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# Coordinated and Joint Ordering Policies for Two Commodity Inventory System

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*We deal with the two commodity continuous review inventory system with two joint reordering policies. The maximum inventory level for the ith commodity is $S_i$ $(i = 1, 2)$ units. It is assumed that demand for the ith commodity is of unit size and the demand time points form a Poisson process with parameter $\lambda_i$, $i = 1, 2$. The reorder level is fixed as $c_i$ for the ith commodity $(i = 1, 2)$. The ordering policy is to place an order for $P_i (= S_i - c_i)$ items for the ith commodity $(i = 1, 2)$ when both the inventory levels are less than or equal to their respective reorder levels. If the total net inventory level drops to a prefixed level $s$, an order will be placed for $Q_i (= S_i - s)$ items for the ith commodity. The lead times are assumed to be negative exponential. Demands that occur during the stock-out periods are assumed to be lost. The joint inventory level for both commodities is obtained in the steady state. Various system performance measures in the steady state are derived.*

***Keywords***: *Two commodity, Joint order, Stochastic lead time, (s, S) policy.*

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## 1. Introduction

A number of practical multi-item inventory systems are concentrated on the coordination of replenishment orders for group of items. Presently, it is very much applicable to run successful businesses and industries. These systems unlike those dealing with single commodity involve more complexities in the reordering procedures. The modeling of multi-item inventory system under joint replenishment has been receiving considerable attention for the past three decades.

For continuous review inventory systems, Ballintify [7] and Silver [17] considered a coordinated reordering policy which is represented by the triplet $(S, c, s)$, where the three parameters $S_i, c_i$ and $s_i$ are specified for each item $i$ with $s_i \le c_i \le S_i$, under the unit sized Poisson demand and constant lead time. In this policy, if the level of the *i*th commodity at any time is below $s_i$, an order is placed for $S_i - s_i$ items and at the same time, for any other item $j(\neq i)$ with available inventory at or below its can-order level $c_j$, an order is placed so as to bring its level back to its maximum capacity $S_j$.

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Subsequently, several articles appeared with models involving the above policy. Also, an article of interest is that of Federgruen et al. [8], dealing with the general case of compound Poisson demands and non-zero lead times. A review of inventory models under joint replenishment is provided by Goyal and Satir [9].

Kalpakam and Arivarignan [10] introduced the $(s, S)$ policy with a single reorder level $s$ defined in terms of the total number of items in the stock. This policy avoids separate ordering for each commodity and hence a single processing of orders for both commodities has some advantages in situations where in procurement is made from the same supplies, items produced on the same machine, or items supplied by the same transport facility.

Krishnamoorthy et al. [12] considered a two commodity continuous review inventory system without lead time. In their model, each demand is for one unit of first commodity or one unit of second commodity or one unit of each commodity 1 and 2, with prefixed probabilities. Krishnamoorthy and Varghese [13] considered a two commodity inventory problem without lead time and with Markov shift in demand for the type of commodity, namely “commodity-1”, “commodity-2” or “both commodity”, using the direct Markov renewal theoretical results. And also for the same problem, Sivasamy and Pandiyan [19] derived various results by the application of a filtering technique.

Anbazhagan and Arivarignan [1] considered a two commodity inventory system with Poisson demands and a joint reorder policy placing fixed ordering quantities for both commodities whenever both inventory levels were less than or equal to their respective reorder levels. Anbazhagan and Vigneswaran [4] extended the work of Anbazhagan and Arivarignan [1] by assuming the set of reorder levels with prescribed probability distribution for reordering. Sivakumar et al. [18] generalized the same work by assuming an arbitrary distribution for the inter-demand times and by having probability $p_i$ for the *i*th commodity $(p_i \geq 0, i = 1,2, p_1 + p_2 = 1)$ and adopted two ordering policies, one being a pair of individual ordering policy and the other being a joint reorder policy.

