Comparison and Supply Chain Optimization for Vendor-Buyer Coordination System

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Here, we consider single vendor-buyer model with multi-product and multi-customer and multi-facility location-production-distribution problem. It is assumed that the players of the supply chain are coordinated by sharing information. Vendor manufactures produce different products at different plants with limited capacities and then distribute the products to the consumers according to deterministic demands. A mixed integer linear fractional programming (MILFP) model is formulated and a solution approach for MILFP is discussed. Product distribution and allocation of different customers along with sensitivity of the key parameters and performance of the model are discussed through a numerical example. The results illustrate that profit achieved by the MILFP model is slightly higher than mixed integer programming (MIP) model. It is observed that increase in the opening cost decreases the profit obtained by both MILFP and MIP models. If the opening cost of a location decreases or increases, the demand and capacity of the location changes accordingly. The opening cost dramatically changes the demand rather than the capacity of the product. Finally, a conclusion is drawn in favor of the MILFP model as a relevant approach in a logistic model searching for the optimum solution.

Keywords: *Vendor and buyer, Coordination, Optimization, Mixed integer linear fractional program, Mixed integer program.*

1. Introduction

Supply Chain Management (SCM) as well as coordination among the members have undergone rapid developments in theory and practice and are considered to be key issues at present. In the literature, the most common definition of a supply chain is a system of suppliers, manufacturers, distributors, retailers, and customers where materials flow downstream from suppliers to customers, and information flows in both directions. SCM is primarily concerned with the efficient integration of suppliers, factories, warehouses and stores so that the merchandise can be produced in the right amounts, and distributed to the right locations and at the right time, as to minimize the total system cost subject to satisfying service requirements.

In addition, in the global competition market, the importance of SCM is increasing daily. Maximizing the profit and minimizing the cost are the main factors which play important roles in the supply chain. Furthermore, it is important to make the model optimal for both consumers and the manufacturer. The design of an optimal distribution network in supply chain has become extensively

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a contemporary enterprise. An efficient supply chain system operates under a strategy to minimize costs by integrating the different functions inside the system and by meeting customer demands in time.

There is an extensive research on SCM dealing with different aspects of the subject. Numerous models in the literature, conceptual as well as quantitative, refer to planning and quantitative aspects of the different business functions of location, production, inventory and transportation. Research has also been done to consider combinations of these areas for optimization. Proposed models include a combination of two or more of these areas for integration. Facility location problems typically being used to design distribution networks, involve determining the sites to install resources, as well as the assignment of potential consumers to resources. Drezner and Hamacher et al. [6] briefly described FLP the location of manufacturing plants, the assignment of warehouses to these plants and finally the assignment of retailers to each warehouse. Other than geographical boundaries, Hung and Kubiak et al. [8] described the location allocation with balancing requirements among the distribution centers (DC). They formulated a bi-level programming model to minimize the total cost of the distribution network and balanced the work load to each DC for the delivery of products for its customer, and finally the model was solved by a genetic algorithm.

Azad and Ameli et al. [1] modeled a two-echelon distribution network considering customer's responsiveness and a hybrid heuristic combining Tabu search and Simulated Annealing (SA) sharing the same Tabu list developed for solving the problem. In addition, Jokar and Seifbarghy [9] explained a two-echelon inventory system, where an independent Poisson demand with constant transportation and lead-time were considered. Finally, an approximate cost function was developed to find the optimal reorder points for given batch sizes in all installations and accuracy was assessed by simulation. Moreover, Nagurney [10] derived a relationship between supply chain network equilibrium and transportation network equilibrium with elastic demands.

In recent years, fractional programming with complex rather than real variables involving analytic functions in the objective and the constraints has found renewed interest. In various applications of nonlinear programming, a ratio of two functions is to be maximized or minimized. In other applications, the objective function involves more than one such ratio. Ratio optimization is commonly called fractional programming. The study of fractional programs with only one ratio has largely dominated the literature in this field until about 1980. Ratios of convex and/or concave functions as well as compositions of such ratios are not convex, in general, even in the case of linear functions. However, they are often generalized convex in some sense, and fractional programming has benefited from advances in generalized convexity and vice versa.

Sabri Ehap and Benita et al. [12] presented a multi-objective multi-product multi-echelon stochastic model that simultaneously addressed strategic and operational planning while taking the uncertainty in demand, and production and supply lead-times into consideration. The authors point out some gaps in the SC literature – most stochastic models presented considered only up to two echelons. Conversely, larger models were mostly deterministic in nature. Moreover, these deterministic models considered only profitability and ignored other performance measures. Another observation is that strategic and operational levels were not considered simultaneously. The main model, presented here, consists of a mixed integer linear programming (MILP) sub-model for the strategic level to determine the optimal number and locations of manufacturing facilities and DCs, and assignment of service regions to DCs. The stochastic operational level sub-model is an extension of the one given in Cohen and Lee et al. [3], considering simultaneous optimization of non-linear production, distribution and transportation costs. An algorithm is presented to integrate these two sub-models to achieve an overall supply chain performance vector. The approach presented here mainly

emphasizes the integration of strategic and operational level decisions while considering demand uncertainty.

Uddin and Sano [13] developed an MIP based vendor-buyer multiple products-consumers, facility selection problem with a price-sensitive linear demand function. They assumed that a coordinating mechanism among the members of supply chain could achieve the optimal solution and the optimal location for the warehouse. Consequently, Uddin and Sano [14] explained an MIP based supply chain with a coordination mechanism consisting of a single vendor and buyer. Instead of a price sensitive linear or deterministic demand function, a price-sensitive non-linear demand function was introduced. They assumed that the production and shipping functions of the vendor were continuously harmonized and occurred at the same rate. These models showed that, using a coordinating mechanism, individual profit as well as coordinated profit could be increased and consumer's purchasing price could be reduced. Gaur and Arora [7] presented a technique for solving a special class of non-linear fractional programs where the both numerator and denominator were separable functions and used the concept of piecewise linear approximation. The problem was then solved by using the Charnes and Cooper [2] transformation method.

On the other hand, Dhaenens-Flipo and Finke [4] considered an integrated production-distribution problem in multi-facility, multi-product and multi-period environment. They formulated a network flow problem with an objective to match products with production lines to minimize the related costs generated randomly and solved using the CPLEX software. Moreover, an MIP model for a production, transportation, and distribution problem was developed to represent a multi-product tri-echelon capacitated plant and warehouse location problem by Pirkul and Jayaraman [11]. They minimized the sum of fixed costs of operating the plants and warehouses, and the variable costs of transporting multiple products from the plants to the warehouses and finally to the customers. In addition, a solution procedure was provided based on the Lagrangian relaxation (LR) to find the lower bound, followed by a heurwastic to solve the problem. There are copious researches on LFP to find the best solution approach. Among these, Charnes and Cooper [2] described a transformation technique to transform LFP into an equivalent linear program. The method is quite simple but needs to solve two transformed model to obtain the optimal solution.

