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Interval type-2 fuzzy backup 2-median location problem on trees

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The backup 2-median location problem on a tree T is to deploy two servers at the vertices such that the expected sum of distances from all vertices to the set of functioning servers is minimum. In this paper, we investigate the backup 2-median location problem on tree networks with trapezoidal interval type-2 fuzzy weights. We first present a new method for comparing generalized trapezoidal fuzzy numbers and then develop it for trapezoidal interval type-2 fuzzy numbers. Then numerical examples are given to compare the proposed methods with other existing methods. Finally, we apply our ranking method to solve the the backup 2-median location problem on a tree network with trapezoidal interval type-2 fuzzy weights.

***Keywords**: Location theory, Backup 2-median, Generalized fuzzy number, Trapezoidal interval type-2 fuzzy number.*

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1. Introduction

The p-median location problem is one of the most studied facility location models. In a p-median problem on a network, the aim is to find p locations for establishing facilities on edges or vertices of the network such that the sum of the weighted distances from the clients to the closest facility becomes minimum. Hakimi in [16] introduced the p-median problem and showed that optimal locations of the facilities exist at the vertices of the network. Kariv and Hakimi [19] showed that the p-median problem on a general network is NP-hard. Benkoczi and Bhattacharya [4] in 2005 proposed an $O(n\log^{p+2}n)$ time algorithm to solve the p-median problem on trees. Goldman [14] presented an algorithm for the 1-median problem on a tree. Drezner in [13] developed optimal solution procedures for the location of $p = 2$ facilities. Schobel and Scholz [29] optimally solved problems with $p = 2,3$ facilities.

In practice, uncertainties can play an influential role. For example, in the location problems, we are usually not sure of the vertex weights, the travel times between vertices, the establishing costs of facilities and the weight modification costs of a network. One of the suitable tools to deal with this uncertain parameter is the fuzzy set theory [40]. The other one is the minmax-regret facility location problem [2, 3] with uncertainties considered. Another model, called a reliability model [30, 31, 32], deals with the situation where servers may sometimes fail, and the clients originally allocated to these servers have to request service from functioning servers. In [36] the authors consider a reliability-based formulation, in which each server may fail with a given probability, for the 2-

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median problem ($V/V/2$) on tree networks. Once a server fails, the other server will take full responsibility for the services, with the assumption that the servers do not fail simultaneously. Note that a server may fail, but there is still a client at the location where the server is allocated. That is, the distance between any pair of vertices does not change when a server fails. Also, it is not required that the servers are deployed at different vertices. Under the assumption that two servers do not fail simultaneously, the backup 2-median problems ask for a pair of vertices which minimizes the expected value of the sum of distances, respectively, from all vertices to the functioning servers.

In this study, we solve the backup 2-median location problem on tree networks with trapezoidal interval type-2 fuzzy vertex weights. In first, we present a new simple, reliable and acceptability method for ranking generalized trapezoidal type-1 fuzzy numbers (GTrT1FNs) and then develop it for the trapezoidal interval type-2 fuzzy numbers (TrIT2FNs). To compare the proposed methods with other exiting methods, we give some numerical examples. Finally, using the proposed method, we solve the backup 2-median location problem on a tree network with interval type-2 fuzzy vertex weights.

2. Preliminaries and proposed ranking approaches for GTrT1FNs and TrIT2FNs

In this section, we briefly review the basic concepts of generalized type-1 fuzzy sets (GT1FSs), type-2 fuzzy sets (T2FSs) and trapezoidal interval type-2 fuzzy sets (TrIT2FSs). Moreover, some properties of them are discussed as well.

2.1. Basic definitions of generalized type-1 fuzzy numbers

Theory of type-1 fuzzy sets (T1FSs) defined by Zadeh in 1965 [40]. A fuzzy number is a special kind of fuzzy set that is bounded, convex, and whose membership function is a piecewise continuous function [15]. In a T1FS, each elements membership grade is a crisp number in the interval [0,1]. A T1FS $\tilde{A}$ on the universal set X can be represented by its membership function, i.e., $\mu_A(x)$, as follows:

$$\tilde{A} = \{(x, \mu_A(x)) | \forall x \in X, \mu_A(x) \in [0,1]\}$$

where $\mu_A(x)$ is the membership degree of an element $x \in X$.

Definition 2.1. *([6]) A generalized type-1 fuzzy number(GT1FN) $\tilde{A} = (a_1, a_2, a_3, a_4; h(A))$ is defined to be a generalized trapezoidal type-1 fuzzy number if its membership function is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{h(A)(x-a_1)}{(a_2-a_1)}, & a_1 \leq x \leq a_2 \\ h(A), & d_2 \leq x \leq a_3 \\ \frac{h(A)(x-a_4)}{(a_3-a_4)}, & a_3 \leq x \leq a_4 \\ o, & o.w. \end{cases}$$

where $h(A)$ is the height of the GT1FN.

Definition 2.2. *([39]) An α-cut of a fuzzy set, $\tilde{A}$ is a crisp set, determined as follows:*

$$\tilde{A}_\alpha = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha, \alpha \in [0,1]\}.$$

In this paper, we define the operation laws of fuzzy numbers based on the t-norm and t-conorm. Then, operation laws are extended to TrIT2FNs. The definition of t-norm and t-conorm are given as follows:

Definition 2.3. *([22, 27]) A t-norm is a function* $T: [0,1] \times [0,1] \rightarrow [0,1]$ *if it satisfies the following conditions:*

1) $\forall x \in [0,1]$, $T(x,1) = x$, $T(x,0) = 0$
2) $\forall x, y \in [0,1]$, $T(x,y) = T(y,x)$
3) *if* $x \leq x'$ *and* $y \leq y'$, *then* $T(x,y) \leq T(x', y')$
4) $\forall x, y, z \in [0,1]$, $T(x, T(y,z)) = T(T(x,y), z)$.