Anbazhagan and Arivarignan [2] analysed a model with a joint ordering policy placing orders for both commodities whenever the total net inventory level droped to a prefixed level $s$. Anbazhagan and Arivarignan [3] also analyzed the model with individual and joint ordering policies. For the individual reorder policy, the reorder level for the *i*th commodity is fixed as $r_i$, and whenever the inventory level of the *i*th commodity falls on $r_i$ an order for $P_i$ $(= S_i - r_i)$ items are placed for that commodity irrespective of the inventory level of the other commodity. A joint reorder policy is used with prefixed reorder levels $s$ and orders of $Q_x^1$ $(S_1 - x)$ and $Q_y^2$ $(S_2 - y)$ items are placed for both commodities by cancelling the previous orders, whenever both commodities have their inventory level droped to a reorder level $s$ $(x + y = s)$. Yadavalli et al. [21] analyzed a model with joint ordering policy and varying order quantities. Anbazhagan et al. [5] considered a two commodity substitutable inventory system with variable ordering quantity. The review article [15] provides an excellent summary of lost sales inventory system.

Stochastic retrial inventory systems were extensively studied in various cases [6, 11, 20]. Zhao and Lian [16] analyzed a queueing-inventory system with two classes of customers. The authors assumed Poisson arrival, with the service times following exponential distributions. Each service uses one item in the attached inventory supplied by an outside supplier with exponentially distributed lead time. Lopez-Herrero [14] considered a continuous review $(s, S)$ inventory model with retrial demands.

Here, a two commodity continuous review inventory system with two different joint orders are considered. The rest of the paper is organized as follows. Section 2 deals with model formulation. Section 3 analyses part of the system as a Markov process and the system performance measures are computed in Section 4. In Section 5, the total expected cost rate is computed. We conclude in Section 6.

## 2. Model Formulation

In this model, we consider a continuous review inventory system with a maximum stock of $S_i$ units for the *i*th commodity $(i = 1,2)$. The demand for the *i*th commodity is of unit size and the time points of demand occurrences form independent Poisson processes each with parameter $\lambda_i$. The coordinated reorder level for the *i*th commodity is fixed at $c_i$ $(s < c_i < S_i/2)$. The coordinated ordering policy is to place an order for $P_i(= S_i - c_i)$ items for the *i*th commodity, when both the inventory levels are less than or equal to their respective reorder levels. The lead time for coordinated ordering policy is assumed to be distributed as negative exponential with parameter $\mu_1(> 0)$. The reorder level for the joint ordering policy is fixed at $s$ for both commodities. If the total net inventory level drops to a prefixed level $s$, an order will be placed for $Q_i(= S_i - s)$ items for the *i*th commodity and the previous order gets cancelled. The lead time for the joint ordering policy is assumed to be distributed as negative exponential with parameter $\mu_2(> 0)$. Demands that occur during stock-out periods are assumed to be lost.

**Notations:**

$[A]_{ij}$ : The element/submatrix at $(i, j)$ th position of $A$.

**0** : Zero matrix.

**e** : A column vector of ones with appropriate dimension.

$E_1$ : $\{0,1,..., S_1\}$.

$E_2$ : $\{0,1,..., S_2\}$.

$E$ : $E_1 \times E_2$.

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases}$$

$$\underset{i=j}{\overset{k}{\Omega}} c_i = \begin{cases} c_j c_{j-1} \cdots c_k, & \text{if } j \ge k \\ 1, & \text{if } j < k. \end{cases}$$

$$\sum_{i=j}^{k} a_i = \begin{cases} a_j + a_{j+1} + \cdots + a_k, & \text{if } j \le k \\ 0, & \text{otherwise.} \end{cases}$$

## 3. Analysis

Let $X_i(t), i = 1, 2,$ denote the inventory level of the *i*th commodity at time $t$. The stochastic process $\{(X_1(t), X_2(t)), t \ge 0\}$ has the state space *E*.

From the assumptions made on the demand and on replenishment processes, it follows that $\{(X_1(t), X_2(t)), t \ge 0\}$ is a Markov process. The infinitesimal generator $A = ((\, a((i, j), (k, l))\,)), \quad (i, j), (k, l) \in E,$ of this process can be conveniently expressed as a block partitioned matrix:

$$A = ((A_{ij})),$$

where,

$$[A]_{ij} = \begin{cases} D_{s+2-i}, & j = i, & i = 0,1,\ldots,s, \\ D_1, & j = i, & i = s+1, s+2, \ldots, c_1 \\ D, & j = i, & i = c_1+1, c_1+2, \ldots, S_1 \\ U, & j = i-1, & i = 1,2,\ldots,S_1, \\ M_{s+1-i}, & j = i+P_1, & i = 0,1,\ldots,s, \\ M, & j = i+P_1, & i = s+1, s+2, \ldots, c_1, \\ N_{s+1-i}, & j = i+Q_1, & i = 0,1,\ldots,s, \\ \mathbf{0}, & \text{otherwise,} & \end{cases}$$

