

## Competitive Pricing in a Supply Chain Using a Game Theoretic Approach

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*We develop a price competition model for a new supply chain that competes in a market comprised of some rival supply chains. The new supply chain has one risk-neutral manufacturer and one risk-averse retailer in which the manufacturer is a leader and retailer is a follower. The manufacturer pays a fraction of the risk cost (caused by demand uncertainty) to the retailer. We apply this competitive model to a real-world case in a supply chain under uncertain environment and obtain the optimal wholesale and retail prices. We show that our obtained prices are better than the existing wholesale and retail prices and admit more profits for both manufacturer and retailer and generally for the entire supply chain. Also, using this case, the effects of risk sensitivity of retailer and fraction of risk cost shared by manufacturer in the total risk cost on the new supply chain's optimal wholesale and retail prices and profits are illustrated.*

**Keywords:** Supply chain management, Price competition, Pricing, Competitive pricing, Risk sensitivity, Game theory.

Manuscript was received on 24/04/2013, revised on 29/07/2013 and accepted for publication on 22/08/2013.

### 1. Introduction

Nowadays, the competition between firms is shifting towards the competition between supply chains (Taylor [20], Wu and Chen [24], Boyaci and Gallego [6], Barnes [4], Zhang [28], Xiao and Yang [26], Ha and Tong [9], Shou et al. [18]). Examples include the competition between Wal-Mart and Kmart, competition between Ford supply chain and GM supply chain for end markets of family cars, sport cars, and trucks (Zhang [28]).

Because of different objectives of supply chain members, conflicts occur within a supply chain and hence, behavior that is locally rational for a member, can be inefficient for the overall supply chain performance. In the supply chain management (SCM) literature, it is well known that coordination among supply chain members will improve the overall supply chain performance but the majority of this literature ignores the competition from other external supply chains and hence, there is no guaranty for improving the supply chain performance in the existence of other coordinated supply chains (Boyaci and Gallego [6]). Hence, we investigate the equilibrium behavior of a new supply chain that tends to entering in the stochastic market consisting of some competing supply chains. This new supply chain consists of one risk-neutral manufacturer and one risk-averse retailer in which, the retailer is a leader and the manufacturer is a follower. We suppose that the manufacturer should pay a fraction of the risk cost of retailer. Today, risk sensitivity has potential effects on performance of supply chain members and can cause inefficiency across the entire supply chain. The risk sensitivity of a retailer towards demand uncertainty has a considerable impact on its decisions.

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Because of the important role of pricing in business behavior, here we suppose that the competition occurs on the basis of product price under a stochastic price-depended demand. To focus on the effects of competition, we consider all model parameters as a common knowledge for supply chain members. How will this supply chain compete in the market? What are the optimal wholesale and retail prices? And, how does the risk sensitivity affect the supply chain members' decisions? Our concern here is to answer these questions.

The remainder of our work is organized as follows. The price competition literature in SCM is reviewed in Section 2. In Section 3, we present our basic model, and derive profit functions of the new supply chain members. In section 4, we give the equilibrium prices of the new supply chain members. An illustrative case study and sensitivity analysis are presented in Section 5. Finally, in Section 6 we summarize our concluding remarks and point out some directions for future research.

## 2. Literature Review

Price competition has been addressed in the SCM literature. At first, it is worth mentioning the work by McGuire and Staelin [15]. They investigate equilibrium supply chain structures for duopoly market, in which two competing manufacturers sell their products through an exclusive retailer. They develop a deterministic model with price competition and product substitutability and no inventory considerations and show that the wholesaler's equilibrium distribution structure (i.e., vertical integration versus decentralized distribution) depends on the degree of product substitutability, which determines the intensity of retail price competition.

Zhang [28] study a supply chain economy that comprises heterogeneous supply chains involving multiple products and competing for multiple markets. He presents a variational inequality formulation of the problem. Qian [17] considers two competitive parallel distribution channels (PDCs), where in each the retailer plays as a leader and moves first, and the manufacturer is a follower with PDC one moving first, and PDC two moving next. Under the deterministic demand, she shows that the second-mover PDC has the advantage. Liu and Wang [13] investigate competition between two supply chains, each being composed of one upstream firm and one downstream firm, and show that supply chain structure and externality should be considered simultaneously when the firms make materials supply strategies. Xiao and Yang [26] develop a price-service competition model of two supply chains under demand uncertainty when each supply chain consists of one risk-neutral supplier and one risk-averse retailer. They find that the higher the risk sensitivity of one retailer is, the lower his optimal service level and retail price will be. Ha and Tong [9] study contracting and information sharing in two competing supply chains with each supply chain having one manufacturer and one retailer. They study this problem using a two-stage game for two different contract types and explain that information sharing is a source of competitive advantage in supply chains' competition. Baron et al. [5] study the Nash equilibrium of two supply chains each being composed of one manufacturer and one retailer by extending the seminal work of McGuire and Staelin [15]. They show that both the traditional Manufacturer's Stackelberg (MS) and the Vertical Integration (VI) strategie are special cases of Nash bargaining on the wholesale price. Shou et al. [18] investigate the competition of two supply chains, each consisting of one retailer and its exclusive supplier under supply uncertainty. They explore the impact of supply uncertainty and chain-to-chain competition on contract choices and supply chain profits. Wu et al. [23] extend the work of Baron et al. [5] to include uncertain demand. They consider joint pricing and quantity decisions in competition between two supply chains each being composed of a manufacturer and a retailer. Anderson and Bao [3] investigate price competition having a linear demand function with deterministic parameters. They assume there are  $n$