Definition 2.4. *([22, 27]) A t-conorm is a function* $S: [0,1] \times [0,1] \rightarrow [0,1]$ *if it satisfies the following conditions:*

1) $\forall x \in [0,1]$, $S(x,0) = x$, $S(x,1) = 1$
2) $\forall x, y \in [0,1]$, $S(x,y) = S(y,x)$
3) *if* $x \leq x'$ *and* $y \leq y'$, *then* $S(x,y) \leq S(x', y')$
4) $\forall x, y, z \in [0,1]$, $S(x, S(y,z)) = S(S(x,y), z)$.

It is also necessary that t-norm and t-conorm are continuous functions. It is well known that a strictly t-norm is generated via an additive generator k as $T(x,y) = k^{-1}(k(x) + k(y))$, and similarly, applied to its dual t-conorm $S(x,y) = l^{-1}(l(x) + l(y))$ with $l(x) = k(1-x)$ [21]. It is noted that an additive generator of a continuous t-norm is a strictly decreasing function $k: [0,1] \rightarrow [0, \infty]$ such that $k(1) = 0$. Now, we define a new function k as $k(x) = \frac{1-x}{x}$. It's easy to prove that k is a strictly decreasing function $k: [0,1] \rightarrow [0, \infty]$ such that $k(1) = 0$. So, t-norms and t-conorms can be obtained as follows:

1) $T(x,y) = k^{-1}(k(x) + k(y)) = \frac{xy}{x+y-xy}$
2) $S(x,y) = l^{-1}(l(x) + l(y)) = \frac{x+y-2xy}{1-xy}$
3) $\lambda x = k^{-1}(\lambda(k(x)) = \frac{\lambda x}{1+(\lambda-1)x}, \forall \lambda \in R$
4) $x^\lambda = l^{-1}(\lambda l(x)) = \frac{x}{\lambda+(1-\lambda)x}, \forall \lambda \in R$

Definition 2.5. *The addition and multiplication operations for arbitrary numbers* $\widetilde{A_1} = (a_{11}, a_{12}, a_{13}, a_{14}, h(A_1))$ *and* $\widetilde{A_2} = (a_{21}, a_{22}, a_{23}, a_{24}, h(A_2))$ *of GTrT1FNs are defined as follows:*

1) $\widetilde{A_1} \oplus \widetilde{A_2} = (\frac{a_{11}+a_{21}-2a_{11}a_{21}}{1-a_{11}a_{21}}; \frac{a_{12}+a_{22}-2a_{12}a_{22}}{1-a_{12}a_{22}}; \frac{a_{13}+a_{23}-2a_{13}a_{23}}{1-a_{13}a_{23}}; \frac{a_{14}+a_{24}-2a_{14}a_{24}}{1-a_{11}a_{21}};$ $min(h(A_1), h(A_2))$.

2) $\widetilde{A_1} \otimes \widetilde{A_2} = (\frac{a_{11}a_{21}}{a_{11}+a_{21}-a_{11}a_{21}}; \frac{a_{12}a_{22}}{a_{12}+a_{22}-a_{12}a_{22}}; \frac{a_{13}a_{23}}{a_{13}+a_{23}-a_{13}a_{23}}; \frac{a_{14}a_{24}}{a_{14}+a_{24}-a_{14}a_{24}}$ $min(h(A_1), h(A_2))$.

3) $\lambda \tilde{A}_1 = (\frac{\lambda a_{11}}{1+(\lambda-1)a_{11}}, \frac{\lambda a_{12}}{1+(\lambda-1)a_{12}}, \frac{\lambda a_{13}}{1+(\lambda-1)a_{13}}, \frac{\lambda a_{14}}{1+(\lambda-1)a_{14}}; h(A_1))$

4) $\tilde{A}_1^{\lambda} = (\frac{a_{11}}{\lambda+(1-\lambda)a_{11}}; \frac{a_{12}}{\lambda+(1-\lambda)a_{12}}; \frac{a_{13}}{\lambda+(1-\lambda)a_{13}}; \frac{a_{14}}{\lambda+(1-\lambda)a_{14}}; h(A_1))$

Theorem 2.6. *The following properties is satisfied for the addition and multiplication of fuzzy numbers:*

1) $\widetilde{A_1} \oplus \widetilde{A_2} = \widetilde{A_2} \oplus \widetilde{A_1}$

2) $\widetilde{A_1} \otimes \widetilde{A_2} = \widetilde{A_2} \otimes \widetilde{A_1}$

3) $(\widetilde{A_1} \oplus \widetilde{A_2}) \oplus \widetilde{A_3} = \widetilde{A_1} \oplus (\widetilde{A_2} \oplus \widetilde{A_3})$

4) $(\widetilde{A_1} \otimes \widetilde{A_2}) \otimes \widetilde{A_3} = \widetilde{A_1} \otimes (\widetilde{A_2} \otimes \widetilde{A_3})$

5) $\lambda\widetilde{A_1} \oplus \lambda\widetilde{A_2} = \lambda(\widetilde{A_1} \oplus \widetilde{A_2})$

6) $\tilde{A}^{\lambda_1+\lambda_2} = \widetilde{A_1}^{\lambda_1}\widetilde{A_2}^{\lambda_2}$

7) $(\widetilde{A_1} \otimes \widetilde{A_2})^{\lambda} = \widetilde{A_1}^{\lambda} \otimes \widetilde{A_2}^{\lambda}$

2.2. The proposed approach to ranking of generalized fuzzy number

Let $\tilde{A}_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4}; h(A_i))$, $i = 1, \ldots, n$, be GTrT1FNs. Set $k = min_{i=1,2,,\ldots,n} a_{i1}$. If $k < 0$ then set

$$\tilde{A}_i = \left(a_{i1} + k, a_{i2} + k, a_{i3} + k, a_{i4} + k; h(A_i)\right), i = 1, \ldots, n.$$

Set $x_{\max} = \max_{1<i<n} a_{i4} + k$ and $x_{\min} = \min_{1<i<n} a_{i1} + k$, see Figure 1.

