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A Markov Model to Determine Optimal Equipment Adjustment in Multi-stage Production Systems Considering Variable Cost

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Our aim is to maximize expected profit per item of a multi-stage production system by determining best adjustment points of the equipments used based on technical product specifications defined by designer. In this system, the quality characteristics of items produced should be within lower and higher tolerance limits. When a quality characteristic of an item either falls beneath the lower limit or lies above the upper limit, it is reworked or classified as scrap, each with its own cost. A function of the expected profit per item is first presented based on equipment adjustment points. Then, the problem is modeled by a Markovian approach. Finally, numerical examples are solved in order to illustrate the proposed model.

***Keywords**: Optimum set of equipment adjustment, Markovian approach, Multi-stage serial production system.*

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1. Introduction

Today, many products are manufactured in multi-stage production systems where raw materials are transformed into final products during some distinct stages. However, only conforming items are allowed to go to the next stage and those that are non-conforming should be reworked or scrapped. To identify non-conforming items in any stage, it is essential to define some limitations on quality characteristics of the product in all stages. If the measured quality characteristics in any stage is higher than an upper specification limit (USL) or lower than a lower specification limit (LSL), then the product is designated as non-conforming. These limitations affect costs associated with production and non-conformity in terms of rework and scrap.

When an item is being produced, the equipment is usually adjusted at the midpoint of the upper and lower specification limits. In this case, assuming rework to be performed on the items with the quality characteristic higher than USL, and those with quality characteristic lower than LSL being scrapped, the probability of an item to be reworked is equal to the one for a scrapped item. However, the cost associated with rework is usually less than that of scrapped items. Therefore, the midpoint of USL and LSL is not an optimal equipment adjustment that minimizes the total cost. The question is how to determine the optimal adjustment point, which enables one to produce items with the minimum costs (Bowling et al. [3]). As a result, the main purpose of this research is to determine the optimal adjustment points of the equipments involved in a multi-stage production system using a Markovian approach.

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The method proposed here is applicable in many industries such as food packing processes and instrument production industries. The other advantage is the ease of its implementation, because the mathematical formulation is based on a uniform distribution that makes it possible to be encoded on a small chip (phase ICs or microcontrollers). It is also possible to set these chips on the equipment such as CNC Lathes and CNC milling machines to improve their intelligence. For example, since such a chip equipment calculates the optimal equipment adjustment and its rework or scrap costs, it is able to reduce production cost. The other distinctive advantage of the proposed method is to determine the amount of raw materials that is wasted after the reworking or the scrapping processes in order to considering them in the model, using the variable cost for a more realistic model.

The remainder of our work is organized as follows. A brief review on relevant works is given in Section 2. The required notations and assumptions are given in Section 3. The Markovian models for a single-stage as well as two-stage production systems are first developed and then a general model for an *n*-stage production system is derived in Section 4. To validate the model, numerical examples are provided in Section 5. Finally, results are discussed and conclusion is made in Section 6.

2. Related Literature

The determination of optimum process mean is one of the most important decision-making problems encountered in industrial applications and has been the topic of research for many years (Hunter and Kartha [8], Nelson [10], Pollock and Golhar [12], Teeravaraprug and Cho [13]). Al-Sultan and Pulak [2] proposed a model considering a production system with two stages in series to find the optimum mean values with a lower specification limit. Bowling et al. [3] employed a Markovian model in order to maximize the total profit associated with a multi-stage serial production system. Further, Pillai and Chandrasekharan [11] modeled the flow of material through a production system as an absorbing Markov chain considering scrapping and reworking. Fallahnehad and Niaki [7] proposed a Markovian model in order to maximize total profit associated with one-stage and two-stage serial production systems.

Abbasi et al. [1] developed a model when the deviation of a quality characteristic in one direction is more costly than in the opposite direction. They showed that the optimal mean of the process is not the middle point of the tolerance limits. Khasawneh et al. [9] considered a production system consisting of a single machine and a single inspection station for items with two-quality characteristics. Fallahnehad and Nasab [6] proposed a Markovian model in order to maximize total profit associated with a production process of items with two quality characteristics. Fallahenzhad and Ahmadi [5] considered a production system with several quality characteristics.

3. Notations and Assumptions

The following is a summary of notations being used:

$E(PR)$ Expected profit of an item.

$E(RV)$ Expected revenue of an item.

$E(PC)$ Expected process cost of an item.

$E(SC)$ Expected scrap cost of an item.

$E(RC)$ Expected rework cost of an item.

