship between $i$th CN and $j$th DR
- $C_{jr}$: Cost of $r$th alternative of $j$th DR
- $t_{jr}$: Development time of $r$th alternative of $j$th DR
- $d_i$: Minimum satisfaction level (%) of $i$th CN
- $CS$: Desired target level of customer’s satisfaction goal
- $CO$: Desired target level of cost budget goal
- $DT$: Desired target level of development time goal

#### Decision variables

- $B_{jr}$: 1, if alternative $r$ of $j$th DR is selected, 0, otherwise ($j\in$DR, $r\in$Alt$_j$)
\[ y_i \quad \text{Level of satisfaction for } i\text{th CN} \]
\[ x_j \quad \text{Fulfillment level of } j\text{th DR} \]

Now, our model can be presented as follows:

\[
\text{Goal 1: } \sum_{j=1}^{N} \sum_{r=1}^{l_j} \bar{c}_{jr} B_{jr} \leq CO \tag{11}
\]
\[
\text{Goal 2: } \sum_{i=1}^{M} w_i y_i \geq CS \tag{12}
\]
\[
\text{Goal 3: } \sum_{j=1}^{N} \sum_{r=1}^{l_j} \bar{c}_{jr} B_{jr} \leq DT \tag{13}
\]
\[
y_i = \frac{\sum_{j=1}^{N} R_{ij} x_j}{\sum_{j=1}^{N} R_{ij}}, \quad \forall i \in \text{CN} \tag{14}
\]
\[
x_j = \frac{1}{\sum_{i=1}^{M} \left( \sum_{r=1}^{l_i} r B_{jr} \right)}, \quad \forall j \in \text{DR} \tag{15}
\]
\[
\sum_{r=1}^{l_j} B_{jr} = 1, \quad \forall j \in \text{DR} \tag{16}
\]
\[
y_i \geq d_i, \quad \forall i \in \text{CN} \tag{17}
\]
\[
0 \leq x_j, y_i \leq 1, \quad \forall j \in \text{DR}, \forall i \in \text{CN} \tag{18}
\]
\[
B_{jr} \in [0,1], \quad \forall j \in \text{DR}, \forall r \in \text{Alt}_j. \tag{19}
\]

Constraint (11) states that the total expenses should not be more than CO’s target level as much as possible. Constraint (12) ensures that the customer’s satisfaction level is preferred to be greater than or equal to the corresponding target level. In constraint (13), we try to prevent the violation of DT’s target value as much as possible. Eq. (14) determines the level of satisfaction for each CN. Eq. (15) guarantees that the fulfillment level of each DR is determined according to the selected alternative. Eq. (16) states that for each DR, only one alternative should be selected. Constraint (17) ensures that the customer’s satisfaction for each CN is greater than \( d_i \).

In order to address different priorities of the three considered goals, a two-level pre-emptive FGP model could be proposed in which customer’s satisfaction goal along with the cost budget goal are placed in the first priority level, whereas the development time goal is involved at the second level. However, on one hand, we cannot precisely specify that how much the goals at the first priority level are more important than the one at the second level. On the other hand, as stated before, such a traditional pre-emptive GP model assumes infinite trade-offs between the different levels of the goal hierarchy. So, in the next subsection, we suggest a flexible goal hierarchy to be able to efficiently incorporate the imprecise priorities into the FGP model.

### 3.3. Flexible Goal Hierarchy

In order to convert our FMILGP model into its equivalent crisp version, we apply a flexible goal hierarchy to form an appropriate achievement function. In this method, the goal importance levels are imprecisely defined and represented by fuzzy relations with appropriate membership functions. Different linguistic terms can be used to express fuzzy importance relations such as “slightly more importance than”, “moderately more important than”, “significantly more important than”, and so on. The achievement function is defined as the sum of achievement degrees of all the goals and degrees of satisfaction of the relative importance relations among them. As the first priority level goals in the considered problem are naturally more important than the second one, we use the linguistic term “significantly more important than” in order to denote the importance relation between the fuzzy goals in the two levels of goal hierarchy. Suppose that \( \mu_i \) and \( \mu_j \) are achievement degrees of the fuzzy goals \( i \) and \( j \), respectively. Then, the membership function of fuzzy binary relation which says that goal \( i \) is “significantly more important than” goal \( j \) (i.e., \( \mu_{R(i,j)} \)) is shown in Figure 3 and expressed by...
A Fuzzy Mixed-integer Goal Programming Model

\[ \mu_i - \mu_j \geq \mu_{R(i,j)}, \quad \mu_{R(i,j)} \geq 0 \]  

