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Gradual Merging of Decision-Making Units to Achieve Pareto Efficiency

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Existing merger approaches in data envelopment analysis merge decision-making units in a single stage, but in practice, merging units may not be possible or affordable at once. We propose a finite multi-stage framework for the gradual merger of decision-making units that incorporates practical constraints. The model determines input and output contributions of the merged units at each stage and constructs a strictly increasing efficiency sequence that converges to a Pareto-efficient state. Each new unit is optimized using both input and output orientation while preserving a uniform return to scale type across stages. The framework is validated through a multi-stage merger application on a subset of Iranian banks, specifically merging several inefficient banks in two stages to ultimately yield a single efficient bank with decreasing returns to scale. Furthermore, a comparative analysis against existing methodologies confirms the superiority and robustness of the proposed framework, demonstrating that it yields highly favorable and consistent results in optimizing banking efficiency.

Keywords*: DEA, efficiency, gradual merger, return to scale, Pareto efficiency.*

1. Introduction

Decision-makers usually merge multiple units within their management scope to optimize resource utilization, efficiency and reduce costs as much as possible [8, 14]. It is accepted that the merger of units must be organized such that the efficiency of the obtained new unit is at least equal to or greater than the efficiency of the merged units, and moreover, the share of inputs and outputs from each merged unit must be clearly identified. In this regard, data envelopment analysis (DEA) is a powerful tool for evaluating performance and facilitating the merger of decision-making units (DMUs) [11, 12]. Several studies have proposed DEA-based models to merge DMUs and improve overall system efficiency. Al-Sharkas et al. [1], using the non-parametric DEA approach, showed that merging units increases system cost and profit efficiency. Lozano and Villa [19] presented DEA models to minimize costs and maximize post-merger profits. Gattoufi et al. [13] proposed a model using Inverse Data Envelopment Analysis (InvDEA) to merge two DMUs and reach a new DMU with a predefined efficiency. Subsequently, Amin et al. [2] presented an InvDEA model for merging a group of DMUs to achieve some new DMUs with predefined efficiencies, and they also used InvDEA to assess the impact of mergers on the market [3]. Amin et al. [5] proposed a flexible InvDEA-based approach for setting input and output targets in the merger of DMUs. Moreover, Amin and Boamah [4] proposed a merger model utilizing InvDEA and cost-efficiency. Soltanifar et al. [23] presented a model for merging DMUs with negative data, and Khoveyni and Eslami [17], using InvDEA, introduced a model for merging DMUs with a two-stage series network structure. Pyra and Siedlecka [20] investigated the potential advantages of merging universities in Poland using DEA models. Lozano and Contreras [18] applied DEA to evaluate the Spanish public university system

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and examined the potential outcomes of university mergers across various regions. Soltanifar et al. [22] proposed a goal programming model based on the InvDEA framework to merge multiple groups of DMUs simultaneously. Recently, Fakharzadeh and Rostamzadeh introduced new non-radial models for mergers and acquisitions to achieve strong efficiency and the most productive scale size without altering the efficient frontier [11].

While all previous merger studies assume a one-stage process for achieving an efficient unit, in reality, it is often difficult or even impossible to reduce inputs and increase outputs to reach an efficient unit in a single stage.

To achieve this aim, the presented paper introduces a new approach to merge units gradually over several finite stages, ultimately leading to a Pareto-efficient unit; in this process, the share of inputs and outputs for each merged unit is determined at each stage. Moreover, the new units obtained from the merger at all stages exhibit the same returns to scale. The proposed method is both input-oriented and output-oriented and guarantees that the optimal inputs and outputs for the new unit are simultaneously achieved.

In this regard, Section 2 presents the required concepts of DEA, Section 3 focuses on the main discussion including the new models and a gradual merger algorithm, Section 4 provides a real example to illustrate the proposed approach, and the conclusion section finishes the study.

2. Some concepts of DEA

DEA researchers have proposed many models to evaluate the decision-making unit (DMU) within the production possibility set (PPS). In this regard, Charnes et al. in 1978 introduced the CCR model to evaluate DMU in PPS with the property of constant returns to scale ($PPS_{(CRS)}$) [7]. Later, Banker et al. in 1984 presented the BCC model, in which the production possibility set has the property of variable returns to scale ($PPS_{(VRS)}$) [6]. In $PPS_{(CRS)}$, outputs change proportionally with inputs. While in $PPS_{(VRS)}$ the units can also exhibit increasing or decreasing returns to scale. In $PPS_{(VRS)}$ with increasing returns to scale, outputs grow faster than inputs; and with decreasing returns to scale, outputs grow more slowly than inputs [9].

Definition 2.1. Let $X_j = (x_{1j}, x_{2j}, \dots, x_{mj}) > 0$ and $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj}) > 0$ be the input and output vectors of DMU_j for $j = 1,2, \dots, n,$, respectively, PPS which is introduced by Banker et al. [6] are displayed as $PS_{(VRS)} = \{(X,Y) | \sum_{j=1}^{n} \lambda_j X_j \leq X, \sum_{j=1}^{n} \lambda_j Y_j \geq Y, 1\lambda = 1, \lambda_j \geq 0\}$.

The multiplicative and covering models of BCC for evaluating $DMU_P = (X_P, Y_P)$ are defined as follows [9]:

$$\left\{ MaxU^tY_p - u_0 \middle| \begin{matrix} U^tY_j - V^tX_j - u_0 \leq 0, j = 1,2, \dots n; \\ V^tX_p = 1, U \geq 1\varepsilon, V \geq 1\varepsilon \end{matrix} \right\}, \tag{1}$$

$$\theta = \{Min(\theta - \varepsilon(1s^- + 1s^+)) | (\theta X_P - s^-, Y_p + s^+) \in PPS_{(VRS)}\}. \tag{2}$$

Definition 2.2. A unit (X, Y) is strongly efficient or Pareto-Koopmans efficient, if there does not exists a unit $(\bar{X}, \bar{Y}) \in PPS_{(VRS)}$, such that $\bar{X} \leq X, \bar{Y} \geq Y$ and $(\bar{X}, \bar{Y}) \neq (X, Y)$.