SP	Selling price of an item.
PC_i	Process cost in stage i .
SC_i	Scrap costs of raw material in stage i .
RC_i	Reworking cost of raw material in stage i .
n	Number of stages.
L_i	Lower specification limit in stage i .
U_i	Upper specification limit in stage i .
a_i	Lower limit of the uniform distribution for quality characteristic in stage i .
b_i	Upper limit of the uniform distribution for quality characteristic in stage i .
T_i	ith natural tolerance of stage i (the distance between a_i and b_i).
P_{ij}	Probability of going from state i to state j in a single stage.
m_{ij}	Expected number of transitions from a non-absorbing state i to another non-absorbing state j before absorption occurs.
f_{ij}	Long-run probability of going from a non-absorbing state i to an absorbing state j .
$\boldsymbol{P}$	Transition probability matrix.
$\boldsymbol{Q}$	A square matrix containing the transition probabilities of going from a non-absorbing state to another non-absorbing state.
$\boldsymbol{R}$	A matrix containing all probabilities of going from a non-absorbing state to an absorbing state (i.e., finished or to scrapped product).
$\boldsymbol{A}$	An identity matrix representing the probability of staying in a state.
$\boldsymbol{O}$	A matrix that shows the probability of escaping an absorbing state (always being zero).
$\boldsymbol{M}$	Fundamental matrix containing the expected number of transitions from any non-absorbing state to any other non-absorbing state before absorption occurs.
$\boldsymbol{F}$	Absorption probability matrix containing long-run probabilities of transition from any non-absorbing state to an absorbing state.

The assumptions to formulate the problem are:

1. Products are produced continuously.
2. All products are 100% inspected with a cost assumed to be part of the operation cost
3. There is a single quality characteristic measured in each stage.
4. If the measured quality characteristic of the product in stage i is lower than L_i or it is higher than U_i , then the product is assumed to be scraped or to be reworked, respectively. Otherwise, the product goes on to the next stage.
5. The quality characteristic of the product in stage i $\left(X_i\right)$ is assumed to be a uniform random variable in the interval $\left[a_i, b_i\right]$.
6. The sequence at which the product is produced in serial stages is fixed. It means that a conforming item that is processed in stage i always goes to stage $i+1$, when $i<n$.
7. The process is assumed statistically under control for the means of the quality characteristics in all stages.

4. Model Development

Consider a multi-stage serial production system in which products are continuously produced. Each production stage of this system involves a machine and an inspection station. The items and their quality characteristics are controlled at the inspection station in each stage. An item is reworked, if its measured quality characteristic of interest falls above U_i , scrapped, if its quality characteristic falls below L_i , a lower specification limit, or accepted, if its measure quality characteristic falls within the specification limits. Therefore, the expected profit per item can be expressed in a general form as follows (Khasawneh et al. [9]):

$$E(PR)=E(RV)-E(PC)-E(SC)-E(RE). \tag{1}$$

Here, we intend to develop a Markovian model to determine the optimum adjustment of the machines in different production stages. The model is first derived for a single-stage system in Section 4.1. Then, it is developed for a two-stage model in Section 4.2. Finally, it is extended for a serial n-stage production system in Section 4.3.

4.1. One-stage Systems

Consider a one-stage production system shown in Fig. 1, where possible states are given in parentheses. When raw material is processed by machine, the product, depending on its measured quality characteristics, lower, and upper specification limits, is classified as rework (state 1), conforming (finished product as state 2,) or scrapped (state 3).

Using the Markovian approach, the transition probability matrix of the one-stage system is:

$$\boldsymbol{P}=\begin{pmatrix} P_{11} & P_{12} & P_{13} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

As seen, $\boldsymbol{P}$ is an absorbing Markov chain with states 2 and 3 being absorbing and state 1 being transient. In this matrix, P_{11} denotes the rework probability of the product, P_{12} is the conforming probability, and P_{13} is the probability of the product being scrap. This value of quality characteristic is assumed to follow a uniform distribution in $[a_1,b_1]$. Thus, the probabilities P_{11}, P_{12} and P_{13} are the probabilities of intervals $[a_1,L_1]$, $[L_1,U_1]$ and $[U_1,b_1]$, respectively.

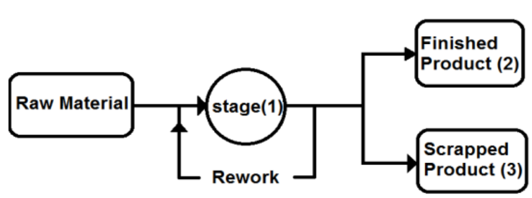

Figure 1. States of a product in a single-stage production system

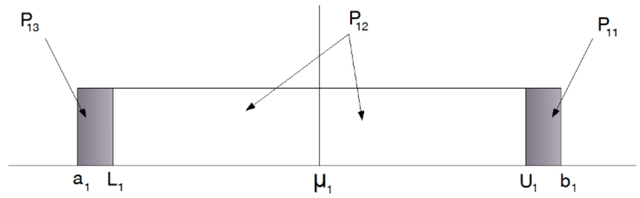

Figure 2. The percent of rework, conforming, and scrap with uniform distributions

It is assumed that if the value of quality characteristic gets to be more than the upper specification limit, then the item can be reworked to an acceptable item, but if the value of quality characteristic gets less than the lower specification limit, then the item is scrapped and it cannot continue to the next stage. Fig. 2 shows these distributions, where μ_1 is the process mean.