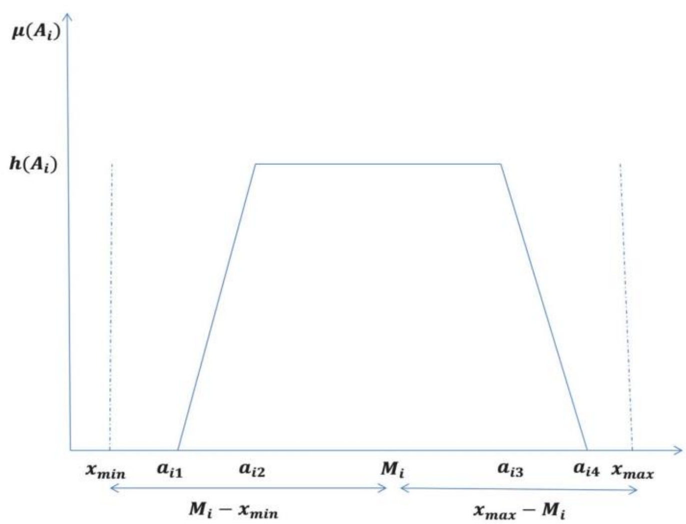

Figure 1: Generalized trapezoidal type-1 fuzzy number

Definition 2.7. *For a group of GTrT1FNs, $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n$, let $M_i = \frac{a_{i1}+a_{i2}+a_{i3}+a_{i4}}{4} + k$ be the mean of $a_{i1}, a_{i2}, a_{i3}, a_{i4}$ for $i = 1,2,\ldots,n$. For each number $\tilde{A}_i$, L_i^+ and L_i^- are the distance between $x_{\min}$ and $x_{\max}$ from M_i, respectively and defined as follows:*

$$L_i^- = \frac{M_i - x_{\min}}{a_{i4} - a_{i1}} h(A_i)$$

$$L_i^+ = \frac{x_{\max} - M_i}{a_{i4} - a_{i1}} h(A_i)$$

For each A_i, however, the distance $x_{\max}$ from M_i is more than distance $x_{\min}$ from M_i.

Definition 2.8. *Let $M_{\min} = \min_{1<i<n} M_i$ and $M_{\max} = \max_{1<i<n} M_i$. For each $\tilde{A}_i, i = 1,\ldots,n$ of GTrT1FN, the transfer coefficient of $\tilde{A}_i$ $i = 1,\ldots,n$ is defined by*

$$\lambda_i = \frac{M_i - M_{\min}}{M_{\max} - M_{\min}}.$$

Now, we define the ranking function of fuzzy numbers $\tilde{A}_i$ as follows:

$$Rank(\tilde{A}_i) = \begin{cases} \dfrac{\lambda_i L_i^-}{1 + (1-\lambda_i) L_i^+}, & if\, M_{\min} \neq M_{\max} \\ \dfrac{L_i^-}{1 + L_i^+}, & if\, M_{\min} = M_{\max}. \end{cases}$$

Definition 2.9. *Let $\tilde{A}$ and $\tilde{B}$ be two GTrT1FNs. We compare $\tilde{A}$ and $\tilde{B}$ as follows:*

i) $Rank(\tilde{A}) \leq Rank(\tilde{B})$ *if and only if* $\tilde{A} < \tilde{B}$ *or* $\tilde{A} \sim \tilde{B}$,

ii) $Rank(\tilde{A}) = Rank(\tilde{B})$ *if and only if* $\tilde{A} = \tilde{B}$,

iii) $Rank(\tilde{A}) \geq Rank(\tilde{B})$ *if and only if* $\tilde{A} > \tilde{B}$ *or* $\tilde{A} \sim \tilde{B}$.

Theorem 2.10. *Consider three arbitrary numbers $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ of GTrT1FNs. The following properties are satisfied.*

i) *If* $\tilde{A} \leq \tilde{B}$ *and* $\tilde{A} \geq \tilde{B}$ *then* $\tilde{A} \sim \tilde{B}$

ii) *If* $\tilde{A} \leq \tilde{B}$ *and* $\tilde{B} < \tilde{C}$ *then* $\tilde{A} < \tilde{C}$

iii) *If* $inf(supp\tilde{A}) > sup(supp\tilde{B})$ *then* $\tilde{A} > \tilde{B}$

iv) *If* $\tilde{A} < \tilde{B}$ *then* $-\tilde{B} < -\tilde{A}$.

Proof. It is easy to verify that the properties $(i) - (ii)$ hold. For the proof of (iii), according to $inf(supp\tilde{A}) > sup(supp\tilde{B})$ we have

$$a_4 \geq a_3 \geq a_2 \geq a_1 = inf(supp\tilde{A}) > sup(supp\tilde{B}) = b_4 \geq b_3 \geq b_2 \geq b_1$$

Hence, $M_{\tilde{A}} \geq M_{\tilde{B}}$ that will be the result $\lambda_{\tilde{A}} = 1$ and $\lambda_{\tilde{B}} = 0$. So $Rank(\tilde{A}) > Rank(\tilde{B})$. For the proof of (iv), we have the following two cases:

1. If $M_{\tilde{A}} \neq M_{\tilde{B}}$ then according to assumption, we have $M_{\tilde{A}} \leq M_{\tilde{B}}$. Thus $M_{-\tilde{B}} \leq M_{-\tilde{A}}$, which can be deduced $-\tilde{B} \leq -\tilde{A}$.

2. Let $M_{\tilde{A}} = M_{\tilde{B}}$. If $\tilde{A} = (a_1, a_2, a_3, a_4; h(A))$ then $-\tilde{A} = (-a_4, -a_3, -a_2, -a_1; h(A))$, $\max(a_4, b_4) = -\min(-a_4, -b_4)$, $\min(a_1, b_1) = -\max(-a_1, -b_1)$, $M_{\tilde{A}} = -M_{-\tilde{A}}$ and $M_{\tilde{B}} = -M_{-\tilde{B}}$.

