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A New Control Chart to Monitor Mean Shifts of Bi-Variate Quality Control Processes

Seyed Taghi Akhavan Niaki[1]
Mohammad Saber Fallah Nezhad[2]

We propose a new approach to monitor an overall mean shift of a bi-variate quality control system. To do this, we first define beliefs on deciding whether the quality characteristics are in out-of-control state. Then, by taking new observations, in an iterative approach we update the belief of each quality characteristics being out-of-control. This task is performed using a recursive method and prior beliefs. Finally, we introduce a statistics in combination with Bivariate Exponentially Weighted Moving Average (BEWMA) statistics to improve the performance of the proposed method. In order to understand the proposed methodology and to evaluate its performance, we perform a simulation study. Moreover, we compare in- and out-of-control average run lengths of the proposed method with the ones from the well-known MCUSUM and MEWMA procedures in different scenarios of mean shifts. The results of the simulation study show that the proposed methodology performs better than the other methods for small shifts of the process mean.

Keywords: *Multivariate statistical quality control; MCUSUM; MEWMA; Average run length.*

1. Introduction and Literature Review

In many situations, the quality of the process can be characterized by a single continuous random variable, which is usually assumed to follow a normal distribution.

[1] Corresponding author, Sharif University of Technology, Department of Industrial Engineering, Tehran Iran. Email: Niaki@Sharif.edu
[2] Department of Industrial Engineering, Sharif University of Technology, Tehran Iran.
Email: Fallahnezhad@mehr.sharif.edu

However, it is increasingly common for processes to be characterized by several, usually correlated, variables [8].

Multivariate control charts are widely used to monitor industrial processes [12]. As the objective of performing multivariate statistical process control is to monitor the process over time, in order to detect any unusual events allowing quality and process improvement, it is essential to track the cause of an out-of-control signal. However, as opposed to univariate control charts, the complexity of multivariate control charts and the cross-correlation among variables make it difficult for analysis of assignable causes to the out-of-control signal. This is the basis for extensive research performed in the field of multivariate control chart since the 1940's, when Hotteling [6] recognized that the quality of a product might depend on several correlated characteristics. However, because of computational complexity, researchers and practitioners did not pursue the multivariate quality control at that time. Now that the development of high-speed computers, the technological advances in industrial control procedures, and the availability of modern data-acquisition equipments have alleviated this problem, many researchers have proposed several multivariate control charts, where each has advantages as well as disadvantages [14].

Early research on multivariate Shewhart charts goes back to Hotelling [6], where he introduced the problem of correlation between the quality characteristics of a process and came up with the well-known T^2 statistic to identify whether the whole process is out-of-control. A major advantage of Hotelling's T^2 statistic is that it is the optimal test statistic for detecting a general shift in the process mean vector for an individual multivariate normal observation [4]. However, the technique has several practical drawbacks. One of the most important ones is that when the T^2 statistic indicates that a process is out of control, it does not provide information on which variable or set of variables is out of control. Moreover, it is difficult to distinguish location shifts from scale shifts, since the T^2 statistic is sensitive to both types of process changes.

Murphy [15] proposed a method to identify the "out-of-control" variables based on discriminant analysis. We can view this quality control method as trying to discriminate between the process of being "in control" or "out-of-control". He divided the complete set of variables into two subsets and then tried to determine which one caused the "out-of-control" signal. Extensions of the Murphy's work are Niaki and Moeinzadeh [17] and Niaki et al. [18], where they developed a statistic and an algorithm for the cause-selecting problem in which the population parameters were not known and were to be estimated.

The principal components analysis is a way of explaining the variance-covariance structure in a multivariate environment by the use of a few linear combinations of the original variables. Jackson [7] gave a detailed description of principal components and its possible use as a multivariate quality control tool. The problem with principal components is that they are not easily interpretable in many cases, and do not have a one-to-one relation with the original variables. Nevertheless, in some cases, depending on the context, they can be very useful.

Doganoskoy et al. [3] proposed the use of the univariate t-statistic for ranking the variables most likely to have changed. Then, to further strengthen the belief that a certain variable has changed, they applied the Bonferroni type interval. The obvious drawback of this method is that it only tells you which variable is most likely to have shifted, which is not conclusive. Also, this method does not allow the user to study the trends.