supply chains competing in the market with substitutable products each having one manufacturer and one retailer. They study the effect of varying the level of price competition on profits of the industry participants. Xiao and Yang [27] develop an information revelation mechanism model of a supply chain facing an outside integrated competitor in price and service competition under demand uncertainty. They assume that the supply chain consists of one risk-neutral manufacturer and one risk-averse retailer and manufacturer partially pays the retailer's risk cost incurred by demand uncertainty. Ha et al. [10] consider supply chains competing with one another and seek to examine how the value of information sharing depends on production diseconomies of scale, information accuracy, competition intensity, and the type of competition. Liu et al. [14] study relations between eco-friendly operations level, competition level and profit of the supply chain partners. Ai et al. [2] develop models for two competing supply chains selling substitutable products. They examine how a full-returns policy affects the decisions of retailers as well as manufacturers. Wu [22] examines the impact of buyback policy on retail price, order quantity and wholesale price in competition of two manufacturer–retailer supply chains and shows buyback strategy can lead to a higher profit than a non-buyback policy. Reviewing price competition literature in SCM, we summarize related studies in Table 1.

**Table 1.** A summary of price competition literature review in SCM

Research	Competing supply chains' partners	Modeling approach
Zhang [28]	$n$ competing supply chains each consisted of its specific number of tiers.	Variational inequality formulation
Qian [17]	two supply chains each consisted of a manufacturer and a retailer.	Game theory
Liu and Wang [13]	two supply chains each consisted of one upstream firm and one downstream firm.	Game theory
Xiao and Yang [26]	two supply chains each consisted of one risk-neutral supplier and one risk-averse retailer.	Game theory
Ha and Tong [9]	two supply chains each consisted of one manufacturer and one retailer.	Game theory
Baron et al. [5]	two supply chains each consisted of one manufacturer and one retailer.	Game theory
Shou et al. [18]	two supply chains each consisted of one retailer and its exclusive supplier.	Game theory
Wu et al. [23]	two supply chains each consisted of a manufacturer and a retailer.	Game theory
Anderson and Bao [3]	$n$ competing supply chains each consisted of one manufacturer and one retailer.	Game theory
Xiao and Yang [27]	a supply chain consisted of one risk-neutral manufacturer and one risk-averse retailer facing an outside integrated competitor.	Game theory
Ha et al. [10]	two supply chains each consisted of one manufacturer selling to one retailer.	Game theory
Liu et al. [14]	two competing supply chains each consisted of one manufacturer selling product to two retailers in common between two supply chains.	Game theory
Ai et al. [2]	two competing supply chains each consisted of one manufacturer selling product to its own retailer.	Game theory
Wu [22]	two manufacturer–retailer supply chains.	Game theory

In Table 1, we can see that in the SCM's price competition literature, game theory is a dominant approach for modeling the problem and deriving optimal decisions under different assumptions. Also, it is seen that the models for price competition problem have been investigated from theoretical points of view.

Here, our new supply chain consists of one risk-neutral manufacturer and one risk-averse retailer and the market demand is considered to be stochastic. In addition, similar to Xiao and Yang [27], we consider a situation that the manufacturer partially pays the retailer's risk cost incurred by demand uncertainty (risk sharing rule). This is common in the real-world, where partners in a supply chain often share risk with one another, as being beneficial to the whole supply chain. Also, the retail price of market (competing supply chains) is considered to be stochastic and all the information is common knowledge to the partners who play a one-shot game within a single period. As the main contribution of our study, we apply the developed competition model to a real-world case in FMCG supply chain under uncertain environment and obtain the optimal wholesale and retail prices. We show our proposed prices to be better than the current wholesale and retail prices in this supply chain and yield more profit to both manufacturer and retailer and generally to the entire supply chain. To the best of our knowledge, no previous SCM work studied price competition with these considerations to apply it for pricing to a real-world case.

### 3. Basic Model

Manufacturer produces products with a unit production cost  $c_m$  (subscript  $m$  represents manufacturer) and sells to market through her retailer. The retailer purchases product from manufacturer with a unit wholesale price  $w$ , and then adds some values to the product with a unit cost  $c_r$  (subscript  $r$  represents retailer), and then determines her retail price. The following notation is used to show the parameters and decision variables in the model.

- $\tilde{a}$ : stochastic market base for supply chain, with mean  $\bar{a} > 0$ , and variance  $\sigma^2$ ;
- $c_m$ : unit production cost of manufacturer;
- $c_r$ : unit cost of adding some values to the product by retailer;
- $d$ : substitutability coefficient of products (the cross-price sensitivity of competitors in the market),  $0 \leq d \leq 1$ ;
- $b$ : self-price sensitivity of retailer,  $0 \leq d \leq b$ ;
- $p$ : retail price of retailer;
- $\tilde{p}$ : stochastic retail price of market (competing supply chains) with mean  $\bar{p} > 0$ , and variance  $\sigma_p^2$ ;
- $w$ : unit wholesale price of manufacturer;
- $\lambda$ : constant absolute risk aversion (CARA) of retailer,  $\lambda \geq 0$ ;
- $t$ : fraction of the risk cost shared by manufacturer in the total risk cost,  $0 \leq t \leq 1$ .