As a result, these probabilities are obtained to be:

$$\begin{aligned} P_{11} &= \int_{U_1}^{b_1} \frac{1}{b_1 - a_1} dx = \frac{b_1 - U_1}{b_1 - a_1}, \\ P_{12} &= \int_{L_1}^{U_1} \frac{1}{b_1 - a_1} dx = \frac{U_1 - L_1}{b_1 - a_1}, \\ P_{13} &= \int_{a_1}^{L_1} \frac{1}{b_1 - a_1} dx = \frac{L_1 - a_1}{b_1 - a_1}. \end{aligned} \tag{3}$$

Analyzing this Markov chain requires the rearrangement of the single-step transition probability matrix in the following form:

$$\boldsymbol{P} = \begin{pmatrix} A & O \\ R & Q \end{pmatrix}. \tag{4}$$

Rearranging the $\boldsymbol{P}$ matrix in the latter form yields the following matrix:

$$\boldsymbol{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ P_{12} & P_{13} & P_{11} \end{pmatrix}. \tag{5}$$

The fundamental matrix $\boldsymbol{M}$, i.e., $\boldsymbol{M = (I - Q)^{-1}}$, where $\boldsymbol{I}$ is the identity matrix, can be obtained as follows (Fallahnezhad [4]):

$$\mathbf{M} = (\mathbf{I} - \mathbf{Q})^{-1} = m_{11} = \frac{1}{(1 - P_{11})}, \tag{6}$$

where m_{11} denotes the expected number of times the transient state 1 is occupied before absorption occurs (i.e., the product is either accepted or rejected), given that the initial state is Now, the long-run absorption probability matrix $\boldsymbol{F}$ can be obtained by

$$\boldsymbol{F} = \boldsymbol{M} \times \boldsymbol{R} = \begin{array}{cc} & \begin{array}{cc} \quad 2 & \qquad 3 \end{array} \\ 1 & \begin{bmatrix} \dfrac{P_{12}}{1-P_{11}} & \dfrac{P_{13}}{1-P_{11}} \end{bmatrix} \end{array}. \tag{7}$$

The two elements of the $\boldsymbol{F}$ (f_{12}, f_{13}) denote the probabilities of an item to be accepted or scraped, respectively. Moreover, in order to derive the expected profit, the expected revenue in Eq. (1) is calculated by multiplying the selling price (SP) by the acceptance probability (i.e., f_{12}) of an item. Besides, the expected production cost of an item is PC_1 and the expected scrap cost is obtained by the scrap cost of raw material used in an item, by SC, multiplied by the absorption probability of a scrapping item (i.e., f_{13}) multiplied by the length of the interval $\left[E\left(X \mid X < L_1\right), L_1\right]$, which measures the amount of quality characteristic that is under the lower specification limit, i.e., $\left(L_1 - E\left(X \mid X < L_1\right)\right)$. Note that when a product enters state 2 or 3, it cannot go back. Hence, all the transitions are performed from state 1. Moreover, in case a product has to be reworked, the expected rework cost for a rework process is $RC_1\left(m_{11}-1\right)$ multiplied by amount of extra raw material above the upper specification limit, i.e., $E\left(X \mid X > U_1\right) - U_1$. Then, the expected profit of an item in a one-stage system is a function of f_{12}, f_{13}, m_{11}, L_1, U_1 and is formulated as:

$$\begin{aligned} E(PR) = {} & SP.f_{12} - PC_1 - SC_1.\, f_{13}\left(L_1 - E\left(X \mid X < L_1\right)\right) \\ & - RC_1\left(m_{11}-1\right)\left(E\left(X \mid X > U_1\right) - U_1\right)\right) \end{aligned} \tag{8}$$

The conditional expectation terms in Eq. (8) are obtained by

$$E\left[X \mid X < L_1\right] = \frac{1}{\Pr\left\{X < L_1\right\}} \int_{a_1}^{L_1} \frac{x}{b_1 - a_1} dx = \frac{1}{2}(L_1 + a_1), \tag{9}$$

$$E\left[X \mid X > U_1\right] = \frac{1}{\Pr\left\{X > U_1\right\}} \int_{U_1}^{b_1} \frac{x}{b_1 - a_1} dx = \frac{1}{2}(b_1 + U_1). \tag{10}$$