Mason et al. [12] proposed a cause-selecting procedure using the decomposition of T^2 statistic. By decomposing T^2 statistic, the user can see the contribution of each variable. This decomposition also allows the user to detect which variable(s) with significant contribution is (are) the cause of deviation. The drawback of this method is its extensive computing needs and its sensitivity to the number of variables.

Multivariate Exponential Weighted Moving Average (MEWMA) charts have been also discussed by Mohebbi and Lakhbir [13], Ryan [23], Wade and Woodall [25], Crowder [2], Lowry et al. [9], Lucas and Saccucci [10], Prabhu and Runger [22], Hawkins [4], Doganaksoy et al. [3], and Marion and Young [11]. The MEWMA control charts use all the observations since the detection of the last special event rather than only the last observation vector as in the Shewhart-type charts. Their advantage over the latter charts is that their average run length is smaller for small shifts in the process mean. In MEWMA category, Lawry et al. [9] presented a multivariate extension of the exponentially weighted moving average (EWMA) control chart, and compared their chart to a multivariate cumulative sum (MCUSUM) control chart based on the average run length (ARL) performance. They concluded that their chart was similar to the MCUSUM chart in detecting a shift in the mean vector of a multivariate normal distribution, and that the ARL performance of the MEWMA chart, as well as the Hotelling's and MCUSUM charts depended on the underlying mean vector and covariance matrix only through the value of the non-centrality parameter. They stated that in order to avoid the potential inertia problems, one should always use the MEWMA and MCUSUM charts in conjunction with the Hotelling's chart. In order to improve the detection of small shifts in multivariate statistical process control, Prabhu and Runger [22] provided some recommendations in the selection of the parameters of a multivariate exponentially weighted moving average control chart.

The properties of MCUSUM control charts are quite similar to those of the MEWMA charts. In this category, Woodall and Ncube [26] proposed methods to approximate parameters of the distribution of the minimum of the run length of the univariate CUSUM charts. For the bivariate normal distributions, they showed that their MCUSUM method worked better than the Hotelling's T^2 procedure. Healy [5] discussed the natural applications of CUSUM procedures to the multivariate normal distribution. Crosier [1] presented the design procedures and the average run length for two MCUSUM quality control procedures. The first MCUSUM procedures reduced each multivariate observation to a scalar, and then formed a CUSUM of the scalars. The second MCUSUM method formed a CUSUM vector directly from the observations. These two procedures were then compared to a multivariate Shewhart chart and the robustness of the procedures was discussed. Pignatiello and Runger [21] considered several approaches for controlling the mean of a multivariate normal process. They compared the performance of these approaches, as well as the performance of their two newly proposed charts, based on the estimated ARL and reported the results.

The multivariate quality control problem can also be considered as an optimization problem to minimize the total cost. Serel and Moskowitz [24] proposed a method to design joint EWMA control charts for mean and variance. They calculated the quality related production costs using Taguchi's quadratic loss function. Nenes and Tagaras [16] considered the economically Bayesian control charts.

Furthermore, Artificial Neural Networks (ANN) have been other tools to detect out-of-control signals and classify the state of multivariate quality control systems; see for example; Niaki and Abbasi [19], Noorossana et al. [20].

We introduce a new approach to control the mean shifts of quality characteristics in bi-variate environments. To do this, in Section 2, we first define the beliefs and explain how to model a multivariate SPC problem by an iterative approach, where we take advantage of Bayesian inference. Then, in Section 3, we introduce a statistics in combination with MEWMA statistics and clarify the approach by which we improve the beliefs. An illustrative example is given in Section 4 to better understand the proposed methodology. In order to evaluate the performance of the proposed procedure in terms of in and out-of-control average run lengths, we perform some simulation studies in Section 5. Finally, the conclusions and recommendations for future research come in Section 6.

2. Beliefs and the Approach of its Improvement

For the sake of simplicity, we assume only one single observation (n=1) on the quality characteristic of interest in each iteration of the data gathering process. For other values of n, we will reach the same conclusion.

