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A Fuzzy Multi-objective Linear Programming Approach for Solving a New Multi-objective Job shop Scheduling with Sequence-dependent Setup Times

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We present a new mathematical model for a bi-objective job shop scheduling problem with sequence-dependent setup times to minimize the weighted mean completion time and the weighted mean tardiness time simultaneously. For solving this multi-objective model, a fuzzy multi-objective linear programming (FMOLP) approach is developed with respect to the overall acceptable degree of the decision maker (DM) satisfaction. Finally, a numerical example is worked through to demonstrate the feasibility in applying the proposed model to job shop scheduling with set up times problem. The proposed model could obtain an effective solution and an overall degree of decision maker satisfaction with the determined objective values.

***Keywords**: Scheduling, Job shop, Sequence-dependent setup times, Multi-objective linear programming, Fuzzy multi-objective linear programming.*

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1. Introduction

Scheduling problem is the allocation of resources to perform a set of activities in a period of time [2].Job shop scheduling problem (JSS) is one of the most complicated combinatorial problems. A JSS problem could be described as follows: we have a set of n jobs need to be operated on a set of m machines [2]. Each job has its own processing route; that is, jobs visit machines in different orders. Each job might need to be performed only on a fraction of m machines, not all of them. The following assumptions are additionally characterized. Each job can be processed by at most one machine at a time and each machine can process at most one job at a time. When the process of an operation starts, it cannot be interrupted before the completion; that is, the jobs are non-preemptive. There is no transportation time between machines; in other words, when an operation of a job finishes, its operation on subsequent machine can immediately begin.

In many real-life situations such as chemical, printing, pharmaceutical, and automobile manufacturing, the setup operations, such as cleaning up or changing tools, are not only often required between jobs but they are also strongly dependent on the immediately preceding process on the same machine [21] (sequence dependent). The consideration of sequence-dependent setup times (SDST) is gaining increasing attention among researchers in recent years. The motivation behind this assumption is to obtain tremendous savings when setup times are explicitly included in scheduling decisions [2]. With respect to the corresponding explanation, we take into account SDSTs in our problem.

A lot of studies have been made on scheduling problems with very different conditions of work and criteria. Most research is dedicated to single-criterion problems. For example, the approximate

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approaches including simulated annealing (SA) [4], tabu search [14, 15] and genetic algorithm (GA) [4, 26] proposed for minimizing the makespan. Roshanaei et al. [17] presented a VNS method to solve job shop scheduling problem (JSP) with sequence-dependent setup times with respect to minimizing the makespan. Naderi et al. [13] used an SA method to solve job- shop scheduling problems with sequence-dependent set-up time that minimizes the makespan. Naderi et al. [12] considered job shop scheduling problem with dependent setup time and preventive maintenance that minimizes the makespan. They proposed two techniques to integrate production planning and preventive maintenance problems. Tavakkoli-Moghaddam et al. [23] presented a hybrid method of SA and electromagnetic- like mechanism to solve job shop scheduling problems with sequence-dependent setup times and availability constraint with respect to minimizing the total weighted tardiness.

It is well known that the optimal solution of single-objective models can be quite different from the models consisting of multiple objectives. However, real world production systems require simultaneous achievement of multiple objective requirements. This means that the academic concentration of objectives in job shop scheduling problem (JSP) must be extended from single to multiple. Recent work on JSP with multiple objectives is summarized as below.

Skawa and Kubota [19] developed a fuzzy programming method for solving job shop scheduling problem with fuzzy processing times and fuzzy due date with a genetic algorithm. Thiagarajan and Rajendran [25] considered a dynamic assembly job shop scheduling problem with some dispatching rules and compared the obtained results with after simulation. Their objective functions are to minimize the sum of the weighted tardiness/earliness and weighted flow time of all the jobs. Low et al. [10] developed a mathematical model for multi-objective job shop scheduling to minimize the total job flow time, total job tardiness, and machine idle time. They considered sequence-dependent setup times and re-entrant operations in their model. At first, they used integer programming to solve the problem according to each objective individually. Then, they employed a multiple-decision-making technique to evaluate three objectives simultaneously. The Pareto archived simulated annealing (PASA) method, a meta-heuristic procedure based on the SA algorithm, was developed in[23] to find non-dominated solution sets for the JSP with the objectives of minimizing the makespan and the mean flow time of jobs. Lei [8] presented a PSO for the multi-objective JSP to minimize makespan and total job tardiness simultaneously. Job-shop scheduling can be converted to a continuous optimization problem by constructing the corresponding relationship between a real vector and a chromosome obtained using the priority rule-based representation method. The global best position selection is combined with crowding-measure-based archive maintenance to design a Pareto archive PSO. The algorithm is capable of producing a number of high-quality Pareto optimal scheduling plans. Sha and Lin [20] proposed a multi objective particle swarm optimization (MOPSO) approach for an elaborate multi-objective job-shop scheduling problem. Their objective function is minimization of makespan, total tardiness, and total machine idle time. Qing et al. [16] presented a hybrid genetic algorithm for an inventory based two-objective job shop scheduling model in which both the makespan and the inventory capacity as objectives were optimized simultaneously.