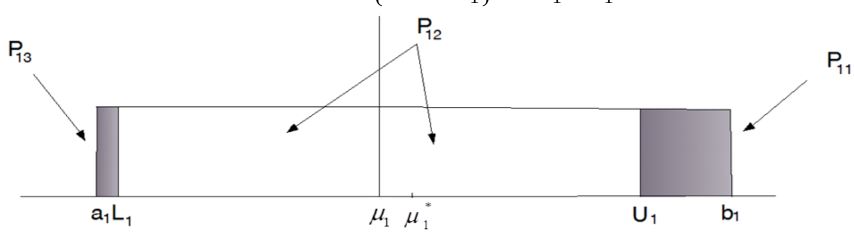

Figure 3. The percent of reworked and scrapped items for uniform distribution along with mean values

Using Eqs. (9) and (10) and substituting m_{11} and f_{12} (from Eq. (6) and (7)) in Eq. (8), the expected profit would be:

$$E(PR) = SP\left(1-\frac{P_{13}}{1-P_{11}}\right) - PC_1 - SC_1\left(\frac{P_{13}}{1-P_{11}}\right)\left[L_1 - \frac{1}{2}(L_1+a_1)\right] - RC_1\left(\frac{P_{11}}{1-P_{11}}\right)\left[\frac{1}{2}(b_1+U_1) - U_1\right] \tag{11}$$

Since the expected profit in Eq. (11) is a function of a_1 and b_1, it is also a function of the process mean (μ_1), because $\mu_1 = \frac{a_1+b_1}{2}$. Hence, the optimal value of μ_1, μ_1^* as the optimal equipment adjustment, can be obtained such that $E(PR)$ is maximized. It should be noticed that the length of the interval $[a_1, b_1]$, that is, $b_1 - a_1$, is assumed to be a constant and known. Thus, after determining μ_1, we can obtain the optimal values of a_1 and b_1.

As shown in Fig. 3, on one hand adjusting the equipment such that products with an optimal mean bigger than the average value of the tolerance limits, i.e., $(L_1+U_1)/2$, are produced tends to reduce the probability of producing scrapped items. On the other hand, due to the use of a uniform distribution, this adjustment tends to increase the probability of producing reworks by exactly the same magnitude. However, the main point is that the cost associated with a reworking process is much less than the one to scrap an item. Consequently, the optimal equipment adjustment may result in considerable long-run savings for a production system.

4.2. Two-stage Systems

Consider a serial two-stage production system shown in Fig. 4, where possible states are given in parentheses. The Markovian model that was developed for a one-stage system in Section 4.1 is extended for a two-stage production system, where it is assumed that the quality of an item produced in the second stage depends on the quality obtained in the first stage. Moreover, it is assumed that the quality characteristics measured in the two stages follow uniform distributions in their ranges between the lower and upper specification limits. Fig. 5 shows this assumption schematically.

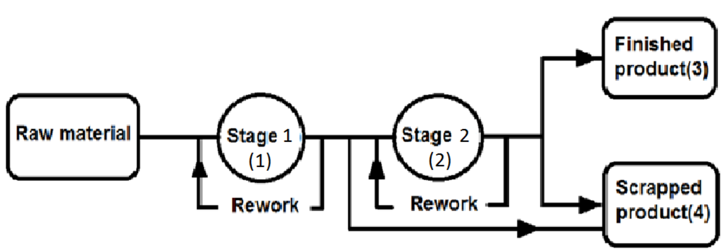

Figure 4. Illustration of item/product state in a two-stage production system

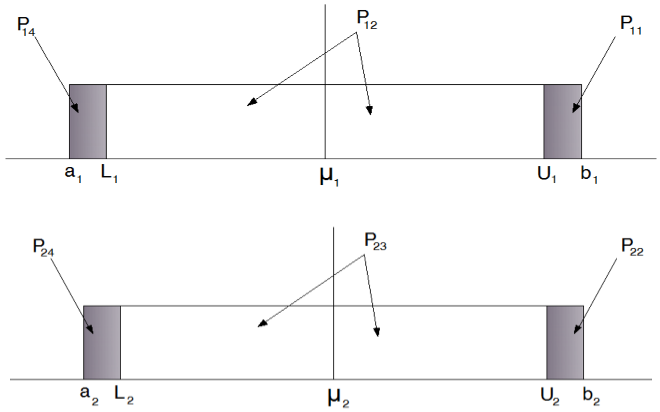

Figure 5. The percent of reworked and scrapped items for uniform distribution in a two-stage system

