

A fuzzy multi objective linear programming model for a supplier selection problem under flexibility conditions

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Supplier selection is one of the main discussions in the Supply Chain. The issue of assigning purchase orders to suppliers that act differently in terms of quality, cost, services, etc. criteria is one of the significant concerns of purchase managers in the supply chain. To adopt an optimal decision in this regard is related to a multi-objective problem that the objectives are contradicting each other and have different importance and priority depending on the location. In practice, the existence of kind of ambiguity in explaining the information related to the problem constraints and complicated. In this regard, the emergence of Fuzzy set theory as a tool to describe such conditions besides presenting question model realistically can help to solve such problems well. Despite the importance of the model with the mentioned structure, unfortunately, few original works have been done in this field. As a result, in this paper, in addition to presenting a new multi-objective Fuzzy model being modelled based on assigning purchase order to suppliers in a supply chain a solution method is introduced based on using Fuzzy linear programming. To clarify solution process modelling and description, a case study is included related to selecting flour supplier for providing industrial bread of Khoshkar factory. The proposed model includes four objective functions:

- 1) Aggregate costs of minimizing type,
 - 2) Services of maximizing type (such as packing, being faithful to promise, factory health, discount, correct transportation, good relationships, honestly, etc.),
 - 3) Flour useful survival of maximizing type (regarding monthly flour buying by the factory),
 - 4) The purchased flour quality of maximizing type (concerning product type).
- Especially in the solution process, a method is determined based on setting weight for each of the objectives concerning the major factory stockholders.*

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1. Introduction

Supply Chain is one of the most application fields of operations research, where the supplier selection decision plays essentially an important role. Furthermore, in the competitive environment, factories and companies focus highly on selecting appropriate suppliers since selecting appropriate supplier helps to reduce purchase costs, to improve final product quality and services. Supplier selecting issue is a Multi-Criteria Decision-Making problem which is included quantitative and qualitative factors. In this problem, many criteria may be contradictory to each other; thus, the process of selection gets complicated and this involves two problems:

- 1) Which suppliers should be selected?
- 2) How much should be bought from each of the selected suppliers?

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Many variants approach and techniques have been proposed for Supplier Selection (SS) model. Some of these studies are based on multi-objective programming models. In particular, recently, the supplier selection models with multiple products and/or suppliers, the fuzzy goals, fuzzy restriction and/or coefficients have been attracted by many interests. Weber et al. (1991) [20] considered 74 publications that discussed supplier selection criteria, and showed that net price is the main criteria for supplier selection. They deduced that SS model is a multi-criteria model and the preference of criteria depends on each purchasing situation .

As a pioneering work, Roa and Kiser [19] and Bache et al. [5] proposed more than 50 criteria for identifying the selection of suppliers. About two last decades, a comprehensive review of criteria for supplier selection is presented which are concluded that the number and the weights of criteria depend on purchasing strategies ([1], [8], [9], [13], [14], [15], [22]). As we know there are just a few studies to organize uncertain information and ambiguity in Selection Criteria (SC) models. For example, in [2] and [3], authors developed a Fuzzy Multi-Objective (FMO) Linear Programming (LP) that enables the decision-makers to allocate variant weights to various criteria in the problem. Furthermore, fuzzy weighted additive model investigated such that different weights to SC is assigned according to the decision-maker's relative importance. In a recent research, Yucel et al. (2011) [21] have been developed a new weighted additive fuzzy programming approach to handle ambiguity and fuzziness supplier. In this paper, we concentrate on a MOSS model in the fuzzy environment as an extension of the model which is proposed by Amid et al. (2006) [2]. In particular, we assume that the objectives in their models are including with fuzziness. Based on the weighted additive fuzzy programming approach which is given by A. Yucel et al. [21] in 2011, we propose a new approach for solving supplier selection in supply chain problem in the fuzzy environment as an extension of Amid et al. model. The model and solution process is explained by solving a real problem which is modeled as a generalized form of supplier selection problem in a fuzzy environment. To clarify the solution process modeling and description, a case study is included related to selecting flour supplier for providing industrial bread of Khoshkar factory in the north of Iran. The proposed model includes four objective functions:

- 1) Aggregate costs of minimizing type,
- 2) Services of maximizing type (such as packing, being faithful to promise, factory health, discount, correct transportation, good relationships, honestly, etc.),
- 3) Flour useful survival of maximizing type (regarding monthly flour buying by the factory),
- 4) The purchased flour quality of maximizing type (concerning product type).

Especially in the solution process, a method is determined based on setting weight for each of the objectives concerning the major factory stockholders. On one hand, regarding the existing traditional context in most provinces in Iran and lack of sufficient supervision over traditional bread supplied in Iran, the Iran ministry of health has decided to raise industrial bread quality as much as possible and because in producing high-quality bread, the consumed flour quality has the basic role, food industry experts evaluate the flour given by each supplier according to the ministry of health standard code and analyze each of the quality indices. Given this information, we deal with solving the model and set the order amount to each one of the suppliers.

This study is outlined as following. The essential definitions and fundamental concept are considered in Section 2. In Section 3, a fuzzy multi-objective linear programming is defined. The main model that is a general multi-objective model for supplier selection and the solution process is given in Section 4 which is more adaptable form in the real world. Finally, we conclude in Section 5.

2. Preliminaries and Fundamental Concepts

This section is related to the fuzzy sets theory and we get some necessary definitions and concepts which are important throughout the paper which is taken from [18].

2.1. Basic definitions of fuzzy sets

We recall that the characteristic function of a crisp set assigns a value of either one or zero to each individual in the universal set, thereby discriminating between members and non-members of the crisp set under consideration. This function can be generalized such that the values assigned to the element of the universal set fall within a specified range and indicate the membership grade of these elements in the set in question. Larger values denote the higher degrees of set membership. Such a function is called membership function and the set defined by it a fuzzy set. The formula definition is given as follows.