Theorem 2.3. Suppose ρ^* and $(\theta^*, s^{-*}, s^{+*})$ are the optimal value of models (1) and (2) in the evaluation of DMU_P respectively, if $\rho^* = 1$ and $(\theta^* = 1, s^{-*} = 0, s^{+*} = 0)$, then DMU_P is Pareto-Koopmans efficient.

Proof. Refer to [9].

Theorem 2.4. Suppose (U^*, V^*, u_0^*) is an optimal solution of (1) in the evaluation of DMU_P, then $H = \{(X,Y) | U^{*t}Y - V^{*t}X - u_0^* = 0\}$ is a supporting hyperplane of $PPS_{(VRS)}$. Moreover, if $DMU_t = (X_t, Y_t) \in H$ $(t \in \{1,2,\dots,n\})$, then DMU_t is Pareto-efficient.

Proof. Refer to [9].

Theorem 2.5. Suppose (U^*, V^*, u_0^*) is an optimal solution of model (1) in the evaluation of $\mathrm{DMU_P}$; in this case

1.If $u_0^* = 0$, then DMU_P has constant returns to scale.
2.If $u_0^* > 0$, then DMU_P has decreasing returns to scale.
3.If $u_0^* < 0$, then DMU_P has increasing returns to scale.
Proof. Refer to [9].

Remark 2.6. In determining the return to scale of DMU_P if model (1) has multiple optimal solutions and (U^*, V^*, u_0^*) is one of its optimal solutions, such that $u_0^* < 0 (u_0^* > 0)$, then in Theorem 2.5, one can replace model (1) with model (3) when DMU_P is efficient, and one can replace model (1) with model (4) when DMU_P is inefficient [9].

$$\left\{ Max\ (Min) u_0 \left| \begin{array}{l} U^t Y_j - V^t X_j - u_0 \le 0, j = 1,2,\dots n; \\ U^t Y_p - u_0 = 1, V^t X_p = 1; \\ u_0 \le 0 (u_0 \ge 0); \\ U \ge 1\varepsilon, V \ge 1\varepsilon; \end{array} \right. \right\} \tag{3}$$

$$\left\{ Max\ (Min) u_0 \left| \begin{array}{l} U^t Y_j - V^t X_j - u_0 \le 0, j = 1,2,\dots n; \\ U^t Y_{\bar{p}} - u_0 = 1, V^t X_{\bar{p}} = 1; \\ u_0 \le 0 (u_0 \ge 0); \\ U \ge 1\varepsilon, V \ge 1\varepsilon; \\ \left(\bar{X}_P, \bar{Y}_P\right) = (\theta^* X_P - s^{-*}, Y_P - s^{+*}). \end{array} \right. \right\} \tag{4}$$

In model (4), $(\theta^*, s^{-*}, s^{+*})$ is the optimal solution of (2) in evaluating of the inefficient DMU_P, and $(\bar{X}_P, \bar{Y}_P) = (\theta^* X_P - s^{-*}, Y_P - s^{+*})$ is its improvement unit.

Corollary 2.7. The returns to scale of each DMU in $PPS_{(VRS)}$ align with its improvement DMU.
Proof. Refer to [9].

3. The gradual merger of units

To enhance company profits and productivity, decision-makers often attempt to merge units under their control. Typically, such mergers are done with the assumption that input and output factors are combined. However, this simple method does not always result in an efficient unit ([11]); even sometimes, merging two efficient units does not necessarily lead to a single efficient one ([11]). In fact, merging units is successful when the obtained new unit is efficient or, ideally, Pareto-efficient. Moreover, sometimes, in the merger process, it is not possible to achieve a Pareto-efficient unit in one stage because reducing inputs and increasing outputs of the merged unit in one stage can be difficult and sometimes impossible in practice. For example, merging two banks in a single stage may result in substantial workforce reductions, leading to employee dissatisfaction and organizational unrest. Hence, to create a Pareto-efficient unit, the decision-makers may need to merge units in several stages so that in all the stages, the return to scale type of all obtained new units would be the same. Therefore, in this section, to merge units, we present an algorithm in which, according to the opinion of the managers the units are gradually merged in several stages, so that, at each stage, the share of the inputs and outputs of each merged unit is determined. In this approach, a finite sequence is formed from the obtained units from the merger, in which the efficient score of each unit is higher than the previous one, and all merged units have one type of return to scale; eventually, this sequence ends to a Pareto-efficient unit.

Suppose that the set K contains units that we incline to merge, and $DMU_M^0 = (X_M^0,\ Y_M^0) = (\sum_{k\in K} X_k, \sum_{k\in K} Y_k)$ is the initial integration of all units in K. We add DMU_M^0 to the set $\{PPS_{(VRS)} - K\}$, and evaluate DMU_M^0 by using model (1). If the optimal value of (1) is equal to one, then DMU_M^0 is Pareto-efficient, and it is a suitable unit for us; then according to Theorem 2.5, its return to scale type is determined. But, if DMU_M^0 is not Pareto-efficient, in the sequel, we introduce a merger algorithm to derive a Pareto-efficient unit through the gradual merger of units in K in several periods.

For this end, assume that the newly formed unit from merging all units in K at period $t \geq 1$ is denoted by $DMU_M^t = (X_M^t,\ Y_M^t) = (x_{iM}^{t-1}\delta_i^t, y_{rM}^{t-1}\partial_r^t)$, such that for $i = 1,2,\ldots,m$ and $r = 1,2,\ldots,s$, $\delta_i^t \in [0,1]$ and $\partial_r^t \in [1,\infty)$ are variables; hence, DMU_M^t dominates DMU_M^{t-1}. By removing the merged units in K from $PPS_{(VRS)}$ and adding DMU_M^f for $f = 1,2,\ldots,t-1$ to the $PPS_{(VRS)}$, the production possibility set at stage t is equal to

$$PPS_{(VRS)}^{t.new} = \{(X^t, Y^t): \sum_{j\in E_M} \lambda_j X_j + \sum_{f=0}^{t-1} \lambda_f X_M^f \leq X^t, \sum_{j\in E_M} \lambda_j Y_j + \sum_{f=0}^{t-1} \lambda_f Y_M^f \geq Y^t, \sum_{j\in E_M} \lambda_j + \sum_{f=0}^{t-1} \lambda_f = 1, \lambda_j \geq 0, \lambda_f \geq 0, j \in E_M, f = 0,1,\ldots,t-1\}, \quad (5)$$

where $E_M = PPS_{(VRS)} - K$. Now, to obtain $DMU_M^t \in PPS_{(VRS)}^{t.new}$ in step $t \geq 1$, so that DMU_M^t achieves a maximum percentage improvement in the i-th input (l_i^t) and the r-th output (b_r^t) compared to DMU_M^{t-1}, while maintaining the same returns to scale as DMU_M^0. We optimize levels of inputs $x_{iM}^{t-1}\delta_i^t (i = 1,2,\ldots,m)$ and outputs $y_{rM}^{t-1}\partial_r^t (r = 1,2,\ldots,s)$, by introducing the following two objective model.