Once again, using the Markovian approach, the transition probability matrix of the two-stage system is

$$\boldsymbol{P}=\begin{bmatrix} P_{11} & P_{12} & 0 & P_{14} \\ 0 & P_{22} & P_{23} & P_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{12}$$

In (12), P_{ii} is the probability of an item to be reworked in stage i and P_{ii+1} is the probability of producing conforming product in stage i . Similar to the one-stage system, $\boldsymbol{P}$ is rearranged as

$$\boldsymbol{M}=\begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} \quad 1 \quad & \quad\quad 2 \quad\quad \end{array} \\ \begin{bmatrix} \dfrac{1}{1-P_{11}} & \dfrac{P_{12}}{(1-P_{11})(1-P_{22})} \\ 0 & \dfrac{1}{1-P_{22}} \end{bmatrix} \end{array} \tag{13}$$

where m_{ii} represents the expected number of times the transient state i , $(i=1,2)$, is occupied before absorption occurs (i.e., accepted or scrapped), given that the initial state is 1.

Moreover, the long-run absorption probability matrix $\boldsymbol{F}$ can then be determined as follows (Bowling et al. [3]):

$$\boldsymbol{F}=\begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} \quad\quad 3 \quad\quad & \quad\quad\quad 4 \quad\quad\quad \end{array} \\ \begin{bmatrix} \dfrac{P_{12}P_{23}}{(1-P_{11})(1-P_{22})} & \dfrac{P_{14}}{(1-P_{11})}+\dfrac{P_{12}P_{24}}{(1-P_{11})(1-P_{22})} \\ \dfrac{P_{23}}{(1-P_{22})} & \dfrac{P_{24}}{1-P_{22}} \end{bmatrix} \end{array}. \tag{14}$$

Consequently, Bowling et al. [3] obtained the expected profit function as

$$\begin{aligned} E(PR) = & \left[SP(1-f_{14})(1-f_{24})\right] - \left[PC_1 + PC_2(1-f_{14})\right] \\ & -\left[SC_1 \,.\, f_{14} + SC_2(1-f_{14})f_{24}\right] \\ & -\left[RC_1(m_{11}-1) + RC_2(m_{22}-1)(1-f_{14})\right] \end{aligned} \tag{15}$$

However, there are some shortcomings in Eq. (15) as follows:

1. Although it is correct to obtain the expected revenue by multiplication of the selling price SP and the probability of producing a conforming item, the probability term $(1-f_{14})(1-f_{24})$ is not correct. This is due to the fact that the absorption probability from state 2 to state 4, i.e., $f_{24} = \frac{P_{24}}{1-P_{22}}$, has already been considered in the calculation to obtain f_{14} (the percent of scrapped items transferred either from state 2 or state 4.) Therefore, the term $(1-f_{24})$ in the derivation of the probability of producing a conforming item is redundant and has to be deleted. In other words, instead of $SP(1-f_{14})(1-f_{24})$, the expected revenue should be $SP(1-f_{14})$.

2. The above shortcoming is also seen in the derivation of the production cost in the second stage, where it is determined by the multiplication of the production cost in the second stage and the probability of the item not going from state 1 to state 4, i.e., $(1-f_{14})$. However, the production cost in the second stage should be obtained for those items entering stage 2. In other words, instead of multiplying the production cost of the second stage by

$$1-f_{14} = \left(1-\left\{\frac{P_{14}}{1-P_{11}} + \frac{P_{12}P_{24}}{(1-P_{11})(1-P_{22})}\right\}\right),$$

it should only be multiplied by $1-\frac{P_{14}}{(1-P_{11})}$.

3. Similar shortcomings are observed in the derivations of the expected scrap cost in stages 1 and 2 as well as the expected rework cost in stage 2.

Hence, the expected profit function is revised as follows. The first two terms, $E(RV)$ and $E(PC)$, have already been revised in the two shortcomings. Moreover, by multiplying the probability of states (either scrapping or reworking) and their unit costs, the expected costs of scrapping and reworking are derived. To be more specific, the expected scrapping cost $E(SC)$ is calculated by multiplying the scrap probability in stage one, $\frac{P_{14}}{1-P_{11}}$, by

