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Optimization of Inventory-Related Decisions in Outsourced Supply Chain Management Using Response Surface Methodology

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This paper presents mixed-integer nonlinear mathematical programming model (MINLPM) for a company operating several stores and handling multiple products. The demand for each customer is characterized using fuzzy logic via triangular numbers, while the replenishment policy for each store and product follows the popular economic order quantity (EOQ) model under backorder. The proposed EOQ formulation considers throughput, dispatch, and budget constraints. The objective is to integrate vendor selection problem with EOQ policy while adopting multi-sourcing strategy. Under this strategy, the ordered quantity for each store and product can be split among one or more vendors. Thus, each store can be replenished for each product from set of selected vendors. This research aims to answer the following questions:

(i) Which vendors should be chosen?
(ii) Which stores should be allocated to the selected vendors for each product?
(iii) What are the optimal values for the inventory decisions?

Several numerical examples were designed and classified into four categories to evaluate the efficiency of the proposed approach. The findings indicate that when more than one vendor is used, the company's total cost can be reduced. As a result, the multi-sourcing strategy proves to be an efficient approach in this context. Moreover, genetic algorithm (GA) provides appropriate solutions within a reasonable computation time. The results demonstrate the practical applicability of the proposed model.

Keywords: *Vendor selection; Economic order quantity; Backorder; Multi-sourcing strategy; Genetic algorithm.*

1. Introduction

Today's outsourcing strategy has attracted considerable attention from researchers. Many studies have examined the importance of outsourcing business processes, activities, and functions to lower costs and risk, as well as to enhance performance, flexibility, and efficiency. Various reasons for outsourcing have been identified [1]: (i) it provides access to specialized technology and operational platforms; (ii) it can reduce staffing levels; (iii) technological advancements have been made it possible to procure highly specialized services. Although outsourcing offers significant benefits, it also entails risks and challenges, such as data/security breaches, loss of business knowledge, poor

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vendor selection, and other issues. As such, selecting appropriate vendors represents one of the most critical decisions for companies engaged in outsourcing [2]. Competitive market conditions create a challenging environment in which organizations must produce high-quality products at lower costs—a goal that is difficult to achieve without reliable and satisfactory vendors. Kumar et al. [3] identified the vendor selection problem (VSP) as complex for five reasons. One reason is that “suppliers may impose constraints on the supplying process, such as minimum or maximum order quantities based on their production capacity.” Dobler et al. [4] further focused that VSP is a critical strategic decision. Therefore, it is essential to investigate the relationship between VSP and order quantity decisions, which are inherently linked to inventory decisions—specifically, when and how much to order. VSP is typically studied using two approaches: single-sourcing and multi-sourcing strategies. Both are important in outsourcing contexts. Single-sourcing, where a customer relies on a single supplier can lead to moral hazard issues and poses several risks in unstable environments. Conversely, multi-sourcing has gained popularity in practice, though it can increase costs due to the complexity of managing multiple suppliers. Nevertheless, Costantino & Pellegrino [5] highlighted the benefits of adopting multi-sourcing strategy under risky conditions for specific issues. Many studies on VSP focus on single-sourcing and aim primarily at selecting suitable vendors, often without integrating inventory decisions—particularly backorder levels—or addressing transportation, throughput, dispatch, and budget constraints. As a result, the integration of VSP with inventory decisions, transportation, and capacity considerations remains understudied, despite offering significant opportunities for improving operational efficiency and cost-effectiveness [6, 7]. Furthermore, while numerous VSP studies concentrate on lot sizing and employ either exact or heuristic methods, there has been limited discussion on the design of meta-heuristic approaches to date. This paper develops mixed-integer nonlinear programming (MINLP) model for multi-store, multi-product purchasing environment, where demand for each store and product is characterized using fuzzy logic. The inventory policy for each store follows the economic order quantity (EOQ) assumptions for each product, including backorders. Notably, EOQ model is widely adopted due to its ease of use [8, 9]. The objective is to integrate VSP with inventory decisions under EOQ framework, incorporating throughput, dispatch, and budget constraints. A multi-sourcing strategy is adopted, allowing each store to be replenished by a set of selected vendors per product. The paper aims to minimize total cost, which comprises VSP-related costs—such as fixed, purchase, and transportation costs—and inventory-related costs, including ordering, backorder, and holding costs. The model builds upon the foundational work of Keskin et al. [6], extending it with additional constraints and assumptions. Genetic algorithm (GA) is developed to solve the proposed model, with response surface methodology (RSM) used to tune its parameters. The results from GA are validated using the branch-and-reduce algorithm available in GAMS software. In short, the main novelties of this paper are as followings:

1- Optimizing the inventory-related decisions using an outsourcing strategy under fuzzy environment;
2- Integrating vendor selection problem and EOQ policy;
3- Developing response surface methodology (RSM) for tuning GA parameters;
4- Designing an efficient GA for fuzzy EOQ policy.

The reminder of this paper is organized in which section (2) contains literature review. Section (3) includes problem description. The mathematical model of the problem is given in section (4). In section (5), GA is developed to solve the problem and its parameters are tuned. In order to evaluate the application of the developed model, several examples are provided. At the end, conclusion and further research comes in section (6).

2. Literature review

Inventory control is essential for any supply chain (SC) participant to ensure seamless system operation. Within the frameworks of Closed-Loop Supply Chains (CLSCs) and Reverse Logistics (RLs), effective inventory management not only enhances product flow but also supports manufacturers' sustainability in an increasingly competitive landscape. CLSC and RL systems facilitate the recovery of used products, conserve natural resources, and contribute to long-term environmental stewardship. CLSC management focuses the recuperation of used items to maximize economic benefits. For various reasons, CLSC concept is increasingly becoming central focus for manufacturers. Most developed countries now enforce legislation requiring businesses to manage their products and packaging after consumer use. Furthermore, growing customer concern about environmental damage from purchases, packaging disposal, and other waste sources has spurred greater adoption of CLSC models. These systems encourage manufacturers to reuse and recycle materials rather than discard them. The vendor selection problem (VSP) has been focal area of research since the 1960s, when Dickson [10] provided an early analysis of VSP systems. Weber et al. [11] later reviewed 74 studies related to VSP published since 1966. More recently, Deshmukh and Chaudhari [12] published a comprehensive paper on supplier selection criteria and methodologies. Based on these studies, three main approaches to vendor selection can be identified [3]:

1- Linear weighting methods;
2- Mathematical programming models;
3- Statistical methods.

The literature related to this research can be categorized into those of the second class, which are related to cost-based rather than vendor selection. Turner [13] developed single objective LP model for British Coal to reduce the total discounted price with regard to the vendor capacity, maximum and minimum order quantities. Degraeve et al. [14] studied the concept of total cost of ownership mathematical programming model as a basis for investigating the VSP models. Kumar et al. [3] developed fuzzy programming approach for VSP with three important goals, which are cost-minimization, quality-maximization and maximization of on-time delivery. Two-stage vendor selection framework as combinatorial optimization model in outsourcing is introduced by Cao and Wang [15]. Wang et al. [16] provided two mathematical models for VSP increasing the total quality level under fuzzy quality, budget, and demand. They utilized chance-constrained programming model and GA based on fuzzy set to solve the models. Díaz-Madroñero et al. [17] stated VSP with fuzzy goals in which the aim were the total ordering costs, the number of rejected products, and the number of late delivered products under several constraints. They designed an interactive method to solve it where fuzzy data are described using S-curve membership functions. Xu and Yan [18] modeled VSP in bi-fuzzy condition and implemented the chance-constrained programming approach to get the crisp model. Particle Swarm Optimization (PSO) is utilized to solve it. Finally, VSP with lean procurement environment is developed by Yu et al. [19]. They considered capacity uncertainty to determine suitable selection policy. In their study, those vendors that can promise tighter delivery schedules are preferable. Pan [20] introduced LP model to determine the number of suppliers for utilizing and purchasing the quantity allocations among the suppliers in multi-sourcing strategy. In their research, transportation, and capacity considerations are not considered. NLP quantity discount procedures under conditions of multi-product, resources limitations are stated by Benton [21], without any transportation cost.

