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Coordination of a Cyclic Three-stage Supply Chain for Fast Moving Consumer Goods

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We focus on a three-stage supply chain problem for fast moving consumer goods including a supplier, a manufacturer and customers. There are different orders over identical cycles, to be processed in production site. The problem is to find a joint cyclic schedule of raw material procurement and job scheduling minimized the total cost comprised of raw material ordering cost and holding cost, production cost, holding cost of finished products, tardiness cost and rejection cost. An integrated mixed integer programing model is proposed and optimal solution of some instances are provided by solving the model.

Keywords: *Supply chain management, Fast moving consumer goods, Inventory control, Mixed integer programming.*

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1. Introduction

Supply chain consists of different stages to transform raw materials placed in a supply site to the final product ready for customer consumption. Integration of various stages of procurement, production and distribution has been one of the most active topics in supply chain management in recent years. This research deals with the coordination of operations in a three stage supply chain focusing on the fast moving consumer goods (FMCG). FMCGs are type of goods which are consumed quickly by the average costumers and need to be replaced frequently. These products usually have a short shelf life, either as a result of high consumer demand (such as toiletries, soft drinks and cleaning products) or due to deteriorating features (such as meat, fruits, vegetables and dairy products).

Traditionally, procurement, manufacturing, sales, and distribution along the supply chain operate independently under supervision of various managers with different and conflicting objectives. Therefore, in practice, coming to a common conclusion on the objectives and the integrated executive plan along the supply chains is a challenging job. Supply chain management is a strategy through which such an integration can be made. We refer the reader to Kanda and Deshmukh [15] as a comprehensive recent literature review on supply chain coordination.

Different cases of integration and coordination in supply chain have been widely investigated in recent years. Coordination of production and distribution is a popular topic in this field. Extensive reviews of integrated production-distribution systems have been provided by Fahiminia et al. [8] and Chen [6]. Pundoor and Chen [20] have dealt with a production-distribution supply chain considering supplier's storage, customer's storage and intermediate warehouse. Different heuristic algorithms have also been presented by Lee [18] for minimizing the total cost of a multi-machine two-stage manufacturing problem including holding cost or tardiness penalty of early or late jobs. An integrated

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production-distribution problem for perishable products is also discussed in Amorim et al. [2] where a multi objective framework is considered to take economic benefits and freshness level of products into consideration. We refer the reader to Lee and Chen [17], Chang and Lee [4], Pundoor and Chen [21], Chen and Pundoor [7] and Raa et al. [22] as more instances among many others.

There are also papers in the literature focusing on material procurement and job scheduling in production sites. Valentinia and Zavanella [26] and Grigoriev et al. [9] can be considered as instances of this subject. As a recent research, Yeung et al. [30] considered a two-echelon supply chain scheduling problem consisted of a supplier, a manufacturer and retailers. The objective of the problem is to minimize the total cost including the raw material storage cost in supplier site and transportation cost for delivering the finished products to the retailers.

In the past decade, a considerable number of studies on logistics scheduling dealt with three stages of procurement, production and distribution. Hall and Potts [12] conducted the first study on multiple-production-stage scheduling with batch delivery in a supply chain. Sawik [23] proposed a mixed integer programming approach for a long-term, integrated scheduling of material manufacturing, material supply and product assembly in a customer driven supply chain. Wang and Cheng [27] studied a three-stage-logistics scheduling problem seeking for an optimal joint schedule for material supply, production scheduling, and job delivery in a way that transport and WIP inventory costs are minimized. More recent results can be found in Wang and Cheng [28], Yu et al. [31], Hajji et al. [10], Jaber et al. [14] and Kolisch [16].

Cyclic scheduling has been also widely investigated in recent years. The problem deals with planning of activities or jobs that have to be identically repeated at regular intervals over an infinite or a long horizon. The problem has various applications and Pundoor and Chen [21], Šcha and Hanzálek [24] and Trautmann and Schwindt [25] can be mentioned as the examples of research made on cyclic scheduling in different industries. Cyclic jobshop problems and cyclic project scheduling problems are known as classic topics in this field. In the cyclic jobshop problem, different types of products must be processed on a set of machines, structured as a jobshop. The problem is to find the order of operations which are iteratively processed on each machine. Agnetis and Pacciarelli [1], Hall et al. [11], Che et al. [5] and Manier and Bloch [19] are examples of the work dealing with cyclic flowshop problem and cyclic robotic scheduling problem. The cyclic project scheduling problem is known also as the PERT-shop scheduling problem or the general cyclic machine scheduling problem. Different variants of the problem have been discussed in the literature such as Hall et al. [13], Wu and Ierapetritou [29] and Castro et al. [3].