$$
\begin{aligned}
&Min \sum_{i=1}^{m} x_{iM}^{t-1}\delta_i^t \\
&Max \sum_{r=1}^{s} y_{rM}^{t-1}\partial_r^t \\
&S.to\colon \sum_{j\in E_M}\lambda_j x_{ij} + \sum_{f=0}^{t-1}\lambda_f x_{iM}^{f} \le x_{iM}^{t-1}\delta_i^t, \qquad i = 1,2,\dots,m; (a) \\
&\sum_{j\in E_M}\lambda_j y_{rj} + \sum_{f=0}^{t-1}\lambda_f y_{rM}^{f} \ge y_{rM}^{t-1}\partial_r^t, \qquad r = 1,2,\dots,s; (b) \\
&\sum_{j\in E_M}\lambda_j + \sum_{f=0}^{t-1}\lambda_f = 1, \qquad \lambda_j \ge 0, j \in E_M, \lambda_f \ge 0, f = 0,1,\dots,t-1;\ (c) \\
&\frac{\sum_{r=1}^{s} u_r^* y_{rM}^{t-1}\partial_r^t - u_0^*}{\sum_{i=1}^{m} v_i^* x_{iM}^{t-1}\delta_i^t} \le 1; (d) \\
&(1 - l_i^t) \le \delta_i^t \le 1, i = 1,2,\dots,m; (e) \\
&1 \le \partial_r^t \le (b_r^t + 1), r = 1,2,\dots,s, (f)
\end{aligned}
\tag{6}
$$

In model (6), $PPS_{(VRS)}^{t.new}$ is characterized by the three constraints labeled (a), (b), and (c). Also, (U^*, V^*, u_0^*) is the optimal solution of model (1) in the evaluation of DMU_M^0 with considering $PPS_{(VRS)}^{1.new}$ as the production possibility set; if model (1) has more than one optimal solution, then model (4) is used instead. According to Theorem 2.4, $H = \{(X,Y)|U^{*t}Y - V^{*t}X - u_0^* = 0\}$ is a supporting hyperplane of $PPS_{(VRS)}^{1.new}$. Now, in order to, the set $PPS_{(VRS)}^{t.new}$ lies entirely below the hyperplane H and $DMU_M^t = (X_M^t,\ Y_M^t) = (x_{iM}^{t-1}\delta_i^t, y_{rM}^{t-1}\partial_r^t) \in PPS_{(VRS)}^{t.new}$ for $t \ge 1$, the relation $\sum_{r=1}^{s} u_r^* y_{rM}^{t-1}\partial_r^t - \sum_{i=1}^{m} v_i^* x_{iM}^{t-1}\delta_i^t - u_0^* \le 0$ must be hold; so, we incorporate condition (d) as $\frac{\sum_{r=1}^{s} u_r^* y_{rM}^{t-1}\partial_r^t - u_0^*}{\sum_{i=1}^{m} v_i^* x_{iM}^{t-1}\delta_i^t} \le 1$ into model (6). In constraints (e) and (f) of model (6), for all *i* and *r*, l_i^t and b_r^t specify the maximum of allowable decrease of *i*-th input and allowable increase of *r*-th output of DMU_M^t at stage *t* relative to stage *(t-1)*, i.e., $0 \le \frac{x_{iM}^{t-1} - x_{iM}^{t-1}\delta_i^t}{x_{iM}^{t-1}} \le l_i^t\ (i = 1,2,\dots,m)$ and $0 \le \frac{y_{rM}^{t-1}\partial_r^t - y_{rM}^{t-1}}{y_{rM}^{t-1}} \le b_r^t\ (r = 1,2,\dots,s)$. For all *i* and *r*, l_i^t and b_r^t are determined by the decision-makers and guarantee that DMU_M^t inputs are lower than the inputs of DMU_M^{t-1} and DMU_M^t outputs are greater than the outputs of DMU_M^{t-1} as well; this leads to that DMU_M^t dominates DMU_M^{t-1} and also $\rho^{*t} \ge \rho^{*t-1} (t \ge 1)$. Consequently, model (6) produces a finite sequence of merged units in *K* with the same return to scale, ultimately leading to a Pareto-efficient unit.

3.1. The obtained results in stage t

Now, suppose for all i and r, δ_i^{*t} and ∂_r^{*t} are the optimal solutions of (6) in stage *t*, then the following results in stage *t* are obtained:

1. The obtained new unit from merger in stage t is $DMU_M^t = (\delta_i^{*t} x_{iM}^{t-1}, \partial_r^{*t} y_{rM}^{t-1})$. Also, for all i and r, if $l_i^t = 1$ and $b_r^t = 1$, then, $DMU_M^t = (\delta_i^{*t} x_{iM}^{t-1}, \partial_r^{*t} y_{rM}^{t-1}) \in H$ and it is located on the efficient frontier; because the i-th input and the r-th output, respectively, are allowed to have maximum decrease and increase at stage t relative to stage $(t-1)$.
2. For $t \geq 1$, $\rho^{*t} = \frac{\sum_{r=1}^{s} u_r^* y_{rM}^{t-1} \partial_r^t - u_0^*}{\sum_{i=1}^{m} v_i^* x_{iM}^{t-1} \delta_i^t} = \frac{\sum_{r=1}^{s} u_r^* y_{rM}^t - u_0^*}{\sum_{i=1}^{m} v_i^* x_{iM}^t}$ is the cross-efficiency score of DMU_M^t under the evaluation of DMU_M^0 ([9]). If $\rho^{*t} = 1 (t \geq 1)$, then $DMU_M^t \in H$ and according to Theorem 2.4, DMU_M^t is Pareto-efficient. Since for $i = 1,2,\ldots,m$ and $r = 1,2,\ldots,s$, $\delta_i^t \in [0,1]$ and $\partial_r^t \in [1,\infty)$, we have $1 \geq \rho^{*t} \geq \rho^{*t-1}$; therefore, the efficiency value of the obtained new unit in stage t is higher than the efficiency value of the obtained unit in stage $(t-1)$ and is closer to the efficient frontier ([9]).
3. In stage t, the share of i-th input and r-th output of each unit $k \in K$ in the merger is $\delta_i^{*t} x_{ik}^{t-1}$ and $\partial_r^{*t} y_{rk}^{t-1}$, respectively.
4. Since the inputs and outputs of all units are positive, and for all i and r, $\delta_i^t > 0$ and $\partial_r^t > 0$, therefore, the values of $\delta_i^{*t} x_{ik}^{t-1}$ and $\partial_r^{*t} y_{rk}^{t-1}$ for all i and r always are positive, hence, surely DMU_k^t will have a share of the merger and the contribution of each merged entity in the merger process will never be zero.

Proposition 3.1. Model (6) in stage t is feasible.

Proof. Since $(x_{iM}^{t-1}, y_{rM}^{t-1}) \in PPS_{(VRS)}^{t.new}$, for $j \in E_M$ and $f = 0,1,\ldots,t-1$, there exists $\bar{\lambda}_j$ and $\bar{\lambda}_f$ such that

$$\begin{aligned} &\sum_{j \in E_M} \bar{\lambda}_j x_{ij} + \sum_{f=0}^{t-1} \bar{\lambda}_f x_{iM}^f \leq x_{iM}^{t-1}, i = 1,2,\ldots,m, \\ &\sum_{j \in E_M} \bar{\lambda}_j y_{rj} + \sum_{f=0}^{t-1} \bar{\lambda}_f y_{rM}^f \geq y_{rM}^{t-1}, r = 1,2,\ldots,s, \\ &\sum_{j \in E_M} \bar{\lambda}_j + \sum_{f=0}^{t-1} \bar{\lambda}_f = 1, \bar{\lambda}_j \geq 0, \bar{\lambda}_f \geq 0, j \in E_M \text{ and } f = 0,1,\ldots,t-1. \end{aligned} \tag{7}$$

Also, according to definition of ρ^{*t-1} in stage $t-1$, we have

$$\rho^{*t-1} = \frac{\sum_{r=1}^{s} u_r^* y_{rM}^{t-1} - u_0^*}{\sum_{i=1}^{m} v_i^* x_{iM}^{t-1}} \leq 1. \tag{8}$$

Relations (7) and (8) show that for $j \in E_M, f = 0,1,\ldots,t-2$, and all i, r, $\left(\bar{\lambda}_j = 0, \bar{\lambda}_f = 0, \bar{\lambda}_{t-1} = 1, \bar{\delta}_i^t = 1, \bar{\partial}_r^t = 1\right)$ is a feasible solution for (6). ■

Proposition 3.2. In model (6), for $i = 1,2,\ldots,m$ and $r = 1,2,\ldots,s$, $(x_{iM}^{t-1}, y_{rM}^{t-1}) (t \geq 1)$ is Pareto-efficient if and only if $\delta_i^{*t} = 1$ and $\partial_r^{*t} = 1$.

Proof. By contradiction, suppose that for $i = 1,2,\ldots,m$ and $r = 1,2,\ldots,s$, $(x_{iM}^{t-1}, y_{rM}^{t-1}) (t \geq 1)$ is not Pareto-efficient, therefore, according to Definition 2.2, there exists $(x_{iM}^t, y_{rM}^t) = (\delta_i^{*t} x_{iM}^{t-1}, \partial_r^{*t} y_{rM}^{t-1}) \in PPS_{(VRS)}$, such that for $\delta_i^{*t} \in [0,1]$ and $\partial_r^{*t} \in [1,\infty)$, $x_{iM}^t \leq x_{iM}^{t-1}, y_{rM}^t \geq y_{rM}^{t-1}$

and $(x_{iM}^{t}, y_{rM}^{t}) \neq (x_{iM}^{t-1}, y_{rM}^{t-1})$. This implies that for some $i \in \{1,2,\dots,m\}$ or $r \in \{1,2,\dots,s\}$, we have $\delta_i^{*t} \neq 1$ or $\partial_r^{*t} \neq 1$, which contradicts the assumption.

Conversely: now, if there exists $z \in \{1,2,\dots,m\}$ such that $\delta_z^{*t} < 1$ and for $i \neq z$ and all r, $\delta_i^{*t} = 1$ and $\partial_r^{*t} = 1$, then $(\delta_i^{*t} x_{iM}^{t-1}, \partial_r^{*t} y_{rM}^{t-1})$ dominates $(x_{iM}^{t-1}, y_{rM}^{t-1})$, which is in contradiction with the Pareto-efficiency of $(x_{iM}^{t-1}, y_{rM}^{t-1})$, hence, for $i = 1,2,\dots,m,$ $\delta_i^{*t+1} = 1$. Similarly, one can prove that for all r, $\partial_r^{*t+1} = 1$. ■

Theorem 3.3. The return to scale of the obtained $DMU_M^t (t \geq 1)$ by model (5) is the same as DMU_M^0.

Proof. Suppose that $\rho^{*t} = 1$, hence, based on the definition of $\rho^{*t} = \frac{\sum_{r=1}^{s} u_r^* y_{rM}^t - u_0^*}{\sum_{i=1}^{m} v_i^* x_{iM}^t}$, we have $\sum_{r=1}^{s} u_r^* y_{rM}^t + \sum_{i=1}^{m} v_i^* x_{iM}^t - u_0^* = 0$, therefore, $DMU_M^t \in H$ and, according to Theorem 2.5, its return to scale type is the same with DMU_M^0.