Fuzzy set theory has found extensive applications in various fields. In 1976, Zimmermann [28] first introduced fuzzy set theory into an ordinary linear programming (LP) problem with fuzzy objective and constraints. Applying the fuzzy decision-making concept of Bellman and Zadeh [3], that study confirmed the existence of an equivalent ordinary LP form. Furthermore, Zimmermann [29] extended his fuzzy linear programming method to a conventional multi-objective linear programming (MOLP) problem. Subsequent studies on fuzzy goal programming were made by

Hannan [6], Leberling [7], Luhandjula [11], and Sakawa [18]. The main differences among these methods result from the types of aggregation operators and membership functions.

Here, we present a new multi-objective job shop scheduling problem with set up times in a fuzzy environment. The weighted mean completion time and the weighted mean tardiness time are to be optimized simultaneously. First, a MOLP model is constructed. Then, we develop a fuzzy multi-objective linear programming (FMOLP) model for solving it by integrating fuzzy sets and objective programming approaches.

2. Mathematical Model

2.1- Problem Description, Assumptions and Notation

This section presents a MILP model for the job shop scheduling problem with setup times. The model characterized here is based on the following hypotheses: The problem has *n* jobs and *m* machines. Each job has its own sequence of operations that must be processed on one specific machine out of the machine set (*m*). All jobs are independent and available for processing at the time 0. We assume infinite intermediate buffer between machines. Each machine can process only one job at a time, and each job can be processed by only one machine at any time. Each job has its own due date. Setup times are sequence-dependent. Transportation time between machines is negligible. Since, in job shops, some jobs might not visit some machines, a machine might be visited by no jobs; in this case, we additionally assume that each machine must be at least visited by one job. To present the mathematical model, the following notations are needed.

- **Indices**

i machine index $\{1, 2, \ldots, m\}$

j jobs index $\{1, 2, \ldots, n\}$

l jobs index $\{0, 1, 2, \ldots, n\}$

- **Parameters**

n number of jobs

m number of machines

P_{ij} processing time of job j on machine i

d_j due date of job j

s_{ilj} set up time of job j on machine i immediately after job l

$$a_{kij} = \begin{cases} 1 & \text{if job } j \text{ must visit machine } i \text{ immediately after machine } k \\ 0 & \text{otherwise} \end{cases}$$

W_j importance factor related to job j

H A large positive number

- **Variables**

c_{ij} Processing time of job j on machine i

T_j Tardiness of job j

$$X_{ilj} = \begin{cases} 1 & \text{if job } j \text{ is processed immediately after job } l \text{ on machine } i, \\ 0 & \text{otherwise.} \end{cases}$$

Note that we introduce a dummy job 0, which precedes the first job on the machines. The problem is modeled as follows:

$$\min\ Z_1 = \frac{\sum_{j=1}^{n} w_j^1 c_j}{\sum_{j=1}^{n} w_j^1} \tag{1}$$

$$\min Z_2 = \frac{\sum_{j=1}^{n} w_j^2 T_j}{\sum_{j=1}^{n} w_j^2} \tag{2}$$

s.t.

$$c_{ij} \geq c_{kj} + p_{ij} + \sum_{l=0}^{n} X_{ilj} \times s_{ilj} - H(1 - a_{ikj}), \qquad \forall j;\ k \neq j, i, k, \tag{3}$$

$$c_{ij} \geq c_{il} + p_{ij} + s_{ilj} - H(1 - X_{ilj}), \qquad \forall\, j;\ l = 1,2,\ldots,n;\ i = 1,2,\ldots,m, \tag{4}$$

$$\sum_{j=1}^{n} X_{i0j} = 1, \qquad i = 1,2,\ldots m, \tag{5}$$

$$\sum_{j=1}^{n} X_{ilj} \leq 1, \qquad i = 1,2,\ldots,m;\ l = 0,1,2,\ldots,n, \tag{6}$$

$$\sum_{l=0}^{n} X_{ilj} = 1, \qquad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n, \tag{7}$$

