Iranian Journal of Operations Research
Vol. 17, No. 1, 2026, pp. 132-144

On the Solution of Multi-Objective Linear Programming Problem with Fuzzy Goals

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In this research, a Multi-Objective Linear Programming model with fuzzy goals is addressed. We review two existing methods to find the solution for the Fuzzy Multi-Objective Linear Programming problem. We discuss some advantages and shortcomings of the solving methods; accordingly, we recommend the preferred model. More- over, we propose a revision of the conventional point of view about the process of finding the solution of these kinds of problems and present the results regarding the two mentioned methods. A numerical example is provided to illustrate our approach.

Keywords*: Fuzzy goals, multi-objective linear programming, Fuzzy efficient solutions, Pareto-optimal solutions.*

1. INTRODUCTION

One of the distinctive characteristics of a real-life problem is its numerical imprecision. In fact, encounter with vagueness in information about parameters of these problems is a common occurrence. According to Zadeh [38] approach, fuzzy quantification plays crucial role for modeling real life problems. The theory of possibility is the cornerstone of Zadeh's proposed technique for such imprecise numerical problems. The theory demonstrated that fuzzy parameters could be represented as fuzzy numbers based upon their possibility distribution.

After the Bellman and Zadeh's article [4] in which they proposed the fuzzy concepts in goals, constraints and decisions, several authors like Inuiguchi and Lodwick [10], followed them. Their mathematical program which has fuzzy constraints and fuzzy objective function was modeled with the use of proposed fuzzy basic concepts. Furthermore, they utilized the properties of α-level sets for investigation into the fuzzy mathematical programming problems. Some reviews on the fuzzy optimization approaches and their applications for some different fields of science have been provided in [3, 17,33].

In fuzzy mathematical programming, the main purpose is to obtain a satisfying solution in a way that the membership degree of the fuzzy decision is maximized. When two or more sources of ambiguity appear at the same time (may be due to the inadequate information available), the membership degree cannot be determined exactly. The concept of intuitionistic fuzzy sets is amongst

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the generalized ordinary fuzzy sets concept which is used in such situations (see e.g. [2, 11, 27, 28]). Categorized Fuzzy Linear Programming (FLP) problems are investigated by Buckley and Feuring [4]. They defined parameters and variables in fuzzy numbers which also called as fully FLP problems. Some researchers have studied these kinds of models under various conditions. For some developments in this field, we refer to [1, 14, 22, 29].

Some attempts have been made to employ ranking function method in the process of finding a solution for an FLP problem, which are as follows:
Applying ranking function method to the solving procedure of a fully FLP problem with triangular fuzzy numbers, yielded an innovative approach, proposed by Kumar *e*t al. [14]. Mishmast Nehi and Hajmohamadi [18] also operated the method of ranking functions for solving the FLP problems. Nasseri and Ebrahimnejad [21] ordered fuzzy numbers using ranking functions in the solving procedure of these kinds of problems. Also, Nasseri *e*t al. [25] proposed an approach based upon the angle of the reference functions to ranking the fuzzy quantities.

Most of the real-world problems have inherent properties that bring forth multiple, conflicting and incommensurate aspects of evaluation. Generally, the evaluation is applicable when the linear programming models with more than one objective function employed to optimize the objective functions. In other words, whether the generalizability of fuzzy objectives reaches the expected preference model or what is the expected aspiration or ambition degree for each of the objectives are parts of the concept of conflicting and non-conflicting proposed by Cohon [6]. The concept of multi-objective optimization was originally introduced by Rangaiah [26]. So far, this issue has greatly been of interest to a wide range of authors. Many researchers applied multi-objective optimization in different fields of science such as transportation planning, financial planning, business management, etc. [33].

Recently, several researchers like Lin [15], Yaghoobi and Tamiz [33] attracted by fuzzy goal programming approach in order to find a solution for the multi objective programming problems with fuzzy parameters. Besides, a Fuzzy Multi-Objective Linear Programming (FMOLP) problem was solved by Lu *e*t al. [16] through an idea of α-fuzzy goal approximate method. Recent study by Cheng *e*t al. [5] demonstrated the use of deviation degree measures and the technique of weighted max-min as a solution for a FMOLP problem.

To solve a FMOLP problem, Mishmast Nehi and Hajmohamadi [18] applied a ranking function method. Pareto-optimal solutions were proposed by Nasseri and Khazaee Kohpar [23] to solve a fuzzy multi-objective linear programming within a supplier selection problem framework. Razmi *e*t al. [27] proposed an approach based on intuitional fuzzy goal programming in order to find Pareto optimal solutions to the problems with multiple objectives.