Let x_{ki} be the observation of the ith quality characteristic (variable), $i=1,2$, at iteration k, $k=1,2,\ldots$. Then, at iteration k of the data gathering process, we define the observation vector $\mathbf{x}_k = \left[x_{k1}, x_{k2}\right]$ and observation matrix $\mathbf{O_k} = (\mathbf{x_1}, \mathbf{x_2}, ..., \mathbf{x}_k)$. The decision-making process at any iteration is in a stochastic space such that we never can surely say that the production process is in an out-of-control state. After taking a new observation, $\mathbf{x}_k$, we define the belief of variable i to be in an out-of-control state as $B_i(\mathbf{O}_k) = B_i(\mathbf{x}_k, \mathbf{O}_{k-1})$. We let large and small values of $B_i(\mathbf{x}_k, \mathbf{O}_{k-1})$ be signals of positive and negative shifts in the mean of ith quality characteristic, respectively. We call this statistics the belief of variable i to be in out-of-control condition given the observation matrix up to iteration k-1 and the observation vector at iteration k. At this iteration, we want to improve the belief of being in an out-of-control state based on the observation matrix $\mathbf{O}_{k-1}$ and the new observation vector $\mathbf{x}_k$. Note that the prior belief of variable i is $B_i(\mathbf{x}_{k-1}, \mathbf{O}_{k-2})$ and we want to update it to the posterior belief as $B_i(\mathbf{x}_k, \mathbf{O}_{k-1})$.

Assuming that the quality characteristics of interest follow a bi-normal distribution with mean vector $\boldsymbol{\mu} = \left[\mu_1, \mu_2\right]$ and covariance matrix $\boldsymbol{\sigma}^2 = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$, at different iterations we use equation (3) to calculate the probability of shifts in the process mean $\boldsymbol{\mu}$. However, in order to update the beliefs at iteration *k*, for $j = 1, 2$, we use the decomposition method of Mason et al. [12] and define,

$$T_{k_1} = \left(\frac{x_{k_1} - E(x_{k_1})}{\sigma_1} \right), \tag{1}$$

$$T_{k_{2.1}} = \left(\frac{x_{k_2} - E(x_{k_2} \mid x_{k_1})}{\sigma_{2.1}} \right). \tag{2}$$

Then, in the case of in-control state, both T_{k_1} and $T_{k_{2.1}}$ are independent standard normal random variables.

Based on the values of T_{k_1} and $T_{k_{2.1}}$, the belief-updating process is proposed next.

3. The Proposed Method

For the initial value setting of the beliefs, assume $B_1(\mathbf{O}_0) = B_2(\mathbf{O}_0) = 0.5$. Then, based on equations (1) and (2), we define,

$$B_1(\mathbf{x}_k, \mathbf{O}_{k-1}) = \frac{B_1(\mathbf{x}_{k-1}, \mathbf{O}_{k-2}) e^{T_{k_1}}}{B_1(\mathbf{x}_{k-1}, \mathbf{O}_{k-2}) e^{T_{k_1}} + B_2(\mathbf{x}_{k-1}, \mathbf{O}_{k-2}) e^{T_{k_{2.1}}}}, \tag{3}$$

and

$$B_2(\mathbf{x}_k, \mathbf{O}_{k-1}) = \frac{B_2(\mathbf{x}_{k-1}, \mathbf{O}_{k-2}) e^{T_{k_{2.1}}}}{B_1(\mathbf{x}_{k-1}, \mathbf{O}_{k-2}) e^{T_{k_1}} + B_2(\mathbf{x}_{k-1}, \mathbf{O}_{k-2}) e^{T_{k_{2.1}}}}, \tag{4}$$

Hence,

$$\frac{B_1(\mathbf{x}_k, \mathbf{O}_{k-1})}{B_2(\mathbf{x}_k, \mathbf{O}_{k-1})} = \frac{B_1(\mathbf{x}_{k-1}, \mathbf{O}_{k-2})}{B_2(\mathbf{x}_{k-1}, \mathbf{O}_{k-2})} e^{T_{k_1} - T_{k_{2.1}}}. \tag{5}$$

If we define $M_k = \dfrac{B_1(\mathbf{x}_k, \mathbf{O}_{k-1})}{B_2(\mathbf{x}_k, \mathbf{O}_{k-1})}$, we will then have,

$$M_k = M_{k-1} e^{T_{k_1} - T_{k_{2.1}}} = \ldots = e^{\sum_{l=1}^{k} \left(T_{l_1} - T_{l_{2.1}} \right)}. \tag{6}$$

Since $\sum_{l=1}^{k} \left(T_{l_1} - T_{l_{2.1}} \right)$ follows a normal distribution with mean zero and variance *2k*, we then conclude that $Ln(M_k)$ follows a normal distribution with mean zero and variance *2k*. Hence, we can define the upper and the lower control limits of $Ln(M_k)$ as

$$\begin{aligned} UCL &= C\sqrt{2k}, \\ LCL &= -C\sqrt{2k}, \end{aligned} \tag{7}$$

where C is the upper $(1-\alpha)\%$ percentile of a standard normal distribution.