Considering the importance of MILFP and its application in SCM, here, a vendor-buyer multi-product, multi-facility, and multi-customer location production problem is formulated as a MILFP which maximizes the ratio of return on investment, and at the same time optimizes location, transportation cost, and the investment. It is assumed that the vendor and buyer of this supply chain are coordinated by sharing information. Furthermore, an MIP model is derived from the same model to determine the sites for vendor and the best allocation for both the buyer and the vendor. Using the suitable transformation of Charnes and Cooper [2], the formulated MILFP is solved by AMPL. Finally, a numerical example along with sensitivity of the opening cost is considered to estimate the performance of the models.

The remainder of our work is organized as follows. In Section 2, a mathematical formulation of the model as MILFP and MIP are presented. The section has four subsections, which describe the concept of mixed integer linear fractional programming problem, notations, assumption, prerequisites and finally the MILFP and MIP models. In Section 3, a numerical example is worked through. In Section 4, the obtained results are discussed. Finally, in Section 5, our conclusions and contributions are summarized.

2. Model Formulation

In this section, we formulate an integrated model that explores the tradeoff among location, transportation cost and distribution considering a multi-product, multi-facility, and multi-customer location-production-distribution system. It is assumed that a logistics center seeks to determine an integrated plan of a set of L locations of the vendor with production capability of m products and n buyer's destinations as shown in Figure 1. In Figure 1, solid arrows represent the commodity flow and dotted arrows stand for the information flow. Each source has an available supply of commodity to distribute in various destinations, and each destination has a forecast demand of commodity to be received by various sources. The coordination contains a set of manufacturing facilities with limited production capacities situated within a geographical area. Each of these facilities can produce one or all of the products in the company's portfolio. The buyer demands for multiple products are to be satisfied from this set of manufacturing facilities. Therefore, the production capacities of these facilities effectively represent the current and potential capacities. Our work focuses on developing MILFP and MIP programs to optimize the capacitated facility location, buyer allocation decisions and production quantities at these locations and satisfy customer demands.
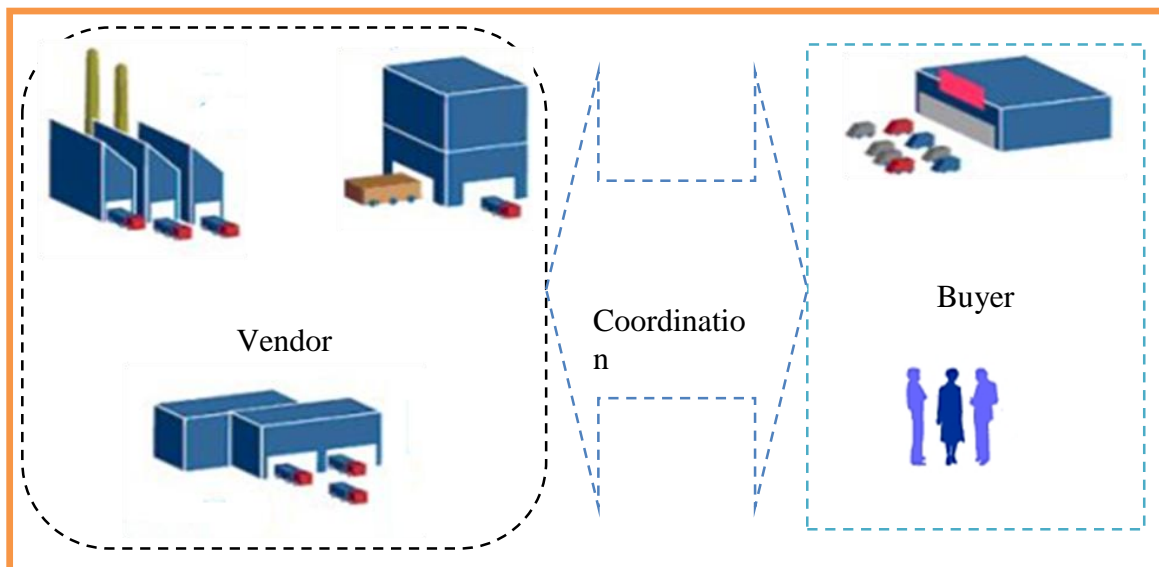

Figure 1. Distribution pattern of a coordinated supply chain

2.1. Mixed integer linear fractional program

Recently, various optimization problems, involving optimization of the ratio of functions, such as time/cost, volume/cost, profit/cost, loss/cost or other quantities measuring the efficiency of the system, have been of wide interest in non-linear programming.

A fractional programming problem is a mathematical programming problem in which the objective is the ratio of two functions and needs to be optimized with respect a set of constraints. If the numerator and denominator of the objective function and the constraint set are all linear, then the fractional programming problem is called a linear fractional programming (LFP) problem. Mathematically, the LFP problem can be represented as:

Optimize (Minimize or Maximize) $Z = \dfrac{C^T x + \alpha}{D^T x + \beta}$

s.t.

$$x \in X = \left\{ x \in R^n : Ax = B, \, x \geq 0 \right\},$$

where

x is the set of decision variables $(n \times 1)$

A is the constraint matrix $(m \times n)$

C and D are the contribution coefficient vectors $(n \times 1)$

B is the constant or resource vector $(m \times 1)$

α and β are scalars, which shows some constant profit and cost, respectively.

n and m are the number of variables and constraints, respectively.

A mixed integer programming (MIP) problem arises when some variables in the model are real valued (can take on fractional values) and some variables are integer valued. When the objective function and the constraint set are all linear, then we have an MIP problem. On the other hand, if the problem is of an LFP type, then it is called a mixed integer linear fractional programming (MILFP) problem.

Charnes and Coorper [2] Transformation Technique:

Numerous methods such as iterative, parametric, genetic and fuzzy are available in the literature to solve an LFP problem. In our work, we use the Charnes and Coorper transformation technique. Charnes and Cooper [2] considered the LFP problem as defined above and assumed that

(1) the feasible region X is non–empty and bounded,

(2) $C^T x + \alpha$ and $D^T x + \beta$ do not vanish simultaneously at x.

Introducing the variable transformation $y = tx,$ where $t > 0$, Charnes and Cooper [2] proved that the LFP problem is reduced to either one of the following two equivalent linear programs:

(EQP):   Maximize $Z_1 = C^T y + \alpha t$

           s.t.

$$Ay + Bt = 0$$
$$D^T y + \beta t = 1$$
$$y \geq 0, t \geq 0,$$

and

(EQN):   Maximize $Z_2 = -C^T y - \alpha t$

           s.t.