In our work here, a cyclic scheduling problem is considered for a supply chain of FMCG where raw material and finished products are highly perishable and manufacturer is not allowed to store them for more than the given shelf lives. It is supposed that the supplier, the manufacturer and the retailers establish a long relationship and are interested in designing an iterative schedule over the cycles. The overall problem is to find a joint cyclic schedule of raw material procurement and job scheduling to minimize the total cost procurement, production and distribution. To the best of our knowledge a cyclic scheduling of a three stage supply chain problem for FMCG has not been considered yet in the literature. The problem under consideration is NP-hard and to solve large scale instances we need to develop effective heuristic methods. However, the current research is dedicated to providing an integrated mixed integer programming (MIP) model of the problem to solve small size instances.

The rest of our work is organized as follows. Section 2 briefly introduces fast moving consumer goods and the problem is described in Section 3. Section 4 presents the MIP model in three sections corresponding to the supply chain stages. The computational results are presented in Section 4 and Section 5 gives our concluding remarks.

2. Fast Moving Consumer Goods

Fast moving consumer goods (FMCG), or consumer packaged goods (CPG), are products which have a quick turnover and are sold quickly at a relatively low cost. These kinds of products are generally produced in large quantities, distributed through retail stores and used up by end customers over a short period of days, weeks, or months. In contrast with FMCG, there are durable goods which are normally replaced over a period of several years. FMCGs are known as the quickest items which leave the supermarket shelves. A wide range of frequently purchased consumer products such as packaged food products, personal care items, detergents and grocery items are placed in the group of FMCG. FMCGs usually have a short shelf life, either as a result of a high market demand or dye to the high deteriorating rate.

3. Problem Description

Each retailer has a constant demand and a preferred due date during the scheduling cycle. The manufacturer takes orders of retailers and schedules jobs on m different parallel machines in a way that meets retailers' demands. The manufacturer is able to produce p different kinds of products using a given amount of the single raw material for each product. Raw material is prepared by an external supplier and is delivered to the production site according to the manufacturer orders. Figure 1 schematically shows a three-stage supply chain.

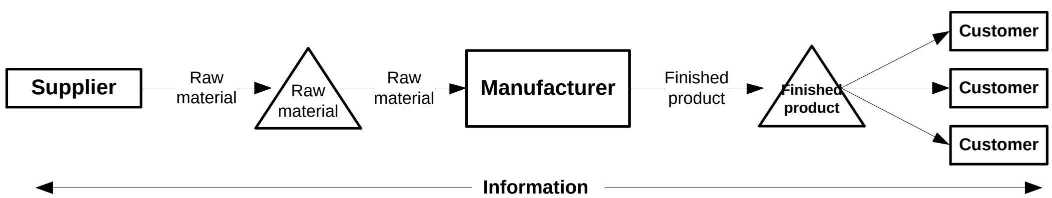

Figure 1. A three-stage supply chain

This problem is modeled from the manufacturer's point of view. The overall problem is to coordinate raw material procurement, job scheduling in production site and the distribution planning such that the total cost comprise raw material ordering cost and holding cost, setup and production costs, products holding and distribution cost and rejecting and earliness/tardiness penalty, is minimized. It is supposed that both raw material and finished products are perishable and the shelf life constraints must be satisfied.

4. Mathematical Model

This section is dedicated to develop an MIP model for the problem. Each stage of supply chain is explained and modeled separately, while the complete MIP model is obtained by merging all the constraints and objectives of the three models of procurement, production and distribution.

Since we develop a cyclic schedule in a cycle of length T, all variables with a temporal index such as X_t satisfy:

$$X_t = X_{T+t}, \qquad t = 0, 1, \dots, T \tag{1}$$

To reduce the number of variables, if in the mathematical model there is a variable with temporal index larger than T, it should be replaced by a variable with smaller index according to (1).

4.1. Procurement Stage

The raw material, required in the production site, is supplied by an external supplier and fixed ordering costs are incurred every time an order is placed. The raw material is perishable with a given shelf life. Thus, holding of raw material in production site also incurs a holding cost.

At procurement stage, we must control the raw material inventory and design a schedule to receive the raw material. To do this, the variable Q_t^R is defined for determining the received batch quantity at time t. Furthermore, to control the inventory level, denoted by y_t^R, the variable U_t^R is used for calculating the consumption of the raw material. U_t^R is a common variable among procurement and production stages and can be considered as the linking variable. In this stage, index R denotes the variables corresponding to the procurement stage. The parameters and variables of this stage are listed in Table 1.