$SC_1\left(L_1 - E\left(X \mid X < L_1\right)\right)$ plus multiplying the scrap probability in stage two, $\left(1-\frac{P_{14}}{1-P_{11}}\right)f_{24}$, by $SC_2\left(L_2 - E\left(X \mid X < L_2\right)\right)$. Similarly, the expected reworking cost is derived by multiplication of the reworking unit cost in stage 1, $RC_1\left(E\left(X \mid X > U_1\right) - U_1\right)$, and the expected number of reworks in this stage, $m_{11}-1$, plus $RC_2\left(E\left(X \mid X > U_2\right) - U_2\right)$ multiplied by the expected number of reworks in stage 2, $m_{22}-1$ that is multiplied by the probability of an item going from stage 1 to stage 2, $\left(1-\frac{p_{14}}{1-p_{11}}\right)$. Therefore, the expected profit per item for a two-stage equipment adjustment problem can be expressed by

$$
\begin{aligned}
E(PR) = & \left[SP\left(1-\left\{\frac{P_{14}}{1-P_{11}}+\frac{P_{12}P_{24}}{(1-P_{11})(1-P_{22})}\right\}\right)\right]-\left[PC_1+PC_2\left(1-\left\{\frac{P_{14}}{1-P_{11}}\right\}\right)\right] \\
& -\left[SC_1\left(L_1-\frac{1}{2}(L_1+a_1)\right)\left(\frac{P_{14}}{1-P_{11}}\right)+SC_2\left(L_2-\frac{1}{2}(L_2+a_2)\right)\left(1-\left\{\frac{P_{14}}{1-P_{11}}\right\}\right)\left(\frac{P_{24}}{(1-P_{22})}\right)\right] \\
& -\left[RC_1\left(\frac{1}{2}(b_1+U_1)-U_1\right)\left(\frac{P_{11}}{1-P_{11}}\right)+RC_2\left(\frac{1}{2}(b_2+U_2)-U_2\right)\left(\frac{P_{22}}{1-P_{22}}\right)\left(1-\left\{\frac{P_{14}}{1-P_{11}}\right\}\right)\right]
\end{aligned}
\tag{16}
$$

The expected profit using the fact that (16) can be transformed into an equation that contains μ_1 and μ_2

$$
\mu_1 = \frac{a_1+b_1}{2} \quad , \quad \mu_2 = \frac{a_2+b_2}{2}. \tag{17}
$$

Then, the decision making problem turns into determining μ_1 and μ_2 such that the expected profit is maximized. Nonlinear optimizing software packages can be used to solve the problem.

4.3. The General *n*-stage System

The general serial *n*-stage production system is depicted in Fig. 6, where possible states are given in parentheses.

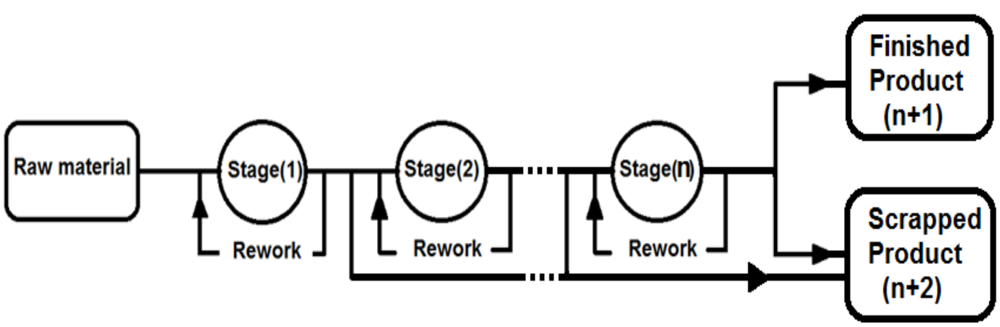

Figure 6. Illustration of item/product state for *n*-stage production system

The transition probability matrix $\boldsymbol{P}$ of this system is:

$$\boldsymbol{P} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ . \\ . \\ . \\ n \\ n+1 \\ n+2 \end{array} \begin{bmatrix} P_{11} & P_{12} & 0 & . & . & . & . & . & 0 & P_{1n+2} \\ 0 & P_{22} & P_{23} & 0 & . & . & . & . & 0 & P_{2n+2} \\ . & 0 & P_{33} & P_{34} & 0 & . & . & . & 0 & P_{3n+2} \\ . & . & 0 & P_{44} & P_{45} & 0 & . & . & 0 & P_{4n+2} \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{nn} & P_{nn+1} & P_{nn+2} \\ 0 & 0 & 0 & 0 & 0 & 0 & . & . & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 1 \end{bmatrix},$$

which is rearranged to

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & . & . & . & m_{1n-1} & m_{1n} \\ 0 & m_{22} & m_{23} & . & . & . & m_{2n-1} & m_{2n} \\ 0 & 0 & m_{33} & . & . & . & m_{3n-1} & m_{3n} \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & m_{n-1n-1} & m_{n-1n} \\ 0 & 0 & 0 & . & . & 0 & 0 & m_{nn} \end{bmatrix}, \tag{18}$$