$$Ay - Bt = 0$$
$$D^T y + \beta t = -1$$
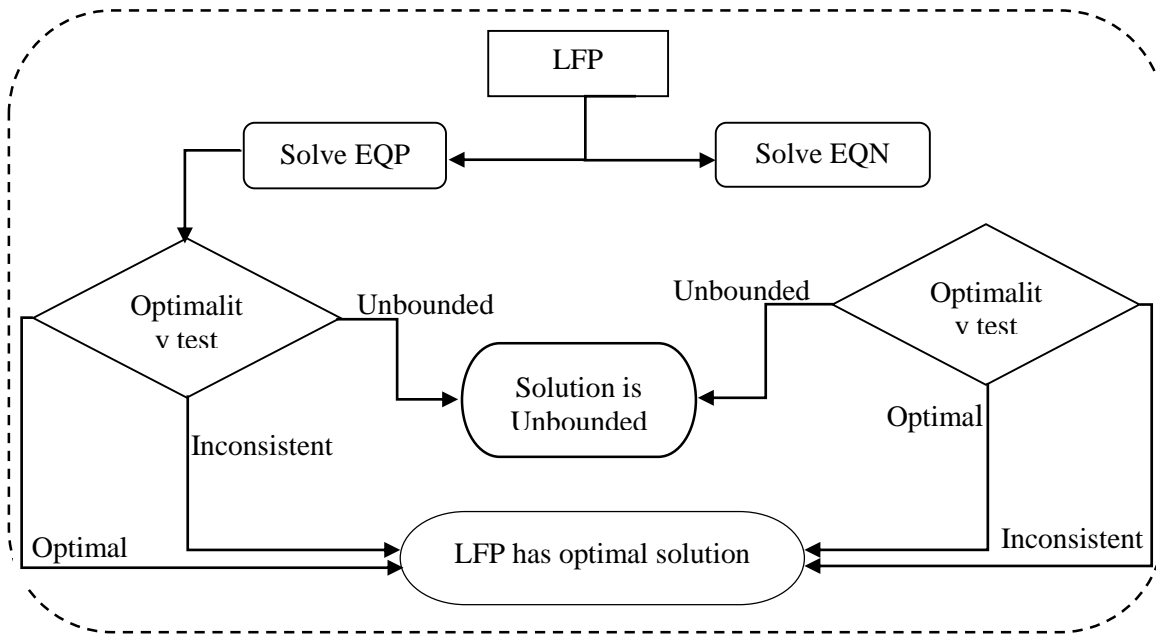$$y \geq 0, t \geq 0.$$

Figure 2. Flowchart for the Charnes and Cooper algorithm

The equivalent positive (EQP) and equivalent negative (EQN) problems are solved by the well known Dantzig and George [5] simplex method. If one of the problems EQP and EQN has an optimal solution (y^*, t^*) and the other is inconsistent, then the LFP problem has an optimal solution, which can be obtained simply by $x^* = y^*/t^*$. If any one of the two problems is unbounded, then the LFP problem is unbounded. So, if one problem is found unbounded, then one can avoid solving the other as described in Figure 2.

2.2. Notations and assumptions

Here, we describe the notations, assumptions, parameters declaration and decision variables for the MILFP based vendor-buyer coordination model. The prerequisite terms require to formulate the model are also briefly explained. The notations are defined as shown in Table 1.

Assumptions

(1) Each manufacturing facility is able to produce all products.
(2) The selling price for a product may vary from buyer to buyer depending on the order size, discount, historical relationships, etc.
(3) The company and buyer agree beforehand on the inventory distribution pattern and the shipping plans are formulated accordingly.

Table 1. Notations for the multi-product multi-customer and multi-facility vendor-buyer system

Index and Parameters

i : index for product, $i = 1, \dots, m$.

j : index for buyer, $j = 1, \dots, n$.

l : index for location of vendor, $l = 1, \dots, L$.

c_{ij} : the price of ith product to jth buyer (\$/unit).

α_l : the fixed cost for opening the vendor at location l (\$).

β : any positive scalar.

c_i^l : the price of a unit of raw material for ith product at lth vendor (\$/unit).

a_i^l : the amount of raw material needed to produce ith product at lth vendor (\$/unit).

t_i^l : unit transportation cost of raw material for ith product at lth vendor (\$/unit).

p_{ij}^l : the production cost of ith product for jth buyer at lth vendor (\$/unit).

h_{ij}^l : unit holding cost of ith product from lth vendor for jth buyer for a given unit of time (\$/unit-time).

c_{ij}^l : the shipment cost of ith product from lth vendor to jth buyer (\$/unit).

d_{ij} : the total demand of ith product by jth buyer (unit).

w_i^l : the capacity for ith product at lth vendor (unit).

t_j^l : the required time for delivery of products from lth vendor to jth buyer (unit).

t_j^{*l} : the time within which product should be delivered from lth vendor to jth buyer (unit).

p : penalty cost for delay in delivery for one unit of demand in one unit of time (\$/unit).

c_j^{*l} : the transportation cost per unit of product from lth vendor to jth buyer (\$/unit).

A binary variable:

$$g_j^l = \begin{cases} 1, \text{if } t_j^l > t_j^{*l}, \\ 0, \text{otherwise} \end{cases}$$

where t_j^l is the required time for delivery of product from lth vendor to jth buyer and t_j^{*l} is the time within which product should be delivered from lth vendor to jth buyer.

Decision variables:

$$y_j^l = \begin{cases} 1, \text{if customer } j \text{ is assigned to manufacturer } l, \\ 0, \text{otherwise} \end{cases}$$

$$x_l = \begin{cases} 1, if \text{ location } l \text{ is used,} \\ 0, \text{otherwise} \end{cases}$$

Q_{ij}^l = the production quantity of product i for buyer j at lth vendor (in units).

2.3. The MILFP model

Here, we formulate the MILFP problem.

Objective function:

$$\text{Maximize } \frac{Z_1}{Z_2}, \tag{1}$$

where

$$Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} Q_{ij}^l c_{ij}$$

$$Z_2 = \sum_{l=1}^{L} x_l \alpha_l + \sum_{l=1}^{L}\sum_{i=1}^{m} c_i^l a_i^l + \sum_{l=1}^{L}\sum_{i=1}^{m} t_i^l \ a_i^l + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} Q_{ij}^l \ p_{ij}^l + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} Q_{ij}^l \ cc_{ij}^l +$$

$$\sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} Q_{ij}^l \ h_{ij}^l / 2 + \sum_{i=1}^{m}\sum_{l=}^{L}\sum_{j=1}^{n} p \ d_{ij} z_j^l \ (t_j^l - t_j^{*l}) \ g_j^l + \sum_{l=1}^{L}\sum_{j=1}^{n} t_j^l \ c_j^{*l}$$

s.t.

$$\sum_{l=1}^{L}\sum_{i=1}^{m} Q_{ij}^l \geq \sum_{i=1}^{m} d_{ij}, \ \forall j \tag{2}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{n} Q_{ij}^l \geq \sum_{j=1}^{n} d_{ij}, \ \forall i \tag{3}$$

$$\sum_{l=1}^{L} Q_{ij}^l \geq d_{ij}, \ \forall i, j \tag{4}$$

$$\sum_{j=1}^{n} Q_{ij}^l \leq w_i^l, \ \forall i, l \tag{5}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{m} Q_{ij}^l \leq \beta \ x_l, \ \forall l \tag{6}$$

$$\sum_{l=1}^{L} y_j^l = 1, \forall j \tag{7}$$

$$Q_{ij}^l, c_{ij}, \alpha_l, d_{ij}, w_i^l, cc_{ij}^l, h_{ij}^l, p_{ij}^l, t_i^l, t_j^l, t_j^{*l}, p, c_j^{*l}, a_i^l, c_i^l \geq 0, x_l, y_j^i \text{ are binary } \quad \forall i, j, l \tag{8}$$

The objective function (1) estimates the ratio of return and investment. Constraints (2) ensure that the total amount of products being manufactured at all plants for a particular buyer is equal to the total demand of that buyer. Similarly, constraints (3) guarantee that the total amount of a particular product being manufactured at all plants for all buyers is equal to the total demand of that product from all buyers. It is important to note here that the first two constraints are stated separately to show better accountability of the total demands from all buyers and for all products respectively. Constraints (4) assure that the total amount of a specific product being manufactured for a particular buyer at all plants is equal to the demand of the specific product from that buyer. Constraints (5) present the capacity constraints. Constraints (6) ensure that a plant is located if and only if there is a demand for any product. Constraints (7) show that each buyer is assigned to exactly one vendor. The constraints (8) are the nonnegative constraints.