Definition 2.1. Let \mathbb{R} be the universal set. \tilde{A} is called a fuzzy set in \mathbb{R} , if \tilde{A} is a set of ordered pairs $\tilde{A} = \{(x, \tilde{A}(x)) \mid x \in \mathbb{R}\}$, where $\tilde{A}(x)$ is membership function of \tilde{A} and assigns to each element $x \in \mathbb{R}$, a real number $\tilde{A}(x)$ in the interval $[0, 1]$.

One of the most important concepts of fuzzy sets is the concept of an α -cut. Given a fuzzy set \tilde{A} defined on \mathbb{R} and any number $\alpha \in [0, 1]$, the α -cut is the crisp set $[\tilde{A}]_{\alpha} = \{x \in \mathbb{R} \mid \tilde{A}(x) \geq \alpha\}$. That is, the α -cut of a fuzzy set \tilde{A} is the crisp $[\tilde{A}]_{\alpha}$ that contains all the elements of the universal set \mathbb{R} whose membership grades in \tilde{A} are greater than or equal to the specified value of α .

Theorem 2.1. A fuzzy set \tilde{A} is called convex if and only if for each $x, y \in \mathbb{R}$ and each $\lambda \in [0, 1]$, $\tilde{A}(\lambda x + (1 - \lambda)y) \geq \min\{\tilde{A}(x), \tilde{A}(y)\}$.

2.2. Fuzzy numbers and fuzzy arithmetic

Among the various types of fuzzy sets, of special significance are fuzzy sets that are defined on the set \mathbb{R} of real numbers. Membership functions of these sets, which have the form $\tilde{A}: \mathbb{R} \rightarrow [0, 1]$ clearly have a quantitative meaning and may, under certain conditions, be viewed as fuzzy numbers or fuzzy intervals. To view them in this way, they should capture our intuitive conceptions of approximate numbers or intervals, such that numbers that close to a given real number" or numbers that are around a given interval of real numbers". Such concepts are essential for characterizing states of fuzzy variables and, consequently, play an important role in many applications, including fuzzy control, decision making, approximate reasoning, optimization, and statistics with imprecise probabilities.

The following theorem is related to the definition of fuzzy sets based on its membership functions.

Theorem 2.2. Let \tilde{A} be a fuzzy set in \mathbb{R} . Then, \tilde{A} is a fuzzy number if and only if there exist a closed interval $[m, n] \neq \emptyset$ such that

$$\tilde{A}(x) = \begin{cases} L(x), & \text{for } x \in (-\infty, m], \\ 1, & \text{for } x \in [m, n], \\ R(x), & \text{for } x \in [n, \infty), \end{cases}$$

where $L: (-\infty, m] \rightarrow [0, 1]$ is monotonic increasing, continuous from the right and $L(x) = 0$ for $x \in (-\infty, w_1]$, $w_1 < m$; $R: [n, \infty) \rightarrow [0, 1]$ is monotonic decreasing, continuous from the left and $R(x) = 0$ for $x \in [w_2, \infty)$, $w_2 > n$.

Each relation on real numbers can be extended on fuzzy numbers directly by applying Zadeh's extension principle, which permits us to define a fuzzy set \tilde{B} with respect to each function $f: X \rightarrow Y$ whose membership function is given as follows,

$$\tilde{B}(y) = f(\tilde{A})(y) = \begin{cases} \sup_{y=f(x)} \tilde{A}(x) & f^{-1}(y) \neq \emptyset \\ 0 & f^{-1}(y) = \emptyset \end{cases}$$

According to the above axiom, each binary operation $*$ on \mathbb{R}^n can be extended to the binary operation $\tilde{*}$ on fuzzy numbers \tilde{A} and \tilde{B} with the following membership function:

$$\tilde{A} \tilde{*} \tilde{B}(z) = \sup_{z=x*y} \min\{\tilde{A}(x), \tilde{B}(y)\}.$$

Definition 2.2. A fuzzy number \tilde{A} is called an LR fuzzy number if its membership function $\tilde{A}: \mathbb{R} \rightarrow [0,1]$ has the following form:

$$\tilde{A}(x) = \begin{cases} L\left(\frac{x - a_1}{a_2 - a_1}\right), & \text{for } a_1 \leq x \leq a_2 \\ 1, & \text{for } a_2 \leq x \leq a_3 \\ R\left(\frac{x - a_3}{a_4 - a_3}\right), & \text{for } a_3 \leq x \leq a_4 \\ 0, & \text{else} \end{cases}$$

where $L(\cdot)$ and $R(\cdot)$ are piecewise continuous functions, $L(\cdot)$ is increasing, $R(\cdot)$ is decreasing and $L(0) = R(0) = 1$. The LR fuzzy number \tilde{A} as described above will be represented as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$. Here L and R are called as the left and right reference functions, a_1 and a_3 are respectively called starting and end points of the at interval, $\alpha = a_2 - a_1$ is called the left spread and $\beta = a_4 - a_3$ is called the right spread.

We denote the all LR fuzzy numbers by $F(\mathbb{R})$. It needs to point out that when $L(x) = 1 + x$; $R(x) = 1 - x$ and $a_2 \leq a_3$, fuzzy number \tilde{A} denotes trapezoidal fuzzy number.

Definition 2.3. A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is called a Trapezoidal Fuzzy Number (TRFN) if its membership function is given by

$$\tilde{A}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2 \\ 1, & \text{for } a_2 \leq x \leq a_3 \\ \frac{a_4 - x}{a_4 - a_3}, & \text{for } a_3 \leq x \leq a_4 \\ 0, & \text{else} \end{cases}$$

Remark 2.1. If $a_3 = a_4 = a$ in trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, we obtain a Triangular Fuzzy Number (TNF), and we denote it $\tilde{A} = (a_1, a, a_4)$.