The total cost of procurement stage consists of purchasing cost, ordering cost and holding cost of the raw material should be minimized in the objective function. The MIP model of this stage, referred as Model 1, is presented in the following.

Table 1. Indices, parameters and variables of procurement stage

Indices:	σ^R: Shelf life of the raw material
t : Time periods ($t = 1, 2, \dots, T$)	M_1 : A large integer number
R : Procurement stage	***Variables:***
Parameters:	Q_t^R :Batch quantity of received raw material in period t
T : Cycle length	φ_t^R : 1 if a batch of raw material is received in period t, and 0, otherwise
h^R : Holding cost of raw material	U_t^R : Resource consumption of the raw material in period t
o^R : Ordering cost of raw material	y_t^R : Raw material inventory level at the beginning of period t
c^R : Unit price of the raw material	

Model 1. *Procurement:*

$$\text{Min } Z = \sum_{t=1}^{T} c^R Q_t^R + \sum_{t=1}^{T} o^R \varphi_t^R + \sum_{t=1}^{T} h^R y_t^R \tag{2}$$

s.t.

$$y_{t+1}^R = y_t^R + Q_t^R - U_t^R \qquad t = 1, \dots, T \tag{3}$$

$$Q_t^R \le M_1 \varphi_t^R \qquad t = 1, \dots, T \tag{4}$$

$$y_t^R \le \sum_{v=t-\sigma^R}^{t-1} Q_v^R \qquad t = 1, \dots, T \tag{5}$$

$$y_t^R,\ Q_t^R,\ U_t^R \ge 0,\ \varphi_t^R \in \{0,1\}.$$

In Model 1, objective function (2) minimizes the total purchase cost, ordering cost, and holding cost of the raw material. Constraint (3) adjusts the inventory level at each time t by considering received batch quantity (Q_t^R) and the raw material consumption (U_t^R). In (3), by setting $t = T$ in the left hand side of the equation, y_{T+1}^R appears based on (1) which must be replaced by y_1^R in solving the model.

Using a large number M_1, ensures that the binary variable φ_t^R is set to 1, whenever Q_t^R is larger than zero. Since Q_t^R cannot exceed the total required raw material for satisfying all the customer orders, we can set the value of M_1 to the total required raw material according to (6), in which D_{ik} denotes the demand of retailer i for product k and α_k is the conversion factor of the raw material to product k:

$$M_1 = \sum_k \sum_i \alpha_k D_{ik}. \tag{6}$$

Eq. (5) satisfies the shelf life constraint by focusing on FIFO method adopted in the raw material inventory system. In a FIFO system, oldest inventory items are consumed first and the remaining items are supplied by the most recent orders. Based on the shelf life limitation, raw material should be consumed in less than σ^R periods. Thus, the inventory level at the beginning of each period cannot exceed the sum of received items during the last σ^R periods. Figure 2 schematically depicts the logic of this constraint.

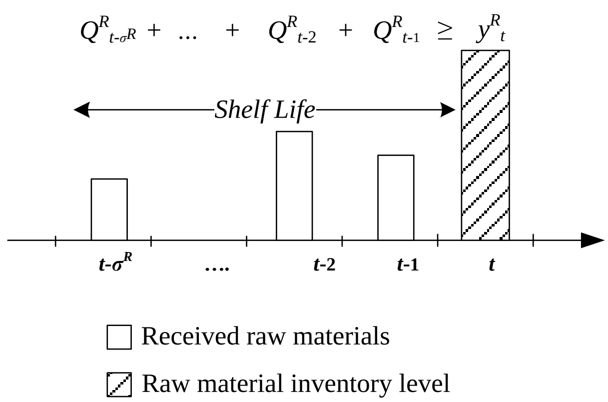

Figure 2. Calculation of the shelf life constraint

4.2. Production Stage

At production stage, m parallel different machines are considered which are capable of producing p different products at constant rates. There is a sequence-independent setup time, per production run for each machine. Production cost, including machine cost, setup cost and finished product holding cost needs to be minimized.

At this stage, we must schedule jobs on machines to produce various products at each period and control the finished products inventory level. In order to do this, Q_{jkt}^P is defined to determine batch quantity of product k, processing on machine j at each period. The inventory level of each product (k), given by y_{kt}^P, is also adjusted considering production plan (Q_{jkt}^P) besides the variable U_{kt}^P that deals with the quantity of the products, delivered to the retailers at each period.

It should be noted that jobs in production site are scheduled by considering availability and consumption of the raw material (U_t^R) which is introduced in Model 1. Furthermore, since the variable U_{kt}^P defines the quantity of products that must be delivered to the costumers, it is considered as the linking variable of production and distribution stages.

