

An EPQ Model for Product Life Cycle (Maturity Stage) with Deteriorating Items and Shortages

C. Krishnamoorthi

A product life cycle is the life span of a product in which the period begins with the initial product specification and ends with the withdrawal from the market of both the product and its support. A product life cycle can be divided into several stages characterized by the revenue generated by the product. This study investigates inventory control policies in a manufacturing system for a single product during the product life cycle, which consists of four stages: introduction, growth, maturity and decline. In all inventory models a general assumption is that products have indefinitely long lives. In general, almost all items deteriorate over time. Often, the rate of deterioration is low and there is little need to consider the deterioration in the determination of the economic lot size. The objective is to derive the cycle time and optimal production lot size to minimize total costs for the product life cycle with deteriorating items. The relevant model is built, solved and some main results on the uniqueness of the solution using rigorous mathematical methods are obtained. Illustrative examples are provided to verify our findings numerically.

Keywords: Inventory, Product life cycle, Growth and maturity, Deteriorating items, Demand and production.

Manuscript received on 26/10/2012 and accepted for publication on 22/5/2013.

1. Introduction

A product life cycle is the life span of a product in which the period begins with the initial product specification and ends with the withdrawal from the market of both the product and its support. A new product is first developed and then introduced into the market. Once the introduction gets successful, a growth period follows with wider awareness of the product and increased sales. The product enters maturity when sales stop growing and demand stabilizes. Eventually, sales may decline until the product is finally withdrawn from the market or re-developed. A product life cycle can be divided into several stages characterized by the revenue generated by the product. The product life cycle concept may apply to a brand or to a category of the product. Its duration may be too short ranging from a few months for a faded item to a century or more for product categories. When the product is introduced, the sales are expected to be low until customers become aware of the product and its benefits.

The concept of a product life cycle (PLC) has occupied a prominent position in the marketing literature as both a forecasting instrument (Kovac et al. [9]) and a guideline for corporate marketing strategy (Levitt [10]) and widely discussed (see the overview by Kotler [8]).

In theory, at least two conflicting definitions can be found for PLC. The first refers to the progress of a product from raw material, through production and use, to its final disposal. The second definition of PLC describes the evolution of a product measured by its sales over time as seen in Fig. 1. Kotler [8] presented the product life cycle concept as a marketing management tool for consumer branded products, by: (1) introduction– the product is put on the market, awareness and acceptance are minimal, (2) growth– the product begins to make rapid sales gain because of the cumulative effects of introductory promotion, distribution and words of mouth influence, (3) maturity– sales growth continues and sales reach and remain on a plateau marked by the level of replacement demand, and (4) decline– sales begin to diminish absolutely as the product is gradually edged out by better products or substitutes.

Cox [4] showed that every product passes through a series of phases in the course of its life, referred to as the product life cycle. The phases that a product goes through during its life cycle are the introduction, growth, maturity and decline stages. The most common product life cycle pattern is the S-shaped logistic curve with four major stages: introduction, growth, maturity and decline. For example, Cox in a study of 258 ethical-drug products, introduced during the years 1955-59, found that six types of life cycle curves were needed to describe the sales patterns of the products. These six product life cycle curves are shown in Fig. 2. It is interesting to note that only 28.3% of the products studied by Cox were described by the traditional S-shaped curve. The sixth life cycle pattern-polynomials of the fourth degree was found to be the most appropriate for the products with 39.1% of the products following the pattern. Buzzell [1] indicated that the introductory period is characterized by heavy promotion aimed at building up primary demand; price is relatively unimportant. During the growth phase, more competition appears and there is an increasing pressure on price. Promotional expenditures decline in relation to sales; there is a shift to competition on the basis of brands and specific features. As the product enters maturity, there appears an increasing product brand competition, promotional expenditures and prices tend to stabilize, manufacturers begin efforts to extend life cycles and new brands may appear. Finally, in the decline phase, further declines in price and promotional expenditures can be expected. Tellis and Crawford [13] gave the product life cycle as a model on the fixed cycle of birth-growth-maturity-death through which higher living organisms pass. The PLC can be analyzed on different levels from the main product type