$$X_{ilj} + X_{ijl} \leq 1, \qquad j = 1,2,\ldots,n-1, \quad l > j, \tag{8}$$

$$c_j \geq c_{ij}, \qquad 1 \leq i \leq m, \quad j = 1,2,\ldots,n, \tag{9}$$

$$T_j \geq c_j - d_j, \qquad j = 1,2,\ldots,n, \tag{10}$$

$$c_{ij} \geq 0\ , T_i \geq 0, \qquad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n, \tag{11}$$

$$X_{ilj} \in \{0,1\}, \qquad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n;\ l = 0,1,2,\ldots,n. \tag{12}$$

Eq. (3) ensures that O_{ij} cannot start before the process of the previous job j completes. Constraint set 4 states that if O_{ij} is processed immediately after O_{il} it cannot begin before O_{il} completes. Constraint 5 specifies that dummy job 0 must have exactly one successor on each machine. Constraint set 6 states that every job must have at most one succeeding job on each machine, constraint 7 ensures that every job is scheduled on each machine once. Constraint set 8 ensures that a job cannot be, at the same time, both predecessor and successor of another job.

Constraint set 9 calculates the completion time of each job. Eq. (10) indicates the relationship between tardiness with the completion time and due date of each job. Finally, constraint sets (11) and (12) define the decision variables.

3. Fuzzy Multi-objective Linear Programming Model (FMOLP)

The crisp MOLP model can be extended to the fuzzy model (FMOLP) using the piecewise linear membership function of Hannan [23] to represent the fuzzy goals of the DM in the MOLP model, together with the fuzzy decision-making of Bellman and Zadeh [21]. In general, a piecewise linear membership function given in [21] can be adopted in order to convert the problem to an ordinary LP problem. The algorithm contains following steps.

Step 1. Specify the degree of membership $f_i(Z_i)$ for several values for each of the objective functions $Z_i, (i = 1,2)$.

Step 2. Setup the piecewise linear membership functions for each $(Z_i, f_i(Z_i)), (i = 1,2)$.

Step 3-1. Convert the membership functions $f_i(Z_i)$ to the following form

$$f_j(Z_j) = \sum_{b=1}^{P_j} \alpha_{jb} \left| Z_j - Y_{jb} \right| + \beta_j Z_j + \theta_j, \quad j = 1,2, \tag{13}$$

where

$$\alpha_{jb} = -\frac{\gamma_{j,b+1} - \gamma_j}{2}, \quad \beta_j = \frac{\gamma_{j,V_{j+1}} + \gamma_{j1}}{2}, \quad \theta_j = \frac{S_{j,V_{j+1}} + S_{j1}}{2}. \tag{14}$$

It is assumed that $f_j(Z_j) = \gamma_{jr} Z_j + S_{jr}$ for each segment $Y_{j,r-1} \le Z_j \le Y_{jr}$, where γ_{jr} denotes the slope and S_{jr} is the y-intercept of the section of the line segment on $[Y_{j,r-1}, Y_{jr}]$ in the piecewise linear function. Hence,

$$\begin{aligned} f_j(Z_j) = &-\left(\frac{\gamma_{j2} - \gamma_{j1}}{2}\right)\left|Z_j - Y_{j1}\right| - \left(\frac{\gamma_{j3} - \gamma_{j2}}{2}\right)\left|Z_j - Y_{j2}\right| - \ldots - \left(\frac{\gamma_{j,V_j+1} - \gamma_{jV_j}}{2}\right)\left|Z_j - Y_{jP_j}\right| \\ &+ \left(\frac{\gamma_{ij,V_{i+1}} + \gamma_{j1}}{2}\right) Z_j + \frac{S_{j,V_j+1} + S_{j1}}{2}\left(\frac{\gamma_{j,b+1} - \gamma_{jb}}{2}\right) \neq 0, \quad j = 1,2, \quad b = 1,2,\ldots,V_j, \end{aligned} \tag{15}$$

$$\gamma_{j1} = \left(\frac{q_{j1} - 0}{Y_{j1} - Y_{j0}}\right), \quad \gamma_{j2} = \left(\frac{q_{j2} - q_{j1}}{Y_{j2} - Y_{j1}}\right), \ldots, \gamma_{j,V_j+1} = \left(\frac{1 - q_{jV_j}}{Y_{j,V_j+1} - Y_{jV_j}}\right),$$

V_i is the numbers of divided points of the *i*th objective function (piecewise linear), and $S_{j,V_{j+1}}$ is the y-intercept of the section of the line segment on $[Y_{j,V_j}, Y_{j,V_{j+1}}]$.