Ghodsypour & O'Brien [22] described MINLP model to solve supplier selection, under conditions of multi-sourcing, multi-criteria, and capacity constraint. They evaluated multiple vendors, single store, stationary demand order sizing without any transportation cost, and backorders. Basnet and Leung [23] studied multi-period inventory lot-sizing scenario and VSP, where there are multi-product and multi-vendor. Each product can be supplied by set of vendors, in which a vendordependent transaction cost is applied for each period. Finally, an enumerative search algorithm and heuristic are designed to solve it, while throughput, dispatch, and budget capacity considerations are not embedded. Multi-objective mathematical optimization model for VSP is defined by Wadhwa and Ravindran [1], where one or more buyers order multi-product among several vendors in multi-sourcing network. Furthermore, they compared several multi-objective optimization approaches to solve the proposed VSP, such as weighted objective, goal programming and compromise programming. Nevertheless, they did not investigate the inventory decisions in their work. According to Kausar et al [13], CLSC is widely recognized as sustainable alternative compared to traditional SCs; however, Inventory control and making pricing decisions remains a complex process. This research examines two-layer SC that incorporates both the forward SC and RSC. The FSC produces new items, whereas RSC handles end of life (EOL) products for remanufacturing purposes. The focus is on refining inventory management, pricing strategies, and waste disposal procedures in order to enhance the sustainability of CLSC operations. The model was solved using environmental optimization run in LINGO and Mathematica software. The enhancement in profitability illustrates the responsiveness of pricing and scheduling choices in CLSCs design with higher rate of remanufacturing. Sensitivity study confirms the model's robustness across several parameter settings. The results provide quantifiable insights into how synchronized inventory and pricing strategies might improve economic performance while fostering sustainability through remanufacturing processes. Hallak et al. [10] developed an inventory model for manufactured-remanufactured items with demand uncertainties. This study aimed to aid in the transition toward GSC by formulating ecologically responsible inventory policies. Mogale et al. [17] developed CLSC network problem with multi-objective optimization. This model considered the various activities related to CLSC network to provide optimal solutions. Furthermore, Maheshwari et al. [33] proposed three-tier SCs model for the reworking and remanufacturing of the used items. This model tried to focus the role of the collection center in the smooth functioning of CLSC and RSC practices. Zhang et al [29] believed that rapid development of digital technologies and intelligent systems has accelerated the growth of e-commerce platforms, intensifying market competition and generating complex strategic challenges for both platforms and manufacturers. This paper develops an integrated game-theoretic framework to optimize logistics strategies in platform-based SCNs while incorporating consumer credit provision and sales channel decisions. The results show that the platform's logistics decision exhibits a threshold-based pattern. Moghadam et al [35], widespread involvement made online basic supply shopping the modern ordinary. The perishable & Fast-Moving Shopper Merchandise (FMCG) SC ought to be balanced to amplify their conveyance capabilities and adjust to the modern trade environment. This study presents the Three-Echelon Open Location-Routing Issue with Time Windows (3E-OLRPTW) with synchronous domestic conveyance and store pickup administrations for optimizing last-mile conveyance operations. A Mixed-Integer Non-Linear Programming (MINLP) detailing and made strides metaheuristic, Crossover Hereditary Calculation (HGA) are created employing customized neighborhood look method. The objective is to play down add up to working costs whereas bookkeeping for the time window and capacity imperatives. Numerical tests are conducted to assess the execution of the created arrangement strategy, comparing it with the made strides cross breed variations of Hereditary Calculation (GA), Counterfeit Bee Colony (ABC), Mimicked Strengthening (SA), and Settler Competitive Calculation (ICA) calculations. Factual tests affirm that HGA calculation beats the benchmarks in terms of arrangement quality and meeting. According to Pu et al [20], long-term disruptions can trigger ripple effects across SC, impacting both upstream and downstream stakeholders. To mitigate the consequences of such bidirectional ripple effects, including financial losses, consumer dissatisfaction, and declines in SC performance, an efficient optimization framework is required to enhance supply chain viability (SCV) through the integration of agility, resilience and sustainability. James & Mandal [32] believed that inadequate capacity and delayed delivery of electronic life support equipment was a major impediment in saving human lives during COVID-19. Capital intensive mass customized electronics and semiconductor manufacturing formed critical raw material for the same. Targeted efficiency achievement fails when variety and flexibility are prioritized in chip production. Design of experiments (DOE) was performed using Taguchi based orthogonal arrays. Signal to noise ratios were used to determine the main effects and robust combination of factor levels for high efficiency. Significant factors were identified from ANOVA for variance-reduction based robustness design. A better solution was created using learning-based fruit fly optimization algorithm and further using hybrid fruit fly grasshopper leap optimization. This algorithm successfully supported the high customization scenario for manufacturing efficiency during pandemic for any pre-set parameters by accelerating learning cycles. In addition, a multifactor particle swarm optimization was also performed for managing dynamic changes in all 31 factors together and the results were compared with previous techniques. The managerial implications and conclusion are explained for the benefit of the electronics industry and academia. Hajipour et al [24] concluded that Product recovery is critical in reducing costs, enhancing profitability, and improving SC responsiveness to customer demands. This study presented a model that optimizes the remanufacturing process using in-house workstations and outsourcing to maximize SC profitability, reduce queue lengths, and ensure machine reliability. The remanufacturing system is modeled as an M/M/m/k queuing system, considering real-world SC constraints such as budget constraints, station capacity, and machine reliability. SC optimization is achieved by maintaining efficiency while examining different remanufacturing policies and pricing strategies. The results show that expanding remanufacturing capacity enhances SC profitability, even with moderate increases in queue length. We provide valuable insights for SC managers aiming to optimize their remanufacturing processes and balance cost, efficiency, and reliability. The contribution of this research in comparison of other papers would be as following:

This paper addresses critical gaps in the current literature on optimization of inventory-related decisions in outsourced SCM. While previous studies often focus on singular aspects such as cost minimization reduction that consists of the fixed management costs related to VSP, the purchasing costs, fixed and distance-based transportation costs from the chosen vendors, and inventory costs such as holding costs, backorders, and fixed ordering costs of the stores , few have approached the problem as comprehensive multi-objective challenge. This study's primary contribution lies in developing an integrated optimization model that simultaneously maximizes profitability, and ensures machine reliability, providing more holistic solution for manufacturing systems [32, 33, 34]. Another key contribution is the inclusion of pricing as decision variable, a feature that enhances the model's relevance to managerial decision-making. This addition allows organizations to not only optimize operational efficiency but also refine their pricing strategies for remanufactured products, addressing a crucial aspect of profitability and market competitiveness. The proposed model fills an important gap in the literature by integrating sustainability considerations into the optimization framework. It highlights the potential of remanufacturing to contribute to circular economy goals by reducing waste and conserving resources. Although environmental metrics are not the primary focus of this study, the framework lays foundation for future research to incorporate such dimensions. In summary, this study contributes to the field by providing multi-dimensional optimization model that aligns theoretical rigor with practical applicability. It addresses the pressing need for comprehensive, adaptable solutions in remanufacturing, making significant step toward bridging the gap between academic research and industrial implementation. Future studies can build upon this foundation by exploring real-world applications and extending the model to include environmental impacts and advanced computational techniques.

3. Problem Statement

The problem of the paper includes three dimensions as bellow details: 1- Vendor selection decisions; 2-store assigning decisions and 3-inventory decisions.