From the above control limits, we derive the upper and lower control limits for $B_1(\mathbf{x}_k, \mathbf{O}_{k-1})$ as:

$$UCL_{Ln(M_k)} = C\sqrt{2k} \quad \text{and} \quad LCL_{Ln(M_k)} = -C\sqrt{2k}$$
$$\Rightarrow -C\sqrt{2k} \le Ln\left(M_k\right) \le C\sqrt{2k}$$
$$\Rightarrow -C\sqrt{2k} \le Ln\left(\frac{B_1(O_k)}{1-B_1(O_k)}\right) \le C\sqrt{2k} \Rightarrow e^{-C\sqrt{2k}} \le \frac{B_1(O_k)}{1-B_1(O_k)} \le e^{C\sqrt{2k}}$$
$$\Rightarrow e^{-C\sqrt{2k}} + 1 \le \frac{1}{1-B_1(O_k)} \le e^{C\sqrt{2k}} + 1$$
$$\Rightarrow \frac{1}{e^{-C\sqrt{2k}}+1} \ge 1 - B_1(O_k) \ge \frac{1}{e^{C\sqrt{2k}}+1}$$
$$\Rightarrow \frac{e^{-C\sqrt{2k}}}{\left(e^{-C\sqrt{2k}}+1\right)} \le B_1(O_k) \le \frac{e^{C\sqrt{2k}}}{\left(1+e^{C\sqrt{2k}}\right)} \ . \qquad (8)$$

In a trial application of the proposed method, we observed that in situations where there are simultaneous mean shifts in both quality characteristics, the method did not show good performance in term of in-control and out-of-control average run length criteria. Accordingly, we combined the proposed method with the bi-variate EWMA control chart to improve its performance. In other words, we let an out-of-control signal be detected, when we observe one of the following two signals,

- $B_1(O_k)$ is out of the interval $\left[\frac{e^{-C\sqrt{2k}}}{\left(e^{-C\sqrt{2k}}+1\right)}, \frac{e^{C\sqrt{2k}}}{\left(1+e^{C\sqrt{2k}}\right)}\right]$, or
- the MEWMA statistics $\left(T^2 \text{ for } \lambda = 0.1\right)$ is more that a threshold value th_m.

The values of C and th_m should be determined such that the first-type-error associated with the proposed method equals to a predetermined value and also to ensure good properties of the method.

4. An Illustrative Example

In order to better understand the proposed methodology and to provide appropriate insight over the range of the belief values, the belief-outcomes of one simulation run are shown in Figure 1. To obtain the belief values, we first generate pairs of independent uniform random variates (R_{1_i}, R_{2_i}) ; $i = 1, 2, ..., k, k+1, k+2, ...$, and use $Z_i = \sqrt{-2\ln(R_{1i})}\cos(2\pi R_{2i})$ to generate standard normal observations. If we define the quality characteristics to be *X* and *Y* random variables, assuming $\rho = 0.5$, at stage *k* of the data gathering process we generate $X_i = Z_i$, $i = 1, 2, \ldots, k$, with mean zero and variance one and Y_i by use of $E(Y_i|X_i) = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(X_i - \mu_X)$ and $\sigma_X^2 = (1-\rho^2)\sigma_Y^2$, where $\mu_Y = 0$ and $\sigma_Y = 1$. In the illustrative example, assuming the X variable is out-of-control, the observations have been generated from a Bi-variate normal distribution with $\mu_X = 0.5, \mu_Y = 0, \sigma_X = 1, \sigma_Y = 1$ and ρ=0.5. Furthermore, the control parameter of the chart (C) have been chosen to be 2.