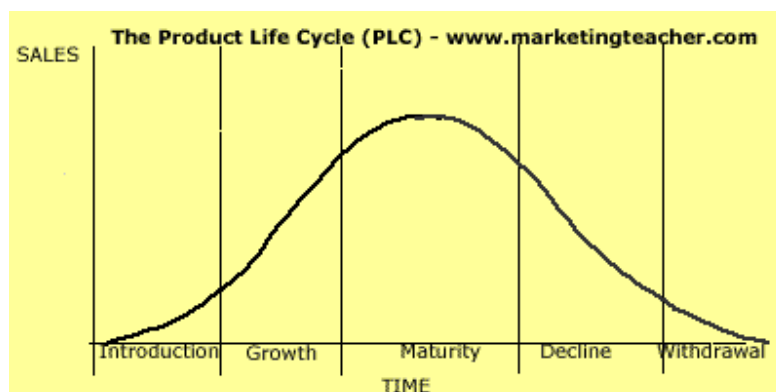


Figure 1. Product life cycle

(product class) down to different product models. The characteristics of the life cycle and its effects on the reverse supply chain have been discussed by Tibben-Lembke [14] without any discussion on the effects on remanufacturing operation. When the historical sales data is known, the data can be used as a basis for forecasting when the products are likely to be returned. Van der Laan [16] presented the robustness of the control parameters of the PUSH and PULL disposal strategies over the different stages of a product life cycle. Umeda et al. [15] presented a model based on empirical data from return rates for remanufacturing of a single use camera and the remanufacturing of a photo copier. In the model, a simple normal distribution function showed to offer sufficient results in predicting returns when using average life as an indicator for timing of returns. The distribution of disposed products $S(t)$ was calculated as the historical sales data $D(t)$ over a limited time frame $D(\tau)\Delta\tau$, distributed as a normal distribution function with a standard deviation (σ) after an average usage time μ . Seo et al. [12] studied an approximate method for computing the preliminary life cycle cost. Learning algorithms trained to use the known characteristics of existing products can perhaps allow the life cycle cost of new products to be approximated quickly during the conceptual design phase without the overheads of defining new product life cycle cost models. In [3], Chi-Yo Huang and Tzeng proposed a forecast methodology for predicting both product life time and nonlinear product life cycle based upon a two-stage, fuzzy, piecewise regression analysis model. Different from the traditional time-based forecast methodology, a generation-based approach was applied, which predicted product life cycle by using the annual fuzzy regression lines, based upon the annual shipments of earlier generation products. Che-Fu Hsueh [2] investigated inventory control policies in a manufacturing system during the product life cycle, and derived the closed-

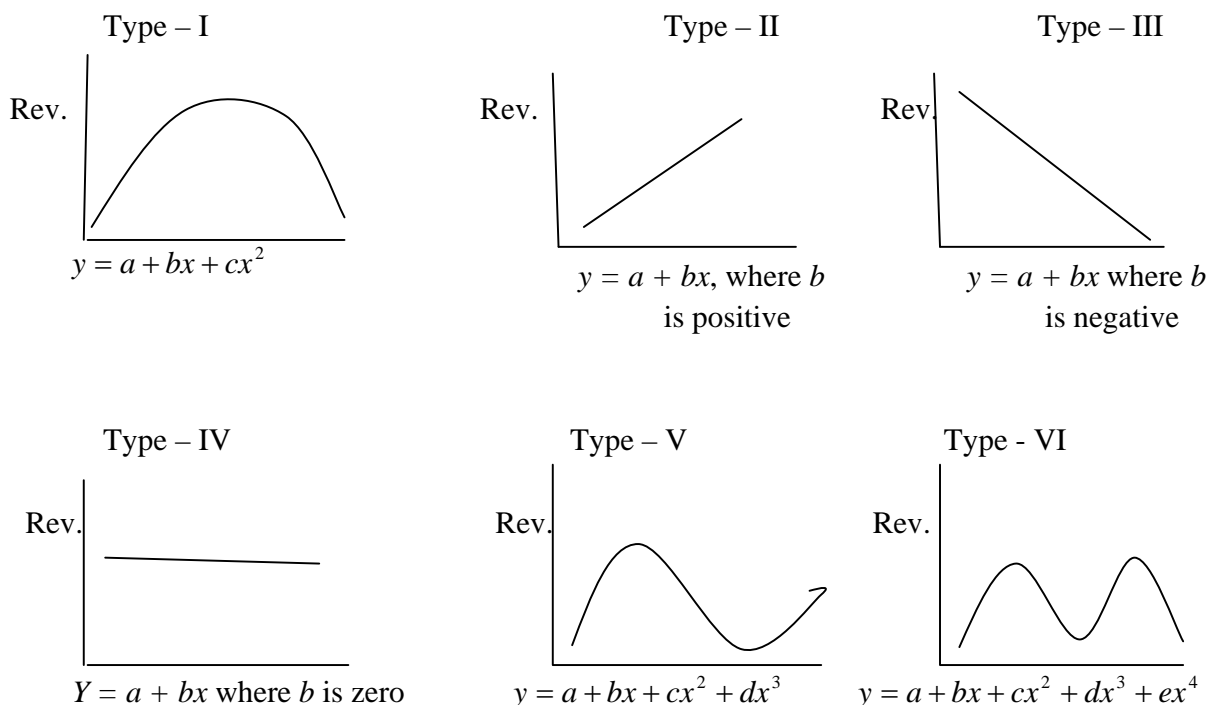


Figure 2. Six Types of Product Life Cycle for Ethical Drugs

form formulas of optimal production lot size, reorder point and safety stock in each phase of product life cycle. Ostlin et al. [11] studied strategies to balance supply and demand for remanufactured product life cycle; This does not present a clear inventory control policy. Krishnamoorthi [5] presented the optimum production lot size and total cost during the product life cycle consisting of introduction, growth, maturity and decline stages with the defective rate considered as a variable of known proportions. Krishnamoorthi [6] developed an optimal production lot size and total cost during the product life cycle consisting of introduction, growth, maturity and decline stages with defective rate considered as a variable of known proportions and shortages permitted. Krishnamoorthi [7] investigated the effect of quality cost on inventory control policies in a manufacturing system during the product life cycle consisting of introduction, growth and decline stages with the defective rate considered as a variable of known proportions. In all inventory models, a general assumption is that products have indefinitely long lives. However, almost all items deteriorate over time. But, often the rate of deterioration is low and there is little need to consider the deterioration in the determination of economic lot size. Here, we consider a dynamic inventory model with deteriorating production in which each of the production, the demand and the deterioration rate as well as all cost parameters are assumed to be general functions of time. The objective is to derive the cycle time and optimal production lot size minimizing total costs for the product life cycle with growth stage and deteriorating items. The remainder of our work is organized as follows. Section 2 presents the assumptions and notations. Section 3 is for problem formulation and a numerical example. Finally, we summarize and conclude in Section 4.