We study the competition though retail price as the only important factor affecting the market demand. Here, we use a linear demand function in which the market demand for retailer is given by

$$\tilde{q} = \tilde{a} - bp + d\tilde{p}, \quad (1)$$

Linear price-dependent demand functions are used in many economics and SCM studies (see, for example, McGuire and Staelin [15], Choi [7], Xiao and Qi [25], Anderson and Bao [3], Sinha and Sarmah [19], Liu et al. [14], Ai et al. [2], Wu [22]) because they are tractable and admit closed-form

solutions. Similar to Anderson and Bao [3], our attention here is focused on equilibrium solutions in the market and the behavior near the equilibrium is important. This can be thought of taking a linearization of a non-linear demand function near the equilibrium point. Also, we generally suppose market demand for new supply chain is decreasing with the supply chain's own retail price and increasing with the competing supply chains' retail price. This is often modeled through the cross-price parameter in a demand curve. Milgrom and Roberts [16] argue that the logarithm of the demand function needs to have increasing differences and show that a linear demand function will satisfy these properties (as will Logit, Cobb-Douglas and constant elasticity of substitution). As in Anderson and Bao [3], here, for convenience, we will use a single substitutability coefficient of two products ( $d$ ) to capture competitive effects for each pair of products. We also suppose that the two product differentiation parameters ( $b$  and  $d$ ) are independent. Also, we suppose that self-price sensitivity is stronger than cross-price sensitivity (Hanssens et al. [11]). Since we do not have negative demand, we assume that

$$\bar{a} - bp \geq -d\bar{p}. \quad (2)$$

According to the above assumptions, profit functions of retailer and manufacturer represented by  $\tilde{\pi}_r$  and  $\tilde{\pi}_m$  are given by

$$\tilde{\pi}_r = (p - w - c_r)(\tilde{a} - bp + d\tilde{p}), \quad (3)$$

$$\tilde{\pi}_m = (w - c_m)(\tilde{a} - bp + d\tilde{p}). \quad (4)$$

Regarding to sensitivity of retailer, here we assume that retailer assesses his utility via the following Mean-Variance value function of his random profit (Agrawal and Sshadri [1], Tsay [21], Gan et al. [8], Lee and Schwarz [12], Xiao and Yang [26-27]):

$$u(\tilde{\pi}_r) = E(\tilde{\pi}_r) - \lambda Var(\tilde{\pi}_r). \quad (5)$$

In (5), the second term is risk cost of retailer, and  $\lambda$  reflects the attitude of retailer towards uncertainty. Eq. (5) means that retailer will make a trade-off between the mean and the variance of his random profit. The larger the  $\lambda$  is, the more conservative the retailer's behavior will be. That is, the higher the retailer's risk sensitivity is, the higher the risk cost will be (Xiao and Yang [26-27]). Regarding to (5), the utility function of retailer is

$$u(\tilde{\pi}_r) = (p - w - c_r)(\bar{a} - bp + d\bar{p}) - (1 - t)\lambda(p - w - c_r)^2(\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p})). \quad (6)$$

Xiao and Yang [27] refer to  $\lambda(p - w - c_r)^2(\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p}))$  as risk cost of retailer. When retailer becomes more conservative, the effect of demand uncertainty on his utility increases. According to risk sharing rule in the paper (fraction  $t$ ), manufacturer will pay a risk subsidy  $t.\lambda(p - w - c_r)^2(\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p}))$  to her retailer. Hence, the last term in (6) is the risk cost borne by the retailer (Xiao and Yang [27]).

Also, in (6),  $cov(\tilde{a}, \tilde{p})$  is the covariance between the stochastic variables  $\tilde{a}$  and  $\tilde{p}$ . We suppose that there is a positive correlation between  $\tilde{a}$  and  $\tilde{p}$  ( $0 \leq cov(\tilde{a}, \tilde{p})$ ). It means that with increasing the competing supply chains' retail price (market retail price), the market base for new supply chain will increase (demand for new supply chain's product will increase) and vice versa. We suppose that the manufacturer is risk-neutral and the expected profit function is

$$E(\tilde{\pi}_m) = (w - c_m)(\bar{a} - bp + d\bar{p}) - t.\lambda(p - w - c_r)^2(\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p})). \quad (7)$$

Here, we assume that the power structure within the supply chain is Retailer's Stackelberg structure which is common in many real-world industries. In this Stackelberg game, the retailer acts as the Stackelberg leader and determines the optimal retail price to maximize his own profit. Then, manufacturer (the Stackelberg follower) chooses the optimal wholesale price and makes a product to fit the determined retail price. This structure is common in some kinds of retailers having more market powers and brand advantages in their supply chains, such as Wal-Mart, K-Mart, etc.

#### 4. Equilibrium Analysis

In this section, we attempt to obtain the equilibrium prices of the manufacturer and retailer in the new supply chain. To do so, we first present two lemmas to establish that the expected profit function of manufacturer and utility function of retailer are concave functions. Then, by using these two lemmas we obtain equilibrium prices. We need to define the followings:

$$B = \sigma^2 + d^2 \sigma_p^2 + 2d \cdot \text{cov}(\tilde{a}, \tilde{p}), \quad (8)$$

$$G = 2(1-t) \cdot \lambda B, \quad (9)$$

$$Q = \frac{\bar{a} + d\bar{p} + c_r b + c_r G}{2b + G}, \quad (10)$$

$$Y = 6b^2 \cdot \lambda \cdot B + 4b \cdot \lambda^2 \cdot B^2 - 5b^2 \cdot \lambda \cdot B \cdot t - 8b \cdot \lambda^2 \cdot B^2 \cdot t + 4b \cdot \lambda^2 \cdot B^2 \cdot t^2 + 2b^3, \quad (11)$$