Suppose a company with geographically dispersed stores, denoted by $i = 1,2,..,I$ and multi-vendor, denoted by $j = 1,2,...,J$ where there are multi-product using$m = 1,2,...,M$. Here, the demand for each store is characterized using fuzzy parameters. Lead-time of the each product is zero and planning horizon is infinite. It is assumed that each vendor can sells each product. The potential vendors can replenish each store with regard to each product. Considering each product, each store can be assigned to multi-vendor if those vendors were chosen. The policy replenishment of each store is EOQ policy under backorder. Besides, EOQ model is formulated under throughput, dispatch, and budget constraints. It is assumed that ordered quantity of store i for each product can be split between one or more vendors. The percent of assignment is introduced for each vendor. The costs related to the proposed VSP are vendor-specific fixed management, transportation and purchasing costs. On the other hand, the costs related to inventory decision are included fixed ordering, holding, and backorder costs. The transportation costs, including fixed and variable costs are considered between the selected vendors and allocated stores. As argued earlier, the objective of this research is integration of VSP and inventory decisions in order to minimize the total inventory cost of the company. As mentioned before, the proposed model originally provided by [6,12,32]. Nevertheless, they provided the case where the company operates under single-sourcing strategy. In other words, only one vendor replenishes each store. Moreover, each store acts under the assumptions of the classical EOQ policy. Besides, in their work, there was no backorder. Therefore, this paper develops the research studied by [6, 12, 32] and compares some of the contributions between our research and their study.

Table 1. Comparisons between our research and [32,6,12]

Contribution	This research	[32,6,12]
Multi-product	✓	
Multi-sourcing strategy	✓	✓
Budget constraint	✓	✓
Backorder	✓	
MINLP	✓	✓
Meta-heuristic algorithm	✓	✓

4. Mathematical Modeling

In order to develop the proposed model of VSP, consider the following notations.

*** Indices**

j Index for vendors ($j = 1,2,...,J$);
i Index for stores ($i = 1,2,...,I$);
m Index for products ($m = 1,2,...,M$);

*** Fuzzy Input Parameters**

$\widetilde{D}_{im}$ Annual fuzzy demand of store i for product m (unit);
$\tilde{h}_{im}$ Inventory holding cost of store i for product m ($/unit);
$\tilde{k}_{im}$ Fixed ordering cost for product m at store i ($);
$\widetilde{\pi_{im}}$ Backorder cost of store i for product m ($/unit);
$\tilde{c}_{jm}$ Per-unit purchasing cost offered by vendor j for product m ($/unit);
$\tilde{d}_{ij}$ Distance between store i and vendor j;
$\tilde{p}_{ij}$ Fixed transportation cost to store i from vendor j($);
$\tilde{r}_{ij}$ Per-mile transportation cost to store i from vendor j($);
$\tilde{f}_j$ Fixed annual cost of managing vendor j($);
$\tilde{P}_{jm}$ Maximum annual throughput capacity of vendor j for product m (unit);
$\tilde{R}_{jm}$ Maximum number of annual dispatches from vendor j for product m;
$\tilde{B}_{im}$ Available budget for purchasing of store i for product m ($);

*** Decision Variables**

Q_{im} Ordered quantity of store i for product m to all vendors (unit);
Q_{ijm} Order quantity of store i to vendor j for product m (unit);
b_{im} Backorder level of store i for product m (unit);
δ_{ijm} Percent of assigned Q_{im} to vendor j;

$$X_{jm} = \begin{cases} 1 \text{ if vendor j for product m is selected,} \\ 0 \text{ otherwise,} \end{cases}$$

$$Y_{ijm} = \begin{cases} 1 & \text{if } \delta_{ijm} > 0, \\ 0 & \text{if } \delta_{ijm} = 0, \end{cases}$$

In figure (1), $X_{1m} = 1$, then the vendor 1 is selected for selling the product m. Also, $Y_{i1m} = 1$ determines store i is assigned to the vendor for purchasing the product m. Moreover, the vendor j is chosen to sell the product m and $m-1$ to store i and $i+1$, respectively. It should be mentioned that the store i is not allowed to purchase the product m from another vendor, except vendor 1 and vendor j. This is because there are only $Y_{ijm} = 1$ and $Y_{i1m} = 1$. Then, the store i is replenished by vendor 1 and vendor j for product m. It can be observed that the company acts under multi sourcing strategy.

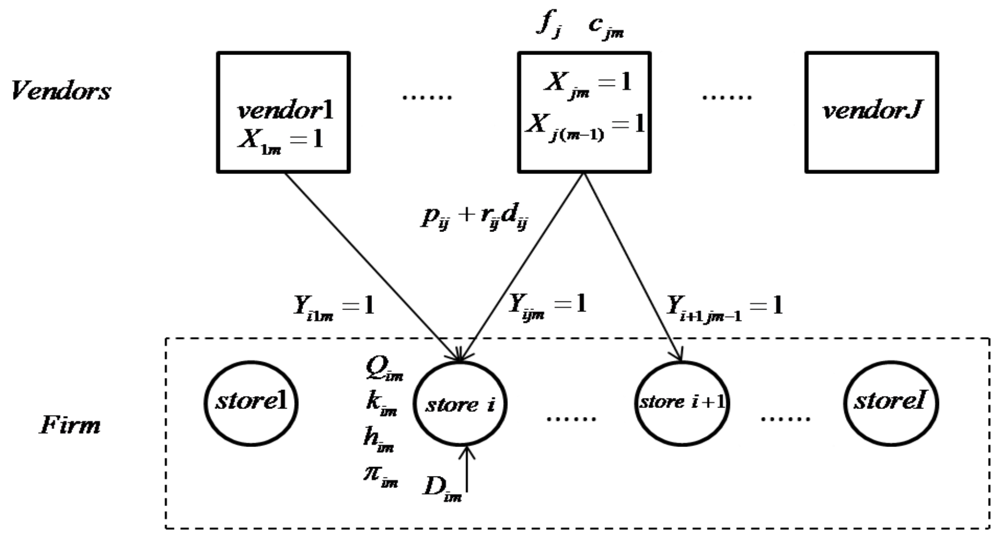

Fig. 1. Problem setting

4.1. Fuzzy numbers

As mentioned before, we utilize the definitions of triangular fuzzy number and membership function and also, definitions of basic arithmetic operation on fuzzy triangular numbers.

Definition 1: A fuzzy number $\tilde{x}$ is a triangular fuzzy number denoted by (x1, x2, x3) where x1, x2 and x3 are real numbers and its membership function $\mu_{\tilde{x}}(a)$ is given below.

$$\mu_{\tilde{x}}(a) = \begin{cases} \dfrac{(a-x_1)}{(x_2-x_1)} & ; \quad \text{for } x_1 \le a \le x_2 \\ \dfrac{(x_3-a)}{(x_3-x_2)} & ; \quad \text{for } x_2 \le a \le x_3 \\ 0 & ; \quad \text{otherwise} \end{cases} \quad (1)$$

Definition 2: A fuzzy number $\tilde{x}$ is a generalized triangular fuzzy number denoted by (x1, x2, x3; w) where x1, x2 and x3 are real numbers and its membership function $\mu_{\tilde{x}}(a)$ is given below.

$$\mu_{\tilde{x}}(a) = \begin{cases} \dfrac{w(a-x_1)}{(x_2-x_1)} & ; \quad \text{for } x_1 \le a \le x_2 \\ \dfrac{w(x_3-a)}{(x_3-x_2)} & ; \quad \text{for } x_2 \le a \le x_3 \\ 0 & ; \quad \text{otherwise} \end{cases} , (2)$$

Definition 3. Let $\widetilde{a}$ = (a1, a2, a3; w1) and $\widetilde{b}$ = (b1, b2, b3; w2) be two generalized triangular fuzzy numbers. Then

(i) $\widetilde{a} \oplus \widetilde{b} = (a_1 + b_1 , a_2 + b_2 , a_3 + b_3; \min.(w_1, w_2))$ (3)