where $m_{ii} = \dfrac{1}{1\text{-}P_{ii}}$. Moreover, the long-run absorption probability matrix $\boldsymbol{F}$ is

$$\boldsymbol{F} = \begin{bmatrix} f_{1n+1} & f_{1n+2} \\ f_{2n+1} & f_{2n+2} \\ f_{3n+1} & f_{3n+2} \\ . & . \\ . & . \\ f_{n-1n+1} & f_{n-1n+2} \\ f_{nn+1} & f_{nn+2} \end{bmatrix}, \tag{19}$$

where $f_{kn+2}=\frac{P_{kn+2}}{1-P_{kk}}+\sum_{i-1}^{n-k}\left\{\frac{P_{n-i+1n+2}}{1-P_{n-i+1n-i+1}}\prod_{j=i}^{n-k}\left(\frac{P_{n-jn-j+1}}{1-P_{n-jn-j}}\right)\right\}$. As a result, the expected profit function of an item can be shown to be

$$E\left(PR\right)=SP\left(1-f_{1n+2}\right)-\left[PC_1+\sum_{i=2}^{n}\left\{PC_i\left(\prod_{j=2}^{i}\left(\frac{P_{j-1j}}{1-P_{j-1j-1}}\right)\right)\right\}\right] \quad (20)$$
$$-\left[SC_1\left(L_1-\frac{1}{2}(L_1+a_1)\right)\left(\frac{P_{1,n+2}}{1-P_{11}}\right)+\sum_{i=2}^{n}\left\{SC_i\left(L_i-\frac{1}{2}(L_i+a_i)\right)\left(\prod_{j=2}^{i}\left(\frac{P_{j-1j}}{1-P_{j-1j-1}}\right)\right)\left(\frac{P_{in+2}}{1-P_{ii}}\right)\right\}\right]$$
$$-\left[RC_1\left(\frac{1}{2}(b_1+U_1)-U_1\right)(m_{11}-1)+\sum_{i=2}^{n}\left\{RC_i\left(\frac{1}{2}(b_i+U_i)-U_i\right)(m_{ii}-1)\left(\prod_{j=2}^{i}\left(\frac{P_{j-1j}}{1-P_{j-1j-1}}\right)\right)\right\}\right]$$

Then, similar to the one-stage and two-stage serial production systems, in order to determine the optimal equipment adjustment, the expected profit function in (20) can be transformed into an equation containing the μ_i terms and can be numerically maximized using an appropriate software package.

5. Numerical Examples

In order to demonstrate the application of the proposed methodology, numerical examples are illustrated in this section: one for a one-stage and the other for a two-stage serial production system.

5.1. An Example of a One-stage System

Consider a one-stage production system with the parameters as follows: $SP=120$, $PC_1=20$, $RC_1=75$, $SC_1=80$, $T_1=6$, $L_1=8$, and $U_1=12$. The length of interval $[a_1,b_1]$ is assumed to be $b_1-a_1=6$. The transition probability matrix can be obtained based on these parameters as

$$\boldsymbol{P}=\begin{pmatrix}P_{11} & P_{12} & P_{13}\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}0.265 & 0.667 & 0.068\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.$$

This problem is solved in the Matlab 2010b software. To this purpose, Eq. (11) is differentiated with respect to μ_1 and then it is set to be equal to zero, which results in two values for the optimal equipment adjustment at $\mu_1^*=10.56$ and $\mu_1^*=19.4395$. Between these two values, $\mu_1^*=10.56$ is the acceptable one, where the objective function (the expected profit) would be equal 76.5586. Then, the optimum amounts of a_1^* and b_1^* are calculated according to the model, 7.56 and 13.56, respectively. Fig. 7 shows the behavior of the concave objective function.

5.2. An Example of a Two-stage System

Consider a two-stage serial system with the parameters given below:

$SP=180,\ PC_1=20,\ PC_2=25,\ RC_1=95,\ RC_2=120,\ SC_1=180,\ SC_2=110,\ T_1=6,$
$T_2=6,\ L_1=8,\ L_2=13,\ U_1=12,$ and $U_2=17$.