and

$$\begin{aligned} X = & -2b \cdot d \cdot \bar{p} \cdot \lambda \cdot B \cdot t - 2a \cdot b \cdot \lambda \cdot B \cdot t + 3b \cdot d \cdot \bar{p} \cdot \lambda \cdot B + 2c_r \cdot b^2 \cdot \lambda \cdot B \cdot t + 4b \cdot \lambda^2 \cdot c_r \cdot B^2 \cdot t \\ & - 2b \cdot \lambda^2 \cdot B^2 \cdot t^2 \cdot c_r - 4 \cdot d \cdot \bar{p} \cdot \lambda^2 \cdot B^2 \cdot t + 2d \cdot \bar{p} \cdot \lambda^2 \cdot B^2 \cdot t^2 - 3c_m \cdot b^2 \cdot \lambda \cdot B \cdot t \\ & - 4c_m \cdot b \cdot \lambda^2 \cdot B^2 \cdot t + 2c_m \cdot b \cdot \lambda^2 \cdot B^2 \cdot t^2 + 2 \cdot a \cdot \lambda^2 \cdot B^2 + d \cdot \bar{p} \cdot b^2 + 3a \cdot b \cdot \lambda \cdot B \\ & - 4a \cdot \lambda^2 \cdot B^2 \cdot t + 2a \cdot \lambda^2 \cdot B^2 \cdot t^2 - 3c_r \cdot b^2 \cdot \lambda \cdot B - 2 \cdot b \cdot \lambda^2 \cdot c_r \cdot B^2 \\ & + 2 \cdot \lambda^2 \cdot B^2 \cdot d \cdot \bar{p} + 3c_m \cdot b^2 \cdot \lambda \cdot B + 2c_m \cdot b \cdot \lambda^2 \cdot B^2 + a \cdot b^2 - c_r \cdot b^3 + c_m \cdot b^3. \end{aligned} \quad (12)$$

**Lemma 1.** The utility function  $u(\tilde{\pi}_r)$  is a concave function on  $p$ , if  $2b + G \neq 0$ .

**Proof.** We have

$$u(\tilde{\pi}_r) = (p - w - c_r)(\bar{a} - bp + d\bar{p}) - (1-t)\lambda(p - w - c_r)^2(\sigma^2 + d^2\sigma_p^2 + 2d \cdot \text{cov}(\tilde{a}, \tilde{p})). \quad (13)$$

Take the first and second derivatives of  $u(\tilde{\pi}_r)$ :

$$\frac{\partial u(\tilde{\pi}_r)}{\partial p} = \bar{a} - 2bp + d\bar{p} + wb + c_r b - 2(1-t)\lambda(p - w - c_r)(\sigma^2 + d^2\sigma_p^2 + 2d \cdot \text{cov}(\tilde{a}, \tilde{p})), \quad (14)$$

$$\begin{aligned} \frac{\partial^2 u(\tilde{\pi}_r)}{\partial p^2} &= -2b - 2(1-t)\lambda(\sigma^2 + d^2\sigma_p^2 + 2d \cdot \text{cov}(\tilde{a}, \tilde{p})) \\ &= -2\left(b + (1-t)\lambda(\sigma^2 + d^2\sigma_p^2 + 2d \cdot \text{cov}(\tilde{a}, \tilde{p}))\right) = -(2b + G). \end{aligned} \quad (15)$$

Since  $\lambda, b, d \geq 0$ ,  $0 \leq t \leq 1$ , and  $0 \leq \text{cov}(\tilde{a}, \tilde{p})$ , we have  $\frac{\partial^2 u(\tilde{\pi}_r)}{\partial p^2} \leq 0$ . From the first-order condition of  $u(\tilde{\pi}_r)$ , we have

$$\frac{\partial u(\tilde{\pi}_r)}{\partial p} = 0, \quad (16)$$

$$\Rightarrow p = \frac{\bar{a} + d\bar{p} + wb + c_r b + 2(1-t)\lambda(w + c_r)(\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p}))}{2(b + (1-t)\lambda(\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p}))}. \quad (17)$$

To have (17) well-defined, we must have

$$2\left(b + (1-t)\lambda(\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p}))\right) = 2b + G \neq 0, \quad (18)$$

Which implies  $\frac{\partial^2 u(\tilde{\pi}_r)}{\partial p^2} < 0$ . Thus, the utility function  $u(\tilde{\pi}_r)$  is a concave function on  $p$  and the solution satisfying the first-order condition for  $u(\tilde{\pi}_r)$  is optimal.  $\square$

**Lemma 2.** The expected profit function  $E(\tilde{\pi}_m)$  is a concave function on  $w$ , if  $2\lambda B - G \neq 0$ .

**Proof.** We have

$$E(\tilde{\pi}_m) = (w - c_m)(\bar{a} - bp + d\bar{p}) - t.\lambda(p - w - c_r)^2 (\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p})). \quad (19)$$

Take the first and second derivatives of  $E(\tilde{\pi}_m)$ :

$$\frac{\partial E(\tilde{\pi}_m)}{\partial w} = \bar{a} - bp + d\bar{p} + 2t.\lambda(p - w - c_r) (\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p})), \quad (20)$$

$$\frac{\partial^2 E(\tilde{\pi}_m)}{\partial w^2} = -2t.\lambda (\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p})) = -(2\lambda B - G). \quad (21)$$

Since  $\lambda, d \geq 0$ ,  $0 \leq t \leq 1$ , and  $0 \leq cov(\tilde{a}, \tilde{p})$ , we have  $\frac{\partial^2 E(\tilde{\pi}_m)}{\partial w^2} \leq 0$ . From the first-order condition of  $E(\tilde{\pi}_m)$ , we have

$$\frac{\partial E(\tilde{\pi}_m)}{\partial w} = 0, \quad (22)$$

$$\Rightarrow w = \frac{\bar{a} - bp + d\bar{p} + 2t.\lambda(p - c_r)(\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p}))}{2t.\lambda(\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p}))}, \quad (23)$$