Now, if $\rho^{*t} < 1$, then there exists q, such that the obtained efficient $DMU_M^{(t+q)} = (x_{iM}^{t+q}, y_{rM}^{t+q})$ via model (6), is an improvement unit of DMU_M^t ($\delta_i^{*t+q-1} = 1$ and $\partial_r^{*t+q-1} = 1$). Consequently, $\rho^{*t+q} = \frac{\sum_{r=1}^{s} u_r^* y_{rM}^{t+q} - u_0^*}{\sum_{i=1}^{m} v_i^* x_{iM}^{t+q}} = 1$ and according to constraint (d), we have $\sum_{r=1}^{s} u_r^* y_{rM}^{(t+q)} - \sum_{i=1}^{m} v_i^* x_{iM}^{(t+q)} - u_0^* = 0$. Therefore, $DMU_M^{t+q} \in H$, and its return to scale is the same as DMU_M^0. As stated in Corollary 2.7, the return to scale of DMU_M^{t+q} is the same as DMU_M^t, which implies the return to scale of DMU_M^t is the same as DMU_M^0. ■

Now, to solve the two-objective problem (6), we assign weights $w_i^t > 0$ and $w_r^t > 0$ to i-th input and r-th output, such that $\sum_{i=1}^{m} w_i^t + \sum_{r=1}^{s} w_r^t = 1$. By doing this, the two-objective model (6) can be transformed into a single-objective problem ([10, 16, 21]). These weights are considered based on the importance of indicators (inputs and outputs) in stage *t*. Since in model (6), v_i^*, x_{iM}^t and δ_i^t for $t \geq 1$ and all i, are positive, we have $\sum_{i=1}^{m} v_i^* x_{iM}^{t-1} \delta_i^t > 0$, therefore the fractional constraint (d) of (6) can be converted into a linear constraint. Accordingly, model (6) is converted to the following model:

$$
\begin{aligned}
&Min\left(\sum_{i=1}^{m} w_i^t x_{iM}^{t-1}\delta_i^t - \sum_{r=1}^{s} w_r^t y_{rM}^{t-1}\delta_r^t\right)\\
&S.to\colon \sum_{j\in E_M}\lambda_j x_{ij} + \sum_{f=0}^{t-1}\lambda_f x_{iM}^f \le x_{iM}^{t-1}\delta_i^t, i = 1,2,\dots,m; (a)\\
&\sum_{j\in E_M}\lambda_j y_{rj} + \sum_{f=0}^{t-1}\lambda_f y_{rM}^f \ge y_{rM}^{t-1}\partial_r^t, r = 1,2,\dots,s; (b)\\
&\sum_{j\in E_M}\lambda_j + \sum_{f=0}^{t-1}\lambda_f = 1, \lambda_j \ge 0, j\in E_M, \lambda_f \ge 0, f = 0,1,\dots,t-1; (c)\\
&\sum_{r=1}^{s} u_r^* y_{rM}^{t-1}\partial_r^t - \sum_{i=1}^{m} v_i^* x_{iM}^{t-1}\delta_i^t \le u_0^*; (d)\\
&(1-l_i^t) \le \delta_i^t \le 1, i = 1,2,\dots,m; (e)\\
&1 \le \delta_r^t \le (b_r^t + 1), r = 1,2,\dots,s. (f)
\end{aligned}
\tag{9}
$$

Now, to gradually merge the units in *K*, based on the above discussions, we present the following algorithm.

Algorithm 3.4.

Step 1: Evaluate $DMU_M^0 = (\sum_{k\in K} X_k, \sum_{k\in K} Y_k)$ in $PPS_{(VRS)}^{1.new}$ by model (1). If DMU_M^0 is efficient, stop, otherwise, considering the optimal solution (U^*, V^*, u_0^*) in model (1) (if model (1) has multiple optimal solutions, then use model (4)) and go to step 2.

Step 2: Put $t = 1$;

Step 3: For all i and r, consider the values for l_i^t, b_r^t, w_i^t and w_r^t;

Step 4: Solve model (9);

Step 5: Let for all *i* and *r*, δ_i^{*t} and ∂_r^{*t} be the optimal solutions of model (9). If $\rho^{*t} = \frac{\sum_{r=1}^{s} u_r^* \partial_r^{*t} y_{rM}^{t-1} - u_0^*}{\sum_{i=1}^{m} v_i^* \delta_i^{*t} x_{iM}^{t-1}} = 1$, then stop and $DMU_M^t = (X_M^t, Y_M^t)$ is Pareto-efficient; also the share of i-th input and *r*-th output of each unit $k \in K$ from the merger is $\delta_i^{*t} x_{ik}^{t-1}$ and $\partial_r^{*t} y_{rk}^{t-1}$, respectively. Otherwise go to step 6;

Step 6: Put $(X_M^t, Y_M^t) \to (X_M^{t-1}, Y_M^{t-1})$ and $t \to t-1$, then go to step 3.

Theorem 3.5. (Convergence). Algorithm 3.4 ends after a finite number of iterations.

Proof. The feasible region of (9) is bounded (in the sequel, we prove this claim), and since for $i = 1,2,\dots,m$ and $r = 1,2,\dots,s$, $\delta_i^{*t} \in [0,1]$ and $\delta_r^{*t} \in [1,\infty)$, we have $0 < \rho^{*t} = \frac{\sum_{r=1}^{s} u_r^* \partial_r^{*t} y_{rM}^{t-1} - u_0^*}{\sum_{i=1}^{m} v_i^* \delta_i^{*t} x_{iM}^{t-1}} =$

$\frac{\sum_{r=1}^{s} u_r^* y_{rM}^t - u_0^*}{\sum_{i=1}^{m} v_i^* x_{iM}^t} \leq 1$, which increases with increasing *t*; finally, for a $t \geq 1$, the unit $DMU_M^t = (X_M^t, Y_M^t)$ lies on the efficient frontier and becomes Pareto-efficient. Therefore Algorithm 3.4 ends after a finite number of iterations.