It can be verified that the arithmetic on LR fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$ and $\tilde{B} = (b_1, b_2, b_3, b_4)_{LR}$ by use of Zadeh's extension principle is as follows:

$$x \geq 0 \quad x \in \mathbb{R}; \quad x\tilde{A} = (xa_1, xa_2, xa_3, xa_4)_{LR}$$

$$x < 0 \quad x \in \mathbb{R}; \quad x\tilde{A} = (xa_4, xa_3, xa_2, xa_1)_{RL}$$

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)_{LR}$$

Remark 2.2. It needs to point out that throughout this paper, for the sake of illustrating the performance of our approach, we only use the trapezoidal fuzzy number, but it is no restrict at all to use this fuzzy number.

2.3. Order on fuzzy numbers

Ranking of fuzzy numbers is an important issue in the study of fuzzy set theory. Ranking procedures are also useful in various applications and one of them will be in the study of fuzzy mathematical programming in later chapters. There are numerous methods proposed in the literature for the ranking of fuzzy numbers, some of them seem to be good in a particular context but not in general. Here, we describe only three simple methods for the ordering of fuzzy numbers.

The first approach is called the k-Preference index approach. This approach has been suggested by Adamo (see in [18]). Let \tilde{A} be the given fuzzy number and $k \in [0,1]$. The k-preference index of \tilde{A} is defined as

$$F_k(\tilde{A}) = \max \{x: \tilde{A}(x) \geq k\}.$$

Now, using this k-preference index, for two fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$, and $\tilde{B} = (b_1, b_2, b_3, b_4)$, $\tilde{A} \preceq \tilde{B}$ with degree $k \in [0,1]$ if and only if $F_k(\tilde{A}) \leq F_k(\tilde{B})$.

The second approach for ranking of fuzzy numbers is based on possibility theory. In fact Dubois and Prade (see in [18]), was first studied the ranking of fuzzy numbers in the setting of possibility theory. To develop this, suppose we have two fuzzy numbers \tilde{A} and \tilde{B} . Then, in accordance with the extension principle of Zadeh, the crisp inequality $x \leq y$ can be extended to obtain the truth value of the assertion that \tilde{A} is less than or equal to \tilde{B} , as follows:

$$T(\tilde{A} \preceq \tilde{B}) = \sup_{x \leq y} (\min \{\tilde{A}(x), \tilde{B}(y)\})$$

This truth value $T(\tilde{A} \preceq \tilde{B})$ is also called the grade of possibility of dominance of \tilde{B} on \tilde{A} and is denoted by $Poss(\tilde{A} \preceq \tilde{B})$. Now define $\tilde{A} \preceq \tilde{B}$ if and only if $Poss(\tilde{B} \preceq \tilde{A}) \leq Poss(\tilde{A} \preceq \tilde{B})$.

The third approach to order of fuzzy numbers is based on ranking functions for comparison of fuzzy numbers. In fact, in this approach a ranking function $R: F(\mathbb{R}) \rightarrow \mathbb{R}$ that maps each fuzzy number into the real line.

Now, by use of the natural order of the real numbers, the fuzzy numbers \tilde{A} and \tilde{B} easily compare as follows:

$$\tilde{A} \preceq \tilde{B} \quad \text{if and only if } R(\tilde{A}) \leq R(\tilde{B});$$

$$\tilde{A} < \tilde{B} \quad \text{if and only if } R(\tilde{A}) < R(\tilde{B});$$

$$\tilde{A} \approx \tilde{B} \quad \text{if and only if } R(\tilde{A}) = R(\tilde{B});$$

where \tilde{A} and \tilde{B} are in $F(\mathbb{R})$. Also we write $\tilde{A} \preceq \tilde{B}$ if and only if $\tilde{B} \succcurlyeq \tilde{A}$.

Several ranking functions have been proposed by researchers to suit their requirements of the problems under consideration. We restrict our attention to linear ranking functions, that is, a ranking function R such that $R(k\tilde{A} + \tilde{B}) = kR(\tilde{A}) + R(\tilde{B})$ for any \tilde{A} and \tilde{B} belonging to $F(\mathbb{R})$ and any $k \in [0,1]$. We consider the linear ranking functions on $F(\mathbb{R})$ as

$$R(\tilde{A}) = c_1 a_1 + c_2 a_2 + c_3 a_3 + c_4 a_4$$

where $\tilde{A} = (a_1, a_2, a_3, a_4)$ and c_1, c_2, c_3 and c_4 are constants, at least one of which is nonzero. For a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, some of these ranking functions is presented in here:

(a) Yager's ranking function (see in [18]):

$$Y_2(\tilde{A}) = \frac{1}{2} \int_0^1 (\inf [\tilde{A}]_\alpha + \sup [\tilde{A}]_\alpha) d\alpha$$

(b) Campos and Munoz's ranking functions (see in [18]):

$$CM_1^\lambda(\tilde{A}) = \int_0^1 (\lambda \inf [\tilde{A}]_\alpha + (1 - \lambda) \sup [\tilde{A}]_\alpha) d\alpha$$

$$CM_2^\lambda(\tilde{A}) = \int_0^1 \alpha (\lambda \inf [\tilde{A}]_\alpha + (1 - \lambda) \sup [\tilde{A}]_\alpha) d\alpha$$