with

$$[U]_{kl} = \begin{cases} \lambda_1, & l = k, \quad k = 0,1,\ldots,S_2 \\ \mathbf{0}, & otherwise, \end{cases}$$

$$[D]_{kl} = \begin{cases} \lambda_2, & l = k-1, \quad k = 1,2,\ldots,S_2 \\ -(\lambda_1 + (1-\delta_{k0})\lambda_2), & l = k, \quad k = 0,1,\ldots,S_2 \\ \mathbf{0}, & otherwise, \end{cases}$$

$$[D_1]_{kl} = \begin{cases} \lambda_2, & l = k-1, \quad k = 1,2,\ldots,S_2 \\ -(\lambda_1 + \lambda_2), & l = k, \quad k = c_2+1, c_2+2,\ldots,S_2 \\ -(\lambda_1 + (1-\delta_{k0})\lambda_2 + \mu_1), & l = k, \quad k = 0,1,\ldots,c_2 \\ \mathbf{0}, & otherwise, \end{cases}$$

for $i = 2,3,\ldots,s+2,$

$$[D_i]_{kl} = \begin{cases} \lambda_2, & l = k-1, \quad k = 1,2,\ldots,S_2 \\ -(\lambda_1 + \lambda_2), & l = k, \quad k = c_2+1, c_2+2,\ldots,S_2 \\ -(\lambda_1 + \lambda_2 + \mu_1), & l = k, \quad k = c_2, c_2-1,\ldots,i,i-1, \\ -(\lambda_1 + (1-\delta_{k0})\lambda_2 + \mu_2), & l = k, \quad k = i-2, i-3,\ldots,1,0 \\ \mathbf{0}, & otherwise, \end{cases}$$

$$[M]_{kl} = \begin{cases} \mu_1, & l = k + P_2, \quad k = 0,1,\ldots,c_2 \\ \mathbf{0}, & otherwise, \end{cases}$$

for $i = 1, 2, \ldots, s + 1,$

$$[M_i]_{kl} = \begin{cases} \mu_1, & l = k + P_2, \quad k = c_2, c_2 - 1, \ldots, i + 1, i \\ \mathbf{0}, & otherwise, \end{cases}$$

and for $i = 1, 2, \ldots, s + 1,$

$$[N_i]_{kl} = \begin{cases} \mu_2, & l = k + Q_2 \quad k = i - 1, i - 2, \ldots, 1, 0 \\ \mathbf{0}, & otherwise. \end{cases}$$

**Steady State Analysis**

It can be seen from the structure of the infinitesimal generator $A$ that the time-homogeneous Markov process $\{(X_1(t), X_2(t)), t \geq 0\}$ on the finite state space $E$ is irreducible, aperiodic and persistent non-null. Hence, the limiting distribution

$$\Phi = \left(\phi^{(S_1)}, \phi^{(S_1 - 1)}, \cdots, \phi^{(0)}\right),$$

with $\varphi^{(m)} = \left(\varphi^{(m, S_2)}, \varphi^{(m, S_2 - 1)}, \ldots, \varphi^{(m,1)}, \varphi^{(m,0)}\right), \quad m = 0, 1, \ldots, S_1,$ where $\phi^{(i,j)}$ denotes the steady state probability for the state $(i, j)$ of the inventory level process, exists and is given by

$$\Phi\tilde{A} = 0 \quad and \quad \sum_{(i,j) \in E}\sum \varphi^{(i,j)} = 1. \tag{1}$$

The first equation of the above yields the following set of equations:

$$\phi^{(i)} D_{s+2-i} + \phi^{(i+1)} U = 0, \qquad i = 0, 1, \ldots, s$$

$$\phi^{(i)} D_1 + \phi^{(i+1)} U = 0, \qquad i = s+1, s+2, \ldots, c_1$$

$$\phi^{(i)} D + \phi^{(i+1)} U = 0, \qquad i = c_1+1, c_1+2, \ldots, P_1-1$$

$$\phi^{(i)} D + \phi^{(i+1)} U + \phi^{(i-P_1)} M_{P_1+s+1-i} = 0, \qquad i = P_1, P_1+1, \ldots, Q_1-1$$