To have (23) well-defined, we must have

$$2t.\lambda(\sigma^2 + d^2\sigma_p^2 + 2d.cov(\tilde{a}, \tilde{p})) = 2\lambda B - G \neq 0, \quad (24)$$

Which implies  $\frac{\partial^2 E(\tilde{\pi}_m)}{\partial w^2} < 0$ . Thus, the utility function  $E(\tilde{\pi}_m)$  is a concave function on  $w$  and the solution satisfying the first-order condition for  $E(\tilde{\pi}_m)$  is optimal.  $\square$

**Proposition.** If  $\bar{a} - bp \geq -d\bar{p}$ ,  $2b + G \neq 0$ ,  $Y \neq 0$ , and  $2\lambda B - G \neq 0$ , then the optimal wholesale and retail prices of new supply chain are

$$w^* = \frac{X}{Y}, \quad (25)$$

$$p^* = Q + \frac{(G+b).X}{(2b+G).Y}. \quad (26)$$

**Proof.** To find equilibrium prices in the new supply chain with Manufacturer's Stackelberg structure, we solve the game using backward induction. Regarding Lemma 1, from the first-order condition for  $u(\tilde{\pi}_r)$ , we have

$$p = Q + \frac{(b+G)w}{2b+G}. \quad (27)$$

By substituting (27) in  $E(\tilde{\pi}_m)$ , we have

$$E(\tilde{\pi}_m) = (w - c_m) \left( \bar{a} - b \left( Q + \frac{(b+G)w}{2b+G} \right) + d\bar{p} \right) - t.\lambda. \left( Q + w \left( \frac{b+G}{2b+G} - 1 \right) - c_r \right)^2 B. \quad (28)$$

From the first-order condition of  $E(\tilde{\pi}_m)$ , (28) we have

$$\frac{\partial E(\tilde{\pi}_m)}{\partial w} = 0, \quad (29)$$

$$\Rightarrow w^* = \frac{X}{Y}. \quad (30)$$

By substituting (30) in (27), we have

$$\Rightarrow p^* = Q + \frac{(G+b).X}{(2b+G).Y}, \quad (31)$$

where for (30) and (31) to be well-defined, we must have  $Y \neq 0$  and  $2b + G \neq 0$ .  $\square$

## 5. An Illustrative Case Study

Solico Group is a large Iranian business group with many active companies. The most famous companies of this group are Kalleh Dairy Co., Kalleh Amol Meat Co., Aris Amol Co., Tehran Meat Products Co., and many others. These companies operate in different industry sectors such as dairy, process meat, ice cream, beverage, dressing, packaging, import and export.

As a case study, the competition between Tehran Meat Products Co. (Solico) and market (consisted of three potential rival supply chains) is investigated here. The supply chain of Solico is composed of a manufacturer of meat products and one exclusive sales organization (retailer). The retailer buys the products from manufacturer and performs all activities concerning with advertising, marketing, transportation and delivering the product to the final market. Here, it is assumed that the retailer adds some values to the product by the unit cost  $c_r$ . Amidst the varied range of products, we have chosen one chicken-based product for this study. These kinds of products have short life cycles and if they are not sold within a certain time (about 24 hours), then they are spoiled. The demand for these products is very sensitive to price appropriate pricing leads to decrease in the spoiled products and increasing the profits of supply chain partners. Hence, the role of pricing is very important for Solico in market competition.

Table 2 shows the optimal wholesale and retail prices in the proposed method (pricing method according to the above proposition) in comparison with the current pricing method in Solico supply chain for a 20-days period. In this table, the unit production cost of manufacturer and the mean of market price vary per day due to the daily variation in the price of chicken. Through interviewing the experts and investigating the related documents, the following values have been considered for parameters



$$\bar{a} = 2000, \sigma = 10, c_r = 50, \sigma_p = 10, \text{cov}(\tilde{a}, \tilde{p}) = 0.6, b=1, d=0.9, t=0.2, \lambda=0.03.$$

Using these values, we have  $B=182.08$ ,  $G=8.739$ , and  $Y=105.69$ . So, the conditions of our proposition are held by having the following inequality

$$2000 - p^* \geq -0.9 \bar{p}. \quad (32)$$

From Table 2, we see that the proposed method is better than the existing method and gains more profit for the manufacturer and retailer. In Table 2,  $\Delta_{E(\tilde{\pi}_m)}\%$ ,  $\Delta_{u(\tilde{\pi}_r)}\%$ , and  $\Delta_{Sc}\%$  are the improvement percentages for the manufacturer, retailer and the entire supply chain, respectively being computed according to the following equations:

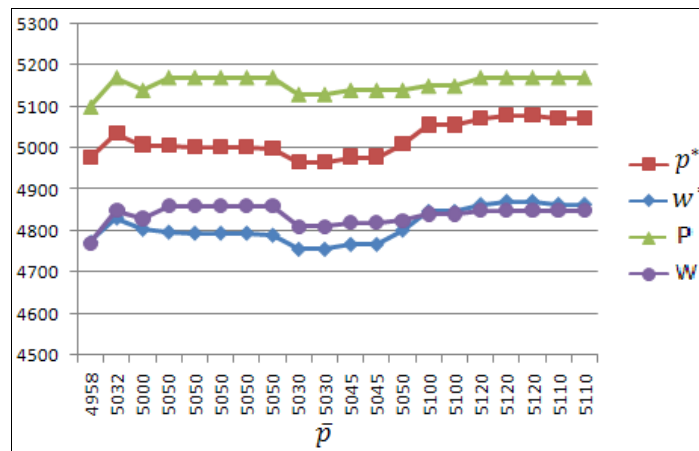
$$\Delta_{E(\tilde{\pi}_m)}\% = \frac{E(\tilde{\pi}_m^*) - E(\tilde{\pi}_m)}{E(\tilde{\pi}_m)} \times 100, \quad (33)$$