2. Assumptions and Notations

Assumptions: The assumption of an inventory model for product life cycle is as follows:

- (i) The demand rate is known, constant and continuous.
- (ii) Items are produced and added to the inventory.
- (iii) The item is a single product; it does not interact with any other inventory item.
- (iv) The production rate is always greater than or equal to the sum of the demand rate.
- (v) Backlogging is not permitted.
- (vi) Setup time for rework process is zero.
- (vii) The other assumptions in classical EPQ model also hold.

Notations: the notations being used in the inventory model are:

- (1) P – Production rate (per unit time)
- (2) D – Demand rate (per unit time)
- (3) θ – Rate of deteriorating items
- (4) Q_1 – On hand inventory level at time T_1
- (5) Q_2 – On hand inventory level at time T_2
- (6) Q_3 – On hand inventory level at time T_3
- (7) Q^* – Optimal size of production run
- (8) C_p – Production cost per unit
- (9) C_h – Holding cost per unit/per unit time
- (10) C_s – Setup cost per setup

- (11) T – Cycle time
- (12) T_i – Unit time in periods i ($i = 1, 2, 3$)
- (13) TC – Total cost.

3. Mathematical Model

In introduction stage, products have to be carefully monitored to ensure that they start to grow. Substantial research and development costs may be high in order to test the market, undergo launch promotion and setup distribution channels. The cycle start at $t=0$. In this stage, inventory is increasing at the rate of P and simultaneously decreasing at the rate of D . Thus, inventory accumulates at the rate of $P - D$ units. Therefore, the maximum inventory level shall be equal to $(P - D)t_1$. In growth stage, more customers become aware of the product and its benefits and additional market segments are targeted. The growth stage is characterized by rapid growth in sales and profits. Profits arise due to an increase in output and possibly better prices. When the product enters growth stage at T_2 , production and demand increase at the rate of “a” times $(P - D)$, i.e., $a(P - D)$, where “a” is a constant. In maturity stage, sales growth continuous and a company achieving its market share goals enjoys the most profitable period. Production and demand increase at the rate of “b” times $(P - D)$, i.e., $b(P - D)$, where “b” is a constant. In decline stage, the market is shrinking, reducing the overall amount of profit that can be shared amongst the remaining competitors. The product becomes technologically obsolete or the customer tastes change. Care should be taken to control the amount of stocks of the product. The inventory level starts to decrease due to demand at a rate D up to time T_3 . In shortage period, shortages starts to accumulate at a rate of B , the inventory level is zero at time t_5 , but shortages accumulate at a rate of D up to time t_5 . Therefore, time t_5 needed to build-up B units of time. The production restarts again at time T at a rate of $P - D$ to

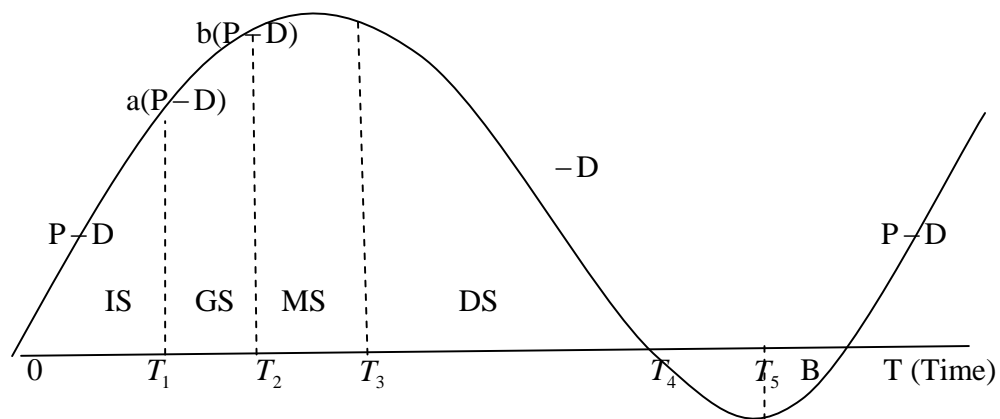


Figure 3. On-hand Inventory Level in Product Life Cycle with Deteriorating Items
 (IS- Introduction Stage, GS- Growth Stage, MS-Maturity Stage, DS-Decline Stage)

recover both the previous shortages in the period t_5 and to satisfy demand in the period T . Time T is needed to consume all units at demand rate Q . The process is repeated. The variation of the underlying inventory system for one cycle is shown in Fig. 3. The production rate of good items is always greater than or equal to demand rate. So, we must have $P \geq D$.