$$\phi^{(i)} D + \phi^{(i+1)} U + H(s-i+P_1)\phi^{(i-P_1)} M_{P_1+s+1-i} +$$

$$H(i-P_1-s-1)\phi^{(i-P_1)} M + \phi^{(i-Q_1)} N_{Q_1+s+1-i} = 0, \qquad i = Q_1, Q_1+1, \ldots, S_1-1$$

and

$$\varphi^{(S_1)} D + \varphi^{(c_1)} M + \varphi^{(s)} N_1 = 0. \qquad (*)$$

After some simplifications, using the above equations, except for the last equation (*), we get

$$\phi^{(i)} = \phi^{(0)} (-1)^i \left( \underset{m=s+2}{\overset{s+3-i}{\Omega}} D_m \right) (U^{-1})^i, \qquad i = 1, 2, \ldots, s+1$$

$$\phi^{(i)} = \phi^{(0)} (-1)^i \left( \underset{m=s+2}{\overset{2}{\Omega}} D_m \right) D_1^{i-s-1} (U^{-1})^i, \qquad i = s+2, s+3, \ldots, c_1+1$$

$$\phi^{(i)} = \phi^{(0)} (-1)^i \left( \underset{m=s+2}{\overset{2}{\Omega}} D_m \right) D_1^{c_1-s} D^{i-c_1-1} (U^{-1})^i, \qquad i = c_1+2, c_1+3, \ldots, P_1$$

$$\phi^{(i)} = \phi^{(0)} \Bigg[ (-1)^i \left( \underset{m=s+2}{\overset{2}{\Omega}} D_m \right) D_1^{c_1-s} D^{i-c_1-1} (U^{-1})^i + (-1)^{i-P_1} \sum_{k=0}^{i-P_1-1} \Bigg\{ H(s+1-k) \left( \underset{m=s+2}{\overset{s+3-k}{\Omega}} D_m \right) +$$

$$H(k-s-2) \left( \underset{m=s+2}{\overset{2}{\Omega}} D_m \right) D_1^{k-s-1} \Bigg\} M_{s+1-k} D^{i-k-P_1-1} (U^{-1})^{i-P_1} \Bigg], \qquad i = P_1+1, P_1+2, \ldots, Q_1$$

$$\phi^{(i)} = \phi^{(0)} \Bigg[ (-1)^i \left( \underset{m=s+2}{\overset{2}{\Omega}} D_m \right) D_1^{c_1-s} D^{i-c_1-1} (U^{-1})^i + (-1)^{i-P_1} \sum_{k=0}^{Q_1-P_1-1} \Bigg\{ H(s+1-k) \left( \underset{m=s+2}{\overset{s+3-k}{\Omega}} D_m \right) +$$

$$H(k-s-2) \left( \underset{m=s+2}{\overset{2}{\Omega}} D_m \right) D_1^{k-s-1} \Bigg\} M_{s+1-k} D^{i-k-P_1-1} (U^{-1})^{i-P_1} + (-1)^{i-P_1}$$

$$\sum_{k=Q_1-P_1}^{i-P_1-1}\left[H(s+1-k)\left(\underset{m=s+2}{\overset{s+3-k}{\Omega}} D_m\right)+H(k-s-2)\left(\underset{m=s+2}{\overset{2}{\Omega}} D_m\right)D_1^{k-s-1}\right]\times$$

$$\{H(s-k)M_{s+1-k}+H(k-s-1)M\}D^{i-k-P_1-1}(U^{-1})^{i-P_1}+$$

$$(-1)^{i-Q_1}\sum_{k=0}^{i-Q_1-1}\left(\underset{m=s+2}{\overset{s+3-k}{\Omega}} D_m\right)N_{s+1-k}D^{i-k-Q_1-1}(U^{-1})^{i-Q_1}\Bigg],\quad i=Q_1+1,Q_1+2,\ldots,S_1.$$