2.4. The MIP model

Here, we formulate the equivalent mixed integer programming problem to estimate the total profit as well as optimal allocation and distribution. The objective function is the difference between return and investment.

Objective function: Maximize $Z_1 - Z_2$
s.t.
Constraints (2)-(8) hold.

3. Solution Approach

In order to solve the formulated MILFP problem, we need to apply a suitable transformation. We apply the Charnes and Cooper transformation to solve the formulated MILFP problem as described in subsection 2.1.

For any non-negative r, let the new decision be redefined as follows:

$$z_l = rx_l, \text{ for } r \geq 0 \text{ and } l = 1,...,L$$
$$z_j^l = ry_j^l, \text{ for } r \geq 0 \text{ and } j = 1,...,n, \ l = 1,...,L$$
$$z_{ij}^l = rQ_{ij}^l, \text{ for } r \geq 0 \text{ and } i = 1,...,m, \ j = 1,...,n, l = 1,...,L$$

Since $r \geq 0$, y_j^l and x_l are binary, z_l and z_j^l are either zero or r. Also, since Q_{ij}^l is nonnegative, the z_{ij}^l are also nonnegative. Therefore, the MILFP problem can be reformulated as two equivalent linear problems as follows:

(EQP):   Maximize $\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} z_{ij}^l c_{ij}$

s.t.

$$\sum_{l=1}^{L}\sum_{i=1}^{m} z_{ij}^l \geq r \sum_{i=1}^{m} d_{ij}, \ \forall j \tag{9}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{n} z_{ij}^l \geq r \sum_{j=1}^{n} d_{ij}, \ \forall i \tag{10}$$

$$\sum_{l=1}^{L} z_{ij}^l \geq r \, d_{ij}, \ \forall i,j \tag{11}$$

$$\sum_{j=1}^{n} z_{ij}^l \leq r \, w_i^l, \ \forall i,l \tag{12}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{m} z_{ij}^l \leq Mz_l, \ \forall l \tag{13}$$

$$\sum_{l=1}^{L} z_j^l = r, \forall j \tag{14}$$

$$\sum_{l=1}^{L} z_l \alpha_l + \sum_{l=1}^{L}\sum_{i=1}^{m} r \; t_i^l a_i^l + \sum_{l=1}^{L}\sum_{i=1}^{m} r \; c_i^l a_i^l + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z_{ij}^l \; p_{ij}^l + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z_{ij}^l \; cc_{ij}^l$$
$$+ \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z_{ij}^l \; h_{ij}^l /2 + \sum_{i=1}^{m}\sum_{l=1}^{L}\sum_{j=1}^{n} p \; d_{ij} z_j^l \; (t_j^l - t_j^{*l}) \; g_j^l + \sum_{l=1}^{L}\sum_{j=1}^{n} r \; t_j^l \; c_j^{*l} = 1 \tag{15}$$

(EQN):   Maximize $\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} - z_{ij}^l c_{ij}$

s.t.

$$\sum_{l=1}^{L}\sum_{i=1}^{m} z_{ij}^l \geq -r \sum_{i=1}^{m} d_{ij}, \; \forall j \tag{16}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{n} z_{ij}^l \geq -r \sum_{j=1}^{n} d_{ij}, \; \forall i \tag{17}$$

$$\sum_{l=1}^{L} z_{ij}^l \geq -r \; d_{ij}, \; \forall i, j \tag{18}$$

$$\sum_{j=1}^{n} z_{ij}^l \leq -r \; w_i^l, \; \forall i, l \tag{19}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{m} z_{ij}^l \leq -Mz_l, \forall l \tag{20}$$

$$\sum_{l=1}^{L} z_j^l = -r, \forall j \tag{21}$$

$$\sum_{l=1}^{L} z_l \alpha_l + \sum_{l=1}^{L}\sum_{i=1}^{m} r \; t_i^l d_i^l + \sum_{l=1}^{L}\sum_{i=1}^{m} r \; c_i^l a_i^l + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z_{ij}^l \; p_{ij}^l + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z_{ij}^l \; cc_{ij}^l$$
$$+ \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z_{ij}^l \; h_{ij}^l /2 + \sum_{i=1}^{m}\sum_{l=1}^{L}\sum_{j=1}^{n} p \; d_{ij} z_j^l \; (t_j^l - t_j^{*l}) \; g_j^{*l} + \sum_{l=1}^{L}\sum_{j=1}^{n} r \; t_j^l \; c_j^{*l} = -1 \tag{22}$$

$$z_{ij}^l, c_{ij}, \alpha_l, d_{ij}, w_j^l, cc_{ij}^l, h_{ij}^l, p_{ij}^l, t_i^l, t_j^l, t_j^{*l}, p, c_j^{*l}, a_i^l, c_i^l, z_l, z_j^l \geq 0, \forall i, i, l. \tag{23}$$

In order to find the solution of the formulated MILFP problem, first the EQP and EQN models are solved by employing AMPL of Bonmin and Couenne. A program was written according to the flowchart illustrated in Figure 2 for AMPL. The program consists of two main parts; the main module contains the actual program and the data file containing data of the various parameters. The formulated MIP model is solved by a branch and bound algorithm deploying AMPL with CPLEX accordingly. Eventually, the solution of the EQN turned to be inconsistent, whereas the solution of the EQP model was optimal. Therefore, by the Charnes and Cooper algorithm it is concluded that the optimal solution of the MILFP problem is obtained by the optimal solution of the EQP problem. The

program was executed on a Pentium IV personal machine with a 1.73 GHz processor and 2.0 GB RAM.

4. Computational Analysis

In order to analyze the effectiveness of the proposed models, a numerical example was worked through. It was assumed that a vendor had 5 locations, with 3 productions for 2 buyers. The deterministic demand of unit products for buyers are (1700, 3500, 2200) and (2300, 1500, 2800), selling per unit prices (in $) of products for buyers are (40, 56, 82) and (42, 58, 75), penalty cost of per unit (in $) products for buyers are (0.50, 0.60, 0.60) and (0.25, 0.40, 0.30), respectively. Table 2 describes additional information regarding the parameters of the MILFP and MIP models.