Also, it can easily conclude that

$$\begin{aligned} L^-(\tilde{A}) = L^+(-\tilde{A}), L^+(\tilde{A}) = L^-(-\tilde{B}) \\ L^-(\tilde{B}) = L^+(-\tilde{B}), L^+(\tilde{B}) = L^-(-\tilde{B}) \end{aligned} \quad (1)$$

On the other hand, $M_{\tilde{A}} - \min(a_1, b_1) = M_{\tilde{B}} - \min(a_1, b_1)$ and $\max(a_4, b_4) - M_{\tilde{A}} = \max(a_4, b_4) - M_{\tilde{B}}$. Thus, it can deduced that

$$L^-(\tilde{A})L^+(\tilde{B}) = L^+(\tilde{A})L^-(\tilde{B}). \quad (2)$$

According to assumption we have

$$\frac{L^-(\tilde{A})}{1 + L^+(\tilde{A})} \leq \frac{L^-(\tilde{B})}{1 + L^+(\tilde{B})} \quad (3)$$

Thus according to (2) and (3) we get

$$L^+(\tilde{B}) \leq L^+(\tilde{A}) \quad (4)$$

So, by adding both sides of (2) and (4), we have

$$L^+(\tilde{B}) + L^+(\tilde{B})L^-(\tilde{A}) = L^+(\tilde{A}) + L^+(\tilde{A})L^-(\tilde{B}) \quad (5)$$

Now we get (5)

$$\frac{L^+(\tilde{B})}{1 + L^-(\tilde{B})} \leq \frac{L^+(\tilde{A})}{1 + L^-(\tilde{A})} \quad (6)$$

According to (6) and (1)

$$\frac{L^-(-\tilde{B})}{1 + L^+(-\tilde{B})} \leq \frac{L^-(-\tilde{A})}{1 + L^+(-\tilde{A})}$$

Thus we get $-\tilde{B} \leq -\tilde{A}$.

∎

Now to show that the proposed method is reliable, we compare some fuzzy numbers using the proposed method and existing methods.

Example 2. 11. *Consider the following sets given in [41, 38]. The ranking results of proposed method and other well-known methods is presented in Table 1.*

set1: $\tilde{A} = (5,6,6,7), \tilde{B} = (5.9,6,6,7), \tilde{C} = (6,6,6,7)$.

set2: $\tilde{A} = (2,4,4,6), \tilde{B} = (1,5,5,6)$.

set3: $\tilde{A} = (0,0.4,0.7,0.8), \tilde{B} = (0.2,0.5,0.5,0.9), \tilde{C} = (0.1,0.6,0.6,0.8)$.

set4: $\tilde{A} = (2,3,3,8), \tilde{B} = (2,3,7,8), \tilde{C} = (2,3,3,10)$.

set5: $\widetilde{A} = (0.3,0.4,0.7,0.8), B = (0.3,0.7,0.7,0.8), C = (0.5,0.7,0.7,0.8)$.

set6: $\tilde{A} = (2,4,4,6), \tilde{B} = (1,5,5,6), \tilde{C} = (3,5,5,6)$.

set7: $\tilde{A} = (1,5,5,5), \tilde{B} = (2,3,5,5)$.

Table1: Ranking results of Example 2.11.

Methods	*Set1*	*Set2*	*Set3*	*Set4*	*Set5*	*Set6*	*Set 7*
Wang(2015) [35]	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B} \sim \tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
Zhang(2014) [41]	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
Yu(2014) [38]	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
Deng(2014) [12]	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
Yu(2013) [37]	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} \sim \tilde{C}$	$\tilde{A} > \tilde{B}$
Phuc(2012) [28]	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
Chou(2011) [8]	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
Proposed method	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{C} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$

Example 2. 12. *Consider the following sets of generalized trapezoidal fuzzy numbers given in [37]. The ranking results of proposed method and other well-known methods is presented in Table 2.*

Set1: $\tilde{A} = (-0.8, -0.6, -0.4, -0.2; 0.35), \tilde{B} = (-0.4, -0.3, -0.2, -0.5; 0.7)$

Set2: $\tilde{A} = (0.2,0.4,0.6,0.8; 0.35), \tilde{B} = (0.1,0.2,0.3,0.4; 0,7)$,

Set3: $\tilde{A} = (1,1,1,1; 0.2), \tilde{B} = (1,1,1,1; 0.4), \tilde{C} = (1,1,1,1, ; 0.6)$, Set4: $\tilde{A} = (-1,3,5,7; 0.5), \tilde{B} = (1,3,5,9; 0.3)$.

Table 2: Ranking results of Example 2.12

Methods	*Set 1*	*Set 2*	*Set3*	*Set 4*
Yu(2014)[38]	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} \sim \tilde{B} \sim \tilde{C}$	$\tilde{A} < \tilde{B}$
Yu(2013)[37]	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} \sim \tilde{B} \sim \tilde{C}$	$\tilde{A} < \tilde{B}$
Chou(2011)[8]	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} \sim \tilde{B} \sim \tilde{C}$	$\tilde{A} < \tilde{B}$
Kumar(2011)[20]	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} \sim \tilde{B} \sim \tilde{C}$	$\tilde{A} < \tilde{B}$
Proposed method	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} \sim \tilde{B} \sim \tilde{C}$	$\tilde{A} < \tilde{B}$

2.3. The basic definitions of type-2 fuzzy number

Zadeh defined type-2 fuzzy sets to be sets with elements that have memberships that themselves are fuzzy sets.

Definition 2. 13. *([24]) A T2FS,* $\tilde{\tilde{D}}$ *on the universal set* U *can be characterized by its membership function* $\mu_{\tilde{D}}$ *and indicated as follows:*

$$\tilde{D} = \{((x,u), \mu_{\tilde{D}}(x,u)) | \forall x \in U, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{D}}(x,u) \leq 1\}$$

So that, the primary membership of x *is the* J_x *in the interval* $[0;1]$ *and the secondary membership function is* $\mu_{\tilde{D}}(x,u)$ *that defines the possibilities of the primary membership.* $\tilde{\tilde{D}}$ *can also be represented as follows:*

$$\tilde{\tilde{D}} = \int_{x \in U} \int_{u \in J_x} \mu_{\tilde{D}}(x,u)/(x,u)$$

where $J_x \subset [0,1]$ *and* $\int\int$ *signified the union over all admissible* x *and* u.