(ii) $\widetilde{a} - \widetilde{b} = (a_1 - b_3 , a_2 - b_2, a_3 - b_1 ; \min.(w_1, w_2))$ (4)

(iii) $k\widetilde{a} = (ka_1, ka_2, ka_3; w_1)$, for $k \geq 0$; (5)

(iv) $k\widetilde{a} = (ka_3, ka_2, ka_1; w_2)$, for $k < 0$; (6)

Finally, the proposed mathematical model is formed as follows:

$$Min \quad z = \sum_{j=1}^{J}\sum_{m=1}^{M} \tilde{f}_j X_{jm} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{M}\left(\widetilde{c_{jm}}\widetilde{D}_{im} + \frac{(\tilde{p}_{ij} + \tilde{r}_{ij}\tilde{d}_{ij})\widetilde{D}_{im}}{Q_{im}}\right)Y_{ijm} + \sum_{i=1}^{I}\sum_{m=1}^{M}\left[\left(\frac{\tilde{h}_{im}}{2Q_{im}}(Q_{im} - b_{im})^2\right) + \left(\frac{\widetilde{\pi_{im}}b_{im}^2}{2Q_{im}}\right) + (\frac{\tilde{k}_{im}\widetilde{D}_{im}}{Q_{im}})\right] \tag{7}$$

Subject to:

$$Q_{im} = \sum_{j=1}^{J} Q_{ijm}, \qquad \forall i = 1, \dots, I, m = 1, \dots, M, \tag{8}$$

$$Q_{ijm} = \delta_{ijm}.Q_{im}, \qquad \forall i = 1, \dots, I, j = 1, \dots, J\ m = 1, \dots, M, \tag{9}$$

$$\sum_{j=1}^{J} \delta_{ijm} = 1, \qquad \forall i = 1, \dots, I, m = 1, \dots, M, \tag{10}$$

$$\delta_{ijm} \leq Y_{ijm}, \qquad \forall i = 1, \dots, I, j = 1, \dots, J\ m = 1, \dots, M, \tag{11}$$

$$\delta_{ijm} \geq \varepsilon Y_{ijm}, \qquad \forall i = 1, \dots, I, j = 1, \dots, J\ m = 1, \dots, M, \tag{12}$$

$$Y_{ijm} \leq X_{jm}, \qquad \forall i = 1, \dots, I, j = 1, \dots, J\ m = 1, \dots, M, \tag{13}$$

$$\sum_{i=1}^{I} \widetilde{D}_{im} Y_{ijm} \leq \tilde{P}_{jm} X_{jm}, \qquad \forall j = 1, \dots, J\ \ m = 1, \dots, M, \tag{14}$$

$$\sum_{i=1}^{I} \frac{\widetilde{D}_{im}}{Q_{im}} Y_{ijm} \leq \tilde{R}_{jm} X_{jm}, \qquad \forall j = 1, \dots, J\ \ m = 1, \dots, M, \tag{15}$$

$$(\tilde{c}_{jm}.Q_{im})Y_{ijm} \leq \tilde{B}_{im}, \qquad \forall i = 1, \dots, I, j = 1, \dots, J, m = 1, \dots, M, \tag{16}$$

$$0 \leq \delta_{ijm} \leq 1, \qquad \forall i = 1, \dots, I, j = 1, \dots, J\ m = 1, \dots, M, \tag{17}$$

$$X_{jm}, Y_{ijm} \in \{0,1\}, \qquad \forall i = 1, \dots, I, j = 1, \dots, J\ m = 1, \dots, M, \tag{18}$$

$$Q_{ijm}, Q_{im}, b_{im} \in \mathbb{R}_+ ; \qquad \forall i = 1, \dots, I, j = 1, \dots, J\ m = 1, \dots, M, \tag{19}$$

It is obvious that the mathematical model is MINLP. The objective provided in Eq. (7) is annual total cost that consists of the fixed management costs related to VSP, purchasing, fixed and distance-based transportation, and inventory costs such as holding costs, backorders, and fixed ordering costs of the stores. Eq. (8) measures the ordered quantity of store i for product m to all vendors. Eq. (9) reveals the ordered quantity must be split between the potential vendors. Eq. (10) provides the annual demand of each store must be satisfied. Eq. (11) and (12) make sure if Y_{ijm} is zero, δ_{ijm} is also zero, and if Y_{ijm} is one, δ_{ijm} must be greater than zero with regard to δ_{ijm} is less than one. In other words, when $Y_{ijm} = 1$, store i is allocated to vendor j for product m. Thus, these equations state our store allocation problem, where ε is less than one. Eq. (13) as a result, this equation describes our VSP. Eq. (14) satisfies the throughput capacity for each vendor while Eq. (15) limit the delivery frequencies from them. Thus, this equation ensures dispatch capacities at vendors. Eq. (16) guarantees available budget for purchasing of stores for each product. Eq. (17) restrict the value of the percent of allocated variable. At the end, Eq. (18) & (19) ensures integrality and non-negativity values for decision variables.

4.2. GA algorithm

The proposed mathematical model is MINLP. It is revealed that MINLP problems belong to NP-hard due to they are the generalization of MILP problems, which are NP-hard. Nevertheless, the MINLPs are so hard to solve [34]. On the other hand, the exact solution methods are complex and not effective to solve such models. In many studies, GAs were designed to solve MINLP problems because they are strong and practical tools to solve MINLP models [35]. As such, GA is utilized to solve the proposed model. The fundamental principle of GAs first was defined by Holland [36]. He encoded the features of problem by chromosomes, where each gene describes feature of the problem. In GA, both the crossover and mutation operators are implemented by predefined probabilities. Generally, GA includes the following steps [38]:

Step 1: Generating an initial population of the chromosomes;

Step 2: Investigating the fitness function for each generated chromosome;

Step 3: Generating new chromosomes by implementing some operators such as reproduction, crossover and mutation on the current chromosomes;

Step 4: Investigating the fitness function for the new population of chromosomes;

Step 5: If termination criteria is satisfied, stopping and returning the best chromosome; otherwise, going to Step 3.

4.2.1. The Chromosome Representation

In GA, chromosome includes a string of genes, which is considered as the coded figure of solution (either appropriate or none appropriate solutions). Note that generating an appropriate chromosome is critical step to implement GA in the solution space of the problem [28]. In this paper, the chromosome is defined under four section by $M \times (2I + J + I \times J)$ matrix. The structure of this chromosome is designed based on the work of [27]. It should be noted that each section is associated with some of our decision variables. Fig. 2 depicts the general form of this structure.

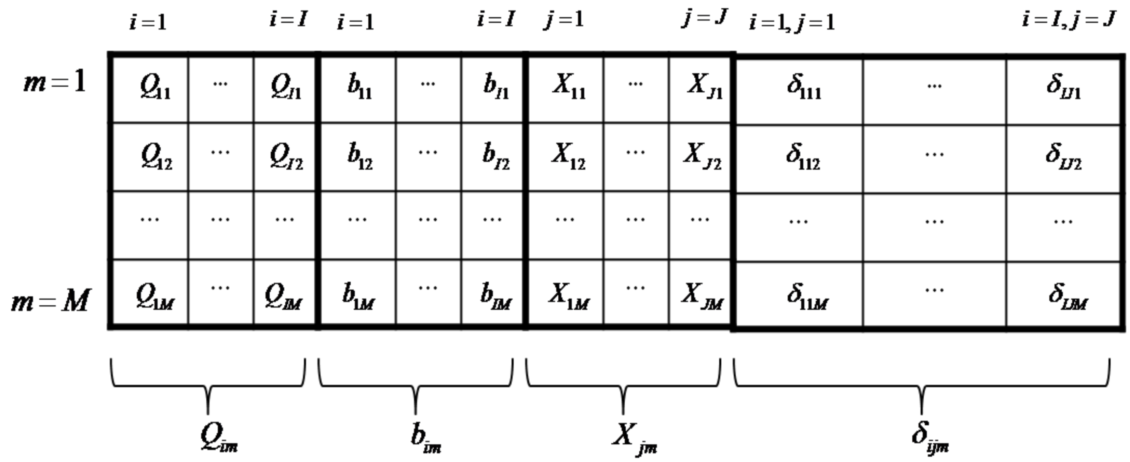

Fig. 2. The chromosome presentation

4.2.2. Investigating and generating an initial population

After that GA is implemented on the problem, fitness value that is the value obtained from the objective function that is required to be allocated for one chromosome, as soon as it is generated. Then, an initial population is generated randomly. Nevertheless, some of them may not be feasible; hence, the generation of the chromosomes is checked via penalty method to make sure generating the feasible chromosomes.