Step 3-2. Introduce the non-negative deviational variables:

$$Z_j + d_{jb}^- - d_{jb}^+ = Y_{jb}, \quad j = 1,2, \quad b = 1,2,\ldots,V_j, \tag{16}$$

where d_{jb}^{+} and d_{jb}^{-} denote the deviational variables at the eth point and Y_{jb} represents the values of the ith objective function at the eth point.

Step 3-3. Substitute expression (17) into expression (15) to get

$$f_j(Z_j) = -\left(\frac{\gamma_{j2}-\gamma_{j1}}{2}\right)(d_{j1}^{-}-d_{j1}^{+}) - \left(\frac{\gamma_{j3}-\gamma_{j2}}{2}\right)(d_{j2}^{-}-d_{j2}^{+}) - \ldots - \left(\frac{\gamma_{j,V_j+1}-\gamma_{jV_j}}{2}\right)(d_{iV_j}^{-}-d_{iV_j}^{+})$$
$$+\left(\frac{\gamma_{ij,V_i+1}+\gamma_{j1}}{2}\right)Z_j + \frac{S_{j,V_j+1}+S_{j1}}{2}, \qquad j=1,2.$$

Table 1. Membership function $f_j(Z_j)$

Z_1	$>Y_{10}$	Y_{10}	Y_{11}	Y_{12}	$\ldots$	$Y_{1,v1}$	$Y_{1,v1+1}$	$<Y_{1,v1+1}$
$f_1(Z_1)$	0	0	q_{11}	q_{12}	$\ldots$	$q_{1,v1}$	1	1
Z_2	$>Y_{20}$	Y_{20}	Y_{21}	Y_{22}	$\ldots$	Y_{2v2}	$Y_{2,v2+1}$	$<Y_{2,v2+1}$
$f_2(Z_2)$	0	0	q_{21}	q_{22}	$\ldots$	$q_{2,v2}$	1	1

Step 4-1. By introducing the auxiliary variable, convert the original fuzzy MOLP problem can be converted to the equivalent ordinary LP form using the minimum operator to aggregate all fuzzy sets [20]. Consequently, the complete ordinary LP form can be formulated as follows:

$$\max \varphi_0 \tag{18}$$
s.t.

$$\varphi_0 \le -\left(\frac{\gamma_{12}-\gamma_{11}}{2}\right)(d_{11}^{-}-d_{11}^{+}) - \left(\frac{\gamma_{13}-\gamma_{12}}{2}\right)(d_{12}^{-}-d_{12}^{+}) - \ldots - \left(\frac{\gamma_{1,v+1}-\gamma_{1v}}{2}\right)(d_{11}^{-}-d_{11}^{+}) +$$
$$\left(\frac{\gamma_{1,v+1}+\gamma_{11}}{2}\right)\left(\frac{\sum_{j=1}^{n} w_j c_j}{\sum_{j=1}^{n} w_j}\right) + \frac{S_{1,v+1}+S_{11}}{2}, \tag{19}$$

$$\varphi_0 \le -\left(\frac{\gamma_{22}-\gamma_{21}}{2}\right)(d_{21}^{-}-d_{21}^{+}) - \left(\frac{\gamma_{23}-\gamma_{22}}{2}\right)(d_{22}^{-}-d_{22}^{+}) - \ldots - \left(\frac{\gamma_{2,v+1}-\gamma_{2v}}{2}\right)(d_{21}^{-}-d_{21}^{+}) +$$
$$\left(\frac{\gamma_{2,v+1}+\gamma_{21}}{2}\right)\left(\frac{\sum_{j=1}^{n} w_j T_j}{\sum_{j=1}^{n} w_j}\right) + \frac{S_{2,v+1}+S_{21}}{2}, \tag{20}$$

$$\frac{\sum_{j=1}^{n} w_j c_j}{\sum_{j=1}^{n} w_j} + d_{1b}^{-} - d_{1b}^{+} = Y_{1b}, \tag{21}$$

$$\frac{\sum_{j=1}^{n} w_j T_j}{\sum_{j=1}^{n} w_j} + d_{2b}^{-} - d_{2b}^{+} = Y_{2b}, \tag{22}$$

$\text{Constraint } (3)-(12).$

Step 4-2. We use the results of the presented model to overcome disadvantages of step 4-1. In this step, the solution is forced to improve, modify, and dominate the one obtained by the "max–min" operator. Also, we add constraints and a new auxiliary objective function to step4-2 in order to achieve at least the satisfaction degree obtained in step (4-1) [24]. Thus,

$$\max \varphi = \varphi_0 + \frac{1}{2}\sum_{i=1}^{2}(\varphi_i - \varphi_0) \tag{23}$$

s.t.