By contradiction, we assume that the feasible set of (9) is unbounded; thus, there exists a vector $d^t \neq 0$, such that for each $x_0 = (\lambda_j, \lambda_f, \delta_i^t, \partial_r^t), j \in E_M, f = 0, 1, \ldots, t-1, i = 1,2,\ldots,m, r = 1,2,\ldots,s$ in the feasible set of (9), the ray $\{x_0 + zd | z \geq 0\}$ is located in the feasible area of (9). According to the constrains (e) and (f) of (9), we have:

$$\frac{(1 - l_i^t) - \delta_i^t}{z} \leq d_i^t \leq \frac{1 - \delta_i^t}{z}, i = 1,2,\ldots,m, \quad (10)$$

$$\frac{1 - \partial_r^t}{z} \leq d_r^t \leq \frac{(b_r^t + 1) - \partial_r^t}{z}, r = 1,2,\ldots,s. \quad (11)$$

In (10) and (11), if $z \to \infty$, then for all i and r, $d_i^t = 0$ and $d_r^t = 0$. Also, according to the constraint (c), we have $\sum_{j \in E_M} d_j^t = \frac{1 - \sum_{j \in E_M} \lambda_j}{z}, d_j^t \geq \frac{\lambda_j}{z}, j \in E_M$ which implies that if $z \to \infty$, then $d_j^t = 0$ for $j \in E_M$. Consequently $d^t = 0$, which is in contradiction with the assumption that $d^t \neq 0$. ■

4. Gradual merger of Iranian banks

In this section, first, we obtain the efficiency of 24 Iranian banks in 2023 based on Iran Banking Institute data [15], Then we apply Algorithm 3.4 from the previous section to the gradual merging of several banks from the set of Iranian banks in 2023, where the list of them is shown in Table 1. Like [13], [4] and [11], the considered inputs and outputs for analyzing in this study are interest expenses (X1), non-interest expenses (X2), interest income (Y1) and non-interest income (Y2). Interest expenses include expenses for deposits and other borrowed funds, expenses of commissions and doubtful receivables, expenses of investment and transactions in foreign currency and income tax cost, non-interest expenses include the costs of personnel salaries, administrative and general expenses, depreciation expenses, and other expenses. Also, interest income includes income from loans and deposits, income from investments and foreign exchange transactions, while, the non-interest income includes service charges on loans and transactions, commissions income, and other operating income. To find the efficiency of Iranian banks, we calculate the efficiency scores of banks by using the BCC model (1), which the results are shown in Table 1.

Table 1. Data and efficiency scores of the Iranian banks in 2023. (Data are based on billion Rials)

Bank number	Interest expenses (X1)	Non-interest expenses (X2)	Interest income (Y1)	Non-interest income (Y2)	Efficiency percent
1	711482.527	45249.549	226356.546	239986.029	100
2	129028.859	9382.970	121179.188	22655.183	100
3	225255	28228	264080	27221	83
4	8490	18990	19470	26692	100
5	2204.575	1007.318	7928.942	2792.595	100
6	60750.948	15063.139	80392.267	8932.198	76

7	304626	132806	405540	31936	89
8	43141.863	8552	76340.561	5241.414	100
9	221868	81376	291551	11728	87
10	74666	8393	60296	13240	72
11	143056	37841	188743	24344	87
12	497797	50153	677474	61924	100
13	27110	12099	41263	10073	80
14	278569	118634	416853	156882	100
15	401543.143	284473.810	547655.540	142832.038	94
16	102146.712	20302.397	121038.468	23277.629	83
17	85922	8193	8362	4960	17
18	241734	42097	226737	86158	100
19	52514	27074.800	76939.300	13842.300	83
20	504991.133	209525.971	716856.560	126437.353	100
21	43312.525	9510	52063.884	3106.59	66
22	42291.475	22703.091	53285.170	11731.826	64
23	10438	19681	28980	18423	100
24	142006.668	20891.127	49078.838	4059.780	22

According to the obtained efficiency scores, among 24 banks, 10 banks are efficient and the rest are inefficient. As mentioned, one way to reduce the number of inefficient units is to merge them. Here, for instance, we merge the inefficient Banks 19, 21, 22, and 24 using the presented Algorithm 3.4 until a Pareto-efficient bank is obtained. The total integration of the inputs and the outputs of these banks are $X_M^0 = (280124.6678, 80179.0181)$ and $Y_M^0 = (231367.1919, 32740.4961)$, respectively. Using model (1), we evaluate $Bank_M^0 = (X_M^0, Y_M^0)$. The optimal solution of (1) is

$$(U^*, V^*, u_0^*) = (2.5747 \times 10^{-6}, 2.22 \times 10^{-16}, 3.3264 \times 10^{-6}, 8.5020 \times 10^{-7}, 0.045776)$$

and the efficiency score of $Bank_M^0 = (X_M^0, Y_M^0)$ is 0.5499.
Therefore, $Bank_M^0 = (X_M^0, Y_M^0)$ is inefficient. Since $u_0^* = 0.045776 > 0$, according to Theorem 2.5, $Bank_M^0 = (X_M^0, Y_M^0)$ has decreasing returns to scale. Also, based on Theorem 2.4,

$$H = \{(X, Y) | 2.5747 \times 10^{-6} y_1 + 2.22 \times 10^{-16} y_1 - 3.3264 \times 10^{-6} x_1 - 8.5020 \times 10^{-7} x_2 - 0.045776 = 0\}$$

is a supporting hyperplane of $PPS_{(VRS)}$ corresponding to $Bank_M^0 = (X_M^0, Y_M^0)$. The efficiency score of $Bank_M^0 = (X_M^0, Y_M^0)$ is lower than the efficiency score of Banks 19, 20, and 21. Now, we apply Algorithm 3.4 to merger gradually Banks 19, 21, 22, and 24 in two stages, and obtain the contribution of each bank in each stage of the merger based on Result 3 of Subsection 3.1.

Notice that, according to Theorem 3.3, in each stage, the return to scale of the obtained new unit is the same as $Bank_M^0$, and it is decreasing. In Table 2, the obtained results from the merger in each stage are shown.

Table 2. Inputs, outputs and efficiency of Banks 19, 21, 22, and 24 after the merger in two stages.