4.2.3 The selection operator

The selection operator presents the opportunity to provide the gene of good chromosome for the next generation. Several selection operators could be utilized to choose the parents. In this research, the roulette wheel selection operator is applied based on which the chromosome selection process in mating pool is based on their probability selection. The probability selection of each chromosome is investigated based on its fitness value.

4.2.4. The crossover operator

Crossover is process that some chromosomes exchange their genes through the breakage and reunion of two chromosomes to make number of children. To explore the solution space of the proposed problem, two-point crossover operator is selected. In this operator, two crossover positions are chosen uniformly at random and then the variables exchanged between the individuals between these points. Afterwards, two new offspring are obtained. It should be noted that this operator is applied in each section of the chromosome.

4.2.5. The mutation operator

Mutation makes an offspring solution by randomly implementing the parent's features. It improves to keep reasonable level of diversity mechanism in the population, and also serves the search by jumping out of local optimal solutions. In this study, an exchange mutation operator is implemented, in which the mutation swaps the value of the two random chosen genes of current solution together. Note that this operator is applied in each section of the chromosome, similar to the crossover operator.

4.2.6. Stopping criterion

The algorithm terminates if either the number of generations is greater than its maximum number or some specified number of generations without any improvement of best-known solution is achieved.

4.2.7. Parameter Tuning

In this section, the Response Surface Methodology (RSM) is developed to tune GA parameters. RSM has both mathematical and statistical approach, which is useful and practical to model and investigate the problems, in which response of interest is affected by several variables while the aim is to optimize this response [29]. Mainly, the first step is to fit first-order model and to conduct test of lack of fit. Nevertheless, as the first-order model was inadequate, second-order model was utilized. The most popular of the second-order model is the central composite design (CCD) [29,40]. In this model, there are 2^{k-1} factorial points (fractional factorial), n_c central points, and $2k$ axial points. The second-order model that is designed in the CCD as follows:

$$E(Y) = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_{ii} X_i^2 + \sum_{i<j}^{k} \sum_{i<j}^{k} \beta_{ij} X_i X_j \tag{20}$$

Where $E(Y)$the expected value of the response variable is, $\beta_0, \beta_i, \beta_{ii}, \beta_{ij}$ are the model parameters, X_i and X_j are the input variables that affect the response Y, and k is the number of factors being examined. In this paper, k factors that affect the response are population size (POPS), the maximum number of generations (MaxG), the crossover probability (Pc), the mutation probability (Pm) and the problem size (ProS). Therefore, k is equal to five and we have 32 points. Three levels of these factors are reported in Table (2).

Table 2. The levels of factors

Parameter	Range	Min	Medium	Max
ProS	3-7	3	5	7
PopS	50-150	50	100	150
MaxG	100-600	100	300	600
Pc	0.5-0.8	0.5	0.65	0.8
Pm	0.1-0.2	0.1	0.15	0.2

In order to investigate GA parameters, three instances with different sizes including 3, 5, 7 products with two vendors and two stores are designed. Then, the values of model parameters are made from the second column in Table 3.

Table 3. Data generation

Parameter	Value in	Value in
$\widetilde{\boldsymbol{D}}_{\boldsymbol{im}}$	510	U(350 , 1400)
$\widetilde{\boldsymbol{h}}_{\boldsymbol{im}}$	5	U(5 , 10)
$\widetilde{\boldsymbol{k}}_{\boldsymbol{im}}$	100	U(75 , 300)
$\widetilde{\boldsymbol{\pi}_{\boldsymbol{im}}}$	7	U(4 , 12)
$\tilde{\boldsymbol{c}}_{\boldsymbol{jm}}$	0.2	U(0.05 , 0.3)
$\widetilde{\boldsymbol{d}}_{\boldsymbol{ij}}$	60	U(10 , 150)
$\widetilde{\boldsymbol{p}}_{\boldsymbol{ij}}$	500	U(425 , 1700)
$\tilde{\boldsymbol{r}}_{\boldsymbol{ij}}$	2	U(0.75 , 3)
$\tilde{\boldsymbol{f}}_{\boldsymbol{j}}$	70,000	U(50,000 ,
$\widetilde{\boldsymbol{P}}_{\boldsymbol{jm}}$	1000	U(800 , 1500)
$\widetilde{\boldsymbol{R}}_{\boldsymbol{jm}}$	25	U(10 , 50)
$\widetilde{\boldsymbol{B}}_{\boldsymbol{im}}$	4000	U(3000 , 5000)

In Table 2, term "U" denotes to the uniform distribution. In The paper proposed by Keskin et al. [6]. All experiments are coded with MATLAB 7.8 (R2018a) software. The experimental results are examined with Minitab 16.2.2 software. Finally, the second-order coefficients ($P < 0.05$) are provided in Table 4.

Table 4. Multiple regression analysis for fitness

Term	Coef	SE Coef	T	P
Constant	9752.100	1325.200	7.362	0.000
ProS	-5164.400	202.200	-25.547	0.000
PopS	19.700	7.100	2.768	0.017
MaxG	-1.200	1.300	-0.867	0.003
ProS*ProS	616.330	18.400	33.547	0.000
ProS*Pc	318.900	96.100	3.310	0.006
ProS*Pm	812.550	288.400	2.817	0.016
PopS*Pc	-10.610	4.800	-2.223	0.046
PopS*Pm	-39.220	14.400	-2.635	0.022

S = 115.365 R-Sq = 99.92% R-Sq (adj) = 99.75%

Table 5. Analysis of variance for fitness

Source	DF	Seq SS	Adj SS	Adj MS	*F*	*P*
Regression	20	172115701	172115701	8605785	646.61	0
Linear	5	122013254	10307990	2061598	154.9	0
Square	5	49599610	49599610	9919922	745.35	0
Interaction	10	502836	502836	50284	3.78	0.016
Residual error	12	159709	159709	13309		
Lack-of-fit	6	79714	79714	13286	1	0.501
Pure error	6	79995	79995	13333		
Total	32	172275410				

The level of significance is 5%. As can be seen, the amount of R2 is 99.92%, and the F-value for the regression is significant at a level of 5% ($P < 0.05$), while the lack of fit was not significant at the 5% ($P > 0.05$). This reveals the good predictability of the model. Besides, the estimated regression of the model fitness is provided in Eq. (15)

$$fitness = 9756.1 - 5164.4(ProS) + 19.7(PopS) - 1.2(MaxG) + 616.33(ProS)^2 + 318.9(ProS)(Pc) + 812.55(ProS)(Pm) - 10.61(PopS)(Pc) - 39.22(PopS)(Pm), \quad (15)$$

Moreover, the tuned values of GA parameters are listed in Table 6, in which the problem size (ProS) is five products.