Let $I(t)$ denote the inventory level of the system at time T . The differential equations describing the system in the interval $(0, T)$ are given by:

$$\frac{dI(t)}{dt} + \theta I(t) = P - D; \quad 0 \leq t \leq T_1 \quad . \quad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = a(P - D); \quad T_1 \leq t \leq T_2 \quad . \quad (2)$$

$$\frac{dI(t)}{dt} + \theta I(t) = b(P - D); \quad T_2 \leq t \leq T_3 \quad . \quad (3)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D; \quad T_3 \leq t \leq T_4 \quad . \quad (4)$$

$$\frac{dI(t)}{dt} = -D; \quad T_4 \leq t \leq T_5 \quad . \quad (5)$$

$$\frac{dI(t)}{dt} = -(P - D); \quad T_5 \leq t \leq T \quad . \quad (6)$$

The boundary conditions are:

$$I(0) = 0; I(T_1) = Q_1; I(T_2) = Q_2, I(T_3) = Q_3, I(T_4) = 0; I(T_5) = B \text{ and } I(T) = 0. \quad (7)$$

The solutions of the above equations are:

$$\text{From equation (1), } I(t) = \frac{P - D}{\theta} [1 - e^{-\theta t}]; \quad 0 \leq t \leq T_1 \quad . \quad (8)$$

$$\text{From equation (2), } I(t) = \frac{a(P - D)}{\theta} (1 - e^{-\theta t}). \quad (9)$$

$$\text{From equation (3), } I(t) = \frac{b(P - D)}{\theta} (1 - e^{-\theta t}). \quad (10)$$

$$\text{From equation (4), } I(t) = \frac{D}{\theta} (e^{\theta(T_4 - t)} - 1) \quad (11)$$

$$\text{From equation (5), } I(t) = -D(T_4 - t). \quad (12)$$

$$\text{From equation (6), } I(t) = (P - D)(T - t). \quad (13)$$

Maximum Inventory Q_1 : The maximum inventory during time T_1 is calculated as follows: From equations (7) and (8), $I(T_1) = Q_1 \Rightarrow \frac{P-D}{\theta}(1 - e^{-\theta T_1}) = Q_1$.

In order to facilitate analysis, we do an asymptotic analysis for $I(t)$. Expanding the exponential functions and neglecting second and higher powers of θ for small value of θ , we have

$$Q_1 = (P - D)T_1. \quad (14)$$

Maximum Inventory Q_2 : The maximum inventory during time T_2 is calculated as follows: From equations (7) and (9), $I(T_2) = Q_2 \Rightarrow \frac{a(P-D)}{\theta}(1 - e^{-\theta T_2}) = Q_2$.

In order to facilitate analysis, we do an asymptotic analysis for $I(t)$. Expanding the exponential functions and neglecting second and higher powers of θ for small value of θ , we have

$$Q_2 = a(P - D)T_2, \quad (15)$$

Maximum Inventory Q_3 : The maximum inventory during time T_3 is calculated as follows: From equations (7) and (10), $I(T_3) = Q_3 \Rightarrow \frac{P-D}{\theta}(1 - e^{-\theta T_3}) = Q_3$.

In order to facilitate analysis, we do an asymptotic analysis for $I(t)$. Expanding the exponential functions and neglecting second and higher power of θ for small value of θ , we have

$$Q_3 = (P - D)T_3. \quad (16)$$

Total Cost: The total cost comprises of the sum of the production cost, ordering cost, holding cost and deteriorating cost. They are grouped together after evaluating the above costs individually:

$$(i) \text{ Production cost per unit time} = DC_p. \quad (17)$$

$$(ii) \text{ Ordering cost per unit time} = \frac{C_0}{T}. \quad (18)$$

$$\begin{aligned} (iii) \text{ Holding cost per unit time} &= \frac{C_h}{T} \left[\int_0^{T_1} I(t)dt + \int_{T_1}^{T_2} I(t)dt + \int_{T_2}^{T_3} I(t)dt + \int_{T_3}^{T_4} I(t)dt \right] \\ &= \frac{C_h}{T} \left[\int_0^{T_1} \frac{P-D}{\theta}(1 - e^{-\theta t})dt + \int_{T_1}^{T_2} \frac{a(P-D)}{\theta}(1 - e^{-\theta t})dt + \int_{T_2}^{T_3} \frac{b(P-D)}{\theta}(1 - e^{-\theta t})dt + \int_{T_3}^{T_4} \frac{D}{\theta}(e^{\theta(T-t)} - 1)dt \right] \\ &= \frac{C_h}{T} \left[\frac{P-D}{\theta} \left(t - \frac{e^{-\theta t}}{-\theta} \right)_0^{T_1} + \frac{a(P-D)}{\theta} \left(t - \frac{e^{-\theta t}}{-\theta} \right)_{T_1}^{T_2} + \frac{b(P-D)}{\theta} \left(t - \frac{e^{-\theta t}}{-\theta} \right)_{T_2}^{T_3} + \frac{D}{\theta} \left(\frac{e^{\theta(T-t)}}{-\theta} - t \right)_{T_3}^{T_4} \right] \end{aligned}$$

$$= \frac{C_h}{T} \left[\frac{P-D}{\theta^2} (\theta T_1 + e^{-\theta T_1} - 1) + \frac{a(P-D)}{\theta^2} (\theta(T_2 - T_1) + e^{-\theta T_2} - e^{-\theta T_1}) \right. \\ \left. + \frac{b(P-D)}{\theta^2} (\theta(T_3 - T_2) + e^{-\theta T_3} - e^{-\theta T_2}) - \frac{D}{\theta^2} (1 - e^{\theta(T_4 - T_3)} + \theta(T_4 - T_3)) \right].$$