Table 2. Parameters of the MILFP model

Parameters	Locations of the vendor				
	1	2	3	4	5
Raw materials (units)	(130,120,130)	(120,180,200)	(150,200,170)	(100,100,100)	(100,100,100)
Trans. cost (input) ($)	(0.3,0.2, 0.3)	(0.2,0.25, 0.2)	(0.5,0.45,0.6)	(0.1,0.1, 0.1)	(0.1,0.1, 0.1)
Production cost ($)	(10,17,15)	(12,12,18)	(14,15,16)	(20,25,30)	(5,10,15)
Holding cost ($)	(1,2,3)	(3,2,2)	(3,4,3)	(5,4,5)	(2,3,1)
Shipping cost ($)	(11,23,36)	(25,27,32)	(13,26,35)	(25,27,32)	(10,15,20)
Capacity (in hundreds) units	(13,12,13)	(12,18,20)	(15,20,17)	(10,10,10)	(10,10,10)
Travel time units	(5,7)	(9,10)	(12,8)	(15,20)	(10,10)
Required delivery time	(5,7)	(10,10)	(12,8)	(15,20)	(10,10)
Obligatory delivery time	(5,7)	(9,10)	(12,8)	(10,10)	(15,20)
Trans. cost ($/unit time)	(0.5,0.7)	(0.6,0.4)	(0.6,0.5)	(1.0,1.2)	(0.5,0.5)

In order to observe the effect of the key parameters, six sets of the vendor's opening costs ($) with same average value such as (50000,30000, 40000,60000, 20000),(40000, 40000, 40000, 40000, 40000), (60000, 30000, 40000, 50000, 20000), (50000, 60000, 40000, 30000, 20000), (50000, 30000, 60000, 40000, 20000) and (50000, 30000, 40000, 20000, 60000) were considered, while all other remaining parameters were kept unchanged as shown in Table 2. Significant findings regarding the numerical example of the proposed MILFP and MIP models as well as the allocations and the distributions of different products for the two buyers are briefly summarized in tables 3 and 4. The values Return on Investment (RI in %) in tables 3 and 4 represent the gap in percentage between the total return on investment incurred by the MILFP and MIP model, that is,

$$RI(\%) = |RI1 - RI2| \times \frac{100}{RI2},$$

where RI 1 and RI 2 are the ratio of return on investment obtained by the MILFP and MIP models, respectively. Finally, in order to estimate effect of the sensitivity of the opening cost parameter, we employed sensitivity on the opening cost ($) for location 3. It was assumed that the opening costs of the vendor located at location 3 were 50, 100, 1000, 2000, 3000, 4000, 5000 and 10000, while all other remaining parameters were kept unchanged.

Figures 3 and 4 describe the optimal allocation of different products for the first case and for both buyers. From the distribution pattern of different products, it is clear that the MILFP model provides the optimal locations of the vendor for buyer 1 as 1, 2 and 5, whereas the MIP provides the optimal locations of the vendor for buyer 1 as 1, 2, 3 and 5. The optimal locations achieved by the MILFP model of the vendor for buyer 2 are 1, 2, 3 and 5. Similarly, the MIP model provides the optimal locations of the vendor for buyer 2 as 1, 2, 3 and 5. Therefore, from the distribution of different products by the MILFP and MIP models, it is recommended that vendor 4 does not remain optimal for the first case.
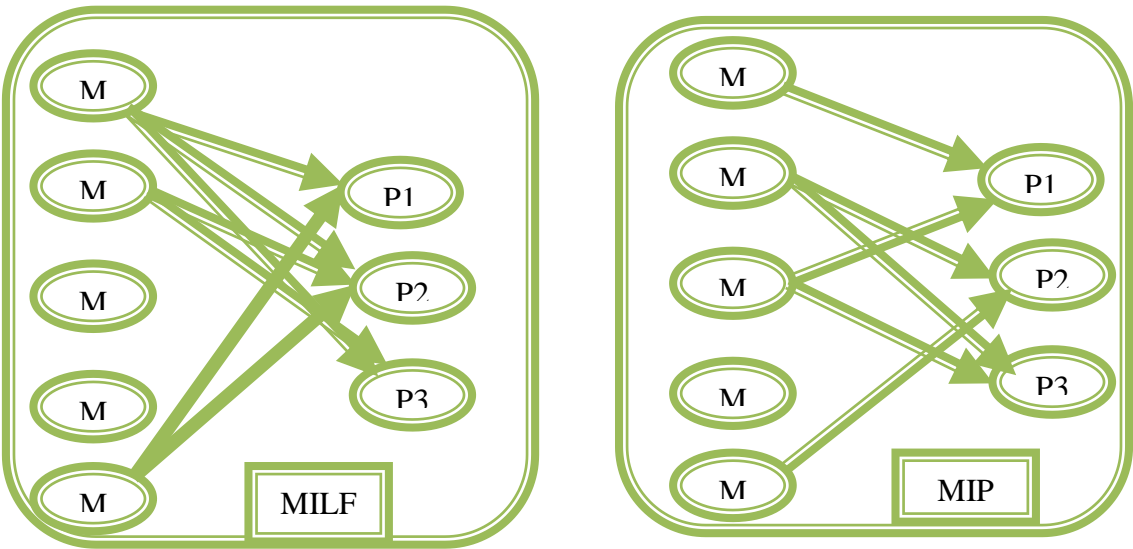
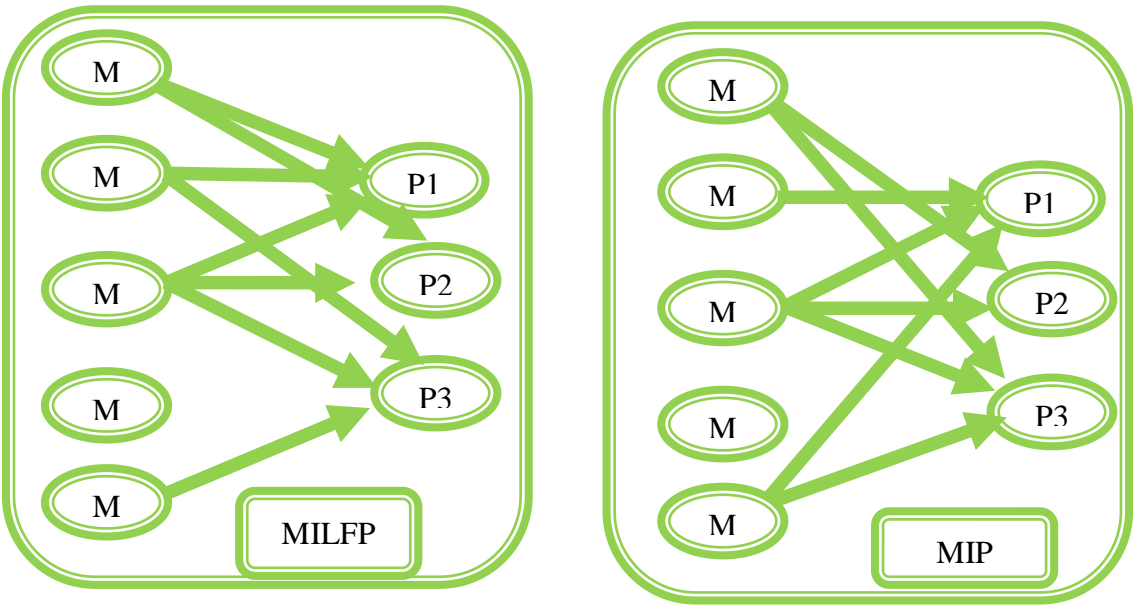**Figure 3.** Allocations for buyer 1 by the MILFP and MIP models