As we mentioned in the above context, the linear ranking function plays its crucial role in ordering the trapezoidal fuzzy numbers being used for testing the optimality (inequality) conditions and making the decision for pivoting. Specially, we use the linear ranking function $Y_2(\tilde{A})$ to illustrate our approach. Then, for the trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$, and $\tilde{B} = (b_1, b_2, b_3, b_4)$, we have ([4], [11], [12], [17], [18]):

$$\tilde{A} \preceq \tilde{B}$$

if and only if

$$R(\tilde{A}) = \frac{1}{4}a_1 + \frac{1}{4}a_2 + \frac{3}{4}a_3 + \frac{1}{4}a_4 \leq R(\tilde{B}) = \frac{1}{4}b_1 + \frac{1}{4}b_2 + \frac{3}{4}b_3 + \frac{1}{4}b_4$$

Remark 2.3. For any trapezoidal fuzzy number \tilde{A} , the relation $\tilde{A} \succeq \tilde{0}$, holds, if there exist $R(\tilde{A}) \geq 0$. We also consider $\tilde{A} \approx \tilde{0}$ if and only if $R(\tilde{A}) = 0$. Thus, without loss of generality, throughout the paper we let $\tilde{0} = (0, 0, 0, 0)$ as the zero trapezoidal fuzzy number.

Definition 2.4. Let $\tilde{A} = (a_1, a_2, a_3, a_4)$, and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. A ordering rule can be used as follows:

1. The symbol “ \preceq ” means that “the left side value of the constraint is equal or less than the right side value in fuzzy sense”.
2. The symbol “ \succeq ” means that “the left side value of the constraint is equal or greater than the right side value in fuzzy sense”.
3. The symbol “ \approx ” means that “the left side value of the constraint is equal to the right side value in fuzzy sense”.

Definition 2.5. A variable whose values are determined in linguistic terms is named a linguistic variable.

This study shows that, decision-makers to investigate the weights of the factors in the fuzzy multi-objective linear model use the linguistic values which are shown in Fig. 2.1.

Assume that $\tilde{A} = (a_1, a_2, a_3, a_4)$, and $\tilde{B} = (b_1, b_2, b_3, b_4)$, be to Trapezoidal Fuzzy Numbers. The distance between these numbers can be calculated by the following rule: (See also in Chen, 2000 [6]).

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{4} [(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + (a_4 - b_4)^2]} \tag{1}$$

Now, suppose that a decision-group has T decision as $t = 1, 2, \dots, T$ and there is also a set of m criteria as $j = 1, 2, \dots, m$ for selecting suppliers. And let the fuzzy rating and importance weight of the t -th decision maker be $\tilde{x}_{ijt} = (a_{ijt}, b_{ijt}, c_{ijt}, d_{ijt})$ and $\tilde{w}_{jt} = (w_{jt1}, w_{jt2}, w_{jt3}, w_{jt4})$, $i = 1, \dots, m$,

$j = 1, \dots, n$, respectively. Hence, the aggregated fuzzy rating (\tilde{x}_{ij}) of alternatives with respect to each criterion can be calculated as:

$$\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$$

where

$$\begin{aligned} a_{ij} &= \min_t \{a_{ijt}\}, & b_{ij} &= \frac{1}{T} \sum_{t=1}^T b_{ijt}, \\ c_{ij} &= \frac{1}{T} \sum_{t=1}^T c_{ijt}, & d_{ij} &= \max_t \{d_{ijt}\}. \end{aligned} \quad (2)$$

Furthermore, the aggregated fuzzy weights (\tilde{w}_j) of each criterion can be calculated as:

$$\begin{aligned} w_{j1} &= \min_t \{w_{jt1}\} & w_{j2} &= \frac{1}{T} \sum_{t=1}^T w_{jt2} \\ w_{j3} &= \frac{1}{T} \sum_{t=1}^T w_{jt3} & w_{j4} &= \max_t \{w_{jt4}\} \end{aligned} \quad (3)$$

Then, based on the weighted approach we consider the fuzzy weights as:

$$\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$$

where the aggregated fuzzy weights (\tilde{w}_j) of each criterion can be calculated as (Chen et al., 2006 [7]):

$$\begin{aligned} w_{j1} &= \min_t \{w_{jt1}\} & w_{j2} &= \frac{1}{T} \sum_{t=1}^T w_{jt2} \\ w_{j3} &= \frac{1}{T} \sum_{t=1}^T w_{jt3} & w_{j4} &= \max_t \{w_{jt4}\} \end{aligned} \quad (4)$$

A proximity coefficient is specified to calculate the weights of each factor for the fuzzy multi-objective linear program.

$$CC_j = \frac{d_j^-}{d_j^+ + d_j^-} \quad j = 1, 2, \dots, m. \quad (5)$$

where d_j^- is a distance to Fuzzy Negative Ideal Rating and d_j^+ is a distance to Fuzzy Positive Ideal Rating which is defined in TOPSIS model (Chen, 2000 [6]).

By normalization to closeness coefficients obtained from (4), final weights (w_j) of each factor can be estimated as:

$$w_j = \frac{CC_j}{\sum_{j=1}^m CC_j} \quad (6)$$

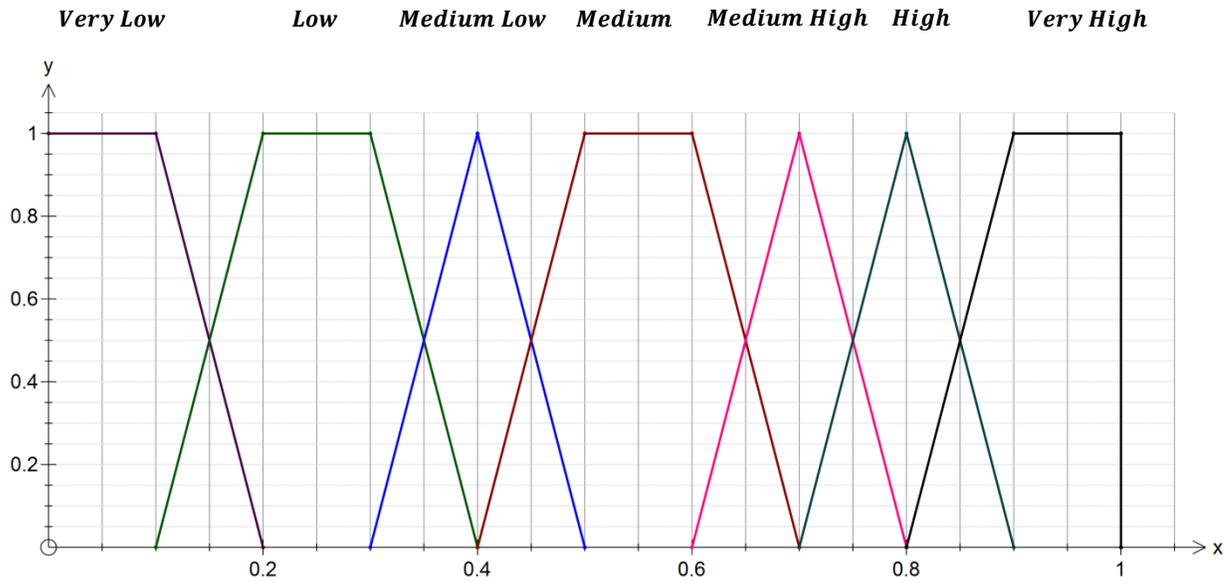


Fig. 2.1. Linguistic variable for importance weight of each factor.

3. Fuzzy Multi-Objective Linear Programming Model

The problem with classical linear programming is to determine the minimum or maximum values of a linear function within the restriction represented by linear inequalities or equations. The formula of the linear model can be presented as follows:

$$\max z = cx \tag{7}$$

$$s. t. Ax \leq b$$

$$x \geq 0.$$

where $x = (x_1, x_2, \dots, x_n)^T$ is a vector of variables.

The problem of optimizing several conflicting objective functions simultaneously under given constraints is called the Multi-Objective Linear Programming problem model:

$$\begin{aligned} \max \quad & f(x) = (f_1(x), f_2(x), \dots, f_q(x))^T \\ s. t. \quad & x \in X = \{x \in \mathbb{R}^n \mid g_j(x) \leq 0, j = 1, 2, \dots, m\} \end{aligned} \tag{8}$$

where $f_1(x), f_2(x), \dots, f_q(x)$ are q distinct objective functions of the decision vector and X is the feasible set of constrained decision.

Definition 3.1. We call x^* an exact optimal solution for (8) if there exists $x^* \in X$ such that $f_i(x^*) \geq f_i(x)$, $i = 1, 2, \dots, k$ for all $x \in X$.

In the operations research literature, such a complete optimal answer that maximizes all multiple-goal functions simultaneously does not always exist when the objective function conflicts with each other. Therefore, instead of a complete optimal answer, the concept of Pareto optimality is used in multi-objective programming.

Definition 3.2. We call $x^* \in X$ a Pareto optimal solution to Problem (8), if there does not exist another $x \in X$ such that $f_i(x^*) \leq f_i(x)$, for all $i = 1, 2, \dots, k$, and $f_j(x^*) < f_j(x)$ for at least one $j = \{1, 2, \dots, k\}$.

Definition 3.3. We call $x^* \in X$ a weak Pareto optimal solution to Problem (8), if there does not exist another $x \in X$ such that $f_i(x^*) < f_i(x)$, for all $i = 1, 2, \dots, k$.

A Fuzzy Multi-Objective Linear Programming is defined as:

$$\begin{aligned} \max \tilde{Z}_r &= \sum_j \tilde{c}_{rj} x_j \\ \text{s. t. } \sum_j \tilde{a}_{ij} x_j &\leq \tilde{b}_i \\ x_j &\geq 0, r \in Q, i \in I \end{aligned} \quad (9)$$

where the symbol " \leq " is defined in Definition 2.4 and $Q = \{1, 2, \dots, q\}$, $I = \{1, 2, \dots, m\}$, \tilde{a}_{ij} and \tilde{c}_{rj} are the trapezoidal fuzzy numbers which are shown as

$$\begin{aligned} \tilde{a}_{ij} &= \langle a_{ij}^{sl}, a_{ij}^{sr}, a_{ij}^l, a_{ij}^r \rangle \\ \tilde{c}_{rj} &= \langle c_{rj}^{sl}, c_{rj}^{sr}, c_{rj}^l, c_{rj}^r \rangle \end{aligned}$$

Definition 3.4. We call $x \in X$ a feasible solution to the Fuzzy Multi-Objective Linear Programming problem (8), if it satisfy in its constraints.

Definition 3.5. We call $x^* \in X$ a Pareto optimal solution to the Fuzzy Multi-Objective Linear Programming problem (8), if there does not exist another $x \in X$ such that $\tilde{z}_r(x) \geq \tilde{z}_r(x^*)$, for all $r = 1, 2, \dots, q$, and $\tilde{z}_j(x) > \tilde{z}_j(x^*)$ for at least one j .