Definition 2. 14. *([25]) The footprint of uncertainty (FOU) is defined by the uncertain bounded area for the primary membership function, which is the outcome of the union of all primary memberships. FOU is explained by the upper membership function (UMF) and the lower membership function (LMF). Although, UMF and LMF are T1FSs which can be expressed as :*

$$FOU(\tilde{\tilde{D}}) = \cup_{\forall x \in U} J_x = \{(x,u): u \in J_x \subseteq [0,1]\}.$$

Definition 2. 15. *([26]) If all the secondary grades are at unity, then a T2FS is called an IT2FS, i.e.,* $\mu_{\tilde{D}}(x,u) = 1$ *for* $\forall x \in U$.

$$\tilde{\tilde{D}} = \int_{x \in U} \int_{u \in J_x} 1/(x,u) = \int_{x \in U} \left(\int_{u \in J_x} 1/u\right)/x$$

Thus, J_x can be represented as

$$J_x = \{(x,u): u \in [\mu_{\widetilde{D}^L}(x,u), \mu_{\widetilde{D}^U}(x,u)]\}.$$

Further, $FOU(\widetilde{\widetilde{D}})$ in Definition 2.14 can also be represented as

$$FOU(\widetilde{\widetilde{D}}) = \cup_{\forall x \in U} [\mu_{\widetilde{D}^L}(x,u), \mu_{\widetilde{D}^U}(x,u)].$$

Definition 2. 16. *An IT2FN,* $\widetilde{\widetilde{D}} = (\widetilde{D}^L, \widetilde{D}^U) = ((d_1^L, d_2^L, d_3^L, d_4^L; h^L)(d_1^U, d_2^U, d_3^U, d_4^U; h^U))$ *is called a TrIT2FN if its membership functions* $\widetilde{D}^L$ *and* $\widetilde{D}^U$ *have the following form:*

$$\mu_{\widetilde{D}^L}(x) = \begin{cases} \frac{x - d_1^L}{d_2^L - d_1^L} h^L, & d_1^L \leq x \leq d_2^L \\ h^L, & d_2^L \leq x \leq d_3^L \\ \frac{x - d_4^L}{d_3^L - d_4^L} h^L, & d_3^L \leq x \leq d_4^L \\ o, & o.w. \end{cases}$$

$$\mu_{\widetilde{D}^U}(x) = \begin{cases} \frac{x - d_1^U}{d_2^U - d_1^U} h^U, & d_1^U \leq x \leq d_2^U \\ h^U, & d_2^U \leq x \leq d_3^U \\ \frac{x - d_4^U}{d_3^U - d_4^U} h^U, & d_3^U \leq x \leq d_4^U \\ o, & o.w. \end{cases}$$

which is shown in Figure 2..

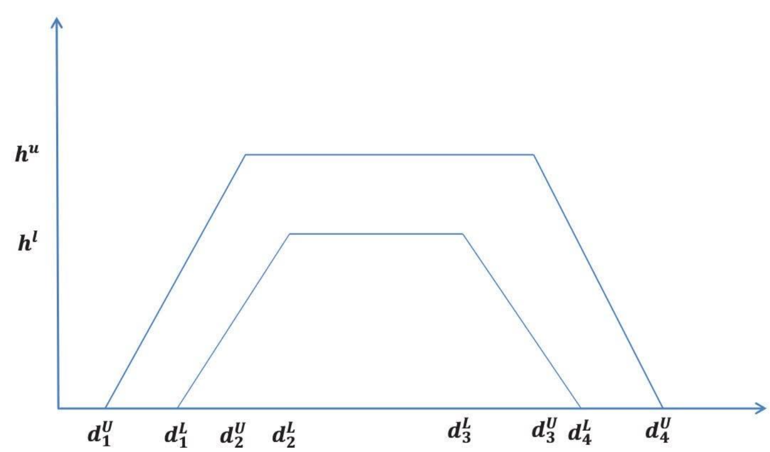

Figure 2: Trapezoidal interval type-2 fuzzy number

Definition 2. 17. *The addition and multiplication operations of two arbitrary numbers* $\widetilde{\widetilde{A_1}}$ *and* $\widetilde{\widetilde{A_2}}$ *of TrIT2FNs can be defined as follows:*

- $\widetilde{\widetilde{A_1}} \oplus \widetilde{\widetilde{A_2}} = ((\frac{a_{11}^L+a_{21}^L-2a_{11}^L a_{21}^L}{1-a_{11}^L a_{21}^L}; \frac{a_{12}^L+a_{22}^L-2a_{12}^L a_{22}^L}{1-a_{12}^L a_{22}^L}; \frac{a_{13}^L+a_{23}^L-2a_{13}^L a_{23}^L}{1-a_{13}^L a_{23}^L}; \frac{a_{14}^L+a_{24}^L-2a_{14}^L a_{24}^L}{1-a_{11}^L a_{21}^L};$ $min(h^L(A_1), h^L(A_2)), (\frac{a_{11}^U+a_{21}^U-2a_{11}^U a_{21}^U}{1-a_{11}^U a_{21}^U}; \frac{a_{12}^U+a_{22}^U-2a_{12}^U a_{22}^U}{1-a_{12}^U a_{22}^U}; \frac{a_{13}^U+a_{23}^U-2a_{13}^U a_{23}^U}{1-a_{13}^U a_{23}^U};$ $\frac{a_{14}^U+a_{24}^U-2a_{14}^U a_{24}^U}{1-a_{11}^U a_{21}^U}; min(h^U(A_1), h^U(A_2))).$