Indices j and k respectively depict machine number and product number, and index P denotes the variables corresponding to the production stage. Definitions of the parameters and variables used at production stage are given in Table 2 and the corresponding MIP model is presented as Model 2.

Table 2. Indices, parameters and variables of production stage

Indices:	c_j^P : Production cost of full capacity utilization of machine j per period
t : Time periods ($t = 1, 2, \dots, T$)	h_k^P : Holding cost of finished product k per unit per period
j : Machines ($j = 1, 2, \dots, m$)	α_k : Conversion factor of the raw material to product k
k : Products ($k = 1, 2, \dots, P$)	σ_k^P : Post production shelf life of final product k
P : Production stage	M_{2k}: A large integer number
Parameters:	***Variables:***
m : Number of machines	Q_{jkt}^P: Batch Quantity of product k produced on machine j in period t
p : Number of products	φ_{jkt}^P: 1 if product k is produced on machine j in period t, and 0, otherwise
K_j^P : Active time of machine j per period	U_{kt}^P : Quantity of product k delivered to retailers in period t
κ_{jk} : Process time of product k on machine j	y_{kt}^P : Finished product inventory level of product k in the beginning of period t
ST_j^P: Setup time of machine j	
SC_j^P: Setup cost of machine j	

Model 2. ***Production:***

$$\text{Min } Z = \sum_k \sum_{t=1}^T h_k^P y_{kt}^P + \sum_{t=1}^T \sum_j c_j^P \sum_k \kappa_{jk} Q_{jkt}^P + \sum_j SC_j^P \sum_k \varphi_{jkt}^P \quad (7)$$

s.t.

$$U_t^R = \sum_k \alpha_k \sum_j Q_{jkt}^P \qquad t = 1, \dots, T \quad (8)$$

$$Q_{jkt}^P \leq M_{2k} \varphi_{jkt}^P \qquad t = 1, \dots, T,\ k = 1, \dots, P,\ j = 1, \dots, m \quad (9)$$

$$y_{kt}^P \leq \sum_{v=t-\sigma_k^p}^{t-1} \sum_j Q_{jkv}^P \qquad t = 1, \dots, T,\ k = 1, \dots, P \quad (7)$$

$$y_{kt+1}^P = y_{kt}^P + \sum_j Q_{jkt}^P - U_{kt}^P \qquad t = 1, \dots, T,\ k = 1, \dots, P \quad (11)$$

$$K_j^P \geq \sum_k \kappa_{jk} Q_{jkt}^P + ST_j^P \sum_k \varphi_{jkt}^P \qquad j = 1, \dots, m \quad (12)$$

$$Q_{jkt}^P,\ y_{kt}^P,\ U_{kt}^P \geq 0,\ \varphi_{jkt}^P \in \{0,1\}.$$

In Model 2, the sum of holding cost, production cost and setup cost are considered to be minimized in objective function (7). Eq. (8) insures that, at each period, the raw material usage in a production stage is equal to the total required raw material in the production stage. Eq. (9), by using a large number

M_{2k}, makes sure that the binary variable φ^P_{jkt} is set to 1, whenever Q^P_{jkt} is larger than zero. Since Q^P_{jkt} cannot exceed the total required product of type k to satisfy all the customers' needs, we can set the value of M_{2k}, according to (13), to the total required product k in each cycle. In the equation, D_{ik} denotes the demand of retailer i for product k:

$$M_{2k} = \sum_i D_{ik} \qquad i = 1, \dots, n. \tag{8}$$

Eq. (10) deals with shelf life constraint of the finished products. Similar to the procurement stage, it is supposed that finished product inventory is organized based on the FIFO system. Due to perishable property, products are not allowed to be stored in the production site for more than the shelf lives (σ^p_k). Thus, the inventory level of each product (k) at the beginning of each period cannot exceed the summation of produced items during the last σ^p_k periods.

Constraints (11) set the inventory level of each product (k) by considering produced items (Q^P_{jkt}) and products which are delivered to the customers at each period (U^P_{kt}). In (11), by setting $t = T$ in the left hand side of the equation, $y^P_{k,T+1}$ appears which according to (1) must be replaced by y^P_{k1}. The machines capacity limitations are also considered in (12) in which the total process time and total setup time are calculated and compared with the available capacity of each machine.

4.3. Distribution Stage

At distribution stage, finished products are first delivered to a central warehouse and then sent in batches to retailers. The retailer's demand for each product is a constant over the cycles and the desired due date of each retailer is known. It must be determined whether accepting an order is economically justifiable and the manufacturer should decide to accept or reject the orders considering the production capacity and satisfying or rejecting costs.