**Figure 3.** Membership function of \( \mu_{R(i,j)} \) for “significantly more important than”

Consequently, to formulate the above importance relation between the two levels of goal hierarchy, we should subject the equivalent crisp model to the following constraints:

\[ \mu_1 = \frac{U_1 - \sum_{j=1}^{N} \sum_{r=1}^{I_j} c_j B_{jr}}{U_1 - CO} \]  

\[ \mu_2 = \frac{(\sum_{i=1}^{M} w_i) L_2}{CS - L_2} \]  

\[ \mu_3 = \frac{U_3 - \sum_{j=1}^{N} \sum_{r=1}^{I_j} t_j B_{jr}}{U_3 - DT} \]  

\[ \mu_1 - \mu_3 \geq \mu_{R(1,3)} \]  

\[ \mu_2 - \mu_3 \geq \mu_{R(2,3)} \]

Where \( L_k \) and \( U_k, k=1, 2, 3 \), represent the admissible tolerances of the considered fuzzy goals, \( \mu_k \), \( k=1, 2, 3 \), is the achievement degree of fuzzy goal \( k \), \( \mu_{R(1,3)} \) and \( \mu_{R(2,3)} \) are the achievement degrees of fuzzy importance relations in the form of goal 1-goal 3 and goal 2-goal 3, respectively. The desired target levels of fuzzy goals as well as their admissible violations for the considered washing machine problem are presented in Table 3. Noteworthied, these values were selected by authors according to the conditions and specifications of the problem.

<table>
<thead>
<tr>
<th>Table 3. Goal’s target levels and their admissible violations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal 1- Cost budget</td>
</tr>
<tr>
<td>( CO )</td>
</tr>
<tr>
<td>14</td>
</tr>
</tbody>
</table>

Accordingly, the achievement function of the crisp model is considered as a convex combination of the sum of achievement degrees of fuzzy goals and the sum of satisfaction degrees of imprecise importance relationship. In this function, \( 0 \leq \lambda \leq 1 \), specified by DM, adjusts the trade-off between the two terms of the aggregated objective function. The smaller the value of \( \lambda \), the more important
satisfying the flexible priority of fuzzy goals while the larger the value of \( \lambda \), the more important achieving the target levels of fuzzy goals.

Now, the equivalent crisp version of our FMILGP model can be formulated as follows:

\[
\begin{align*}
\text{Max } z &= \lambda \left( \sum_{k=1}^{3} \mu_k \right) + (1 - \lambda) \left( \mu_{R(1,3)} + \mu_{R(2,3)} \right) \\
\text{s.t.} \quad &\text{Constraints (14)-(19), (21)-(25)} \\
&\mu_1, \mu_2, \mu_3 \leq 1 \\
&\mu_{R(1,3)}, \mu_{R(2,3)} \geq 0
\end{align*}
\]  

(26)  

(27)  

(28)

With the proposed flexible priority structure, DM may simultaneously account for hierarchical levels of goals and quantify the importance of goals. Moreover, the computational efficiency of the resolution procedure could be enhanced since the two-level pre-emptive structure of the goals is aggregated in a single formulation.

4. Computational Results

In this section, we implement the proposed FMILGP approach to select the best set of alternatives among the possible combinations of DR alternatives in the given washing machine development problem. The crisp model was developed by the GAMS modeling language and solved by the CPLEX solver on a computer with 2.4 GHz processor and 1 GB RAM. The results including the decision variables as well as achievement degrees of fuzzy goals and satisfaction degrees of fuzzy binary relations, when changing the adjusting parameter \( \lambda \) in the range \([0,1]\), are presented in Table 4.