$$\varphi_0 \le \varphi_i \le -\left(\frac{\gamma_{12}-\gamma_{11}}{2}\right)(d_{11}^{-}-d_{11}^{+}) - \left(\frac{\gamma_{13}-\gamma_{12}}{2}\right)(d_{12}^{-}-d_{12}^{+}) - \ldots - \left(\frac{\gamma_{1,v+1}-\gamma_{1v}}{2}\right)(d_{11}^{-}-d_{11}^{+}) + \left(\frac{\gamma_{1,v+1}+\gamma_{11}}{2}\right)\left(\frac{\sum_{j=1}^{n} w_j c_j}{\sum_{j=1}^{n} w_j}\right) + \frac{S_{1,v+1}+S_{11}}{2}, \tag{24}$$

$$\varphi_0 \le \varphi_i \le -\left(\frac{\gamma_{22}-\gamma_{21}}{2}\right)(d_{21}^{-}-d_{21}^{+}) - \left(\frac{\gamma_{23}-\gamma_{22}}{2}\right)(d_{22}^{-}-d_{22}^{+}) - \ldots - \left(\frac{\gamma_{2,v+1}-\gamma_{2v}}{2}\right)(d_{21}^{-}-d_{21}^{+}) + \left(\frac{\gamma_{2,v+1}+\gamma_{21}}{2}\right)\left(\frac{\sum_{j=1}^{n} w_j t_j}{\sum_{j=1}^{n} w_j}\right) + \frac{S_{2,v+1}+S_{21}}{2}, \tag{25}$$

$$\frac{\sum_{j=1}^{n} w_j T_j}{\sum_{j=1}^{n} w_j} + d_{1b}^{-} - d_{1b}^{+} = Y_{1b}, \tag{26}$$

$$\frac{\sum_{j=1}^{n} w_j c_j}{\sum_{j=1}^{n} w_j} + d_{2b}^{-} - d_{2b}^{+} = Y_{2b}, \tag{27}$$

$$\text{Constraint } (3)-(12). \tag{28}$$

Step 5. Execute and modify the interactive decision process. If the DM is not satisfied with the initial solution, the model must be changed until a satisfactory solution is found. Figure 1 shows a block diagram of the FMOLP method.