Figure 4. Allocations for buyer 2 by the MILFP and MIP Models

From the sensitivity analysis, it appears that by the MILFP model for buyer 1 the vendors located at location 3 do not remain optimal, except for the second case. In addition, the MILFP model has no

optimal product distribution from location 4 for buyer 1 for all the six cases as depicted in Table 3. In the same way, for the entire six cases, the vendor located at location 4 is not profitable for buyer 2 by the MILFP model as shown in Table 4. Similarly, by the MIP model for both buyers, the not selected vendor is located at location 4 for all the cases. Therefore, the results of these algorithms indicate that vendors 1, 2, 3 and 5 should be located to satisfy the buyers' demands and vendor 4 could be removed without loss of optimality. Furthermore, it may be concluded that the optimal solutions obtained by MILFP problem are as good as the ones due to the MIP problem. In addition, for the six cases, all the differences of the return on investment for both solutions are less than 0.94% as displayed in Table 4.

Figures 5 and 6 describe the average demands of different products achieved by the MILFP and MIP models for buyer 1. By both MILFP and MIP models, the highest demand of the product for buyer 1 is product 2, which is followed by product 3 and product 1 as shown in figures 5 and 6. The MILFP and MIP models satisfy the optimal demand of buyer 1 by the manufactures located at the location points 1, 2, 3 and 5. The MILFP model illustrates that vendor located at locations 1 is profitable for all the three products. The MIP model illustrates that locations 1 and 3 are profitable for all the three products. Furthermore, both MILFP and MIP models describe that vendor 4 is not in any way optimum for buyer 1 for all the three products.
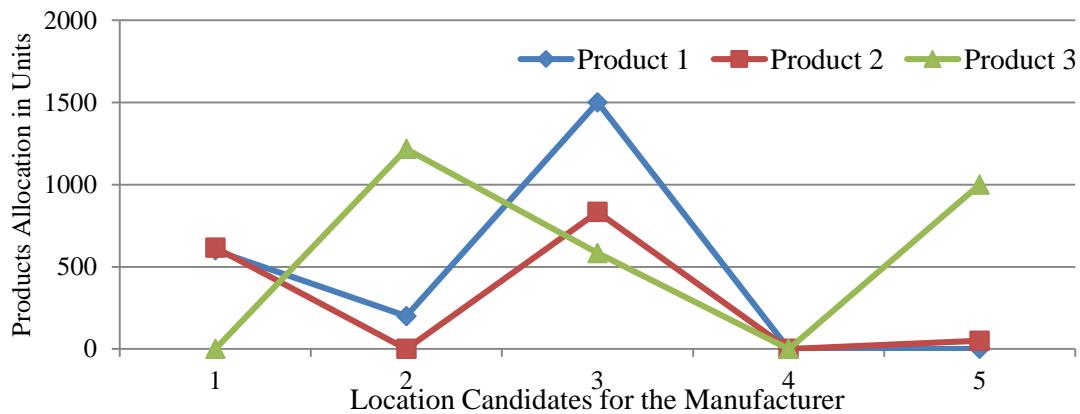

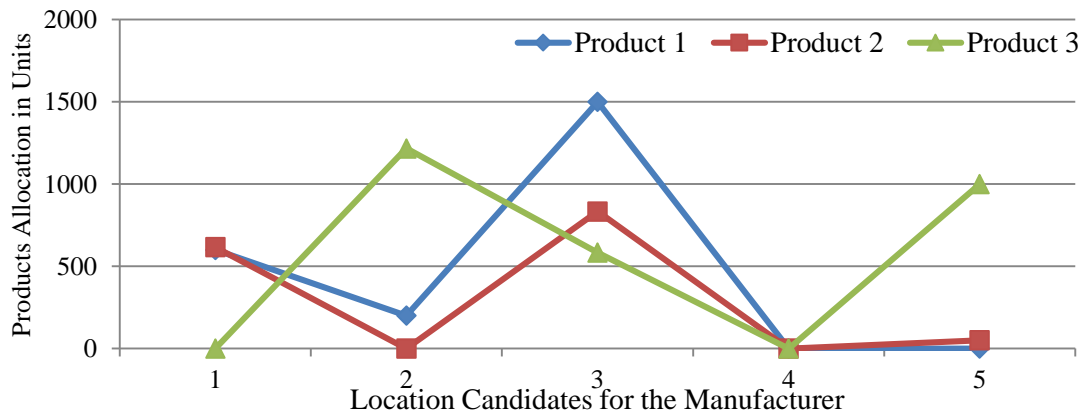**Figure 5.** Demands for different products at different locations for buyer 1 by the MILFP model

Figure 6. Demands for different products at different locations for buyer 1 by the MIP model

Figures 7 and 8 depict the average demands of different products obtained by the MILFP and MIP models for buyer 2. By both MILFP and MIP models, the maximum demand for product of buyer 2

is for product 2 which is followed by product 3 and product 1 as shown in figures 7 and 8. The MILFP and MIP models give the optimal demand of buyer 2 by the manufactures located at the location points 1, 2, 3 and 5. The MILFP model illustrates that vendor located at 3 is profitable and can satisfy the optimal demand of all the three products. The MIP model illustrates that locations 3 and 5 are profitable for all the three products. Furthermore, both MILFP and MIP models show that vendor 4 is not in any way profitable for buyer 2 for all the products.
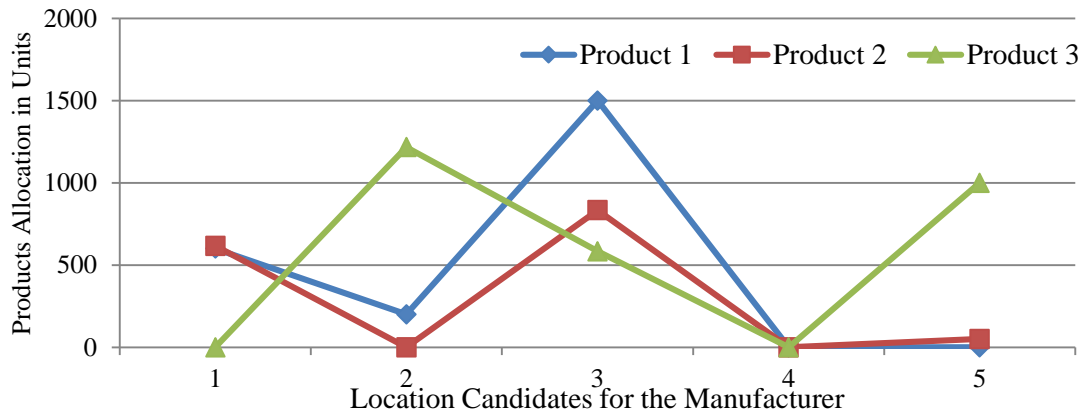