Here, $\phi^{(0)}$ can be obtained by solving

$$\phi^{(S_1)}D+\phi^{(c_1)}M+\phi^{(s)}N_1=0 \text{ and } \Phi e=1,$$

that is,

$$\phi^{(0)}\Bigg(\Bigg[(-1)^{S_1}\left(\underset{m=s+2}{\overset{2}{\Omega}} D_m\right)D_1^{c_1-s}D^{P_1-1}(U^{-1})^{S_1}+(-1)^{c_1}\sum_{k=0}^{Q_1-P_1-1}\Bigg\{H(s+1-k)\left(\underset{m=s+2}{\overset{s+3-k}{\Omega}} D_m\right)+$$

$$H(k-s-2)\left(\underset{m=s+2}{\overset{2}{\Omega}} D_m\right)D_1^{k-s-1}\Bigg\}M_{s+1-k}D^{S_1-k-P_1-1}(U^{-1})^{c_1}+(-1)^{c_1}$$

$$\sum_{k=Q_1-P_1}^{S_1-P_1-1}\left[H(s+1-k)\left(\underset{m=s+2}{\overset{s+3-k}{\Omega}} D_m\right)+H(k-s-2)\left(\underset{m=s+2}{\overset{2}{\Omega}} D_m\right)D_1^{k-s-1}\right]\times$$

$$\{H(s-k)M_{s+1-k}+H(k-s-1)M\}D^{S_1-k-P_1-1}(U^{-1})^{c_1}+$$

$$(-1)^{s}\sum_{k=0}^{S_1-Q_1-1}\left(\underset{m=s+2}{\overset{s+3-k}{\Omega}} D_m\right)N_{s+1-k}D^{S_1-k-Q_1-1}(U^{-1})^{s}\Bigg]D+$$

$$\left\{(-1)^{c_1}\left(\underset{m=s+2}{\overset{2}{\Omega}} D_m\right)D_1^{c_1-s-1}(U^{-1})^{c_1}\right\}M+\left\{(-1)^{s}\underset{m=s+2}{\overset{3}{\Omega}} D_m(U^{-1})^{s}\right\}N_1\Bigg)=0,$$

and

$$\phi^{(0)}\Bigg[I+\sum_{i=1}^{s+1}(-1)^{i}\left(\underset{m=s+2}{\overset{s+3-i}{\Omega}} D_m\right)(U^{-1})^{i}+\sum_{i=s+2}^{c_1+1}(-1)^{i}\left(\underset{m=s+2}{\overset{2}{\Omega}} D_m\right)D_1^{i-s-1}(U^{-1})^{i}+$$

$$\sum_{i=c_1+2}^{P_1}(-1)^{i}\left(\underset{m=s+2}{\overset{2}{\Omega}} D_m\right)D_1^{c_1-s}D^{i-c_1-1}(U^{-1})^{i}+\sum_{i=P_1+1}^{Q_1}\left((-1)^{i}\left(\underset{m=s+2}{\overset{2}{\Omega}} D_m\right)D_1^{c_1-s}D^{i-c_1-1}(U^{-1})^{i}+\right.$$

$$(-1)^{i-P_1}\sum_{k=0}^{i-P_1-1}\Big\{H(s+1-k)\Big(\mathop{\Omega}_{m=s+2}^{s+3-k}D_m\Big)+$$

$$H(k-s-2)\Big(\mathop{\Omega}_{m=s+2}^{2}D_m\Big)D_1^{k-s-1}\Big\}M_{s+1-k}D^{i-k-P_1-1}(U^{-1})^{i-P_1}\Big)+$$

$$\sum_{i=Q_1+1}^{S_1}\Big((-1)^i\Big(\mathop{\Omega}_{m=s+2}^{2}D_m\Big)D_1^{c_1-s}D^{i-c_1-1}(U^{-1})^i+(-1)^{i-P_1}\sum_{k=0}^{Q_1-P_1-1}\Big\{H(s+1-k)\Big(\mathop{\Omega}_{m=s+2}^{s+3-k}D_m\Big)+$$

$$H(k-s-2)\Big(\mathop{\Omega}_{m=s+2}^{2}D_m\Big)D_1^{k-s-1}\Big\}M_{s+1-k}D^{i-k-P_1-1}(U^{-1})^{i-P_1}+$$

$$(-1)^{i-P_1}\sum_{k=Q_1-P_1}^{i-P_1-1}\Big[H(s+1-k)\Big(\mathop{\Omega}_{m=s+2}^{s+3-k}D_m\Big)+H(k-s-2)\Big(\mathop{\Omega}_{m=s+2}^{2}D_m\Big)D_1^{k-s-1}\Big]\times$$

$$\{H(s-k)M_{s+1-k}+H(k-s-1)M\}D^{i-k-P_1-1}(U^{-1})^{i-P_1}+$$

$$(-1)^{i-Q_1}\sum_{k=0}^{i-Q_1-1}\Big(\mathop{\Omega}_{m=s+2}^{s+3-k}D_m\Big)N_{s+1-k}D^{i-k-Q_1-1}(U^{-1})^{i-Q_1}\Big)\Big]e=1.$$

## 4. System Performance Measures

In this section, we derive some stationary performance of the system. Using these measures, we can construct the total expected cost per unit time.