Expanding the exponential functions and neglecting second and higher powers of θ for small value of θ , we have

$$\text{Holding cost per unit time} = \frac{C_h}{T} \left[\frac{P-D}{\theta^2} \left\{ \frac{\theta^2 T_1^2}{2} \right\} + \frac{a(P-D)}{\theta^2} \left(\frac{\theta^2}{2} (T_2^2 - T_1^2) \right) \right. \\ \left. + \frac{b(P-D)}{\theta^2} \left(\frac{\theta^2 (T_3^2 - T_2^2)}{2} \right) + \frac{D}{\theta^2} \left\{ \frac{\theta^2 (T_4 - T_3)^2}{2} \right\} \right] \\ = \frac{C_h}{2T} [(P-D)T_1^2 + a(P-D)(T_2^2 - T_1^2) + b(P-D)(T_3^2 - T_2^2) + D(T_4 - T_3)^2]. \quad (19)$$

(iv) Deteriorating cost per unit time:

$$\text{Deteriorating cost} = \frac{\theta C_P}{T} \left[\int_0^{T_1} I(t) dt + \int_{T_1}^{T_2} I(t) dt + \int_{T_2}^{T_3} I(t) dt + \int_{T_3}^{T_4} I(t) dt \right] \\ = \frac{\theta C_P}{T} \left[\int_0^{T_1} \frac{P-D}{\theta} (1 - e^{-\theta t}) dt + \int_{T_1}^{T_2} \frac{a(P-D)}{\theta} (1 - e^{-\theta t}) dt + \int_{T_2}^{T_3} \frac{b(P-D)}{\theta} (1 - e^{-\theta t}) dt + \int_{T_3}^{T_4} \frac{D}{\theta} (e^{\theta(T-t)} - 1) dt \right].$$

Expanding the exponential functions and neglecting second and higher powers of θ , for small value of θ , we have

$$= \frac{\theta C_P}{2T} [(P-D)T_1^2 + a(P-D)(T_2^2 - T_1^2) + b(P-D)(T_3^2 - T_2^2) + D(T_4 - T_3)^2]. \quad (20)$$

$$\text{(v) Shortage cost} = \frac{C_S}{T} \left[\int_{T_4}^{T_5} I(t) dt + \int_{T_5}^T I(t) dt \right] \\ = \frac{C_S}{T} \left[\int_{T_4}^{T_5} D(t - T_4) dt + \int_{T_5}^T (P-D)(T-t) dt \right] \\ = \frac{C_S}{2T} [D(T_5 - T_4)^2 + (P-D)(T - T_5)^2] \\ = \frac{C_S}{2T} \left[D \left(\frac{P-D}{P} T - \frac{P-D}{P} T_4 \right)^2 + \frac{D(P-D)}{P} (T - T_4)^2 \right] \\ = \frac{C_S}{2T} \left[\frac{D(P-D)}{P} (T - T_4)^2 + \frac{D(P-D)}{P} (T - T_4)^2 \right] \\ = \frac{D(P-D)C_S}{TP} (T - T_4)^2. \quad (21)$$

From equations (12) and (13),

$$I(T_5) = B \Rightarrow -D(T_4 - T_5) = B, \text{ that is, } D(T_5 - T_4) = B$$

$$I(T_5) = B \Rightarrow (P - D)(T - T_5) = B, \text{ that is, } (P - D)(T - T_5) = B$$

$$(P - D)(T - T_5) = D(T_5 - T_4).$$

Therefore,

$$T = \frac{P}{P - D}T_5 - \frac{D}{P - D}T_4 \text{ and } T_5 = \frac{P - D}{P}T + \frac{D}{P}T_4, \quad (22)$$

TC = Purchase cost + Ordering cost + (Holding cost + Deteriorating cost)

$$= DC_p + \frac{C_0}{T} + \frac{(C_h + \theta C_p)}{2T} \left[(P - D)T_1^2 + a(P - D)(T_2^2 - T_1^2) \right. \\ \left. + b(P - D)(T_3^2 - T_2^2) + D(T_4 - T_3)^2 \right] + \frac{D(P - D)C_s}{TP} (T - T_4)^2. \quad (23)$$

Let $T_1 = \alpha T_3$, $T_2 = \beta T_3$ and $T_3 = \gamma T_4$. (24)

Therefore,

TC =

$$DC_p + \frac{C_0}{T} + \frac{(C_h + \theta C_p)}{2T} \left[(P - D)\alpha^2 T_4^2 + a(P - D)(\beta^2 - \alpha^2)T_4^2 \right. \\ \left. + b(P - D)(\gamma^2 - \beta^2)T_4^2 + D(1 - \gamma)^2 T_4^2 \right] + \frac{D(P - D)C_s}{TP} (T - T_4)^2. \quad (25)$$