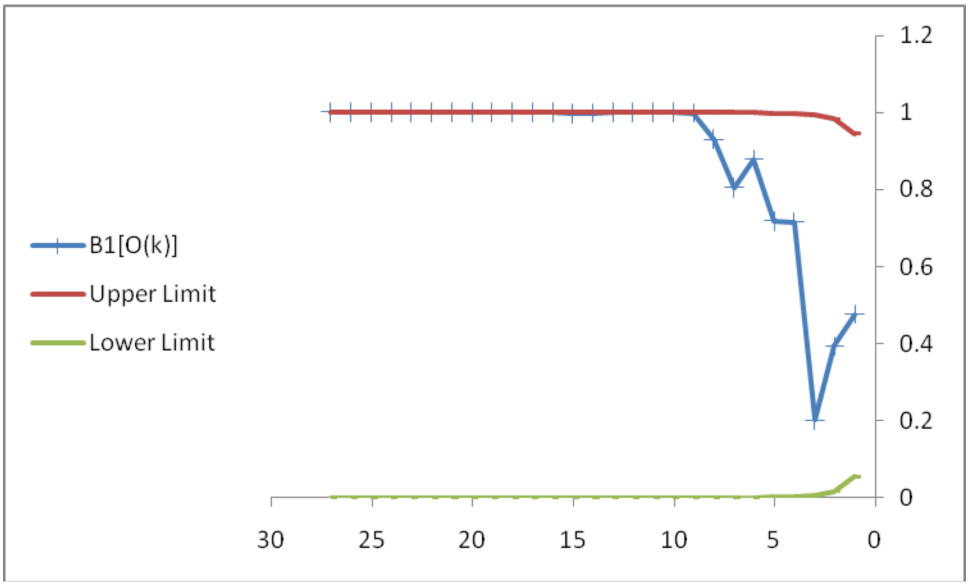

Figure 1: The belief-outcomes of one simulation run

In Figure 1, it can be seen that the out-of-control belief of X starts with 0.5 and reaches 1 after 19 observations and thereafter.

Next, we evaluate the performance of the proposed method and compare it with the ones from the well-known MCUSUM and MEWMA procedures in bi-variate normal cases.

5. Performance Evaluation

The performance evaluation of the proposed method is carried out using simulation. In each replication of the simulation study, we first generate correlated standard normal deviates using the methods as described in Section 4. Then, using equations (3) and (4) we update the beliefs $B_i(O_k)$ and evaluate $Ln(M_k)$. In cases where either $B_1(O_k)$ is out of the interval $\left[\frac{e^{-C\sqrt{2k}}}{\left(e^{-C\sqrt{2k}}+1\right)}, \frac{e^{C\sqrt{2k}}}{\left(1+e^{C\sqrt{2k}}\right)}\right]$ or MEWMA statistics is more than th_m, an out-of-control signal is observed.

In 20000 independent replications, for an intended ARL_0 of 320, the approximate threshold value of the MCUSUM, MEWMA methods are estimated 9.85 and 3, with ARL_0 of 314, 323, respectively. For $C=2$, to achieve ARL_0=330, the threshold value of the MEWMA method, th_m, of the proposed method using trial-and-error becomes 11.
For the comparison study, we estimate the ARL_1 values of the proposed method as well as the MEWMA and MCUSUM procedures by 20000 independent replications in each of the different scenarios of mean shifts. The shifts are given in multiples of the process standard deviations and are shown in the first column of Table 1. The remaining columns of Table 1 show the ARL_1 values of the methods under consideration along with the estimated standard deviations of the run lengths shown in parentheses.

The results of Table 1 show that the proposed method performs better in small shifts of the process mean.

Furthermore, the results of the simulation study on the out-of-control run lengths of the MEWMA, MCUSUM, and the proposed method for smaller shifts of the process mean are summarized in Table 2. Although the MEWMA and MCUSUM have been the most powerful methods in detecting small shifts of process means, the results of Table 2 shows that the proposed method performs better in all scenarios of small shifts in the process mean. In these cases, if a small shift occurs in the mean of one of the variables, the belief of that variable being out-of-control converges to one fast. This leads to a good performance of the proposed recursive method in detecting small mean-shifts.