The parameters and variables corresponding to the distribution stage are listed in Table 3. One of the main decision variables in this stage is delivery time of product batch to the retailer which is denoted by Γ_i , for $i = 1, 2, \dots, n$. At the distribution stage, it is necessary to calculate earliness/tardiness of satisfied orders. In scheduling literature, the earliness or tardiness of a job is measured by $|\mathrm{d_i} - \Gamma_\mathrm{i}|$. Since we consider a cyclic schedule, in order to determine the gap between due date and reception date we also need to check the next or previous cycle for small gaps.

Table 3. Indices, parameters and variables of distribution stage

Indices:	δ_i : Earliness/tardiness penalty for retailer i
t : Time periods ($t = 1, 2, \dots, T$)	λ_{ik} : Penalty of rejecting order of retailer i for product k
i : Retailers ($i = 1, 2, \dots, n$)	
D : Distribution stage	***Variables:***
Parameters:	γ_{it} : 1 if a product batch is received by retailer i at period t, and 0, otherwise
n : Number of retailers	
c^D_i: Transportation cost of delivering a batch to retailer i	Γ_i : The reception time of a product batch by the retailer i
D_{ik}: Demand of retailer i for product k	***Due date variables***:
d_i : Desirable due date of retailer i	P^D_i : Earliness/tardiness penalty related to retailer i
	z^+_i, z^-_i: Integer nonnegative variables
	v^+_i, v^-_i: Integer nonnegative variables
	u_i : Binary variables

As a numerical example, Figure 3 depicts a seven-period-cycle in which there is a retailer with $d_i = 2$ and $\Gamma_i = 6$. In a single cycle, the gap is equal to 4. However, by considering the next cycle the gap decreases to 3.

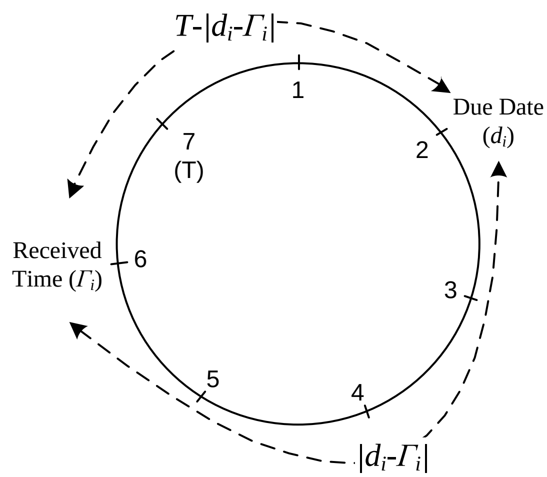

Figure 3. Calculation of cyclic earliness/tardiness

Therefore, we consider $min\{|d_i - \Gamma_i|, T - |d_i - \Gamma_i|\}$ as the gap between due date and reception date, and so earliness/tardiness penalty is calculated by $\sum_i \delta_i(min\{|d_i - \Gamma_i|, T - |d_i - \Gamma_i|\} \times \sum_k D_{ik})$, which is obviously nonlinear. We know that for all $x, y > 0$, $\min\{x, y\} = \frac{|x+y|-|x-y|}{2}$. Using this equality,

$$\min\{|d_i - \Gamma_i|, T - |d_i - \Gamma_i|\} = \frac{|T| - \left|T - 2|d_i - \Gamma_i|\right|}{2} = \frac{T - \left|T - 2|d_i - \Gamma_i|\right|}{2}. \quad (9)$$

We add two pairs of integer nonnegative variables $z_i^{\pm}$ and $v_i^{\pm}$ as follows:

$$z_i^+ - z_i^- = d_i - \Gamma_i \rightarrow |d_i - \Gamma_i| = z_i^+ + z_i^- \quad (10)$$

$$v_i^+ - v_i^- = T - 2(z_i^+ + z_i^-) \rightarrow \left|T - 2|d_i - \Gamma_i|\right| = v_i^+ + v_i^- \quad (11)$$

Therefore, the earliness/tardiness penalty could be written as follows:

$$\text{Earliness/Tardiness Penalty } = \sum_i \delta_i(\min\{|d_i - \Gamma_i|, \ T - |d_i - \Gamma_i|\} \times \sum_k D_{ik}) = \sum_i \delta_i \left(\frac{T - \left|T - 2|d_i - \Gamma_i|\right|}{2} \times \sum_k D_{ik}\right) = \sum_i \delta_i \left(\frac{T - v_i^+ - v_i^-}{2} \times \sum_k D_{ik}\right) = \sum_i \frac{\delta_i}{2}\left((T - v_i^+ - v_i^-) \times \sum_k D_{ik}\right). \quad (12)$$

We should not consider the earliness/tardiness penalty in case of rejecting the orders. Based on this fact, the earliness/tardiness penalty is presented in Model 3.