Table 6. Optimal values of GA parameters

Parameters	Value
POPS	150
MaxG	600
Pc	0.71
Pm	0.18

5. Computational results

To evaluate the applicability of proposed model, 16 problems under different size of vendors, stores, and products are considered in this paper. Here, to investigate the effect of vendor, store, and product variety on the objective function and computation time, these problems are classified into four classes and then are randomly designed based on the given information by[6,32]. Thus, this research utilizes their case study. The values of the parameters for each problem are received through Table 3 (the last column). Now, to examine the performance of GA, these problems are solved using GAMS/BARON 23.5 software on an Intel(R), core (TM) i7, 3.23 GHz lap top with 512 Mb RAM. The Branch-And-Reduce Optimization Navigator (BARON) algorithm developed by [32], is an algorithm for the global solution of nonlinear both NLP and MINLP. They stated that this algorithm can reduce the root-node relaxation gaps by up to 100% and expedites the solution process often by several orders of magnitude. BARON implements algorithms of branch-and-bound type enhanced with a variety of constraint propagation and duality techniques to decrease the ranges of variables in the course of algorithm [32]. GA is applied for three independent runs for each example. The best objective function value of these problems and related CPU times are considered. Then, the obtained solutions of GA and BARON are compared together, and then results are listed in Table 7.

Table 7. Comparison of BARON and GA

Class	Problem	Vendor	Store	Product	BARON		GA		Deviation	
					z ($)	CPU time(s)	Best z($)	CPU time(s)	% Dev of Obj. function	% Dev of CPU time
1	1	2	2	1	884,645	0.180	890,442	19	0.655	10455.556
	2	2	2	2	901,984	1	995,727	53	10.393	5200
	3	2	2	3	1,046,498	4	1,055,113	94	0.823	2250
	4	2	2	4	1,417,948	108	1,420,300	166	0.166	53.704
2	5	3	3	1	818,257	5	823,989	22	0.701	340
	6	3	3	2	1,024,060	49	1,027,541	67	0.340	36.735
	7	3	3	3	1,364,750	207	1,370,227	222	0.401	7.246
	8	3	3	4	2,787,128	876	2,790,058	1,697	0.105	93.721
3	9	4	1	2	269,874	3	275,444	25	2.064	733.333
	10	4	2	2	572,010	9	578,689	58	1.168	544.444
	11	4	3	2	800,267	68	804,005	145	0.467	113.235
	12	4	4	2	1,336,156	841	1,338,565	1,127	0.180	34.007
4	13	1	2	1	4,390,181	1	4,399,477	24	0.212	2300
	14	3	2	1	338,993	2	341,511	36	0.743	1700
	15	4	2	1	271,886	2	274,050	43	0.796	2050
	16	5	2	1	231,746	3	231,856	51	0.048	1600

Now, to examine the results of GA regarding to the results of the BARON, two-quality measure, the percent deviation of objective function and CPU time are defined based on the following equations:

$$\%Deviation_{Obj.} = \frac{z_{GA} - z_{BARON}}{z_{BARON}} \times 100 \quad (21)$$

$$\%Deviation_{CPU.} = \frac{CPU_{GA} - CPU_{BARON}}{CPU_{BARON}} \times 100 \quad (22)$$

The percent deviation of both objective function and CPU time for each problem has been computed. The first class includes problem 1 to 4. The number of the vendor and store are equal to two. The purpose of this class is to investigate the effect of product variety on the objective function and computation time for both two-solution methods, where the number of the vendors and stores are fixed. It can be seen that when there is one product, BARON and GA earn the lowest value in this class for both the objective function and CPU time. However, when the number of product increases from one to two products, the objective function obtained from BARON and GA increase from \$884,645 to \$901,984 and \$890,442 to \$995,727, respectively. In addition, CPU times are relatively increased and maximum values of percent deviation of objective function and CPU time in this class are 10.393 and 10455.556, respectively. Thus, it can be concluded that increasing the number of products can lead to an increase in objective function and computation time. In addition, taking into account the percent deviation of objective function, it can be observed that GA has relatively good performance while the computation time is relatively poor in this class. Fig. 3 and 4 graphically illustrate objective function values and CPU times obtained from BARON and GA, and company results obtained in this class.

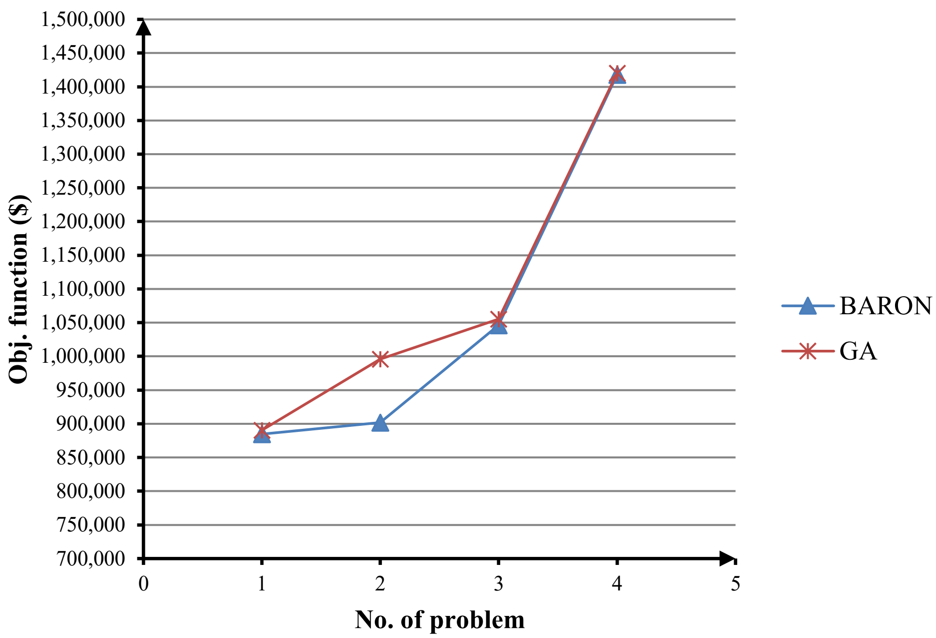

Fig. 3. Objective function values obtained from BARON and GA in class 1

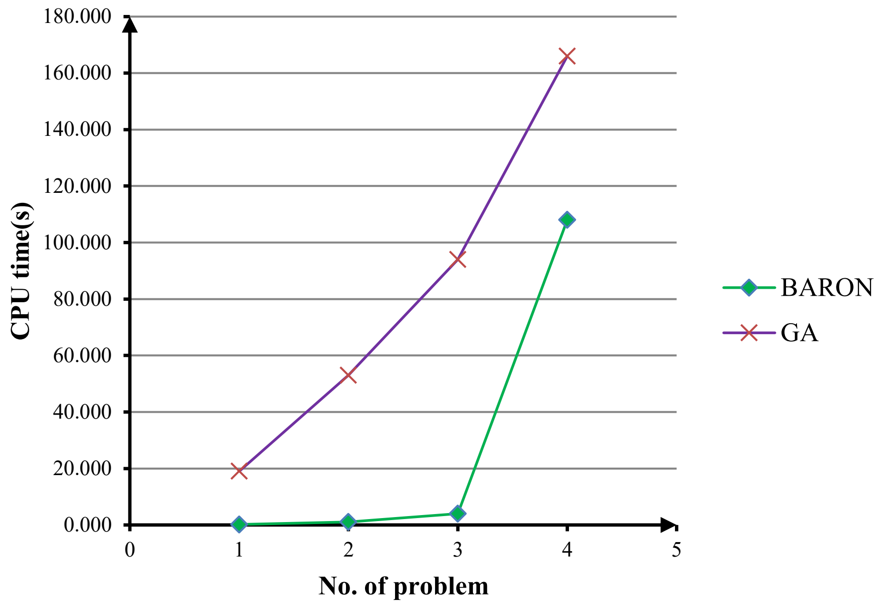

Fig. 4. CPU times obtained from BARON and GA in class 1

The second class contains problem 5 - 8. The number of the vendor and store are equal to three. The objective of this class is to investigate the effect of product variety on the objective function and computation time for both two-solution methods, where the number of the vendors and stores are fixed and increased from two to three. In this class, like the previous class, when there is one product, BARON and GA earn the lowest value in this class for both the objective function and CPU time. However, these values are lower than precious class with regard to one product. Like the previous class, when the number of product increases from one to two products, the objective function obtained from BARON and GA increase from $818,257 to $1,024,060 and from $823,989 to $1,027,541, respectively. It should be mentioned that this increase is higher than previous class. The maximum values of percent deviation of objective function and CPU time in this class are 0.701 and 340. Therefore, taking into account this and previous class, it can be seen that increasing number of products can lead to an increase in objective function and computation time value. In this class, GA has relatively good performance in objective function and computation time. Fig. 5 and 6 graphically illustrate objective function values and CPU times obtained from BARON and GA in this class.