Table 1. The results of ARL_1 (SDRL) study in bi-variate normal processes

Mean shifts	In-control and out-of-control average run lengths		
	Proposed Method	MCUSUM	MEWMA
$(0,0)$	318.47(424.10)	314.84(328.00)	323.01(319.00)
$(1.0\sigma_X,.0)$	5.68(4.17)	10.35(7.94)	9.30(2.10)
$(2.0\sigma_X,0)$	2.18(1.25)	2.92(1.13)	4.16(0.74)
$(3.0\sigma_X,0)$	1.40(0.64)	1.71(0.59)	2.84(0.55)
$(0,1.0\sigma_Y)$	8.06(5.16)	15.13(12.05)	11.37(5.18)
$(1.0\sigma_X,1.0\sigma_Y)$	5.98(2.39)	5.72(3.50)	6.50(2.18)
$(2.0\sigma_X,1.0\sigma_Y)$	3.04(1.26)	2.52(0.97)	3.68(0.71)
$(3.0\sigma_X,1.0\sigma_Y)$	1.76(0.77)	1.64(0.51)	2.64(0.53)
$(0,2.0\sigma_Y)$	3.03(1.74)	3.52(1.66)	4.76(1.31)
$(1.0\sigma_X,2.0\sigma_Y)$	3.80(1.27)	2.85(1.18)	4.03(1.08)
$(2.0\sigma_X,2.0\sigma_Y)$	2.87(0.80)	1.99(0.69)	3.00(0.66)
$(3.0\sigma_X,2.0\sigma_Y)$	2.01(0.69)	1.46(0.52)	2.35(0.51)
$(0,3.0\sigma_Y)$	1.82(0.93)	2.02(0.69)	3.12(0.69)
$(1.0\sigma_X,3.0\sigma_Y)$	2.51(0.93)	1.87(0.64)	2.89(0.63)
$(2.0\sigma_X,3.0\sigma_Y)$	2.38(0.58)	1.56(0.54)	2.49(0.54)
$(3.0\sigma_X,3.0\sigma_Y)$	2.00(0.42)	1.24(0.47)	2.00(0.33)

6. Conclusions and Recommendations for Future Research

We introduced a new approach to control the mean vector of a bivariate quality control process. To do this, we first defined the belief and explained how to model a bivariate SPC problem by an iterative approach involving the beliefs. Second, we clarified the approach by which we improved the beliefs. Third, taking advantage of a bivariate MEWMA method, we explained the decision-making process of mean shift detections in bi-variate quality control environment. Fourth, we provided a numerical example along with its graph to show how we updated the beliefs. Finally, in order to better-understand the proposed method and to evaluate its performance in terms of in-control and out-of-control average run lengths, we performed some simulation studies. The results of the simulation studies showed that the proposed method worked better than the two well-known methods in term of out-of-control average run lengths in small scenarios of process mean-shifts.

Table 2. The performance of the proposed method for small shifts

Mean shifts	Out-of-control average run lengths		
	Proposed method	MCUSUM method	MEWMA method
$(0.1\sigma_X, 0)$	180.1(223.4)	303.1(300.1)	249.3(240.1)
$(0.2\sigma_X, 0)$	75.5(85.2)	237.8(236.1)	141.1(131.2)
$(0.3\sigma_X, 0)$	39.7(41.3)	169.2(166.2)	80.1(70.2)
$(0, 0.1\sigma_Y)$	195.8(269.1)	292.3(295.3)	227.1(221.3)
$(0.1\sigma_X, 0.1\sigma_Y)$	89.5(104.1)	270.1(269.1)	188.9(178.2)
$(0.2\sigma_X, 0.1\sigma_Y)$	45.4(46.6)	215.2(215.1)	118.1(108.3)
$(0.3\sigma_X, 0.1\sigma_Y)$	27.2(26.1)	157.1(156.9)	72.7(61.1)
$(0, 0.2\sigma_Y)$	79.8(92.3)	219.3(215.1)	118.2(108.1)
$(0.1\sigma_X, 0.2\sigma_Y)$	44.4(45.2)	204.4(201.3)	105.4(93.2)
$(0.2\sigma_X, 0.2\sigma_Y)$	29.2(27.1)	167.2(165.1)	79.1(68.2)
$(0.3\sigma_X, 0.2\sigma_Y)$	19.5(16.3)	126.1(123.9)	54.1(44.7)
$(0, 0.3\sigma_Y)$	38.4(39.1)	144.3(142.1)	64.1(63.9)
$(0.1\sigma_X, 0.3\sigma_Y)$	25.6(17.0)	136.2(132.2)	59.3(49.1)
$(0.2\sigma_X, 0.3\sigma_Y)$	20.7(10.1)	116.1(112.7)	50.1(39.6)
$(0.3\sigma_X, 0.3\sigma_Y)$	14.9(10.3)	93.8(90.1)	39.1(30.33)

While the examples of the simulation studies contain correlation of 0.5 between variables, future research may contain cases with different values of correlations. Moreover, we may apply some other functions to determine the beliefs. In addition, the proposed method can be developed for general MEWMA control charts when the number of quality characteristics is more than two. Also, a simulation study for different values of C and th_m is a good case for research.

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