- $\widetilde{\widetilde{A_1}} \otimes \widetilde{\widetilde{A_2}} = (\frac{a_{11}^L+a_{21}^L-2a_{11}^L a_{21}^L}{1-a_{11}^L a_{21}^L}; \frac{a_{12}^L+a_{22}^L-2a_{12}^L a_{22}^L}{1-a_{12}^L a_{22}^L}; \frac{a_{13}^L+a_{23}^L-2a_{13}^L a_{23}^L}{1-a_{13}^L a_{23}^L}; \frac{a_{14}^L+a_{24}^L-2a_{14}^L a_{24}^L}{1-a_{14}^L a_{24}^L};$ $min(h^L(A_1), h^L(A_2)), (\frac{a_{11}^U+a_{21}^U-2a_{11}^U a_{21}^U}{1-a_{11}^U a_{21}^U}; \frac{a_{12}^U+a_{22}^U-2a_{12}^U a_{22}^U}{1-a_{12}^U a_{22}^U}; \frac{a_{13}^U+a_{23}^U-2a_{13}^U a_{23}^U}{1-a_{13}^U a_{23}^U};$ $\frac{a_{14}^U+a_{24}^U-2a_{14}^U a_{24}^U}{1-a_{11}^U a_{21}^U}; min(h^U(A_1), h^U(A_2))).$

- $\lambda\ \tilde{\tilde{A}}_1^\lambda = ((\frac{\lambda a_{11}^L}{1+(\lambda-1)a_{11}^L}, \frac{\lambda a_{12}^L}{1+(\lambda-1)a_{12}^L}, \frac{\lambda a_{13}^L}{1+(\lambda-1)a_{13}^L}, \frac{\lambda a_{14}^L}{1+(\lambda-1)a_{14}^L};\ h^L(A_1)),$ $,(\frac{\lambda a_{11}^U}{1+(\lambda-1)\,a_{11}^U}, \frac{\lambda a_{12}^U}{1+(\lambda-1)\,a_{12}^U}, \frac{\lambda a_{13}^U}{1+(\lambda-1)\,a_{13}^U}, \frac{\lambda a_{14}^U}{1+(\lambda-1)\,a_{14}^U};\ h^U(A_1)))$

- $\tilde{\tilde{A}}_1^\lambda = ((\frac{a_{11}^L}{\lambda+(1-\lambda)a_{11}^L}, \frac{a_{12}^L}{\lambda+(1-\lambda)a_{12}^L}, \frac{a_{13}^L}{\lambda+(1-\lambda)a_{13}^L}, \frac{a_{14}^L}{\lambda+(1-\lambda)a_{14}^L};\ h^L(A_1)),$ $(\frac{a_{11}^U}{\lambda+(1-\lambda)\,a_{11}^U}, \frac{a_{12}^U}{\lambda+(1-\lambda)\,a_{12}^U}, \frac{a_{13}^U}{\lambda+(1-\lambda)\,a_{13}^U}, \frac{a_{14}^U}{\lambda+(1-\lambda)\,a_{14}^U};\ h^U(A_1)))$

2.4. The proposed approach to ranking of TrIT2FN

In this subsection we extend the presented ranking function in Section 2.2 for TrIT2FNs. Let

$$\tilde{\tilde{D}}_i = (\tilde{D}_i^L, \tilde{D}_i^U) = ((d_{i1}^L, d_{i2}^L, d_{i3}^L, d_{i4}^L; h_i^L)(d_{i1}^U, d_{i2}^U, d_{i3}^U, d_{i4}^U; h_i^U)) \qquad i = 1,2,\dots,n$$

are TrIT2TFNs. set $k = min_{i=1,2,\dots,n}(d_{i1}^U)$. $Rank(\tilde{\tilde{D}}_i)$ is defined as

$$Rank(\tilde{\tilde{D}}_i) = \frac{Rank(\tilde{D}_i^L) + Rank(\tilde{D}_i^U)}{2}$$

Definition 2.18. *Let $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ be two TrIT2FNs . We compare $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ as follows:*

i) $Rank(\tilde{\tilde{A}}) \leq Rank(\tilde{\tilde{B}})$ *if and only if* $\tilde{\tilde{A}} < \tilde{\tilde{B}}$ *or* $\tilde{\tilde{A}} \sim \tilde{\tilde{B}}$,

ii) $Rank(\tilde{A}) = Rank(\tilde{B})$ *if and only if* $\tilde{\tilde{A}} = \tilde{\tilde{B}}$,

iii) $Rank(\tilde{\tilde{A}}) \geq Rank(\tilde{\tilde{B}})$ *if and only if* $\tilde{\tilde{A}} > \tilde{\tilde{B}}$ *or* $\tilde{\tilde{A}} \sim \tilde{\tilde{B}}$.

Theorem 2.19. *Consider arbitrary numbers $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ of TrIT2FNs. The following properties are satisfied.*

i) *If* $\tilde{\tilde{A}} \leq \tilde{\tilde{B}}$ *and* $\tilde{\tilde{A}} \geq \tilde{\tilde{B}}$ *then* $\tilde{\tilde{A}} \sim \tilde{\tilde{B}}$

ii) *If* $\tilde{\tilde{A}} \leq \tilde{\tilde{B}}$ *and* $\tilde{\tilde{B}} < \tilde{\tilde{C}}$ *then* $\tilde{\tilde{A}} < \tilde{\tilde{C}}$

iii) *If* $inf\left(supp\tilde{\tilde{A}}\right) > sup\left(supp\tilde{\tilde{B}}\right)$ *then* $\tilde{\tilde{A}} > \tilde{\tilde{B}}$

iv) *If* $\tilde{\tilde{A}} < \tilde{\tilde{B}}$ *then* $-\tilde{\tilde{B}} < -\tilde{\tilde{A}}$.

In the following we compare the proposed method with some well-known methods.

Example 2.20. *Consider the following sets of TrIT2FNs given in* Abdullah *et al [1]). The ranking results of proposed method and other well-known methods is presented in Table* 3.

$\tilde{\tilde{D}}_1 = ((0.6268,0.7895,0.7895,0.8955; 1), (0.7082,0.7896,0.7896,0.8426; 0.9)),$
$\tilde{\tilde{D}}_2 = ((0.5347,0.7099,0.7099,0.8404; 1,), (0.6225,0.7101,0.7101,0.7753; 0.9)),$
$\tilde{\tilde{D}}_3 = ((0.5263,0.7200,0.7200,0.8697; 1), (0.6231,0.7199,0.7199,0.7948; 0.9)),$
$\tilde{\tilde{D}}_4 = ((0.5967,0.7533,0.7533,0.8529; 1), (0.6752,0.7535,0.7535,0.8032; 0.9)),$
$\tilde{\tilde{D}}_5 = ((0.4940,0.6879,0.6879,0.8440; 1,1), (0.5909,0.6878,0.6878,0.7659; 0.9,0.9)).$

Tabel 3: Ranking results of Example 2.20.