<table>
<thead>
<tr>
<th>( \lambda ) Time (s)</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>( y_3 )</th>
<th>( y_4 )</th>
<th>( y_5 )</th>
<th>( \mu_1 )</th>
<th>( \mu_2 )</th>
<th>( \mu_3 )</th>
<th>( \mu_{R(1,3)} )</th>
<th>( \mu_{R(2,3)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.83,1] 0.159 1.00 0.667 0.333 1.00 1.00 0.883 0.667 0.876 0.897 0.791 0.500 0.993 0.314 0.186 0.680</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.83,1] 0.152 1.00 0.667 0.667 1.00 0.667 0.845 0.667 0.845 0.841 0.823 0.658 0.785 0.372 0.286 0.413</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As observed, the selected value of DR alternatives for \( \lambda \in [0,0.83] \) is 96%, 46 db, 39 min, 85%, and 0.6% for washing quality, noise level, washing time, rinsing quality, and clothes damage rate, respectively. The result for \( \lambda \in [0.83,1] \) is 96%, 46 db, 36 min, 85%, 0.8%, which means that the washing time and clothes damage rate get poorer values. As an expected outcome, the solution obtained with a higher \( \lambda \) value has a higher sum of achievement degrees of fuzzy goals whereas by decreasing \( \lambda \), the importance relations are weighted more. In other words, customer’s satisfaction goal gets a higher achievement degree, while achievement degree of the development time goal tends to decrease. The DM can establish a suitable trade-off between the solutions with a higher sum of the achievement degrees and those which may be more interesting in terms of better importance relations among the goals. It is worth noting that although the cost budget goal is also in the first priority level, the cost budget goal tends to decrease when reducing the development time goal. This is due to the data preparation stage in which the given cost budgets of DR alternatives have no inconsistency with the corresponding development times.

As observed, although it may always not be so, the considered problem is not very sensitive to value of \( \lambda \) so that changing \( \lambda \) in the range \([0,0.83]\) does not alter the outcomes. Accordingly, in the following experiments, equal weights are assigned to relative priority relations and achievement degrees of fuzzy goals, i.e., \( \lambda = 0.5 \).
The manager would naturally like to show how the other two goals (i.e., customer’s satisfaction and development time) do change when target level of the cost budget goal (i.e., \( CO \) and \( U_1 \)) are given different values in a certain range. Therefore, \( CO \) value is gradually increased from 13.5 (the least value for achieving the minimum customer’s satisfaction) to 16.5 (the most possible value) by increments of 0.5, while its admissible tolerance is fixed to 15\%. Results are given in Table 5 and Figures 4 to 5. Notably, the \( U_1 \) value for the last three records of Table 5 is assumed to be 17.6 (i.e., cost of the most expensive combination of DR alternatives), meaning that the corresponding admissible tolerances are inevitably less than 15\%. As confirmed by Table 5 and Figure 4, customer’s satisfaction (\( \sum_{i=1}^{M} w_i y_i \)) does not constantly improve when increasing the cost budget. In other words, although the customer’s satisfaction tends to improve when the cost budget is increased to 15.5, it starts reduction after the point. The reason for this trend is that we fix the desired target level of customer’s satisfaction goal (CS) to 0.9, and therefore, the obtained value for \( \mu_2 \) will be greater than 1 if the customer’s satisfaction goal for these values of cost budget can exceed 0.9. On the other hand, as shown in Figure 5, development time (\( \sum_{j=1}^{N} \sum_{r=1}^{I_j} t_{jr} B_{jr} \)) is not sensitive to cost budget up to 16, but, after this point, we will see an erratic trend. Accordingly, it seems that if we change the cost budget in the range \([14, 15.5]\), the other two goals will also be in their appropriate conditions simultaneously.

### Table 5. Sensitivity analysis for \( CO \) target values

<table>
<thead>
<tr>
<th>CO</th>
<th>( U_1 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( \sum_{i=1}^{M} w_i y_i )</th>
<th>( \sum_{j=1}^{N} \sum_{r=1}^{I_j} t_{jr} B_{jr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.5</td>
<td>15.4</td>
<td>1.000</td>
<td>0.667</td>
<td>0.667</td>
<td>1.000</td>
<td>0.667</td>
<td>0.842</td>
<td>19.070</td>
</tr>
<tr>
<td>14.0</td>
<td>16.0</td>
<td>1.000</td>
<td>0.667</td>
<td>0.333</td>
<td>1.000</td>
<td>1.000</td>
<td>0.884</td>
<td>19.216</td>
</tr>
<tr>
<td>14.5</td>
<td>16.5</td>
<td>1.000</td>
<td>0.667</td>
<td>0.333</td>
<td>1.000</td>
<td>1.000</td>
<td>0.884</td>
<td>19.216</td>
</tr>
<tr>
<td>15.0</td>
<td>17.1</td>
<td>1.000</td>
<td>0.667</td>
<td>0.333</td>
<td>1.000</td>
<td>1.000</td>
<td>0.884</td>
<td>19.216</td>
</tr>
<tr>
<td>15.5</td>
<td>17.6</td>
<td>1.000</td>
<td>0.667</td>
<td>1.000</td>
<td>1.000</td>
<td>0.667</td>
<td>0.899</td>
<td>19.305</td>
</tr>
<tr>
<td>16.0</td>
<td>17.6</td>
<td>0.667</td>
<td>1.000</td>
<td>0.667</td>
<td>1.000</td>
<td>1.000</td>
<td>0.859</td>
<td>19.300</td>
</tr>
<tr>
<td>16.5</td>
<td>17.6</td>
<td>0.333</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.832</td>
<td>18.835</td>
</tr>
</tbody>
</table>