4. The Multi-Objective supplier selection model

A Multi-objective (MO) model for the Supplier Selection (SS) problem can be presented as follows:

$$\begin{aligned} \min \quad & Z_1, Z_2, \dots, Z_f, \\ \min \quad & \tilde{Z}_{f+1}, \tilde{Z}_{f+2}, \dots, \tilde{Z}_p, \\ \max \quad & Z_{p+1}, Z_{p+2}, \dots, Z_q, \\ \max \quad & \tilde{Z}_{q+1}, \tilde{Z}_{q+2}, \dots, \tilde{Z}_t \end{aligned} \quad (10)$$

s. t.

$$x \in X_d, \quad X_d = \{x | g(x) \leq b_r, r = 1, 2, \dots, m\}$$

where the objective function in the form of minimum are the negative objectives or criteria-like cost and the objective function in the form of maximum are the positive objectives or criteria such that they will define as: $Z_j = \sum_{i=1}^n c_i^j x_i$, $j = 1, \dots, f$ and $j = p + 1, \dots, q$ and $\tilde{Z}_j \approx \sum_{i=1}^n \tilde{c}_i^j x_i$, $j = f + 1, \dots, p$ and $j = q + 1, \dots, t$. X_d is the set of feasible solutions which satisfy the constraint such as buyer demand, supplier capacity and so on.

The proposed model here for supplied selection problems is as follows:

$$\min \tilde{Z}_1 \approx \sum_{i=1}^n \tilde{P}_i x_i \tag{11}$$

$$\max Z_2 = \sum_{i=1}^n F_i x_i \tag{12}$$

$$\max Z_3 = \sum_{i=1}^n S_i x_i \tag{13}$$

$$\min \tilde{Z}_4 \approx \sum_{i=1}^n \tilde{Q}_i x_i \tag{14}$$

s. t.

$$\sum_{i=1}^n x_i \geq D, \tag{15}$$

$$x_i \leq C_i, \quad i = 1, 2, \dots, n, \tag{16}$$

$$x_i \geq 0, \quad i = 1, 2, \dots, n, \tag{17}$$

where the above-appeared parameters are defined as follows:

D : demand over period,

C_i : capacity of i th-supplier,

\tilde{P}_i : per unit net purchase cost from supplier i ,

S_i : the percentage of service level of i th-supplier (packing, honesty, on-time delivery, hygiene, etc.),

F_i : percentage of the quality level of i th-supplier,

x_i : the number of units purchased from the i th-supplier,

\tilde{Q}_i : the number of useful persistence of i th-supplier

n : number of suppliers.

Four objective functions are explained as follows:

The objective function (11): Net price,

The objective function (12): Quality,

The objective function (13): Service

Objective function (14): Useful persistence

So that is formulated to minimize total monetary cost and distance, maximize total quality, useful persistence and service level of purchased items, respectively. Constraint (15) ensures that demand is satisfied and also constraint (16) means that the order quality of each supplier should be equal or less than its capacity and finally constraint set (17) prohibits negative orders.

4.1. The fuzzy supplier selection model

In this subsection, a supplier selection problem with multi-objective is demonstrated and then a new solving process for this kind of decision-making problem is proposed. So, we first define the mentioned model as follows:

$$\begin{aligned} \min Z_M &= \sum_{i=1}^n c_{Mi} x_i & M &= 1, 2, \dots, f \\ \min \tilde{Z}_G &= \sum_{i=1}^n \tilde{c}_{Gi} x_i & G &= f + 1, f + 2, \dots, p \\ \max Z_U &= \sum_{i=1}^n c_{Ui} x_i & U &= p + 1, p + 2, \dots, q \\ \max \tilde{Z}_S &= \sum_{i=1}^n \tilde{c}_{Si} x_i & S &= q + 1, \dots, t \end{aligned} \tag{18}$$

where we must find a vector x as $x = (x_1, x_2, \dots, x_n)^T$ which minimizes the objective function Z_M and \tilde{Z}_G and maximize the objective function Z_U and \tilde{Z}_S with and constraints:

$$x \in X_d, \quad X_d = \{x | g(x) = \sum_{i=1}^n a_{ri} x_i \leq b_r, r = 1, 2, \dots, m, x \geq 0\}.$$

where a_{ri} and b_r based on their modality are crisp or fuzzy values.

Now we may solve every problem which is given in (18) by using fuzzy linear programming ([10], [11], [12], [16], [17]). In this study, authors formulated a kind of fuzzy linear program to a multi-objective linear program while every objective function is separated into the maximum and minimum form of the new objective functions as following:

$$Z_M^+ = \max Z_M, \quad x \in X_d, \quad Z_M^- = \min Z_M, \quad x \in X_d, \quad (19)$$

$$Z_U^+ = \max Z_U, \quad x \in X_d, \quad Z_U^- = \min Z_U, \quad x \in X_d, \quad (20)$$

$$(\tilde{Z}_G)_\alpha^+ = \max \sum_{i=1}^n (\tilde{c}_{Gi})_\alpha^U x_i, \quad x \in X_d, \quad (\tilde{Z}_G)_\alpha^- = \min \sum_{i=1}^n (\tilde{c}_{Gi})_\alpha^L x_i, \quad x \in X_d, \quad (21)$$

$$(\tilde{Z}_S)_\alpha^+ = \max \sum_{i=1}^n (\tilde{c}_{Si})_\alpha^U x_i, \quad x \in X_d, \quad (\tilde{Z}_S)_\alpha^- = \min \sum_{i=1}^n (\tilde{c}_{Si})_\alpha^L x_i, \quad x \in X_d. \quad (22)$$

where $\alpha = \min\{\mu_{\tilde{c}_{Gi}}(x), \mu_{\tilde{c}_{Si}}(x)\}$.