$$\Delta_{u(\tilde{\pi}_r)}\% = \frac{u(\tilde{\pi}_r^*) - u(\tilde{\pi}_r)}{u(\tilde{\pi}_r)} \times 100, \quad (34)$$

$$\Delta_{Sc}\% = \frac{Sc^* - Sc}{Sc} \times 100. \quad (35)$$

In Table 2, within a 20-days period, applying the proposed method, in the average manufacturer's profit, the retailer's profit, and the profit of entire supply chain (sum of profits of manufacturer and retailer which is represented by  $Sc$ ) are respectively improved by 9.66%, 107.74% and 12.15% in comparison with the existing method.

Figure 1 shows the comparison between the prices of two methods versus the mean of market price within the 20-days period. This figure shows that the optimal retail price is less than the current retail price but in the case of wholesale price, for high values of  $\bar{p}$ , the optimal wholesale price is greater than the current wholesale price.

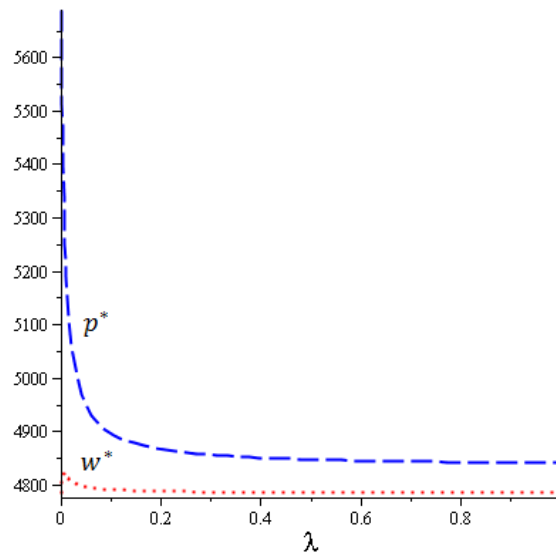


**Figure 1.** Comparison between prices of two methods versus mean of market price

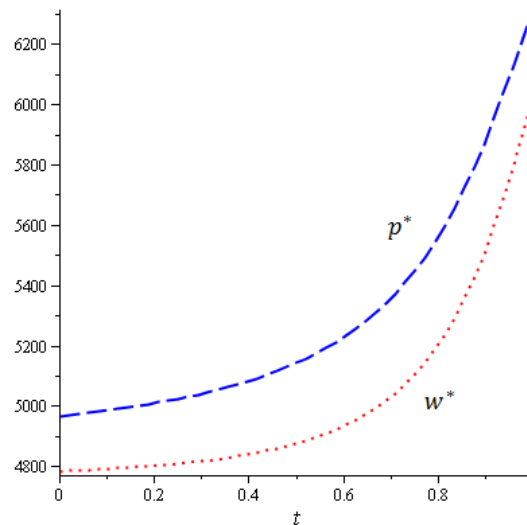
**Table 2.** Comparison between the proposed and existing methods in pricing

Day	$c_m$	$\bar{p}$	Current Method					Proposed Method					Difference		
			w	p	$E(\tilde{\pi}_m)$	$u(\tilde{\pi}_r)$	SC	$w^*$	$p^*$	$E(\tilde{\pi}_m^*)$	$u(\tilde{\pi}_r^*)$	$Sc^*$	$\Delta_{E(\tilde{\pi}_m)}\%$	$\Delta_{u(\tilde{\pi}_r)}\%$	$\Delta_{Sc}\%$
1	3100	4958	4770	5100	2189223	38814	2228037	4773.2	4975.8	2461585	125060	2586646	12.44	222.2	16.09
2	3150	5032	4850	5170	2230318	48308	2278627	4831.6	5034.9	2486321	126317	2612638	11.47	161.47	14.65
3	3125	5000	4830	5140	2244948	58193	2303141	4804.6	5007.8	2480647	126029	2606676	10.49	116.56	13.17
4	3065	5050	4860	5170	2394273	62093	2456366	4797.7	5005.7	2639794	134114	2773908	10.25	115.98	12.92
5	3060	5050	4860	5170	2401148	62093	2463241	4795.2	5003.5	2647495	134505	2782001	10.26	116.61	12.94
6	3060	5050	4860	5170	2401148	62093	2463241	4795.2	5003.5	2647495	134505	2782001	10.26	116.61	12.94
7	3060	5050	4860	5170	2401148	62093	2463241	4795.2	5003.5	2647495	134505	2782001	10.26	116.61	12.94
8	3050	5050	4860	5170	2414898	62093	2476991	4790.3	4999.0	2662933	135290	2798223	10.27	117.88	12.96
9	3000	5030	4810	5130	2448928	58622	2507551	4756.4	4966.6	2712633	137815	2850449	10.76	135.08	13.67
10	3000	5030	4810	5130	2448928	58622	2507551	4756.4	4966.6	2712633	137815	2850449	10.76	135.08	13.67
11	3010	5045	4820	5140	2633908	93553	2727461	4768.2	4978.5	2718097	138092	2856190	3.19	47.6	4.72
12	3010	5045	4820	5140	2633908	93553	2727461	4768.2	4978.5	2718097	138092	2856190	3.19	47.6	4.72
13	3075	5050	4825	5140	2382030	65447	2447477	4802.6	5010.2	2624424	133333	2757757	10.17	103.72	12.67
14	3120	5100	4840	5150	2402948	78993	2481941	4847.6	5055.2	2624424	133333	2757757	9.21	68.79	11.11
15	3120	5100	4840	5150	2402948	78993	2481941	4847.6	5055.2	2624424	133333	2757757	9.21	68.79	11.11
16	3135	5120	4850	5170	2386528	69692	2456221	4864.2	5071.9	2629030	133567	2762598	10.16	91.65	12.47
17	3150	5120	4850	5170	2434651	69692	2364958	4871.6	5078.6	2606039	132399	2738439	10.19	89.97	12.47
18	3150	5120	4850	5170	2434651	69692	2364958	4871.6	5078.6	2606039	132399	2738439	10.19	89.97	12.47
19	3145	5110	4850	5170	2356803	67262	2424066	4864.6	5071.4	2599925	132089	2732014	10.31	96.37	12.7
20	3145	5110	4850	5170	2356803	67262	2424066	4864.6	5071.4	2599925	132089	2732014	10.31	96.37	12.7
Ave	3086	5061	4838	5154	2400007	66358	2452427	4813.3	5020.8	2622473	133234	2755707	9.66	107.74	12.15