Figure 7. Demands for different products at different locations for buyer 2 by the MILFP model
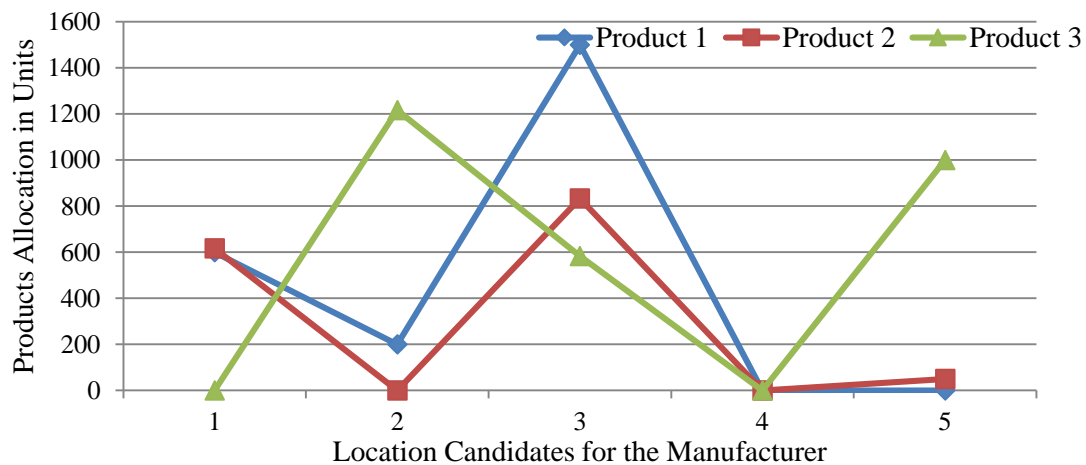

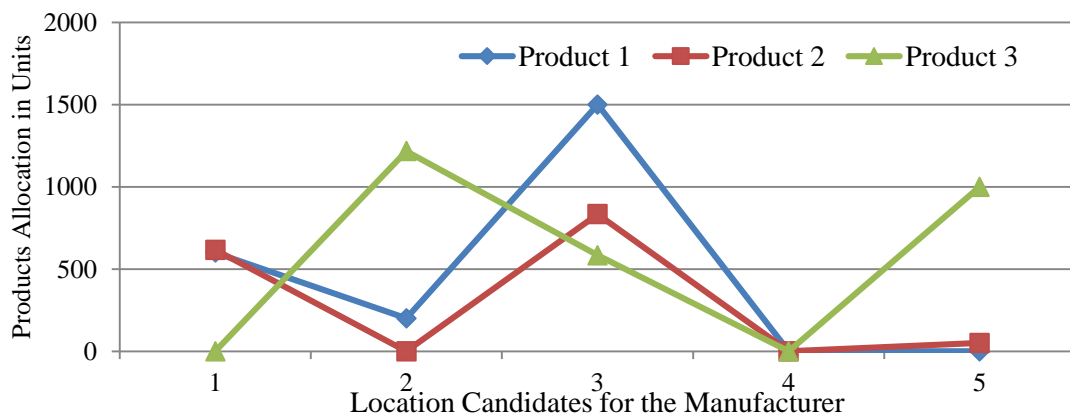**Figure 8.** Demands for different products at different locations for buyer 2 by the MILFP model

Figure 9. Demands for different products at different locations for buyer 2 by the MIP model

Table 3. Sensitivity of opening cost for buyer 1

MILFP model						MIP model					
Loc. 1	Loc. 2	Loc. 3	Loc. 4	Loc. 5	RI 1	Loc. 1	Loc. 2	Loc. 3	Loc. 4	Loc. 5	RI 2
(700,700,1300)	(0,1800,900)	(0,0,0)	(0,0,0)	(1000,1000,0)	1.15	(1300,0,0)	(0,1800,2000)	(400,700,200)	(0,0,0)	(0,1000,0)	1.15
(700,0,1300)	(0,1800,200)	(0,1000,700)	(0,0,0)	(1000,700,0)	1.07	(1300,0,0)	(0,1800,2000)	(400,1000,200)	(0,0,0)	(0,700,0)	1.06
(700,700,1300)	(0,1800,900)	(0,0,0)	(0,0,0)	(1000,1000,0)	1.14	(1300,0,0)	(0,1800,2000)	(400,700,200)	(0,0,0)	(0,1000,0)	1.14
(700,700,1300)	(0,1800,900)	(0,0,0)	(0,0,0)	(1000,1000,0)	1.11	(1300,0,0)	(0,1800,2000)	(400,700,200)	(0,0,0)	(0,1000,0)	1.11
(700,700,1300)	(0,1800,900)	(0,0,0)	(0,0,0)	(1000,1000,0)	1.12	(1300,0,0)	(0,1800,2000)	(400,700,200)	(0,0,0)	(0,1000,0)	1.12
(700,700,1300)	(0,1800,900)	(0,0,0)	(0,0,0)	(1000,1000,0)	1.09	(1300,1200,200)	(0,1800,2000)	(400,500,0)	(0,0,0)	(0,0,0)	1.10