Methods	*The final ranking*
Abdullah *et al (2017))[1]*	$\tilde{\tilde{D}}_1 > \tilde{\tilde{D}}_4 > \tilde{\tilde{D}}_3 > \tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_5$
Chen and Lee(2010)[5]	$\tilde{\tilde{D}}_1 > \tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_3 > \tilde{\tilde{D}}_5 > \tilde{\tilde{D}}_4$
Kabassi(2009)[18]	$\tilde{\tilde{D}}_1 > \tilde{\tilde{D}}_4 > \tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_3 > \tilde{\tilde{D}}_5$
Proposed method	$\tilde{\tilde{D}}_1 > \tilde{\tilde{D}}_4 > \tilde{\tilde{D}}_3 > \tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_5$

Example 2.21. *Consider the following sets of TrIT2FNs presented in [33]. The ranking results of proposed method and other methods is given in Table 4.*

$\tilde{\tilde{D}}_1 = ((0.6425,0.7979,0.7979,0.8907; 1), (0.7202,0.7979,0.7979,0.8443; 0.9000)),$
$\tilde{\tilde{D}}_2 = ((0.6421,0.8105,0.8105,0.9035; 1), (0.7263,0.8105,0.8105,0.8570; 0.9000)),$
$\tilde{\tilde{D}}_3 = ((0.6701,0.8099,0.8099,0.8799; 1), (0.7400,0.8099,0.8099,0.8449; 0.9000)),$

Table 4: Ranking results of Example 2.21.

Methods	*The final ranking*
Wang et. al (2015) [33]	$\tilde{\tilde{D}}_3 > \tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_1$
Hu et.al. (2013) [17]	$\tilde{\tilde{D}}_3 > \tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_1$
Chen and Wang (2013) [7] $\alpha = 1$	$\tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_1 > \tilde{\tilde{D}}_3$
Wang et. al. (2012) [34]	$\tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_1 > \tilde{\tilde{D}}_3$
Chen et. al (2012) [10]	$\tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_1 > \tilde{\tilde{D}}_3$
Chen and Lee(2010) [5]	$\tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_1 > \tilde{\tilde{D}}_3$
Chen and Lee(2010) [11]	$\tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_3 > \tilde{\tilde{D}}_1$
Lee and Chen (2008) [23]	$\tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_3 > \tilde{\tilde{D}}_1$
Proposed method	$\tilde{\tilde{D}}_3 > \tilde{\tilde{D}}_2 > \tilde{\tilde{D}}_1$

3. The interval type-2 fuzzy backup 2-median problem on trees

In this section, we are concerned with the problem of deploying two servers in a uncertain tree network, where each server may fail with a given probability. Once a server fails, the other server will take full responsibility for the services. Here, we assume that the servers do not fail, simultaneously. In the uncertain backup 2-median problem, we want to deploy two servers at the vertices such that the expected sum of distances from all vertices to the set of functioning servers is minimum. Assume that $T = (V, E)$ is an undirected tree with the vertex set $V = \{v_1, v_2, \dots, v_n\}$ and the edge set $E = \{e_1, e_2, \dots, e_n\}$. For each edge, the positive length is assigned and for each vertex $v_i \in V$ the trapezoidal interval type-2 fuzzy weight $\tilde{\tilde{w}}(v_i)$ is assigned. For two vertices $v, u \in V$ define $\Pi(u, v) = (U, V)$ such that

$$U = \{x \in V: d(x,u) \le d(x,v)\} \quad , \quad V = \{x \in V: d(x,v) \le d(x,u)\}.$$

Also, suppose that ρ_1 and ρ_2 are failure probabilities. The backup 2-median location problem asks for a pair of vertices (m_1, m_2) which minimizes the objective function

$$\begin{aligned} \min \quad & \psi(m_1,m_2) = (1-\rho_1)\Big(\sum_{v\in V_1} \tilde{\tilde{w}}(v) d(v,m_1) + \rho_2 \sum_{v\in V_2} \tilde{\tilde{w}}(v) d(v,m_1)\Big) \\ & \qquad + (1-\rho_2)\Big(\sum_{v\in V_1} \tilde{\tilde{w}}(v) d(v,m_2) + \rho_1 \sum_{v\in V_2} \tilde{\tilde{w}}(v) d(v,m_1)\Big) \qquad (7) \\ \text{s.t.} \quad & m_1, m_2 \in V \\ & (V_1, V_2) = \Pi(m_1, m_2) \end{aligned}$$

In the following we consider the backup 2-median problem on uncertain trees with the same failure probabilities. We use the new ranking method and the expressed algorithms in [36] in order to rewrite Algorithm 1 and sub-Algorithms 2 and 3 to find the location of a backup 2-median with trapezoidal interval type-2 fuzzy vertex weights. For this purpose, we have the following assumptions,

We rooted the input tree T at the median m. Suppose that $\{s_1, s_2\}$ is the 2-median of T. For an edge $e = (x, y) \in E$ let x be the parent of y, also let $X(e)$ and $Y(e)$ be the rooted subtrees at x and y, respectively. Assume that $\tilde{\tilde{w}}(x) = \tilde{\tilde{w}}(x) + \rho\tilde{\tilde{w}}(T(y))$ and $\tilde{\tilde{w}}(y) = \tilde{\tilde{w}}(y) + \rho\tilde{\tilde{w}}(T - T(y))$. The weights of all the other vertices and the lengths of all edges remain unchanged. The rooted subtree at u of $X(e)$ is denoted by $X(u(e))$. Similarly, $Y(v(e))$ is the rooted subtree of $Y(e)$ at v. In the following, let V_U and V_L be the vertex sets of $X(e)$ and $Y(e)$, respectively. We rewrite objective function of problem (7) as the edge-dependent cost function

$$\psi'_\rho(e) = D(V_L, m_L) + D(V_U, m_U) + \rho D(V_L, m_U) + \rho D(V_U, m_L)$$

where (m_L, m_U) is the pair of medians of obtained subtrees of removing the edge e and $D(V, u) = \sum_{v\in V} \tilde{\tilde{w}}(v) d(u, v)$.