Now, let us assume that all fuzzy coefficients are in the form of trapezoidal \tilde{P} as which is demonstrated by the foursome $\tilde{P} = (p_1, p_2, p_3, p_4)$ with membership function

$$\mu_{\tilde{P}}(p) = \tilde{P}(p) = \begin{cases} 0, & p \leq p_1, \\ \frac{p - p_1}{p_2 - p_1}, & p_1 \leq p \leq p_2 \\ 1, & p_2 \leq p \leq p_3 \\ \frac{p_4 - p}{p_4 - p_3}, & p_3 \leq p \leq p_4 \\ 0, & p \geq p_4 \end{cases}$$

Then the α -cut of \tilde{P} can be expressed by the following interval:

$$[\tilde{P}]_\alpha = [(\tilde{P})_\alpha^L, (\tilde{P})_\alpha^U] = [p_1 + (p_2 - p_1)\alpha, p_4 - (p_4 - p_3)\alpha].$$

Note that, if $p_2 = p_3$ then \tilde{P} is reduced to the triangular fuzzy number, specified by (p_1, p_2, p_2, p_4) , and if $p_1 = p_2 = p_3 = p_4$, then \tilde{P} is reduced to a common real number.

Using the interval expression, the problem represented by (21) and (22) can be written as:

$$(\tilde{Z}_G)_\alpha^+ = \max \sum_{i=1}^n (\tilde{c}_{Gi})_\alpha^U x_i = \max \sum_{i=1}^n (c_{Gi}^{(4)} - (c_{Gi}^{(4)} - c_{Gi}^{(3)})\alpha) x_i, \quad x \in X_d, \quad (23)$$

$$(\tilde{Z}_G)_\alpha^- = \min \sum_{i=1}^n (\tilde{c}_{Gi})_\alpha^L x_i = \min \sum_{i=1}^n (c_{Gi}^{(1)} + (c_{Gi}^{(2)} - c_{Gi}^{(1)})\alpha) x_i, \quad x \in X_d,$$

$$(\tilde{Z}_S)_\alpha^+ = \max \sum_{i=1}^n (\tilde{c}_{Si})_\alpha^U x_i = \max \sum_{i=1}^n (c_{Si}^{(4)} - (c_{Si}^{(4)} - c_{Si}^{(3)})\alpha) x_i, \quad x \in X_d, \quad (24)$$

$$(\tilde{Z}_S)_\alpha^- = \min \sum_{i=1}^n (\tilde{c}_{Si})_\alpha^L x_i = \min \sum_{i=1}^n (c_{Si}^{(1)} + (c_{Si}^{(2)} - c_{Si}^{(1)})\alpha) x_i, \quad x \in X_d.$$

Note that we have obtained the optimal values $Z_M^-, Z_U^+, (\tilde{Z}_G)_\alpha^-$ and $(\tilde{Z}_S)_\alpha^+$ through solving the associated multi-objective problems where we consider a single objective each time and clearly the mentioned condition and $x \in X_d$ means that solutions must satisfy constraints while X_d is the set of all optimal solutions through solving as a single objective.

Based on the above approach, since for every objective function Z_j , its value linearly changes from Z_j^- to Z_j^+ , it may be considered as a type of fuzzy number with a linear membership function $\mu_{z_j}(x)$. Now, it is obvious that by considering the fuzzy goal and constraints assumptions in Problem (18), we may rewrite it as follows:

Find a decision vector x to satisfy:

$$\begin{aligned} Z_M &= \sum_{i=1}^n c_{Mi}x_i \leq Z_M^0, \quad M = 1, 2, \dots, f, \\ \tilde{Z}_G &= \sum_{i=1}^n \tilde{c}_{Gi}x_i \leq Z_G^0, \quad G = f + 1, f + 2, \dots, p, \\ \tilde{Z}_U &= \sum_{i=1}^n c_{Ui}x_i \geq Z_U^0, \quad U = p + 1, p + 2, \dots, q, \\ \tilde{Z}_S &= \sum_{i=1}^n \tilde{c}_{Si}x_i \leq Z_S^0, \quad S = q + 1, q + 2, \dots, t, \end{aligned} \tag{25}$$

s. t.

$$\begin{aligned} \tilde{g}_r(x) &= \sum_{i=1}^n a_{ri}x_i \leq b_r, \quad r = 1, 2, \dots, h \text{ (for fuzzy constraints).} \\ g_p(x) &= \sum_{i=1}^n a_{pi}x_i \leq b_p, \quad p = h + 1, h + 2, \dots, m \text{ (for classical constraints)} \end{aligned}$$

$$x_i \geq 0, \quad i = 1, 2, \dots, n,$$

where the symbols “ \leq ” and “ \geq ” is defined as well as Definition 2.4, and also Z_M^0 , Z_l^0 , \tilde{Z}_G^0 and \tilde{Z}_S^0 are the aspiration levels that the decision-maker wants to reach.

Now, we are in a place to define the membership functions, based on preference or satisfaction which are the linear form of membership for minimization goals (Z_M, Z_G) and maximization goals (Z_U, Z_S) are given as follows:

$$\mu_{Z_M}(x) = \begin{cases} 1 & Z_M(x) \leq Z_M^-(x) \\ \frac{Z_M^+(x) - Z_M(x)}{Z_M^+(x) - Z_M^-(x)} & Z_M^-(x) \leq Z_M(x) \leq Z_M^+(x) \\ 0 & Z_M(x) \geq Z_M^+(x) \end{cases} \tag{26}$$

$M = 1, 2, \dots, f$.

$$\mu_G^\alpha(Z_G)(x) = \begin{cases} 1 & Z_G^\alpha(x) \leq Z_G^{\alpha-}(x) \\ \frac{(\tilde{Z}_G)_\alpha^+(x) - \sum_{i=1}^n (\tilde{c}_{Gi})_\alpha^L x_i}{(\tilde{Z}_G)_\alpha^+(x) - (\tilde{Z}_G)_\alpha^-(x)} & Z_G^{\alpha-}(x) \leq Z_G^\alpha(x) \leq Z_G^{\alpha+}(x) \\ 0 & Z_G^\alpha(x) \geq Z_G^{\alpha+}(x) \end{cases} \tag{27}$$