**Figure 4.** Customer’s satisfaction versus cost budget target values
For better understanding of the effect of changing CO values on customer’s satisfaction, as the most critical goal of QFD, we ran a single objective MILP in which customer’s satisfaction is assumed to be the only objective function while development time as well as cost budget are posed as constraints. To do this, the right-hand side of the development time constraint (as it has the least important in reality) is fixed at 20.5 (the maximum possible value) in all the runs, whereas the CO value of budget constraint is gradually increased in different runs from the minimum up to the maximum possible value. The results for the objective function are given in Table 6. A comparison of the table with the previous one confirms the efficiency of the proposed multi-objective GP model in establishing a reasonable trade-off between several conflicting goals. In fact, reaching satisfaction levels of more than 90% in the single objective model is only possible when providing and expending significantly more values of the cost budget and development time, while the multi-objective GP model tries to establish a compromise between the three goals since cost budget and development time are actually considered as criteria not constraints in the design optimization. Consequently, through the proposed FGP approach, significant budget and time savings are obtainable in lieu of a little reduction in customer’s satisfaction. This is really an appreciated step in a comprehensive optimization of product design and development. Obviously, the same discussion can be performed for the fulfillment level of DRs. The proposed multi-objective approach is thus closer to reality than the single objective one assuming that both criteria are constraints.

<table>
<thead>
<tr>
<th>CO</th>
<th>x₁</th>
<th>x₂</th>
<th>x₃</th>
<th>x₄</th>
<th>x₅</th>
<th>( \sum_{i=1}^{M} w_i y_i )</th>
<th>( \sum_{j=1}^{N} \sum_{r=1}^{l_j} t_{jr} B_{jr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>infeasible</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>14.0</td>
<td>1.000</td>
<td>0.667</td>
<td>0.333</td>
<td>1.000</td>
<td>0.667</td>
<td>0.785</td>
</tr>
<tr>
<td>3</td>
<td>14.5</td>
<td>1.000</td>
<td>0.667</td>
<td>1.000</td>
<td>0.333</td>
<td>1.000</td>
<td>0.785</td>
</tr>
<tr>
<td>4</td>
<td>15.0</td>
<td>1.000</td>
<td>0.667</td>
<td>0.333</td>
<td>1.000</td>
<td>1.000</td>
<td>0.884</td>
</tr>
<tr>
<td>5</td>
<td>15.5</td>
<td>1.000</td>
<td>0.667</td>
<td>0.333</td>
<td>1.000</td>
<td>1.000</td>
<td>0.884</td>
</tr>
<tr>
<td>6</td>
<td>16.0</td>
<td>1.000</td>
<td>0.667</td>
<td>0.667</td>
<td>1.000</td>
<td>1.000</td>
<td>0.941</td>
</tr>
<tr>
<td>7</td>
<td>16.5</td>
<td>1.000</td>
<td>0.667</td>
<td>0.667</td>
<td>1.000</td>
<td>1.000</td>
<td>0.941</td>
</tr>
</tbody>
</table>
In order to assess the impact of considering fuzziness to capture the involved uncertainties as well as efficiency of the flexible goal hierarchy, we compare the results of fuzzy model with those of deterministic one. To do this, we formulate and solve the deterministic form of proposed model through a traditional two-level pre-emptive MILGP in which the first priority level goals are assumed to be infinitely more important than the second one. To provide the input data for the deterministic model, the most possible values of fuzzy parameters are applied. Also, a crisp AHP approach, rather than the fuzzy one, is used to determine the relative importance of CNs. The normalized relationships among CNs and DRs are also studied in certain environment. The corresponding two-level pre-emptive MILGP is formulated as follows:

**First level:**

\[
\begin{align*}
\text{Min} & \quad \frac{d_1}{\max\ (\text{cost})} + d_2 \\
\text{s.t.} & \quad \text{Constraints (14)-(19)} \\
& \quad \sum_{j=1}^{N} \sum_{r=1}^{I_j} c_{jr}B_{jr} - d_1 = CO \\
& \quad (\sum_{i=1}^{M} w_i y_i) + d_2 = CS \\
& \quad d_1, d_2 \geq 0,
\end{align*}
\]

where \(d_2\), in the objective function of first level, is a dimensionless quantity. So, to normalize and appropriate for the sum, we divide \(d_1\) by \(\max\ (\text{cost})\). Also, we assume that both goals in (29) belonging to the first priority level are of the same importance while it may not be so.