Figure 2 illustrates the effects of the constant absolute risk aversion of retailer ( $\lambda$ ) on the optimal wholesale and retail prices in the supply chain when  $t=0.2$ . In this figure, we observe that generally the optimal retail and wholesale prices of the new supply chain decrease with the CARA of retailer. Also, Figure 3 shows the effect of the fraction of the risk cost shared by manufacturer in the total risk cost ( $t$ ) on the optimal wholesale and retail prices when  $\lambda = 0.03$ . From this figure, we know that in general the optimal wholesale and retail prices of the new supply chain increase when fraction of risk cost shared by manufacturer in the total risk cost ( $t$ ) increases.



**Figure 2.** Optimal wholesale and retail prices versus constant absolute risk aversion of retailer when  $t=0.2$

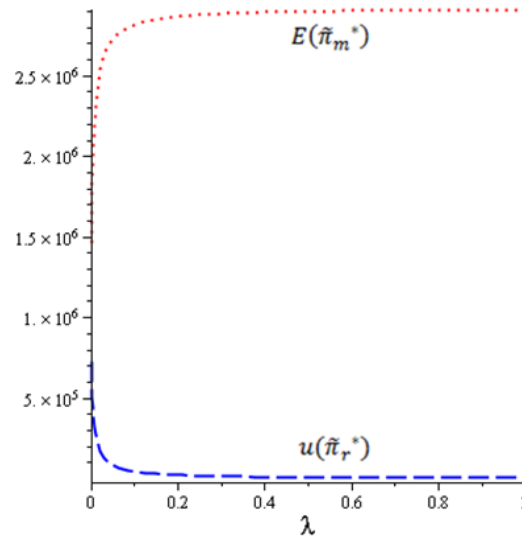


**Figure 3.** Optimal wholesale and retail prices versus fraction of risk sharing when  $\lambda = 0.03$

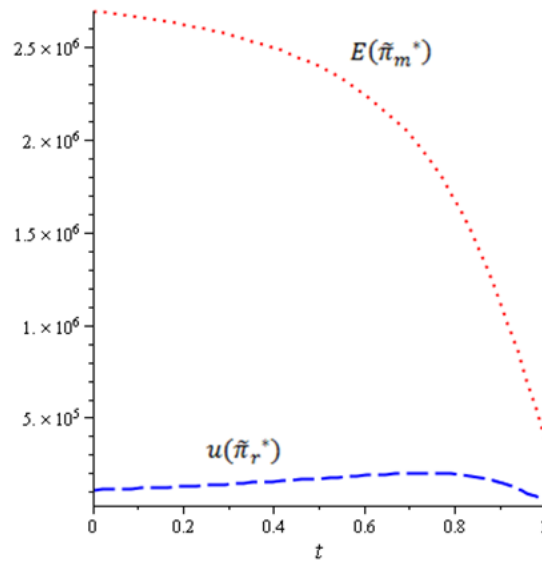
Figures 4 and 5 describe how optimal expected profit of manufacturer and optimal utility of retailer depend on  $\lambda$  and  $t$ . From these figures, we know that in general, the constant absolute risk aversion of retailer ( $\lambda$ ) and the fraction of risk cost shared by manufacturer in the total risk cost ( $t$ ) carry

significant weights on the decisions of the new supply chain members and appropriate determination of these parameters can increase coordination within the supply chain.

Figure 4 shows that the optimal expected profit of manufacturer increases with the CARA but the optimal utility of retailer decreases with the risk sensitivity of her retailer. In Figure 5, the optimal expected profit of manufacturer decreases with the risk-sharing fraction but the optimal utility of retailer first increases and then decreases with  $t$ .



**Figure 4.** Optimal profit of new supply chain partners versus  $\lambda$  when  $t=0.2$



**Figure 5.** Optimal profit of new supply chain partners versus  $t$  when  $\lambda = 0.03$

Similar to Xiao and Yang [26], here we did not consider sensitivity analysis of the substitutability coefficient of the two products because increase in this parameter will result in aggregate demand amplification.

## 6. Conclusions

Price competition is a major aspect of supply chain competition in the real-world. Here, we explored price competition of a new supply chain competing in an stochastic market with demand uncertainty environment. The new supply chain consisted of one risk-neutral manufacturer and one risk-averse retailer in which the manufacturer was a leader and the retailer was a follower. The manufacturer should pay a fraction of the risk cost of retailer. We supposed that all the information was common knowledge for partners in the new supply chain and the retail price of market was stochastic.