Stage 1 (t=1)	$l_1^1 = 0.25, l_2^1 = 0.05, b_1^1 = 0.10, b_2^1 = 0.08, and\ w^1 = (0.25,0.25,0.25,0.25)$				
Bank number	Input X1 after the merger	Input X2 after the merger	Output Y1 after the merger	Output Y2 after the merger	Efficiency percent after merger
19	39385.5000	25721.0600	84633.2300	14949.6840	100
21	32484.3938	9034.5000	57270.2724	3355.1172	94
22	31718.6062	21567.9366	58613.6870	12670.3722	85
24	106505.0010	19846.5707	53986.7218	4384.5624	29
$Bank_M^1$	210093.5010	76170.0673	254503.9112	35359.7358	84
Stage 2 (t=2)	$l_1^2 = 0.70, l_2^2 = 0.80, b_1^2 = 0.50, b_2^2 = 0.80, and\ w^2 = (0.40,0.20,0.10,0.30)$				
Bank number	Input X1 after the merger	Input X2 after the merger	Output Y1 after the merger	Output Y2 after the merger	Efficiency percent after merger
19	33619.4628	5144.2120	84633.2300	26909.4312	100
21	27728.6785	1806.9000	57270.2724	6039.2110	100
22	27075.0022	4313.5873	58613.6869	22806.6699	100
24	90912.6689	3969.3141	53986.7218	7892.2123	52
$Bank_M^2$	179335.8124	15234.0134	254503.9111	63647.5244	100

In the *t*-th stage, for $t = 1, 2$, we consider values for $l_1^t, l_2^t, b_1^t, b_2^t$, and the weight vector w^t arbitrary. According to these arbitrary values, after two stages, a Pareto-efficient bank from the merger of Banks 19, 21, 22, and 24 is obtained. In Stage 1, the optimal solution of model (9) is $(\delta_1^{*t}, \delta_2^{*t}, \partial_1^{*t}, \partial_2^{*t}) = (0.75, 0.95, 1.1, 1.08)$ and $\rho^{*1} = 0.84$; therefore, according to Result 2 of Subsection 3.1, $Bank_M^1$ with efficiency 84 % from this merger is obtained, whose efficiency score is higher than the efficiency scores of Banks 19, 21, 22, and 24 before the merger.
In Stage 2, the optimal solution of model (9) is $(\delta_1^{*2}, \delta_2^{*2}, \partial_1^{*2}, \partial_2^{*2}) = (0.85, 0.20, 1, 1.8)$, and $\rho^{*2} = 1$. Thus, $Bank_M^2$ lies on the hyperplane H and, according to Result 2 of Subsection 3.1, it is an efficient bank.
As shown in Figur 1, the inputs and outputs of the obtained new banks in each stage compared to the previous stage decreased and increased, respectively. Based on Result 3 of Subsection 3.1, in each stage, for $k \in \{19, 21, 22, 24\}$, the share of the *i*-th input and *r*-th output of each bank k in this merger is $\delta_i^{*t} x_{ik}^{t-1}$ and $\partial_r^{*t} y_{rk}^{t-1}$, respectively, these values are shown in Table 2.

The results also show that in each stage, the efficiency percent of each of Banks 19, 21, 22, and 24 after the merger has increased compared to the previous stage. Moreover, according to Theorem 3.3, the returns to scale of $Bank_M^1$, $Bank_M^2$ and $Bank_M^0$ are the same, and they all have decreasing returns to scale.

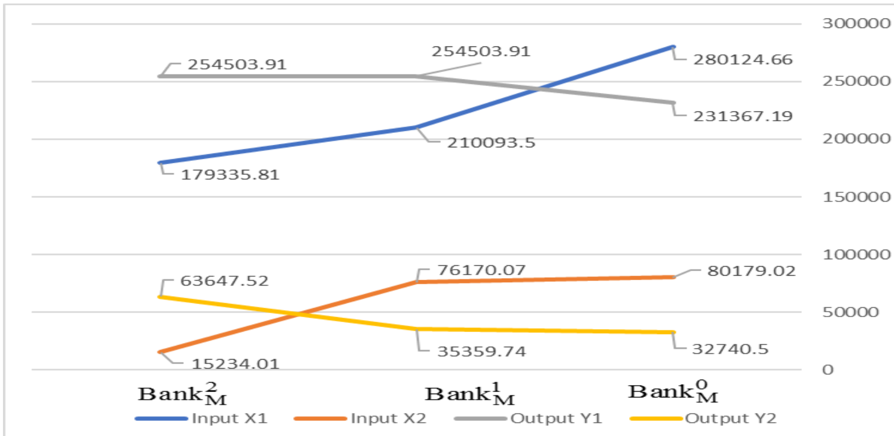

Figure 1. Comparison of the inputs and outputs of the obtained new banks from the merger in two stages.

In the merger process of the mentioned banks, if we want to reach a Pareto-efficient bank in the first stage ($t=1$), as stated in Result 1 of Subsection 3.1, we must consider $l_1^1 = l_2^1 = b_1^1 = b_2^1 = 1$ in model (9), (see Table 3 for the corresponding results).

Table 3. Inputs, outputs and efficiency of Banks 19, 21, 22, and 24 after the merger in the first stage (t=1).

Stage 1 (t=1)	$l_1^1 = l_2^1 = b_1^1 = b_2^1 = 1$ and $w = (0.25,0.25,0.25,0.25)$				
Bank number	Input X1 after the merger	Input X2 after the merger	Output Y1 after the merger	Output Y2 after the merger	Efficiency percent after merger
19	52514.0000	2888.8812	127203.7447	27684.6000	100
21	43312.5250	1014.7170	86077.2194	6213.1800	100
22	42291.4749	2422.4198	88096.3715	23463.6522	100
24	142006.6680	2229.0833	81142.0429	8119.5600	53
$Bank_M^1$	280124.6679	8555.1013	382519.3785	65480.9922	100

Here, we consider the weights of inputs and outputs as the same, $w = (0.25,0.25,0.25,0.25)$. In this merger, the optimal solution of model (9) is $(\delta_1^{*1}, \delta_2^{*1}, \partial_1^{*1}, \partial_2^{*1}) = (1, 0.1067, 1.6533, 2)$, and $\rho^{*1} = 1$. In the first stage, $Bank_M^1$ with 100% efficiency is obtained. Based on Result 3 of Subsection 3.1, the contribution of each input and output of Banks 19, 21, 22, and 24 after the merger has been determined. Moreover, the results show that the efficiency percent of each of Banks 19, 21, 22, and 24 after the merger has increased compared to their initial efficiency percent. Furthermore, according to Theorem 3.3, the returns to scale of $Bank_M^1$ and $Bank_M^0$ are the same, and they both have decreasing returns to scale. Note that in the above mergers, no bank's share in the merger was zero, whereas this condition was violated in previous merger models such as [13] and [4].