$G = f + 1, f + 2, \dots, p$,

$$\mu_{Z_U}(x) = \begin{cases} 1 & Z_U(x) \geq Z_U^+(x) \\ \frac{Z_U(x) - Z_U^-(x)}{Z_U^+(x) - Z_U^-(x)} & Z_U^-(x) \leq Z_U(x) \leq Z_U^+(x) \\ 0 & Z_U(x) \leq Z_U^-(x) \end{cases} \tag{28}$$

$U = p + 1, p + 2, \dots, q$,

$$\mu_S^\alpha(Z_S)(x) = \begin{cases} 1 & Z_S^\alpha(x) \geq Z_S^{\alpha+}(x) \\ \frac{\sum_{i=1}^n (\tilde{c}_{Si})_\alpha^U x_i - (\tilde{Z}_S)_\alpha^-(x)}{(\tilde{Z}_S)_\alpha^+(x) - (\tilde{Z}_S)_\alpha^-(x)} & Z_S^{\alpha-}(x) \leq Z_S^\alpha(x) \leq Z_S^{\alpha+}(x) \\ 0 & Z_S^\alpha(x) \leq Z_S^{\alpha-}(x) \end{cases} \tag{29}$$

$$S = q + 1, q + 2, \dots, t.$$

The linear membership function for the fuzzy constraints is given as:

$$\mu_{gr}(x) = \begin{cases} 1 & g_r(x) \leq b_r \\ \frac{(b_r + d_r) - g_r(x)}{d_r} & b_r \leq g_r(x) \leq b_r + d_r \\ 0 & g_r(x) \geq b_r + d_r \end{cases} \quad (30)$$

$$r = 1, 2, \dots, h.$$

d_r is the subjectively chosen constants expressing the limit of the admissible violation of the r th inequalities constraints (tolerance interval).

4.2. Decision-making operators

First, the max-min operator is discussed which was used by Zimmermann (see in [18]) for fuzzy multi-objective models. To find the optimal solution x^* in the above fuzzy model (24), it is equivalent to solve the following crisp mode:

$$\begin{aligned} & \max \quad \lambda & (31) \\ & \text{s. t.} \\ & \lambda \leq \mu_{Z_M}, \\ & \lambda \leq \mu_G^\alpha(Z_G), \\ & \lambda \leq \mu_{Z_U}, \\ & \lambda \leq \mu_S^\alpha(Z_S), \\ & x \in X_d, \\ & x \geq 0, \\ & \lambda \in [0, 1]. \end{aligned}$$

In the above problem, there are two (variants types) additional unknown parameters: The first parameter, α , which denotes the first parameter, α , which denotes the satisfaction degree of the fuzzy parameters of the model and the second one is λ , which denotes the compromise between the variants objective. However, various approaches can be proposed to solve this problem, model (30) can parametrically solve based on the following algorithm:

Step 1. Define $\epsilon =$ step length, $k =$ iteration counter $= 0$.

Step 2. Set $\alpha_k = 1 - k\epsilon$, then solve the problem represented by (30) to obtain λ_k and x_k .

Step 3. If $|\lambda_k - \alpha_k| \leq \tau$ then let $\delta = \min\{\alpha_k, \lambda_k\}$ and $x = x_k$ and go to step 4. If $\alpha_k - \lambda_k > \epsilon$, then let $k = k + 1$ and go to step 2. If the step size is too large, let $\epsilon = \frac{1}{2}\epsilon$, $k = k$ and go to step 2.

Step 4. Output $\delta, \alpha_k, \lambda_k$ and x .

Now, we conclude that this is a linear problem (for a given α ,) which can be solved easily by the convenient solver which is appeared in the literature. In particular, we see that while the value of α is gradually decreased, the value of λ increases steadily. And therefore, the optimal solution is obtained at $\alpha^* = \lambda^*$.

Thus, the convex operator as a weighted additive approach is stated that enables the decision-makers to assign the different weights to various criteria. Finally, the weighted additive model for supplier selection problem is expressed as follows ([2], [3], [9]):

$$\max \sum_{j=1}^t w_j \eta_j + \sum_{r=1}^h \beta_r \gamma_r \quad (32)$$

s. t.

$$\eta_j \leq \alpha$$

$$\eta_j \leq \lambda_j$$

$$\lambda_j \leq \mu_{z_j}(x), j = 1, 2, \dots, t, \text{ (for all objective functions).}$$

$$\gamma_r \leq \mu_{g_r}(x), r = 1, 2, \dots, h, \text{ (for fuzzy constraints).}$$

$$g_p(x) \leq b_p, p = h + 1, h + 2, \dots, m, \text{ (for deterministic constraints).}$$

$$\lambda_j, \gamma_r \in [0, 1],$$

$$\sum_{j=1}^t w_j + \sum_{r=1}^h \beta_r = 1, w_j, \beta_r \geq 0,$$

$$x_i \geq 0, i = 1, 2, \dots, n.$$

where membership functions $\mu_{z_j}(x)$ and $\mu_{g_r}(x)$ are considered for fuzzy each objective and constraint. Furthermore, w_j and β_r are the weighting coefficients which are obtained from (2)-(5) represent the relative importance of fuzzy goals and constraints.

5. Conclusion

This study focused on solving a supplier selection problem in a fuzzy environment. A multi-objective fuzzy model has been proposed to solve the mentioned problem. In particular, the solution process has been presented as a numerical algorithm. As we saw in the last section, the proposed model and its solving process can suitably use to solve a similar model and the related problems which are appeared in the real world. The suggested model is essentially an extension of the model which is introduced by Amid et al. [2], where their method and the improved method in [3] can not solve the generalized model in the generalized form.

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