Table 4. Sensitivity of opening cost for buyer 2

| MILFP model					MIP model					RI (%)
Loc. 1	Loc. 2	Loc. 3	Loc. 4	Loc. 5	Loc. 1	Loc. 2	Loc. 3	Loc. 4	Loc. 5	
(600,500,0)	(200,0,1100)	(1500,1000,700)	(0,0,0)	(0,0,1000)	(0,1200,300)	(200,0,0)	(1100,300,1500)	(0,0,0)	(1000,0,1000)	0.0
(600,1200,0)	(200,0,1800)	(1500,0,0)	(0,0,0)	(0,300,1000)	(0,1200,300)	(200,0,0)	(1100,0,1500)	(0,0,0)	(1000,300,1000)	0.94
(600,500,0)	(200,0,1100)	(1500,1000,700)	(0,0,0)	(0,0,1000)	(0,1200,300)	(200,0,0)	(1100,300,1500)	(0,0,0)	(1000,0,1000)	0.0
(600,500,0)	(200,0,1100)	(1500,1000,700)	(0,0,0)	(0,0,1000)	(0,1200,300)	(200,0,0)	(1100,300,1500)	(0,0,0)	(1000,0,1000)	0.0
(600,500,0)	(200,0,1100)	(1500,1000,700)	(0,0,0)	(0,0,1000)	(0,1200,300)	(200,0,0)	(1100,300,1500)	(0,0,0)	(1000,0,1000)	0.0
(600,500,0)	(200,0,1100)	(1500,1000,700)	(0,0,0)	(0,0,1000)	(0,0,1100)	(1200,0,0)	(1100,1500,1700)	(0,0,0)	(0,0,0)	0.91
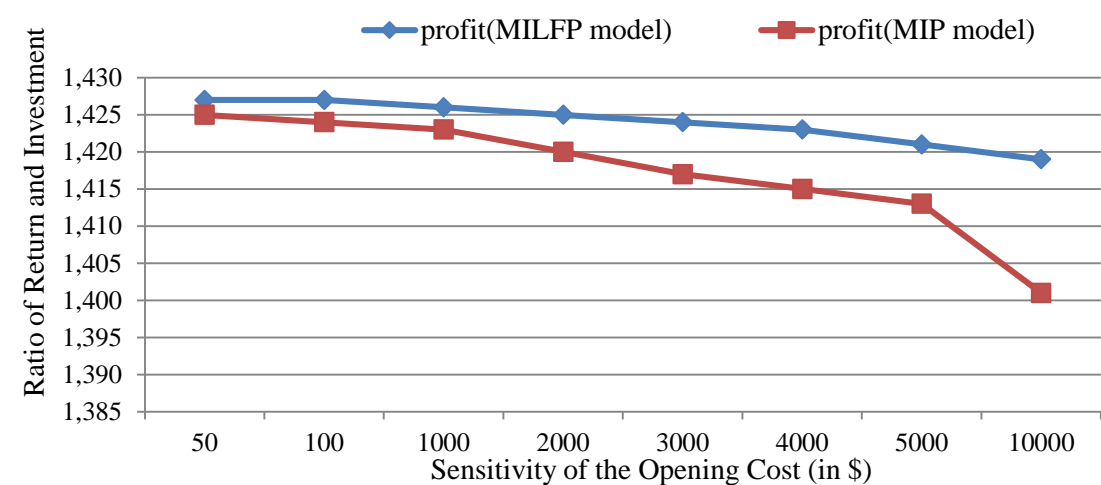
Figure 9. Comparison between return and investment obtained by the MILFP and MIP models
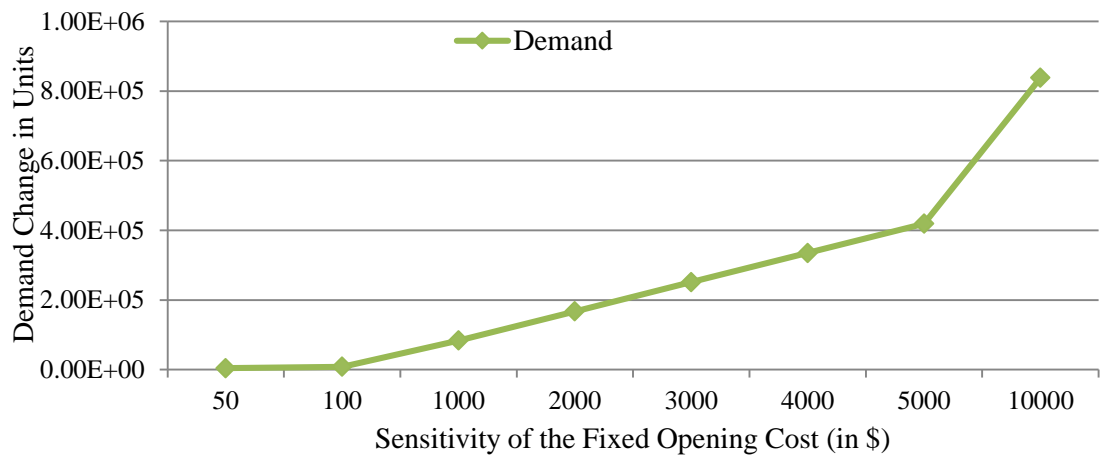

Figure 10. Effect of sensitivity analysis of fixed opening cost on demand
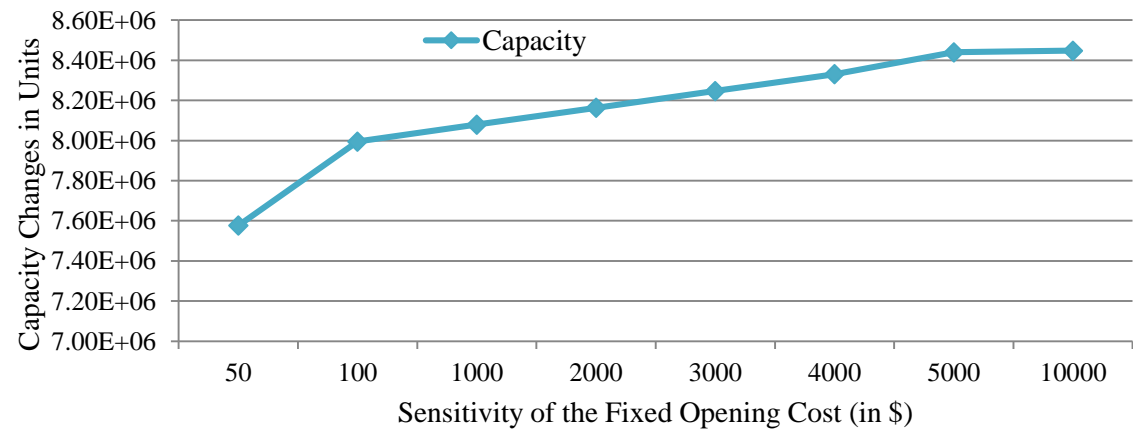

Figure 11. Effect of sensitivity analysis of fixed opening cost on capacity

Figure 9 describes the sensitivity of the opening cost on the total ratio of return on investment obtained in different cases by the MILFP and MIP models. The proportion of return and investment obtained by both MILFP and MIP models are almost the same. In addition, in all cases the profit achieved by the MILFP model is slightly higher than that of the MIP model as shown in Figure 9, due to the nature of the MILFP and MIP models. The sensitivity of the opening cost demonstrates that in all the cases, increase in the opening cost decreases the profit by both MILFP and MIP models, since the additional cost increases the investment as well as cost. Figures 10 and 11 illustrate the influence of the opening cost on demand and capacity of each location. If the opening cost of a location decreases or increases, the demand and capacity of that location change accordingly. The opening cost changes the demand more dramatically than the capacity of the product. This can be interpreted that by additional opening cost, additional advertisement and promotion can be offered which increase the demand. Similarly, increase in the opening cost, which also concerns the reconstruction and expansion activities, turns to increase the capacity can be increased by increasing the opening cost.

5. Conclusion

An MILFP based model was developed for the coordinated supply chain and, using a suitable transformation, the model was solved by AMPL. The formulated model also maximizes the ratio of return on investment. It was assumed that the vendor and buyer of the supply chain were coordinated by sharing information regarding their status. Furthermore, in order to demonstrate the significances of the MILFP model, an MIP based model was also formulated. Our significant findings are summarized as follows.

First, the illustrated numerical example apparently showed that both MILFP and MIP model provide very similar distribution patterns for the integrated multi-product, multi-facility, and multi-buyer location production supply chain network, which is worthwhile for the developed MILFP model. Second, the optimal locations of the warehouses obtained by both models, were quite similar and rejected the same locations. The optimal demands for different products by the buyer were almost analogous for both MILFP and MIP models. The differences of the ratio of the return on investment achieved by both models were less than 0.94%. Moreover, from the sensitivity analysis of the opening cost, it was concluded that opening cost was a momentous factor to increase and decrease the demand and capacity of a vendor, respectively. Moreover, the fixed opening cost had a negative influence on the total profit. Therefore, the MILFP model could be a relevant approach in a logistic model to find the optimal manufacturer as well as optimal distribution with profit maximization and cost minimization.

Additional work may be needed to extend our work to be more realistic. In a future work, these models might be applied to calibrate and validate the latest surveyed data, which might require additional assumptions.

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