**Expected Inventory Level**

And let $MI_1$ denote the expected inventory level of the first commodity in the steady state, given by

$$MI_1=\sum_{i=1}^{S_1}i\left(\sum_{j=0}^{S_2}\phi^{(i,j)}\right), \tag{2}$$

and let $MI_2$ denote the expected inventory level of the second commodity in the steady state given by

$$MI_2=\sum_{j=1}^{S_2}j\left(\sum_{i=0}^{S_1}\phi^{(i,j)}\right). \tag{3}$$

**Mean Reorder Rate**

Let $MCR$ denote the mean coordinated reorder rate for both commodiies in the steady state, given by

$$MCR = \sum_{i=0}^{s_1} \lambda_2 \phi^{(i,s_2+1)} + \sum_{i=0}^{s_2} \lambda_1 \phi^{(s_1+1,i)}, \tag{4}$$

and let $MJR$ denote the mean joint reorder rate for both commodities in the steady state, given by

$$MJR = \sum_{i=0}^{s} \lambda_1 \phi^{(s+1-i,i)} + \sum_{i=0}^{s} \lambda_2 \phi^{(i,s+1-i)}. \tag{5}$$

**Mean Shortage Rate**

Let $MS_1$ denote the mean shortage rate of the first commodity, given by

$$MS_1 = \lambda_1 \sum_{j=0}^{S_2} \phi^{(0,j)}, \tag{6}$$

and let $MS_2$ denote the mean shortage rate of the second commodity, given by

$$MS_2 = \lambda_2 \sum_{i=0}^{S_1} \phi^{(i,0)}. \tag{7}$$

## 5. Total Expected Cost Rate

To compute the total expected cost per unit time (total expected cost rate), we consider the following costs:

$h_1$ : the inventory carrying cost per item per unit time of first commodity.

$h_2$ : the inventory carrying cost per item per unit time of second commodity.

$c_{s1}$ : setup cost per coordinated order.

$c_{s2}$ : setup cost per joint order.

$c_{sh1}$ : shortage cost per unit per unit time of first commodity.

$c_{sh2}$ : shortage cost per unit per unit time of second commodity.

The long run total expected cost rate is given by

$$TEC(S_1, S_2, s, h_1, h_2, c_{s1}, c_{s2}, c_{sh1}, c_{sh2}) =$$

$$h_1 MI_1 + h_2 MI_2 + c_{s1} MCR + c_{s2} MJR + c_{sh1} MS_1 + c_{sh2} MS_2.$$

Substituting the values from $(2)-(7)$ in $TEC(S_1, S_2, s, h_1, h_2, c_{s1}, c_{s2}, c_{sh1}, c_{sh2})$, we get

$$TEC(S_1, S_2, s, h_1, h_2, c_{s1}, c_{s2}, c_{sh1}, c_{sh2}) = h_1 \sum_{i=1}^{S_1} i \left( \sum_{j=0}^{S_2} \varphi^{(i,j)} \right) + h_2 \sum_{j=1}^{S_2} j \left( \sum_{i=0}^{S_1} \varphi^{(i,j)} \right)$$

$$+ c_{s1} \left( \sum_{i=0}^{s_1} \lambda_2 \phi^{(i, s_2+1)} + \sum_{i=0}^{s_2} \lambda_1 \phi^{(s_1+1, i)} \right)$$

$$+ c_{s2} \left( \sum_{i=0}^{s} \lambda_1 \phi^{(s+1-i, i)} + \sum_{i=0}^{s} \lambda_2 \phi^{(i, s+1-i)} \right) + c_{sh1} \sum_{j=0}^{S_2} \lambda_1 \phi^{(0,j)} + c_{sh2} \sum_{i=0}^{S_1} \lambda_2 \phi^{(i,0)}.$$

## 6. conclusion

We discussed coordinated and joint ordering policies for a two-commodity stochastic inventory system with Poisson demand and exponential lead times. Demands occuring during stock out periods were assumed to be lost. We derived the joint probability distribution of the inventory levels in the steady state. We also derived the stationary measures of the system performances.

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