**Second level:**

\[
\begin{align*}
\text{Min} & \quad d_3 \\
\text{s.t.} & \quad \text{Constraints (14)-(19)} \\
& \quad \sum_{j=1}^{N} \sum_{r=1}^{I_j} t_{jr} B_{jr} - d_3 = DT \\
& \quad \sum_{j=1}^{N} \sum_{r=1}^{I_j} c_{jr}B_{jr} \leq CO + d_1 \\
& \quad (\sum_{i=1}^{M} w_i y_i) \geq CS - d_2 \\
& \quad d_3 \geq 0,
\end{align*}
\]

where \(d_k\), \(k=1, 2, 3\), is the unwanted deviation from the desired target level of goal \(k\).

The above two models must be sequentially solved. At first, the mathematical model related to the first level is solved to find the best values of unwanted deviations from the corresponding goals’ aspiration levels, i.e., \(d_1\) and \(d_2\). Thereafter, these values are imposed to the model of the second level through constraints (35) and (36). The solution of the second level is used as the final solution of the MILGP model. The deterministic results compared to the fuzzy ones with \(\lambda=0.5\) are presented in Table 7. As observed in the last three columns of Table 7, although the customer’s satisfaction in the fuzzy model is slightly (1%) less than the one in the crisp model, the obtained values for cost budget and development time objectives in the fuzzy environment are significantly better than those in a deterministic condition. Moreover, fuzzy solution is naturally more robust than the deterministic one, because the former is optimized according to the support of all fuzzy parameters. In fact, the fuzzy solution remains valid when the uncertain parameters accept their values in a wide range due to a disorder in the real condition, whereas the deterministic solution must be validated repeatedly by a sensitivity analysis.
Table 7. A comparison between deterministic and fuzzy results

<table>
<thead>
<tr>
<th></th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
<th>y1</th>
<th>y2</th>
<th>y3</th>
<th>y4</th>
<th>y5</th>
<th>[N \sum_{j=1}^{1} \sum_{r=1}^{I_j} c_{j,r} B_{j,r}]</th>
<th>[N \sum_{i=1}^{M} w_i y_i]</th>
<th>[N \sum_{j=1}^{I_j} \sum_{r=1}^{t_{j,r}} B_{j,r}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>1.00</td>
<td>0.67</td>
<td>0.67</td>
<td>1.00</td>
<td>1.00</td>
<td>0.91</td>
<td>0.67</td>
<td>0.92</td>
<td>0.92</td>
<td>0.88</td>
<td>15.800</td>
<td>0.892</td>
<td>19.500</td>
</tr>
<tr>
<td>Fuzzy</td>
<td>1.00</td>
<td>0.67</td>
<td>0.33</td>
<td>1.00</td>
<td>1.00</td>
<td>0.88</td>
<td>0.67</td>
<td>0.88</td>
<td>0.90</td>
<td>0.79</td>
<td>15.000</td>
<td>0.884</td>
<td>19.216</td>
</tr>
</tbody>
</table>

5. Concluding Remarks

We developed a new fuzzy mixed-integer linear goal programming model to determine an optimal solution from a given set of alternatives of design requirements in QFD. In the proposed model, we aimed to maximize an aggregate function of the achievement degrees of three conflicting objectives including cost, customer’s satisfaction and development time. We proposed a flexible goal hierarchy in which the sum of achievement degrees of fuzzy goals and satisfaction degrees of the priority relations between them were taken simultaneously. Also, to determine relative importance of each customer’s need, a fuzzy AHP approach was proposed. The results inspired by a washing machine development problem in the literature were compared with both single objective and hard pre-emptive GP models. By several experiments, it was shown that the proposed approach could be useful for QFD planning process. With the proposed flexible priority structure, decision maker may take into account hierarchical levels of goals and quantify importance of the goals at the same time. The proposed method depends on two critical inputs including: (1) target values, admissible tolerances and relative importance of the fuzzy goals, and (2) controllable parameter \( \gamma \). In fact, this approach can easily be matched with the characteristics of various QFD problems by using the other goals’ target values and admissible violations, displacing the goals across the hierarchy or changing the linguistic terms for their relative importance, or applying other proper values of \( \gamma \).

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References