Partially differentiate the total cost (25) with respect to T_4 :

$$\frac{\partial}{\partial T_4} (TC) = \frac{(C_h + \theta C_p)T_4}{T} \left[(P - D)\alpha^2 + a(P - D)(\beta^2 - \alpha^2) \right. \\ \left. + b(P - D)(\gamma^2 - \beta^2) + D(1 - \gamma)^2 \right] - \frac{2D(P - D)C_s}{TP} (T - T_4) = 0.$$

$$\frac{\partial^2}{\partial T_4^2} (TC) = \frac{C_h + \theta C_p}{T} \left[(P - D)\alpha^2 + a(P - D)(\beta^2 - \alpha^2) \right. \\ \left. + b(P - D)(\gamma^2 - \beta^2) + D(1 - \gamma)^2 \right] + \frac{2D(P - D)C_s}{TP} > 0.$$

With simplifications,

$$T_4 = \frac{2D(P - D)C_s T}{P(C_h + \theta C_p) \left\{ (P - D)\alpha^2 + a(P - D)(\beta^2 - \alpha^2) \right\} + 2D(P - D)C_s \\ + b(P - D)(\gamma^2 - \beta^2) + D(1 - \gamma)^2}. \quad (26)$$

Let us assume $A = (P - D)\alpha^2 + a(P - D)(\beta^2 - \alpha^2) + b(P - D)(\gamma^2 - \beta^2) + D(1 - \gamma)^2$.

Therefore,

$$T_4 = \frac{2D(P - D)C_s T}{P(C_h + \theta C_p)A + 2D(P - D)C_s},$$

$$\text{Total cost} = DC_p + \frac{C_0}{T} + \frac{(C_h + \theta C_p)}{2T} A + \frac{D(P - D)C_s}{TP} (T - T_4)^2.$$

Partially differentiate the total cost (24) with respect to T :

$$\frac{\partial}{\partial T} = \frac{-C_0}{T^2} - \frac{(C_h + \theta C_p)T_4^2 A}{2T^2} + \frac{D(P - D)C_s (T^2 - T_4^2)}{PT^2} = 0$$

$$\frac{\partial^2}{\partial T^2} = \frac{2C_0}{T^3} + \frac{(C_h + \theta C_p)T_4^2 A}{T^3} + \frac{D(P - D)C_s (T^2 - T_4^2)}{T} > 0$$

$$2D(P - D)C_s T^2 = 2PC_0 + P(C_h + \theta C_p)AT_4^2 + 2D(P - D)C_s T_4^2$$

$$T^2 \left[2D(P - D)C_s - \frac{4D^2(P - D)^2 C_s^2}{P(C_h + \theta C_p)A + 2D(P - D)C_s} \right] = 2PC_0,$$

$$T^2 = \frac{C_0 [2D(P-D)C_S + P(C_h + \theta C_p)A]}{(C_h + \theta C_p)D(P-D)C_S A}$$

Therefore,

$$T = \sqrt{\frac{C_0 [2D(P-D)C_S + P(C_h + \theta C_p)A]}{(C_h + \theta C_p)D(P-D)C_S A}} \quad (27)$$

Note: If $T = \frac{Q}{D}$ then $Q = TD$. (28)

Numerical example: Let us consider the cost parameters as follows:

$P = 5000$ units, $D = 4500$ units, $C_h = 10$, $C_p = 100$, $C_0 = 100$, $\theta = 0.01$ to 0.10 , $a = 2$, $b = 3$, $\alpha = 0.8$,

$\beta = 0.9$, $\gamma = 0.9$

Optimum solution:

Cycle times: $T = 0.2200$, $T_1 = 0.0832$, $T_2 = 0.0951$, $T_3 = 0.1070$, $T_4 = 0.1189$, $T_5 = 0.1290$,

Optimum Quantity $Q^* = 989.83$, $Q_1 = 41.62$, $Q_2 = 95.15$, $Q_3 = 160.56$, $B = 45.46$,

Production cost = 450,000, Setup cost = 454.62, Holding cost = 223.47, Shortage cost = 208.81,

Deteriorating cost = 22.35, Total cost = 450909.25.

In Table 1, looking at rate of deteriorating items and optimum quantity and cycle time T , it is concluded that when the rate of deteriorating items increases, the optimum quantity and cycle time decrease. Also from of rate of deteriorating item with setup cost, holding cost, deteriorating cost, shortage cost and total cost, it is concluded that when the rate of deteriorating items increases, the holding cost decreases but setup cost, deteriorating cost, shortage cost and total cost increase.

Sensitivity Analysis: The total cost functions are the real solutions for which the model parameters are assumed to be static values. It is reasonable to study the sensitivity, i.e., the effect of making changes in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analysis of various system parameters is required to observe whether the current solutions remain unchanged, the current solutions become infeasible, etc.