Model 3. *Cyclic due date:*

$$\text{Min } \sum_i P_i^D \tag{13}$$

s.t.

$$P_i^D \le T\sum_t \gamma_{it} \qquad i = 1, \dots, n \tag{14}$$

$$P_i^D \ge \sum_i \frac{\delta_i}{2}\left((T - v_i^+ - v_i^-) \times \sum_k D_{ik}\right) - M(1 - \sum_t \gamma_{it}) \qquad i = 1, \dots, n \tag{15}$$

$$P_i^D, z_i^+, z_i^-, v_i^+, v_i^- \ge 0, \gamma_{it}, u_i \in \{0,1\}.$$

In Model 3, P_i^D is a variable that calculates the earliness/tardiness penalty of retailer i and is to be minimized via objective function (18). Constrain (19) is considered to ensure that a rejection decision on a retailer's order incurs no earliness/tardiness penalty. The coefficient T is considered in this equation to set the maximum possible earliness\tardiness of the orders to the cycle length T. On the other hand, constraint (20) adjusts the earliness/tardiness penalty of satisfied order according to (17). The constraints and objective function of the model are embedded in Model 4 as the complete MIP model of the distribution stage.

Model 4. *Distribution:*

$$\text{Min } Z = \sum_i P_i^D + \sum_i (1 - \sum_t \gamma_{it}) \sum_k \lambda_{ik} D_{ik} + \sum_i c_i^D \sum_t \gamma_{it} \tag{21}$$

s.t.

$$U_{kt}^P = \sum_i D_{ik}\gamma_{it} \qquad t = 1, \dots, T, \; k = 1, \dots, P \tag{22}$$

$$\Gamma_i = \sum_t t\gamma_{it} \qquad i = 1, \dots, n \tag{23}$$

$$\sum_t \gamma_{it} \le 1 \qquad i = 1, \dots, n \tag{24}$$

$$d_i - \Gamma_i = z_i^+ - z_i^- \qquad i = 1, \dots, n \tag{25}$$

$$T - 2(z_i^+ + z_i^-) = v_i^+ - v_i^- \qquad i = 1, \dots, n \tag{26}$$

$$v_i^+ \le Tu_i \qquad i = 1, \dots, n \tag{27}$$

$$v_i^- \le T(1 - u_i) \qquad i = 1, \dots, n \tag{28}$$

$$P_i^D \le T\sum_t \gamma_{it} \qquad i = 1, \dots, n \tag{29}$$

$$P_i^D \ge \sum_i \frac{\delta_i}{2}\left((T - v_i^+ - v_i^-) \times \sum_k D_{ik}\right) - M(1 - \sum_t \gamma_{it}) \qquad i = 1, \dots, n \tag{30}$$

$$\Gamma_i, P_i^D, z_i^+, z_i^-, v_i^+, v_i^- \ge 0 \;, \quad \gamma_{it}, u_i \in \{0,1\}.$$

The objective function (21) in Model 4 minimizes earliness/tardiness penalty, order rejecting penalty and transportation cost. Constraint (22) guarantees that, in each period, delivered items are equal to the satisfied orders for each product type. It can be considered as a linking constraint between production and distribution stages. Reception times are adjusted by (23) and (24).

Constraints (25)-(30) deal with the earliness or tardiness of the satisfied orders, where constraints (25) and (26) are driven from (15) and (16) to adjust the variables $z_i^{\pm}$ and $v_i^{\pm}$. Constraints (27) and

(28) guarantee that at least one of v_i^+ and v_i^- are zero and concerning (15), T is considered as the maximum value of $v_i^{\pm}$. Constraints (29) and (30) are also embedded in the model to adjust earliness tardiness penalty according to Model 3.

The complete MIP model of the three-stage supply chain is obtained by merging all the constraints of the three models of procurement, production and distribution and the summation of their objectives provides the objective function of the model as minimizing the whole cost of the supply chain.

5. Computational Results

We conduct numerical experiments by investigating some random instances. To generate a random instance, parameters of the problem are classified in three groups called constant parameters, random parameters and complexity indices. Constant parameters defining the problem size include cycle length, number of machines, number of products and number of retailers. These parameters are fixed for different groups of instances. Constant parameters and the proposed levels are presented in Table 4.