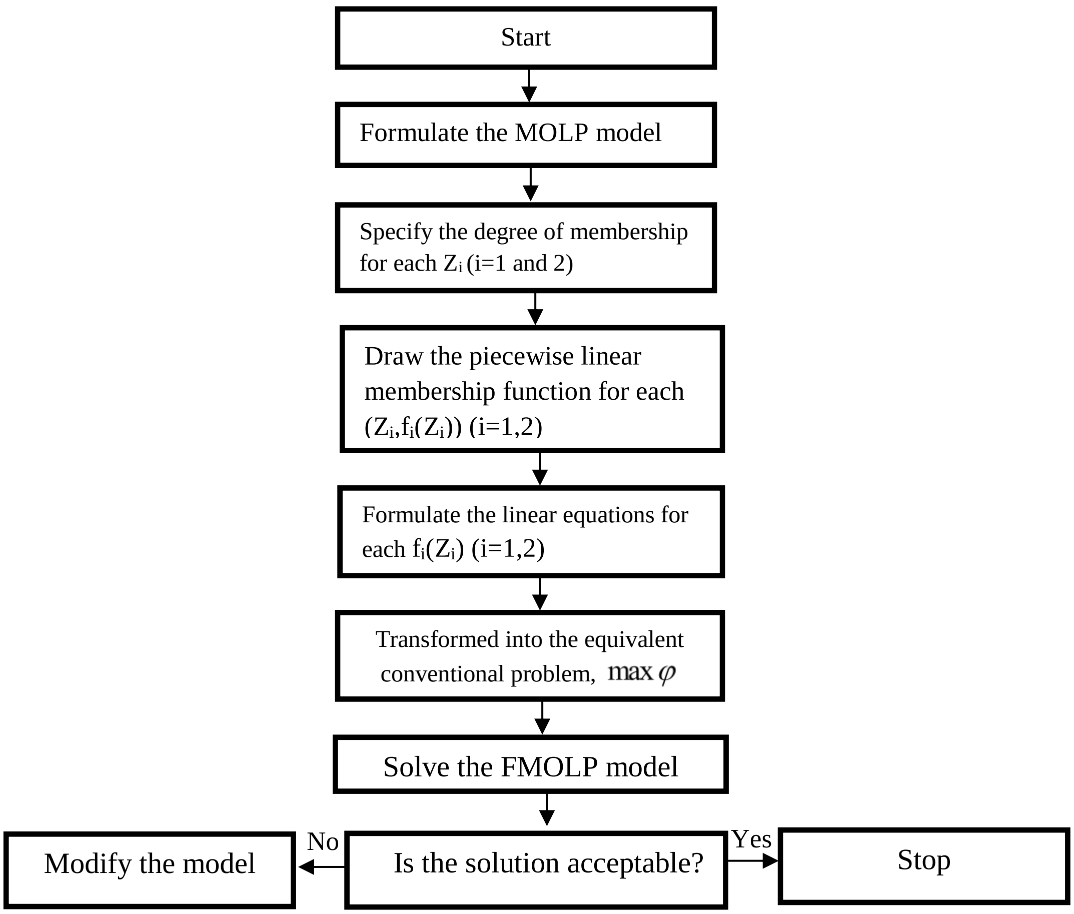


Figure 1. The block diagram for the interactive FMOLP model

4. Numerical Examples

4.1. Assumption for Numerical Example

- The processing times (p_{ij}) are integers and are generated uniformly from the interval [1 -99];
- The due dates (d_j) are generated based on the TWK method [5] as $d_j = r_j + c\sum_{i=1}^{n} p_{ij}$. Here, we consider $r_j = 0$ and the value of c is set to 1.5.

• Setup times are uniformly generated in the interval $[0.2\bar{p}, 0.3\bar{p}]$, where $\bar{p}$ is the mean processing time.
• The weights of jobs in each objective function are uniformly distributed in the interval $(1,20)$.
Table 2 and 3 summarize the basic data for the numerical example.

4.2. The FMOLP Model Formulation

First, we determine the initial solutions for each objective function by using the conventional LP model. The results are obtained by $Z_1 = 416.731, Z_2 = 39.81$. Then, we formulate the FMOLP model by using the initial solutions and the MOLP model was presented in Section 3. Table 4 give the piecewise linear membership functions of proposed model. Figs. 2 and 3 illustrate the corresponding shapes of the piecewise linear membership functions for the example.

The complete FMOLP model of the 5-jobs and 4 machines numerical example is given below.

$$\max \varphi = \varphi_0 + \frac{1}{2}\sum_{i=1}^{2}(\varphi_i - \varphi_0) \tag{29}$$

s.t.

$$\varphi_0 \le \varphi_1 \le -0.005(d_{11}^- - d_{11}^+) - 0.0025(d_{12}^- - d_{12}^+) - 0.0175 \times \left\{ \frac{\sum_{j=1}^{n} w_j^1 c_j}{\sum_{j=1}^{n} w_j^1} \right\} + 8.25, \tag{30}$$

$$\varphi_0 \le \varphi_2 \le -0.0025(d_{21}^- - d_{21}^+) - 0.0175 \times \left\{ \frac{\sum_{j=1}^{n} w_j^2 T_j}{\sum_{j=1}^{n} w_j^2} \right\} + 1.65, \tag{31}$$

$$\left\{ \frac{\sum_{j=1}^{n} w_j^1 C_j}{\sum_{j=1}^{n} w_j^1} \right\} + d_{11}^- - d_{11}^+ = 440, \tag{32}$$

$$\left(\frac{\sum_{j=1}^{n} w_j^1 C_j}{\sum_{j=1}^{n} w_j^1} \right) + d_{12}^- - d_{12}^+ = 420, \tag{33}$$

$$\left\{\frac{\sum_{j=1}^{n} w_j^2 T_j}{\sum_{j=1}^{n} w_j^2}\right\} + d_{12}^- - d_{12}^+ = 80, \tag{34}$$

$$c_{ij} \geq c_{kj} + p_{ij} + \sum_{k=0}^{n} X_{ilj} \times s_{ilj} - H(1 - a_{ikj}), \qquad \forall j;\ k \neq j;\ i,k = 1,2,\ldots,m, \tag{35}$$

$$c_{ij} \geq c_{il} + p_{ij} + s_{ilj} - H(1 - X_{ilj}), \qquad \forall j;\ l = 1,2,\ldots,n;\ i = 1,2,\ldots,m, \tag{36}$$