Table 1. Variation of Rate of Deteriorating Items with Inventory and Total Cost

θ	T	Q	Product-ion cost	Setup cost	Holding cost	Deterio-rating cost	Shortage cost	Total cost
0.01	0.2200	989.83	450000	454.62	223.47	22.35	208.81	450909.25
0.02	0.2149	967.27	450000	465.23	201.22	40.24	223.76	450930.45
0.03	0.2106	947.76	450000	474.80	182.26	54.68	237.86	450949.60
0.04	0.2068	930.72	450000	483.50	165.95	66.38	251.17	450966.99
0.05	0.2035	915.69	450000	491.43	151.79	75.90	263.74	450982.86
0.06	0.2005	902.34	450000	498.71	139.42	83.65	275.63	450997.41
0.07	0.1979	890.38	450000	505.40	128.54	89.98	286.87	451010.80
0.08	0.1955	879.63	450000	511.58	118.92	95.13	297.53	451023.16
0.09	0.1933	869.89	450000	517.31	110.35	99.32	307.64	451034.61

Table 2. Effect of Demand and Cost Parameters on Optimal Policies

Parameters		Optimum values									Total cost
		T	Q	T_1	T_2	T_3	Q_1	Q_2	Q_3	B	
θ	0.01	0.2200	989.83	0.0832	0.0951	0.1070	41.63	95.15	160.56	45.46	450909.25
	0.02	0.2149	967.27	0.0781	0.0893	0.1004	39.05	89.25	150.61	46.52	450930.46
	0.03	0.2106	947.76	0.0736	0.0841	0.0946	36.79	84.08	141.89	47.48	450949.60
	0.04	0.2068	930.72	0.0696	0.0795	0.0894	34.78	79.51	134.17	48.35	450966.99
	0.05	0.2035	915.69	0.0659	0.0754	0.0848	33.00	75.42	127.28	49.14	450982.86
C_0	80	0.1967	885.33	0.0745	0.0851	0.0957	37.23	85.10	143.61	40.66	450813.26
	90	0.2087	939.03	0.0789	0.0903	0.1015	39.49	90.26	152.32	43.13	450862.59
	100	0.2200	989.83	0.0832	0.0951	0.1070	41.63	95.15	160.56	45.46	450909.25
	110	0.2307	1038.14	0.0873	0.0998	0.1123	43.66	99.79	168.40	47.68	450953.63
	120	0.2409	1084.30	0.0912	0.1042	0.1173	45.60	104.23	175.89	49.80	450996.03
C_h	8	0.2328	1047.61	0.0961	0.1099	0.1236	48.07	109.88	185.42	42.96	450859.10
	9	0.2258	1016.23	0.0892	0.1019	0.1147	44.60	101.94	172.03	44.28	450885.62
	10	0.2200	989.83	0.0832	0.0951	0.1070	41.63	95.15	160.56	45.46	450909.25
	11	0.2149	967.27	0.0781	0.0891	0.1004	39.05	89.25	150.61	46.52	450930.45
	12	0.2106	947.76	0.0735	0.0841	0.0946	36.79	84.08	141.89	47.48	450949.60
C_p	80	0.2211	994.77	0.0844	0.0964	0.1085	42.19	96.43	162.72	45.24	360904.73
	90	0.2205	992.28	0.0838	0.0958	0.1078	41.90	95.78	161.63	45.35	405907.00
	100	0.2200	989.83	0.0832	0.0951	0.1070	41.63	95.15	160.56	45.46	450909.25
	110	0.2194	987.41	0.0827	0.0945	0.1063	41.35	94.52	159.50	45.57	495911.47
	120	0.2189	985.04	0.0822	0.0939	0.1056	41.08	93.90	158.64	45.68	540913.67
C_s	8	0.2322	1045.11	0.0788	0.0901	0.1014	39.42	90.11	152.07	53.82	450861.15
	9	0.2255	1014.77	0.0121	0.0928	0.1044	40.60	92.81	156.61	49.27	450886.90
	10	0.2200	989.83	0.0832	0.0951	0.1070	41.63	95.15	160.56	45.46	450909.25
	11	0.2153	968.94	0.0850	0.0972	0.1093	42.52	97.20	164.02	42.22	450928.85
	12	0.2114	951.19	0.0866	0.0990	0.1114	43.32	99.01	167.08	39.42	450946.19

Observations:

1. With the increase in rate of deteriorating items (θ), total cost increases but cycle time, optimum quantity, cycles times (T, T_1, T_2, T_3) and optimum quantity, buffer stock and maximum inventory (Q_1, Q_2, Q_3) decrease.
2. With the increase in setup cost per unit (C_0), optimum quantity (Q^*), maximum inventory Q_1, Q_2 and Q_3 , cycle times (T, T_1, T_2, T_3), buffer stock and total cost increase.
3. With the increase in holding cost per unit (C_h), optimum quantity (Q^*), maximum inventory Q_1, Q_2 and Q_3 , cycle times (T, T_1, T_2, T_3) decrease but total cost increases.
4. Similarly, other cost parameters, production cost and shortage cost can also be observed from Table 2.