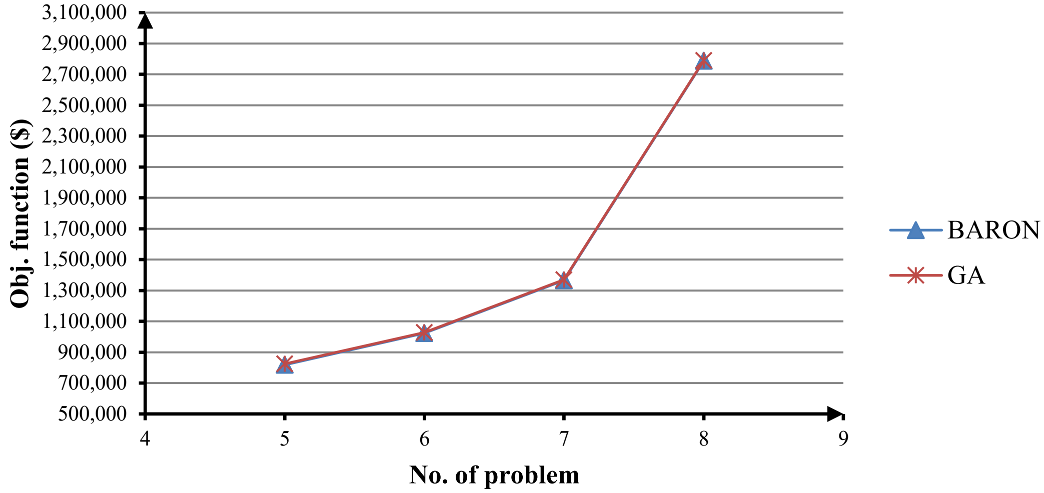

Fig. 5. Objective function values obtained from BARON and GA in class 2

For class 2 problem, BARON acts as the definitive validator. Its job is easier, and it's more likely to find and prove global optimum quickly.

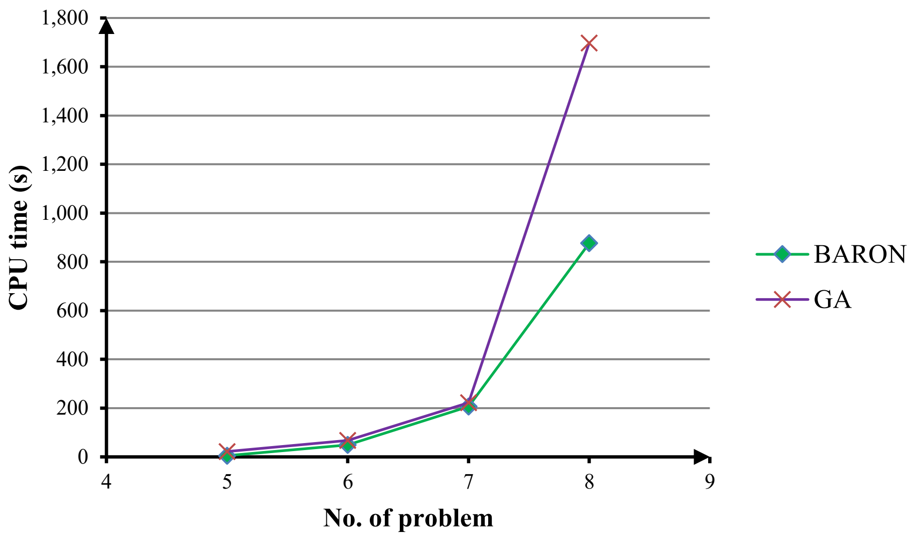

Fig. 6. CPU times obtained from BARON and GA in class 2

The third class includes problem 9 to 12. Unlike previous two classes, number of vendor and product are equal to four and two, respectively. Hence, we can investigate the influence of store variety where the number of vendors and products are fixed. In problem 9, it can be observed that BARON and GA earn lowest value for both objective function and CPU time. In this class, when number of store increases from one to two stores, the objective function obtained from BARON and GA increase from \$269,874 to \$572,010 and \$275,444 to \$578,689. Although, this change also leads to increase, it is less effective than raising number of product in increasing objective function and CPU time. Fig. 7 shows objective function obtained from BARON and GA, while Fig. 8 shows CPU time obtained from BARON and GA in this class. It can be seen that GA has relatively good performance because the maximum values of percent deviation of objective function values and CPU times are 2.064 and 733.333, respectively.

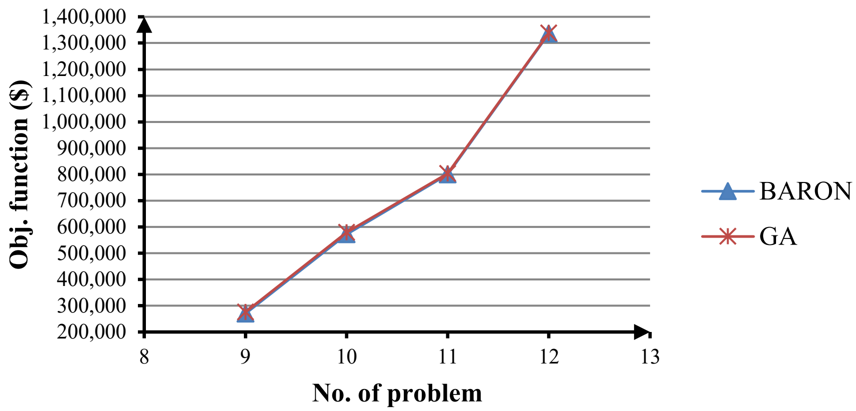

Fig. 7. Objective function values obtained from BARON and GA in class 3

When comparing BARON and GA for Class 3 problem, you are comparing proof with performance. BARON gives you the confidence of a mathematical guarantee. The GA gives you the speed of a practical, heuristic search. The best choice depends on whether your project requires absolute certainty or a fast, effective solution.

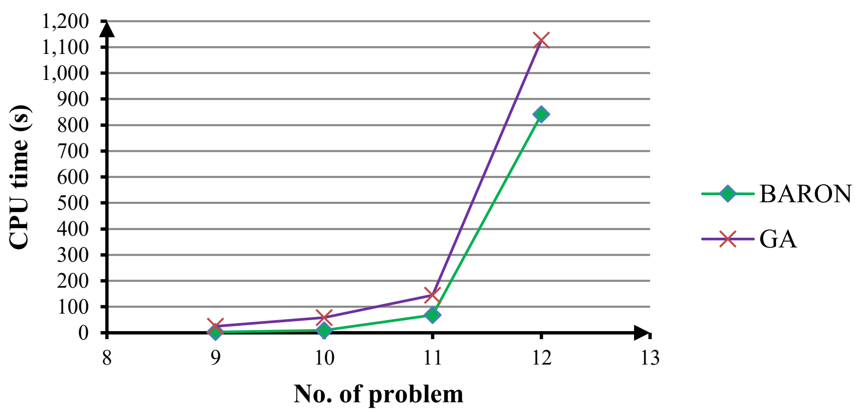

Fig. 8. CPU times obtained from BARON and GA in class 3

Finally, the fourth class contains problem 13 to 16. Unlike the previous classes, the number of store and products are equal to two and one, respectively. Then, it can be analyzed the influence of vendor variety, where number of stores and products are fixed. This class is very important because the role of multi-sourcing strategy is determined. In the beginning, unlike previous classes, BARON and GA earn highest value for objective function. When number of vendor increases from one to three vendors, the objective functions obtained from BARON and GA decrease from $4,390,181 to $338,993 and $4,399,477 to $341,511. This reduction is very noticeable, and it can be interpreted that competition between three vendors has led to decline in total cost. In other words, when there is more than one potential vendor for replenishing stores, the total cost of company can be minimized. Then, with regard to results of this class, the multi-sourcing strategy is an efficient strategy in our study. Fig.9 and Fig. 10 give an insight into the outputs of GA. Therefore, the results concompany applicability of the proposed model and solution approach.