The length of the interval $[a_1,b_1]$ and $[a_2,b_2]$ are assumed to be 6. Based on these parameters, the transition probability matrix is obtained to be

$$\boldsymbol{P}=\begin{pmatrix} P_{11} & P_{12} & 0 & P_{14} \\ 0 & P_{22} & P_{23} & P_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}=\begin{pmatrix} 0.26467 & 0.66667 & 0 & 0.06867 \\ 0 & 0.2620 & 0.66667 & 0.71334 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

This problem is solved by the Matlab 2010b software. To this purpose, Eq. (16) is differentiated with respect to μ_1 and μ_2 and set to zero

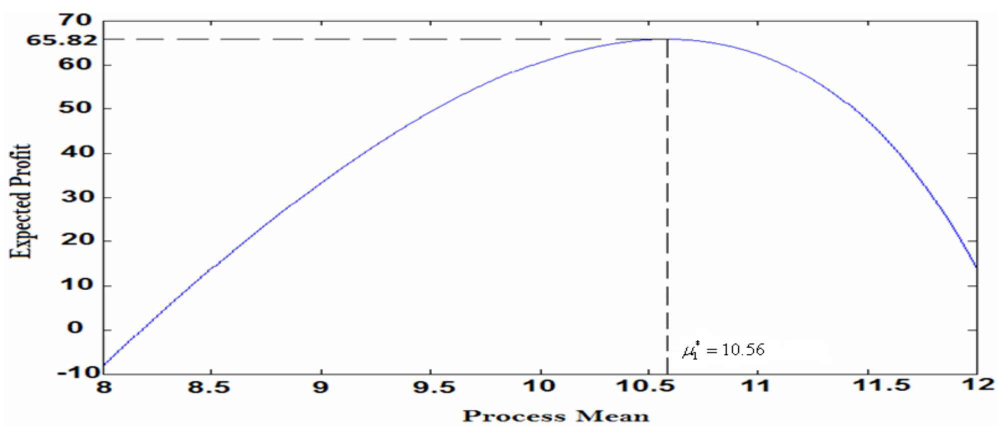

Figure 7. Expected profit function for a one-stage system

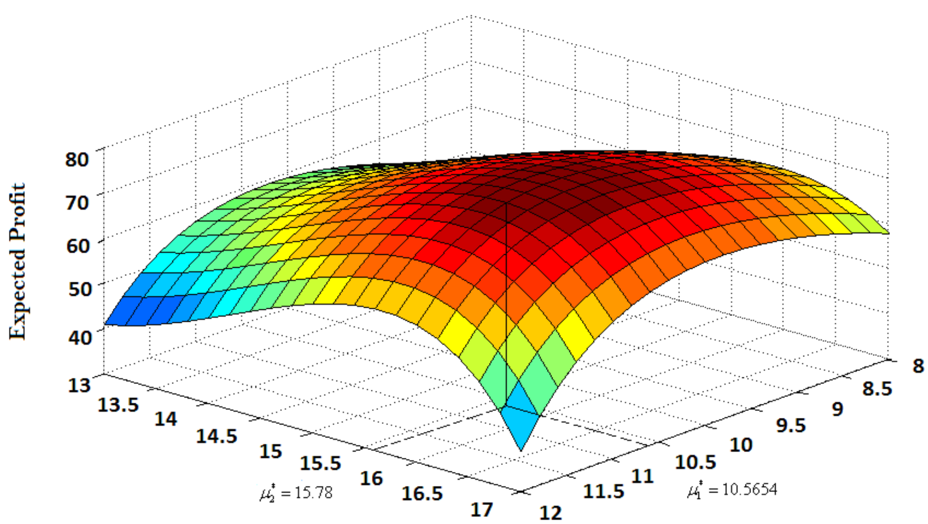

Figure 8. Expected profit function for a two-stage system

which results in two values for μ_1, $\mu_1^* = 10.5654$ and $\mu_1^* = 19.4346$, and two values for μ_2, $\mu_2^* = 15.78$ and $\mu_1^* = 24.2190$, for the optimal equipment adjustment. Among these four values, $\mu_1^* = 10.5605$ and $\mu_2^* = 15.781$ are the acceptable ones where the value of objective function (expected profit) would be equal to 81.9634. So, the optimum amounts of a_1^*, a_2^*, b_1^* and b_2^* are calculated according to the model, 7.5654, 12.7810, 13.5654 and 18.781, respectively.

6. Conclusions and Directions for Future Research

Our aim was to determine the best set of equipment adjustment for multi-stage serial production systems using the Markovian approach. Based on production, scrap, and rework costs of an item, the work started by developing a model for a one-stage production system in order to determine the optimal adjustment of the equipment such that the expected profit per item was maximized. Then, we developed a model for the two-stage serial system, and finally ended up developing a general model for an *n*-stage serial production system. Numerical examples were illustrated to demonstrate the proposed methodology. In our work, quality characteristics of the product were assumed to follow uniform distributions in the ranges of specified limits, which is a reasonable assumption for many production processes. Future research may involve other probability distributions. The second assumption was that the scrap and rework costs were proportional to the deviation of the corresponding quality characteristics with respect to lower and upper specification limits. Since the amount or length of raw materials that are transformed into waste after reworking or scrapping is not constant, this assumption is reasonable as well. However, future research may investigate these costs as random variables.

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