$$\sum_{j=1}^{n} X_{i0j} = 1, \qquad i = 1,2,\ldots m, \tag{37}$$

$$\sum_{j=1}^{n} X_{ilj} \leq 1, \qquad i = 1,2,\ldots,m;\ l = 0,1,2,\ldots,n, \tag{38}$$

$$\sum_{l=0}^{n} X_{ilj} = 1, \qquad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n, \tag{39}$$

$$X_{ilj} + X_{ijl} \leq 1, \qquad j = 1,2,\ldots,n-1\ ,\ l > j, \tag{40}$$

$$c_j \geq c_{ij}, \qquad j = 1,2,\ldots,n;\ i = 1,2,\ldots,m, \tag{41}$$

$$T_j \geq c_j - d_j, \qquad j = 1,2,\ldots,n, \tag{42}$$

$$c_{ij}, T_i, d_{1b}^-, d_{1b}^+, d_{2b}^-, d_{2b}^+, \varphi \geq 0, \qquad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n, \tag{43}$$

$$X_{ilj} \in \{0,1\}, \qquad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n;\ l = 0,1,2,\ldots,n \tag{44}$$

Table 2. Parameters for the example: processing time, due date, weighted completion time, weighted tardiness

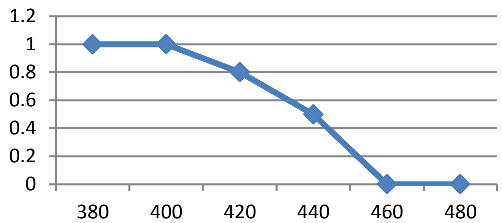
Job (j)	Production sequence	Processing time	Due date	Weighted obj(1)	Weighted obj(2)
1	4,2,1,3	32,59,5,22	165	2	4
2	1,4,3,2	66,53,94,63	424	7	6
3	4,2,1,3	57,76,67,74	409	9	7
4	1,3,2,4	93,91,82,53	488	5	8
5	4,3,2,1	26,54,13,65	233	3	4

Table 3. Sequence-dependent setup times

job	**M$_1$**					**M$_2$**					**M$_3$**					**M$_4$**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
0	17	17	14	13	13	12	12	13	13	12	13	13	16	15	14	13	13	15	16	16
1	-	13	17	14	17	-	12	12	13	14	-	13	15	16	14	-	16	16	17	12
2	15	-	16	13	14	16	-	12	12	17	13	-	17	14	14	13	-	17	14	14
3	16	12	-	17	16	14	17	-	16	12	12	12	-	16	14	12	12	-	16	14
4	15	14	15	-	16	14	14	13	-	14	14	15	15	-	17	14	15	15	-	17
5	13	12	15	15	-	14	17	14	16	-	13	15	16	16	-	13	15	16	16	-

Table 4. Membership functions for the example

Z_1	>460	460	440	420	400	<400
$f_1(Z_1)$	0	0	0.5	0.8	1	1
Z_2	100	100	80	60	40	<40
$f_2(Z_2)$	0	0	0.4	0.7	1	1



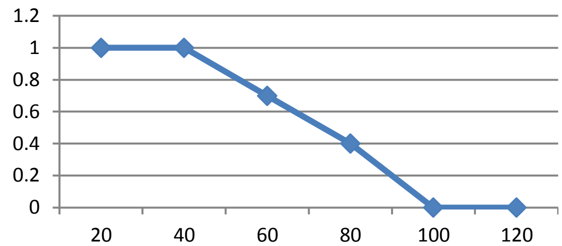


Figure 2. Shape of membership function $(Z_1, f_1(Z_1))$ **Figure3.** Shape of membership function $(Z_2, f_2(Z_2))$

4.3. Output Solutions

The LINGO computer package was used to run this FMOLP model, obtaining the results for the objectives as Z_1 = 416.731, Z_2=39.81 and the overall degree of satisfaction with the DM's multiple fuzzy goals as 0.85. Table 5 presents solutions for each decision variable and the best sequence of jobs on each machine.

For instance, if the DM does not accept the initial overall degree of satisfaction of 0.85 as given in the example, then the DM may try to adjust this φ value by taking account of relevant information in order to obtain a set of output solutions for making the decision. Two scenarios for the example are carried out in order to implement the FMOLP model by manipulating different alternatives and analyzing the sensitivity of decision parameters based on the preceding numerical example. This shows the process of modifying the initial overall degree of satisfaction.

Scenario 1: Set (Z_1, f_1 (Z_1)) to their original values in the numerical example and vary (Z_2, f_2 (Z_2)). Table 6 presents the data and results of implementing the Scenario 1.