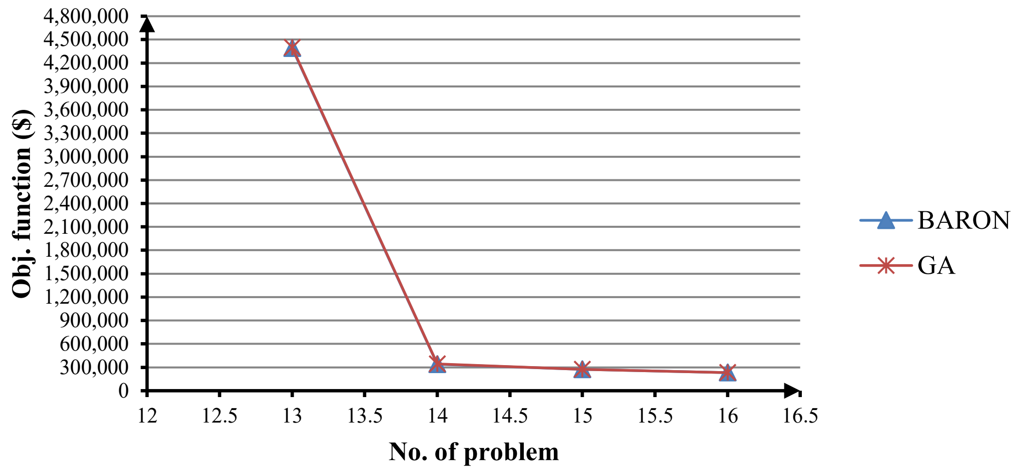

Fig. 9. Objective function values obtained from BARON and GA in class 4

For a Class 4 problem, the relationship between BARON and GA flips. GA is no longer just "fast alternative"; it is often the only viable tool for solving actual problem. BARON's role, if it has one at all, is to provide theoretical benchmark by solving simplified, idealized version of problem. You are not just comparing two solvers; you are often comparing solution to the real problem with a solution to a model of the problem.

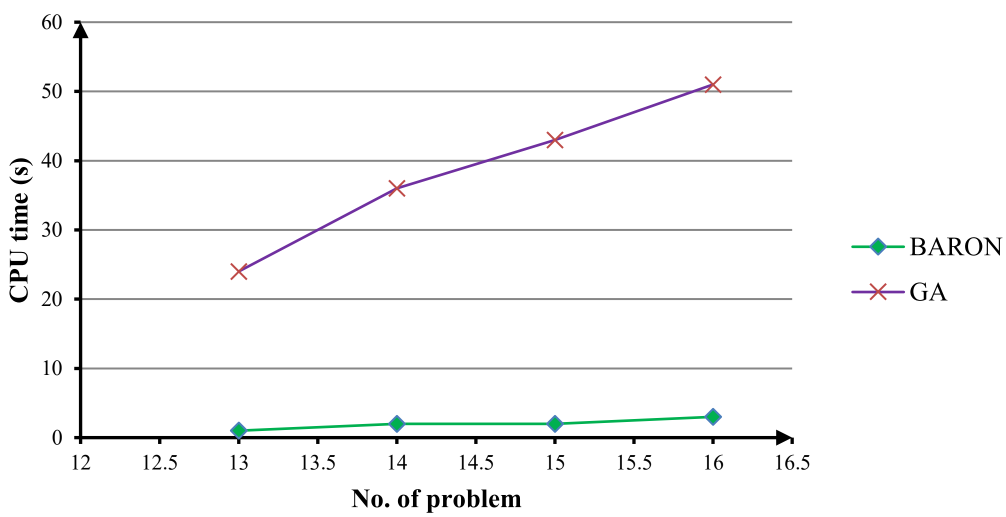

Fig. 10. CPU times obtained from BARON and GA in class 4

A long GA runtime is often sign that you are successfully tackling the true complexity of your problem, while any BARON runtime is likely spent on simplified shadow of that problem.

7. Conclusion

This paper developed mathematical programming model for VSP from potential set vendor in which there were multi-store with fuzzy demand and multi-product. It was assumed that each store operated under EOQ policy with backorder and multiple constraints. These constraints included throughput, dispatch, and budget constraints. The model originally proposed by [6, 32, 25]. In which the multi-sourcing strategy and other feature are considered. This research integrated VSP in the proposed EOQ model where the multi-sourcing strategy is established. The purpose of this research was to choose set of vendors for each product and to determine the optimal value of the related decision variables to minimize total cost. As the proposed model was NP-hard, GA was developed to solve and RSM is applied to tune GA parameters. At the end, some problems were presented and classified into four classes in order to describe sufficiency of proposed strategy and solution method. The results showed when there is more than one potential vendor for replenishing the stores, the total cost of the company can be minimized. Therefore, in this study, the multi-sourcing strategy is an efficient strategy. In addition, GA obtains good results within reasonable computational time. The computational results confirmed applicability of the proposed and solution method. The proposed MINLP model effectively integrates vendor selection with EOQ policy, incorporating fuzzy logic for demand estimation and constraints like throughput, dispatch, and budget. By adopting multi-sourcing strategy, the model demonstrates that splitting orders across multiple vendors can significantly reduce the company's total costs, making it practical and efficient approach. Numerical instances validate model's performance, with Genetic Algorithm (GA) delivering reliable results in reasonable timeframe, highlighting its applicability for real-world inventory management scenarios. The research on optimizing inventory-related decisions in outsourced SCM using Response Surface Methodology (RSM) has yielded significant results that can profoundly impact efficiency and cost reduction in SCs. Key findings of this research are as follows:

1- Optimal Modeling: The use of RSM allows for precise and comprehensive modeling of variables influencing inventory decisions. These models help managers understand the complex relationships between factors such as lead time, holding costs, and demand variability;
2- Cost Reduction: The results indicate that optimizing inventory levels through RSM can significantly reduce associated holding and ordering costs. This improvement in costs can lead to increased profit margins and ultimately enhance the financial performance of the organization;
3- Service Level Improvement (SLI): The developed models are capable of predicting and optimizing inventory levels in way that enhances service levels and increases customer satisfaction. By reducing stakeouts, organizations can ensure higher likelihood of meeting customer needs;
4- Responsiveness to Market Changes: One of the key advantages of RSM is its ability to simulate and predict SC's response to rapid market changes. This capability allows managers to quickly adapt to shifts in demand or supply conditions;
5- Practical Recommendations: Based on the findings, it is recommended that organizations utilize RSM as decision-making tool in inventory management. This approach can contribute to enhancing overall SC performance and increasing competitiveness.

In summary, this research highlights importance of optimizing inventory decisions in outsourced SCM using advanced techniques, which can lead to improved efficiency, cost savings, and better service delivery. Overall, this research underscores the benefits of multi-sourcing and advanced optimization techniques in enhancing operational efficiency. For future work extensions, the following issues are recommended:

1- It is worthwhile to examine another objective function such as service level;
2- Explore the integration of machine learning and artificial intelligence (AI) with RSM to enhance predictive accuracy and decision-making in dynamic environments;
3- Develop RSM frameworks that incorporate real-time data from IoT devices and sensors for more adaptive inventory management strategies.

Disclosure statement

The author report there are no competing interests to declare.

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