Table 4. Constant parameters and the corresponding proposed levels

Description		Levels		
Cycle length	T	5	7	10
Number of machines at the factory	m	3	5	7
Number of products	p	4	6	8
Number of retailers	n	5	8	10

Random parameters which are determined from given distribution functions are presented in Table 5. These parameters are independently generated for each instance and by determining the values of parameters dealing with costs and operation times form the problem specifications. Complexity indices consider the interaction of random parameters which are supposed to have effects on the hardness or complexity of the instances. Capacity strength and cost strength are the two complexity indices to be described later.

A software, written in C# 4.0 and run under Visual Studio 2010, systematically generates all the instances. Samples are solved via ILOG-IBM CPLEX 12.3 by applying concert technology in C# framework. The code is run on a PC with a 2.27 GHz Intel Core 2 Duo Processor and 2.00 GBytes memory.

5.1. Investigation of Instance Size

Here, various groups of instances are generated by considering different levels for constant parameters. These experiments are conducted in order to determine different instance sizes, which are solvable by CPLEX. In this experiment, the random parameters are generated according to Table 5 and the constant parameters are set at different levels according to Table 4. Different combinations of constant parameters are considered and in each group, we generate and test 10 different instances. A summary of the results is presented in Table 6.

Table 5. Random parameters and the corresponding default distribution

Stage	Parameter		LB	UB	Distribution
Procurement	Holding cost	h^R	1	10	Uniform
	Ordering cost	o^R	1	100	Uniform
	Unit price	c^R	1	10	Uniform
	Shelf life	σ^R	1	T	Uniform
Manufacturing	Active time	K_j^P	50	100	Uniform
	Process time	κ_{jk}	1	10	Uniform
	Setup time	ST_j^P	1	10	Uniform
	Setup cost	SC_j^P	1	100	Uniform
	Production cost	c_j^P	1	10	Uniform
	Holding cost	h_k^P	1	10	Uniform
	Conversion factor	α_k	1	10	Uniform
	Shelf life	σ_k^P	1	T	Uniform
Distribution	Transport cost	c_i^D	1	100	Uniform
	Due date	d_i	0	T	Uniform
	Earliness/tardiness penalty	δ_i	1	10	Uniform
	Demand	D_{ik}	1	10	Uniform
	Reject Penalty	λ_{ik}	1	100	Uniform

Table 6. Computational results for the instances with different sizes

ID	*T*	*m*	*p*	*n*	CPU Time			Optimal solution
					Ave.	Max	Min	(%)
1	5	3	4	5	2.5	10.2	0	100
2	5	5	6	8	704.6	3458	2	100
3	5	5	8	8	339.1	825.6	2.9	70
4	5	7	8	10	873.8	3905.1	4.8	60
5	7	3	4	5	63.9	325.7	0.2	100
6	7	5	6	5	713.6	4188.2	0.4	90
7	7	5	6	8	483.3	2142.4	3.3	60
8	7	5	6	10	1181.9	3026.8	0.9	50
9	7	5	8	8	785.3	1566.5	4	20
10	7	5	8	10	----	----	----	0
11	7	7	8	10	---	---	---	0
12	10	3	4	5	462.7	3566.7	1.4	80
13	10	5	6	8	656.2	656.2	656.2	10

The CPU time column shows the average, maximum and minimum process time of all instances, which are solved optimally. In this experiment, some instances are encountered with the out of memory error and their optimal solutions are not accessible in the experiments. The last column in Table 6 reveals the number of instances, which are solved optimally. Based on the results, we classified the instances in three groups as below. The result of this classification is shown in Table 7.

- Small: Problems in which at least 80 percent of instances are solved optimally.
- Medium: Problems in which between 40 to 70 percent of instances are solved optimally.
- Large: Problems in which at most 30 percent of instances are solved optimally.

5.2. Investigation of Complexity Indices

Here, we take the third parameter group into consideration. Two indices are defined in this group as follows:

- ***Capacity strength:*** This parameter, calculated by (31), deals with the equilibrium of the production capacity of production site and the required capacity for satisfying all the customer's demand.

$$\text{Capacity strength} = \frac{\sum_k \bar{\kappa}_k \sum_i D_{ik^*}}{\sum_j K_j^P}, \qquad (16)$$

where
K_j^P = Active time of machine j per period
$\bar{\kappa}_k$ = Average required time to produce one unit of product k on all machines
D_{ik}= Demand of retailer i for product k.