Scenario 2: Set (Z_2, f_2 (Z_2)) to their original value in the numerical example and vary (Z_1, f1 (Z_1)). Table 7 presents the data and results of implementing the Scenario 2.

Table 8 summarizes the result of the implementation of the above two scenarios. The results of scenarios 1 and 2 show that the specific degree of membership for each of the objective functions strongly affects the overall level of satisfaction and the solutions. This fact has two significant implications. First, the most important task of the DM is to specify the rational degree of membership for each objective function; second, the DM may flexibly revise the range of values of the degree of membership to yield satisfactory solutions.

Furthermore, the multiple-objective job shop scheduling with set up times problem was solved using the ordinary single-objective LP model .Table9 compares the results of the proposed FMOLP method with the single-objective LP model and Zimmermann method [22] and Wang and Liang's approach[26] with the proposed FMOLP method. Application of LP-1 to minimize the weighted mean completion time (Z_1) yields an optimal value of 416.73. Application of LP-2 to minimize the weighted mean tardiness (Z_2) yields an optimal value of 39.24. Alternatively, using Zimmermann method [22] with linear membership functions to simultaneously minimize the weighted mean completion time and the weighted mean tardiness, the results are Z_1 = 426.23, Z_2 = 64.34, and the overall degree of DM's satisfaction is 0. 57 and using Wang and Liang's approach [29], the results are Z_1 = 429.63, Z_2 = 54.34, and the overall degree of DM's satisfaction is 0.77 In contrast, the proposed FMOLP method yields Z_1 = 429.65, Z_2 = 41.36, and overall degree of DM's satisfaction is equal to 0.845.

Table 5. FMOLP model for the example

job	**C_i**	**T_i**
1	256	91
2	471	47
3	464	55
4	471	0
5	276	43

Table 6. Data of scenario1

Z_1		>480	460	440	420	400	<400
$f_1(Z_1)$		0	0	0.5	0.8	1	1
Run1	Z_2	>80	80	60	40	20	<20
	$f_2(Z_2)$	0	0	0.4	0.7	1	1
Run2	Z_2	>90	90	70	50	30	<30
	$f_1(Z_1)$	0	0	0.4	0.7	1	1
Run3	Z_2	>100	100	80	60	40	<40
	$f_2(Z_2)$	0	0	0.4	0.7	1	1
Run4	Z_2	>110	110	90	70	50	<50
	$f_2(Z_2)$	0	0	0.4	0.7	1	1
Run5	Z_2	>120	120	100	80	60	<60
	$f_2(Z_2)$	0	0	0.4	0.7	1	1

Table 7. Data of scenario2

Z_2		90	90	70	50	30	<30
$f_2(Z_2)$		0	0	0.4	0.7	1	1
Run1	Z_1	>440	440	420	400	380	<380
	$f_1(Z_1)$	0	0	0.5	0.8	1	1
Run2	Z_2	>450	450	430	410	390	<390
	$f_1(Z_1)$	0	0	0.5	0.8	1	1
Run3	Z_2	>460	460	440	420	400	<400
	$f_2(Z_2)$	0	0	0.5	0.8	1	1
Run4	Z_2	>470	470	450	430	410	<410
	$f_1(Z_1)$	0	0	0.5	0.8	1	1
Run5	Z_2	>480	480	460	440	420	<420
	$f_1(Z_1)$	0	0	0.5	0.8	1	1

Table 9. Solution comparisons

	LP-1	LP-2	Wang and Liang's Method	Zimmerman[3]	FMOLP
Objective Function	Min Z_1	Min Z_2	Max φ	Max φ	Max φ
φ	100	100	0.77	0.56	0.84
Z_1	416.73	439.65	426.65	426.23	429.65
Z_2	93.51	39.24	55.34	64.34	41.36

5. Conclusion

In most real-world job shop scheduling problems, the DM must simultaneously handle conflicting objectives that usually govern the use of the constrained resources within organizations. Here, job shop scheduling with setup times were investigated. The weighted mean completion time and the weighted mean tardiness time were minimized simultaneously. Moreover, a systematic framework was proposed to facilitate the decision-making process, enabling the DM interactively to modify the membership functions of the objectives until a satisfactory solution was obtained. The computational results showed that the method achieved lower objective functions and higher satisfaction degrees. The results were compared with the Wang and Liang's approach to verify the proposed method. Accordingly, FMOLP approach for solving real-world multi-objective Job shop scheduling problem in a fuzzy environment. The FMOLP was based on Hannan'sapproach, which implicitly assumes that the minimum operator is the proper representation of the human DM who combines fuzzy statements by logical 'and' operator.

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