Algorithm 1 Finds a backup 2-median of the uncertain tree T and with failure probability $0 \leq \rho \leq 1$ of each server

find the median m of T and call it m
call T^* as a rooted tree at m
using Algorithm 2 compute M_L
using Algorithm 3 compute M_U
set $opt := \infty$
for $i = 1, \ldots, n-1$ **do**
    if $\psi'_\rho(e_i) \leq \text{opt}$ **then**
        $opt := \psi'_\rho(e_i)$
        $i^* := i$
    end if
end for
if $\psi'_\rho(e_i) > \psi'_\rho(m, m)$ **then**
    return (m, m)
else
    return $(M_L[i^*], M_U[i^*])$
end if

Algorithm 2 Finds an array L of lower medians for all edges

for each v **do**
compute $\tilde{\tilde{w}}(T_v)$
end for
set $(v_1, v_2, \ldots, v_n)$ as arrangement of vertex set in post order
for $i = 1, \ldots, n-1$ **do**
set e_i as the edge between v_i and its parent
if v_i is a leaf **then**
$M_L[i] := v_i$
else
set $\lambda = \tilde{\tilde{w}}(T_{v_i}) + \rho\tilde{\tilde{w}}(T - T_{v_i}$
call the child of v_i as u such that $\tilde{\tilde{w}}(T_u) = \max_{x \in child(v_i)} \tilde{\tilde{w}}(T_x)$
    set median of $Y((v_i, u))$ as g
    While $\tilde{\tilde{w}}(T_g) < \frac{\lambda}{2}$ and $g \neq v_i$ **do**
        set g as parent of g
    end while
    Set $L[i] := g$
end if
end for
return L

Algorithm 3 Finds an array U of upper medians for all edges

for each v **do**
  compute $\tilde{\tilde{w}}(T_v)$
end for
call the heaviest branch of m as T_{v_1}
call the second heaviest branch of m as T_{v_2}
call the median of T_{v_1} as m_{v_1}
call the median of T_{v_2} as m_{v_2}
call the paths between the root, m_{v_1} and m_{v_2}, respectively $Path_1$ and $Path_2$.
arrange the edges in arbitrary order as $(e_1, e_2, \dots, e_{n-1})$
for $i = 1, \dots, n-1$ **do**
  if $e_i \in T_{v1}$ **then**
    the lowest vertex $v \in Path_2$ with $\tilde{\tilde{w}}(T_v) > \frac{\tilde{\tilde{w}}(X(e_i))}{2}$ set in $M_U[i]$
  else
    the lowest vertex $v \in Path_1$ with $\tilde{\tilde{w}}(T_v) > \frac{\tilde{\tilde{w}}(X(e_i))}{2}$ set in $M_U[i]$
  end if
end for
return U

Example 3.1. In order to illustrate the above procedure, consider the uncertain tree given in Figure 3, where vertex weights are TrIT2FNs. The vertex weights are given in Table 5, also let $\rho = \frac{1}{2}$. By using the proposed ranking method, we conclude that v_2 is a 1-median of the given tree. We call the rooted tree at v_2, tree T^*. Using Algorithm 2 we get $M_L = [v_3, v_5, v_4, v_7, v_1, v_6]$. By using Algorithm 3 we also get $M_U = [v_2, v_2, v_2, v_2, v_2, v_2]$. Finally, according to Algorithm 1 we conclude that (v_6, v_2) is a backup 2-median location of the given tree.

Table 5: The interval type-2 fuzzy vertex weights of T

vertex(i)	$\tilde{\tilde{w}}_i^L$	$\tilde{\tilde{w}}_i^U$
v_1	*(0.2, 0.4, 0.6, 0.8; 0.6)*	*(0, 0.3, 0.7, 0.9; 1)*
v_2	*(0.1, 0.3, 0.5, 0.8; 0.6)*	*(0.1, 0.2, 0.7, 1; 1)*
v_3	*(0.2, 0.5, 0.7, 0.9; 0.6)*	*(0, 0.3, 0.7, 1; 1)*
v_4	*(0.4, 0.5, 0.7, 0.8; 0.6)*	*(0.3, 0.5, 0.8, 0.9; 1)*
v_5	*(0.3, 0.6, 0.7, 0.8; 0.6)*	*(0.3, 0.4, 0.8, 0.9; 1)*
v_6	*(0.5, 0.6, 0.6, 0.8; 0.6)*	*(0.4, 0.5, 0.7, 0.9; 1)*
v_7	*(0.3, 0.5, 0.6, 0.8; 0.6)*	*(0.1, 0.5, 0.7, 1; 1)*

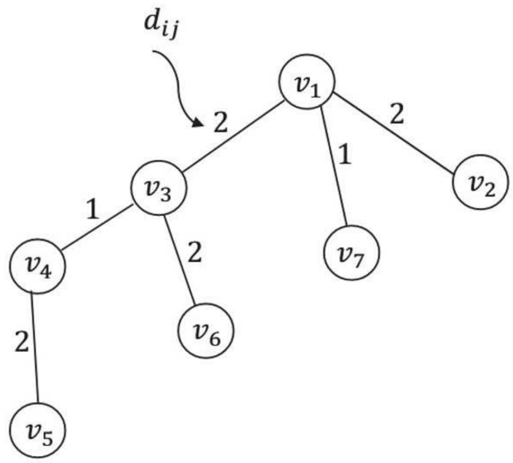

Figure 3: The uncertain tree T with vertex *weights* $\tilde{\tilde{w}}(v_i)$

4. Conclusion

In this paper, first we presented a new ranking function for GTrT1FNs and then we developed it for TrIT2FNs. Using these methods, we can compare fuzzy numbers together. Then by using the new proposed ranking function we proposed an algorithm for solving the backup 2-median location problem with trapezoidal interval type-2 fuzzy numbers.

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