Special Cases:

If the production system is considered to be ideal, that is, no deterioration is assumed, the value of θ is set to zero. In that case, equations (13) and (14) reduce to the classical economic production quantity model as follows:

$$TC = DC_p + \frac{C_0}{T} + \frac{C_h}{2T} \left[(P-D)T_1^2 + a(P-D)(T_2^2 - T_1^2) + b(P-D)(T_3^2 - T_2^2) + D(T_4 - T_3)^2 \right] + \frac{D(P-D)C_s}{TP} (T - T_4)^2.$$

Therefore,

$$T_4 = \frac{2D(P-D)C_s T}{PAC_h + 2D(P-D)C_s},$$

and

$$T = \sqrt{\frac{C_0 [2D(P-D)C_s + PC_h A]}{C_h D(P-D)C_s A}}.$$

Optimum solution:

Cycle times: $T = 0.2258, T_1 = 0.0892, T_2 = 0.1019, T_3 = 0.1147, T_4 = 0.1274, T_5 = 0.1373,$

Optimum Quantity $Q^* = 1016.10, Q_1 = 44.60, Q_2 = 101.94, Q_3 = 172.03, B = 44.28,$

Production cost = 450,000, Setup cost = 442.81, Holding cost = 249.86,

Shortage cost = 192.95, Total cost = 450885.62.

5. Conclusions

In all inventory models, a general assumption is that products have indefinitely long lives. In general, almost all items deteriorate over time. Often, the rate of deterioration is low and there is little need to consider the deterioration in determination of economic lot size. Here, a dynamic inventory model was considered with deteriorating production in which each of the production, the demand and the deterioration rates as well as all cost parameters were assumed to be general functions of time. The objective was to determine cycle time and optimal production lot size to minimize total cost for the product life cycle with growth stage and deteriorating items. The relevant model was built, solved and some main results about the uniqueness of the solution with the use of rigorous mathematical methods were obtained. Illustrative examples were provided. The needed programs were coded in Microsoft Visual Basic 6.0. Our work can be extended as follows:

- (a) Most production systems today are multi-stage systems, where the defective items and scrap can be produced in each stage. The defectives and scrap proportion for a multi-stage system can be different in different stages. Taking these factors into consideration, our work may be extended to a multi-stage production process.
- (b) Traditionally, inspection procedure's incurring cost is an important factor to identify the defectives in order to scrap and remove them from the finished goods inventory. For better production, the placement and effectiveness of inspection procedures are required. So, inspection cost can be included in developing extended models.
- (c) The demand of a product may decrease with time owing to the introduction of a new product which is either technically superior or more attractive or cheaper. On the other hand, the demand

for new product increases. Therefore, demand rate may be varied with time, and thus variable demand rate can be used to develop a new model.

The proposed models can assist the manufacturer and retailer in accurately determining the optimal quantity, cycle time and the inventory total cost. Moreover, the proposed inventory model can be used in inventory control of certain items such as food items, fashionable commodities, stationary stores and others.

References

- [1] Buzzell, R.D. (1972), *Marketing a Contemporary Analysis*, 2nd Edition, McGraw Hill, New York, pp. 374.
- [2] Che-Fu Hsuch (2010), Inventory control for stochastic reverse logistics with product life cycle, *Proceedings of the world congress on Engineering*, London, Vol.III, UK.
- [3] Chi-Yo Huang and Tzeng, G.H. (2008), Multiple generation product life cycle predictions using a novel two-stage fuzzy piecewise regression analysis method, *Technological Forecasting and Social Change*, 75, 12-31.
- [4] Cox, Jr., W.E. (1967), Product life cycles as marketing models, *Journal of Business*, 40, 375-384.
- [5] Krishnamoorthi, C. (2012), An inventory model for product life cycle with maturity stage and defective items, *Opsearch*, 49(3), 209-222.
- [6] Krishnamoorthi, C. (2012), An economic production lot size model for product life cycle (maturity stage) with defective items and shortages, *Opsearch*, 49(3), 253-270.
- [7] Krishnamoorthi, C. (2011), The effect of quality cost on inventory model for product life cycle with growth stage and defective items, *International Journal of Pure and Applied Mathematics*, 71, 1-17.
- [8] Kotler, P. (1967), *Marketing Management: Analysis, Planning and Control*, First Ed., Prentice Hall, USA.
- [9] Kovac, F.J. and Dague, M.F. (1972), Forecasting by product life cycle analysis, *Research Management*, 15, 66-72.
- [10] Levitt, T. (1965), Exploit the product life cycle, *Harvard Business Review*, 43, 81-94.
- [11] Ostlin, J., Sundin, E. and Bhorkman, M. (2009), Product life cycle implications for manufacturing strategies, *Journal of Cleaner Production*, 17, 999-1009.
- [12] Seo, K.K., Park, J.H., Jang, D.S. and Wallace, D. (2002), Approximate estimation of the product life cycle cost using artificial neural networks in conceptual design, *The International Journal of Advanced Manufacturing Technology*, 19, 461-471.
- [13] Tellis, G. and Crawford, C.M. (1981), An evolutionary approach to growth theory, *Journal of marketing*, 45(4), 125-132.
- [14] Tibben-Lembke, R.S. (2002), Life after death: reverse logistics and the product life cycle, *International Journal of Physical Distribution & Logistics Management*, 32(3), 223-244.
- [15] Umeda, U., Kondoh, S. and Takashi, S. (2005), Proposal of marginal reuse rate for evaluating reusability of products, *International Conference on Engineering Design*, Melbourne, pp. 15-18.
- [16] Van der Laan, E. and Salomon, M. (1997), Production planning and inventory control with remanufacturing and disposal, *European Journal of Operational Research*, 102, 264-278.