- ***Cost strength:*** In order to decide on accepting or rejecting an order, the reject penalty must be investigated in comparison with the corresponding production cost. The ratio of average producing cost to reject penalty is considered as the cost strength index, to reveal whether an order is likely to be rejected or accepted.

Table 7. Classification of instances with different sizes

Scale	*T*	*m*	*p*	*n*	CPU Time (Ave.)	Optimal solution (%)
Small	5	3	4	5	2.5	100
	7	3	4	5	63.9	100
	5	5	6	8	704.6	100
	7	5	6	5	713.6	90
	10	3	4	5	462.7	80
Medium	5	5	8	8	339.1	70
	7	5	6	8	483.3	60
	5	7	8	10	873.8	60
	7	5	6	10	1181.9	50
Large	7	5	8	8	785.3	20
	10	5	6	8	656.2	10
	7	7	8	10	---	0
	7	5	8	10	----	0

5.2.1. Analyzing Capacity Strength

To investigate the impact of capacity strength index on complexity of the problem, some random instances in small group are generated in which the value of the index is defined using the uniform distributions in six levels between 10^{-3} and 10^{3} according to Table 8. For each level, five instances are generated and solved. Table 9 shows the results. All the instances are successfully solved. Table 9 confirms that the capacity strength index affects the complexity of instances and for the middle level of this parameter, solving the problem needs more computation time than the other levels. In addition, the number of satisfied orders in the optimal solution increases for large capacity strengths.

Table 8. Different levels of capacity strength/cost strength

level	Capacity strength
1	$U[100,1000]$
2	$U[10,100]$
3	$U[1,10]$
4	$U[0.1,1]$
5	$U[0.01,0.1]$
6	$U[0.001,0.01]$

Table 9. Computational results for various capacity strengths levels

CS	CPU Time			Nodes	Number of satisfied orders
	Ave.	Max	Min		
U [100, 1000]	0.06	0.09	0.03	0.00	0.00
U [10 , 100]	579.03	1736.95	0.05	392196.00	1.67
U [1 , 10]	928.64	2780.14	0.13	880730.67	3.00
U [0.1 , 1]	4.50	10.56	1.30	1804.67	6.33
U [0.01 ,0.1]	4.37	6.96	0.64	1235.33	7.33
U [0.001 , 0.01]	2.41	4.37	1.42	1023.00	7.33

5.2.2. Analyzing Cost Strength

A similar experiment is also run in order to analyze the effect of cost strength index on complexity of instances. Here, random test problems are generated in which the value of the cost strength index is determined from the uniform distributions in six levels between 10^{-3} and 10^{3} according to Table 8. For each level, five instances are generated and solved. Table 10 shows the results. All the instances are successfully solved. Table 10 confirms that the cost strength index also affects the complexity of instances and for the middle level of this index, solving the problem needs more computating time than other levels. Also, the number of satisfied orders in the optimal solution increases for large capacity strengths.

Table 10. Computational results for various cost strength levels

Cost strength	CPU Time			Nodes	Number of satisfied orders
	Ave.	Max	Min		
U [100, 1000]	0.07	0.08	0.06	0	0.00
U [10 , 100]	0.06	0.08	0.05	0	0.00
U [1 , 10]	465.40	1346.06	2.85	335696	6.67
U [0.1 , 1]	319.44	909.14	22.89	97143	8.00
U [0.01 ,0.1]	670.68	652.26	55.71	142698	8.00
U [0.001 , 0.01]	229.37	670.68	8.25	125645	8.00

6. Conclusion and Further Research

We studied coordination of cyclic FMCG supply chain in order to minimize total supply, production and distribution costs. We assumed that during each cycle there were fixed orders from customer, with determined amount of resource to be processed on m identical parallel machines. Raw material and finished products are supposed to be perishable and manufacturer is not allowed to hold them more than the product dependent shelf lives. We investigated the problem from the manufacturer's point of view and proposed a mixed integer model to minimize the total cost. Random instances were generated and solved by ILOG-IBM CPLEX 12.3.

Computational results confirmed that almost all small instances were solved within a reasonable CPU process times while for medium and large groups the solver needed more CPU process times, while in some cases CPLEX was not able to solve them due to the "out of memory" error. Furthermore, experiments revealed the impact of capacity strength and cost strength indices in complexity of the instances. For both indices, the middle range of the parameter showed to increase the complexity level while small and large levels of the parameters were solved in less CPU times. The problem being NP-hard, it would be of interest to develop heuristic and metaheuristic methods to provide fast and effective solutions. On the other hand, to evaluate efficiency of the results an improvement of our work may be obtained by providing lower bounds of the instances. In addition, considering different production scheduling environments could be of interest.

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