We applied this model to a real-world case in an FMCG supply chain and obtained optimal wholesale and retail prices. This is the first time that price competition model in supply chains with such considerations was developed and applied to obtaining optimal prices in a real-world case. We showed our proposed prices to be better than the current wholesale and retail prices in this supply chain and gain more profit for both manufacturer and retailer and generally for the entire supply chain. Also, we found that the risk sensitivity of retailer and the fraction of risk cost shared by manufacturer in the total risk cost carried significant weights on the decisions of the new supply chain's members.

There are several future research directions that can be suggested. First, extension of this model to the case with risk-averse manufacturer is challenging and interesting. Secondly, extension of our work to a multi-period problem with repeated game can be studied. Finally, considering different types of demand functions such as iso-elastic or logit can be interesting.

## References

- [1] Agrawal, V. and Seshadri, S. (2000), Risk intermediation in supply chains, *IIE Transactions*, 32, 819-831.
- [2] Ai, X., Chen, J., Zhao, H. and Tang, X. (2012), Competition among supply chains: Implications of full returns policy, *International Journal of Production Economics*, 139, 257-265.
- [3] Anderson, E.J. and Bao, Y. (2009), Price competition with integrated and decentralized supply chains, *European Journal of Operational Research*, 200, 227-234.
- [4] Barnes, D. (2006), Competing Supply Chains are the Future, *Financial Times*, November 8.
- [5] Baron, O., Berman, O. and Wu, D. (2008), Bargaining in the Supply Chain and Its Implication to Coordination of Supply Chains in an Industry, Working Paper, Joseph L. Rotman School of Management, University of Toronto.
- [6] Boyaci, T. and Gallego, G. (2004), Supply chain coordination in a market with customer service competition, *Production and Operations Management*, 13, 3-22.
- [7] Choi, S.C. (1991), Price competition in a channel structure with a common retailer, *Marketing Science*, 10, 271-296.
- [8] Gan, X.H., Sethi, S.P. and Yan, H.M. (2005), Channel coordination with a risk-neutral supplier and a downside-risk-averse retailer, *Production and Operations Management*, 14, 80-89.
- [9] Ha, A.Y. and Tong, S. (2008), Contracting and information sharing under supply chain competition, *Management Science*, 54, 701-715.
- [10] Ha, A.Y., Tong, S. and Zhang, H. (2011), Sharing demand information in competing supply chains with production diseconomies, *Management Science*, 57, 566-581.

- [11] Hanssens, D.M., Parsons, L.J. and Schultz, R.L. (2001), *Market Response Models: Econometric and Time Series Analysis*, 2nd edition, Kluwer Academic Publishers, Boston.
- [12] Lee, J.Y. and Schwarz, L.B. (2007), Lead time reduction in a (Q, r) inventory system: An agency perspective, *International Journal of Production Economics*, 105, 204-212.
- [13] Liu, L. and Wang, L. (2007), Management strategy of materials supply for manufacturing organizations, *Computers & Industrial Engineering*, 53, 326-349.
- [14] Liu, Z., Anderson, T.D. and Cruz, J.M. (2012), Consumer environmental awareness and competition in two-stage supply chains, *European Journal of Operational Research*, 218, 602-613.
- [15] McGuire, T.W. and Staelin, R. (1983), An industry equilibrium analysis of downstream vertical integration, *Marketing Science*, 2, 161-191.
- [16] Milgrom, P. and Roberts, J. (1990), Rationalizability, learning, and equilibrium in games with strategic complementarities, *Econometrica*, 58, 1255-1277.
- [17] Qian, Y. (2006), Competitive parallel distribution channels with move sequence, *Journal of Systems Science and Systems Engineering*, 15, 114-123.
- [18] Shou, B., Huang, J. and Li, Z. (2009), Managing Supply Uncertainty under Chain-to-Chain Competition, Working Paper, City University of Hong Kong.
- [19] Sinha, S. and Sarmah, S.P. (2010), Coordination and price competition in a duopoly common retailer supply chain, *Computers & Industrial Engineering*, 59, 280-295.
- [20] Taylor, D.A. (2003), *Supply Chains: A Management Guide*, Pearson Education, Boston.
- [21] Tsay, A.A. (2002), Risk sensitivity in distribution channel partnerships: Implications for manufacturer return policies, *Journal of Retailing*, 78, 147-160.
- [22] Wu, D. (2013), Coordination of competing supply chains with news-vendor and buyback contract, *International Journal of Production Economics*, 144, 1-13.
- [23] Wu, D., Baron, O. and Berman, O. (2009), Bargaining in competing supply chains with uncertainty, *European Journal of Operational Research*, 197, 548-556.
- [24] Wu, O.Q. and Chen, H. (2003), Chain-to-Chain Competition under Demand Uncertainty, Working Paper, Stephen M. Ross School of Business, University of Michigan.
- [25] Xiao, T. and Qi, X. (2008), Price competition, cost and demand disruptions and coordination of a supply chain with one manufacturer and two competing retailers, *The International Journal of Management Science*, 36, 741-753.
- [26] Xiao, T. and Yang, D. (2008), Price and service competition of supply chains with risk-averse retailers under demand uncertainty, *International Journal of Production Economics*, 114, 187-200.
- [27] Xiao, T. and Yang, D. (2009), Risk sharing and information revelation mechanism of a one-manufacturer and one-retailer supply chain facing an integrated competitor, *European Journal of Operational Research*, 196, 1076-1085.
- [28] Zhang, D. (2006), A network economic model for supply chain versus supply chain competition, *International Journal of Management Science*, 34, 283-295.