4.1. Comparison of merging methods for Banks 17 and 24

To compare the proposed model (9) with the previously introduced models of [13] and [4], we merge Banks 17 and 24 in a single step under identical conditions in order to obtain an efficient bank. Under the same conditions for all the mentioned models, the parameters are set as follows:

$w = (1,1,1,1)$ and $\bar{\theta} = 1$ in model (9); $\bar{\theta} = 1$ in [13]; and $c_1 = 1, c_2 = 1$ and $\bar{C} = 1$ in [4].

The total integration of the inputs and outputs of Banks 17 and 24 are

$$X_M^0 = (227928.668, 29084.127) \text{ and } Y_M^0 = (57440.838, 9019.78).$$

In evaluating $Bank_M^0$ by model (1), the optimal solution and the efficiency score of $Bank_M^0$ are

$$(U^*, V^*, u_0^*) = (2.2181 \times 10^{-6}, 8.47 \times 10^{-6}, 2.6824 \times 10^{-6}, 1.34 \times 10^{-5}, 0.02187)$$

and 0.1819, respectively. $Bank_M^0$ is inefficient, and since $u_0^* = 0.02187 > 0$, according to Theorem 2.5, $Bank_M^0$ has decreasing returns to scale. The results of this merger using the aforementioned models that are presented in Table 4.

Table 4. Merging of Banks 17 and 24 using [13], [4] and (9) models.

Type of model	Inputs after the merger	Outputs after the merger
[13] and [4]	Bank17= (0,0), Bank24= (31715,9637), Bank M= (31715,9637)	Bank17= (8362,4960), Bank24= (49079.8,4060.78), Bank M= (57441.8, 9020.78)
model (9)	Bank17= (16265.03,8193), Bank24= (26881.86,20891.13), Bank M= (43146.89,29084.13)	Bank17= (13210.29,4960), Bank24= (77534.75,4059.78), Bank M= (90745.04,9020.78)

Bank M from the merger, are lower than the total combined inputs of Banks 17 and 24 (i.e., $X_M^0 = (227928.668, 29084.127)$). However, the first output of Bank M obtained from model (9) (i.e., 90745.04) is greater than the total integration of the outputs of Banks 17 and 24 (i.e., 57440.838). In contrast, outputs of the obtained Bank M by using the models of [13] and [4] (i.e., 57441.8 and 9020.78) are equal to the total combined outputs of Banks 17 and 24 (i.e., 57441.8 and 9020.78).

In the models of [13] and [4], the contribution of each input of Bank 17 in the merger is zero; in other words, Bank 17 has no share of inputs in this merger. This result is not reasonable, because we believe that each merged unit should have a nonzero contribution percentage in the merger. Also, in the models of [13] and [4], the required reduction of the inputs of Banks 17 and 24 to achieve Bank M is very large, which may render the merger infeasible. In contrast, in the proposed model (9), with the optimal solution $(\delta_1^{*1}, \delta_2^{*1}, \partial_1^{*1}, \partial_2^{*1}) = (1, 0.1067, 1.6533, 2)$, according to Result 2 of Subsection 3.1, the contribution of the i-th input and the r-th output of Banks 17 and 24 are calculated the contribution of the i-th input and the r-th output of Banks 17 and 24 are calculated by, $\delta_i^{*1} x_{ik}^0$ and $\partial_r^{*1} y_{rk}^0$ respectively, for k ∈ {17,24}. Here, δ_i^{*1} is the contribution ratio of the i-th input of Banks 17 and 24, and ∂_r^{*1} is the contribution ratio of the r-th output of Banks 17 and 24. In model (9), the required input reductions and output increases for Banks 17 and 24 to form Bank M are reasonable, thereby making the merger feasible.

In this part of comparison, the superior performance of the proposed technique in this paper, relative to some existing methods such as [13], [4], [22] and [11], is discussed. Some explanatory notes are as follows:

1. In some mergers, the models of [13] and [4] are infeasible (see [11]), however, as shown in Theorem 3.1, the proposed approach ensures the feasibility of the presented merger model.

2. Sometimes in the merger process, it is not possible to achieve a Pareto-efficient unit in a single stage, because reducing inputs and increasing outputs simultaneously for the merged unit can be difficult or even impossible. Unlike the previously proposed methods, such as [13], [4], [22] and [11], the new method merges units gradually over several stages to achieve Pareto efficiency.

3. Unlike the methods of [13], [4] and [22], in the proposed method the merger is both input-oriented and output-oriented; that is, in each stage, a new unit with optimal inputs and outputs is obtained, and the share of each merged entity in the process is never zero. In contrast, as shown in Subsection 4.1, in [13] and [4], the contribution of some units in the merger may be zero.

4. The new unit obtained from the merger by the proposed method, is Pareto-efficient, whereas in the methods of [13] and [4], a weakly efficient unit may result (see [11]).

5. Unlike the previously mentioned merger methods, the proposed approach explicitly considers the returns to scale of units during the merger process.

5. Conclusion

In some organizations and institutions, for various reasons, including increasing the efficiency of the units, managers decide to merge two or more units, such that they aggregate the inputs and the outputs together and create a new unit. But in this merger, the new unit is not always efficient, and its efficiency percent is not always higher than the efficiency score of the merged units. However, managers expect to get Pareto-efficient unit from the merger process. But, sometimes, it is not possible to achieve a Pareto-efficient unit in one stage; because reducing inputs and increasing outputs of the merged unit in one stage can be difficult and sometimes impossible. In this paper, we present a new approach for gradually merging units, where units are merged based on a sequence of units in finite stages, and eventually, it ends with a Pareto-efficient unit. In this regard, the share of the inputs and outputs of each unit in each stage from the merger is determined, such that these values are not zero. Also, the obtained units from the merger in each stage have the same return of scale Property. In this study, the presented approach is applied to the merger of several banks of the set of Iranian banks; the results show the application of this method. The presented method can be used to merge units that have a network structure as well. Also, the presented method can be used to merge units with imprecise, interval, and negative data.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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