For the resulting compromise solution in multi-objective optimization methods, the pareto-optimality is a crucial factor due to guarantee the logicality of the designed solution. Jim´*e*nez and Bilbao [13] found out that even if the compromise solution obtained by use of any of procedure that satisfies the condition of fuzzy efficiency, it does not necessarily satisfy the condition of pareto-optimality in multi-objective linear programming problems with fuzzy goals. Then they received help from the customary goal programming approach to propose their algorithm. Also, Wu *e*t al. [35] proposed a new simplified two-phase approach with the change of membership functions for the fuzzy goals to find a pareto-optimal solution for FMOLP without relying on the results of the Guu and Wus two-phase approach which provides potential information for decision makers.
Nasseri and Bavandi [20] investigated multi-choice linear programming problems in a fuzzy-random hybrid uncertainty environment. They developed a model in which multi-choice parameters were represented as fuzzy-random variables and transformed into an equivalent deterministic form using interpolation techniques and the expected value concept. Nasseri et al. [24] used a multi-parameter method to solve a fuzzy multi-objective model. Singh and Yadav [31] presented a two-stage framework for solving fuzzy multi-objective linear programming problems with interval type II triangular fuzzy coefficients. In the first stage, the uncertainty of the parameters was solved by

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converting the interval type II fuzzy numbers into definite values, and in the second stage, the resulting multi-objective problem was solved using the proposed approach. Shrivastava et al. [30] proposed a two-stage method for solving fuzzy multi-objective linear programming problems with interval type-2 triangular fuzzy numbers. In the first stage, the uncertainty of the model is handled by converting interval coefficients into crisp numbers and using the "acceptability index" for constraints. In the second stage, the defuzzified/crisp multi-objective problem is solved through a fuzzy mathematical programming approach, the efficiency of which has been demonstrated in production planning and diet planning. Mishmast Nehi and Sargolzaei [19] studied the interval type II fuzzy multi-objective linear programming problem in which all parameters and coefficients were represented as interval type II triangular fuzzy numbers. They presented an algorithm for solving this class of problems using the weighted sum method and the concept of nearest interval approximation.

In this study, we deal with a multi-objective linear programming model with fuzzy goals. We review two existing methods to find the solution for this problem. We discuss some advantages and shortcomings of the solving methods; accordingly, we recommend the preferred model. Moreover, we propose a revision of the conventional point of view about the process of finding the solution of these kinds of problems and present the results regarding the two mentioned methods. A numerical example is provided to illustrate our approach. In the second section of this paper, we give some essential preliminaries about the multi-objective linear programming in fuzzy environment. The third section reviews the two existing approaches for solving these models. Performance evaluation of the methods and a comparison between them are also presented in this section. In the fourth section, the new approach is provided and a numerical example is also presented. The last section is the conclusion.

2. Multi-objective linear programming in fuzzy environment

2.1. The multi-objective linear programming

Let a Multi-Objective Linear Programming (abbreviated by MOLP) problem with k objective functions be as follows:

$$\begin{aligned} \min\ & Z(x)=(Z_1(x),Z_2(x),\ldots,Z_k(x))=\Big(\sum_{j=1}^{n} c_{1j}x_j, \sum_{j=1}^{n} c_{2j}x_j, \ldots, \sum_{j=1}^{n} c_{kj}x_j\Big) \\ & s.t.\quad \sum_{j=1}^{n} a_{ij}x_j \geq b_i,\quad i=1,\ldots,m \\ & \qquad\quad x_j \geq 0,\quad j=1,\ldots,n, \end{aligned} \tag{1}$$

Or

$$\begin{aligned} \min\ & Z(x)=(c_1x, c_2x, \ldots, c_kx) \\ & s.t.\quad Ax \geq b, \\ & \qquad x \geq 0. \end{aligned}$$

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2.2. The multi-objective linear programming with fuzzy goals

Generally, considering ambiguous of the opinion of decision maker (DM), it is commonly presumed that decision maker may face with fuzzy targets for each of the objective functions. Now, suppose that DM states imprecise aspiration levels such as “the target Z_l(x) should be substantially less than or equal to some value g_l”. Hence model (1) can be written as follows:

$$\begin{aligned} & Find\ \ x \\ s.t. \quad & Z_l(x) \preccurlyeq g_l,\ \ \forall l \in \{1,\ldots,k\}, \\ & Ax \geq b, \\ & x \geq 0 \end{aligned} \tag{2}$$

that is known as Fuzzy Multi-Objective Linear Programming (abbreviated by FMOLP) models. These kinds of statements on aspiration levels can be represented by a linear related membership function as follows:

$$\mu_l(Z_l(x)) = \begin{cases} 1 & \text{if } Z_l(x) \leq g_l, \\ 1 - \dfrac{Z_l(x) - g_l}{d_l} & \text{if } g_l < Z_l(x) < g_l + d_l, \\ 0 & \text{if } Z_l(x) \geq g_l + d_l, \end{cases} \tag{3}$$

For $l = 1,\ldots,k$ where$g_l + d_l$ are the tolerance margins provided by DM, who is desiring to accept them.

Here we present some fundamental notations and basic definitions which play an important role in describing our main results (for details see [9, 10, 17]).

Definition 1. A decision plan $x \in R^n$ is said to be a feasible solution to the MOLP model (1) or the FMOLP model (2) if x satisfies the constraints of (1) or (2), respectively.

Definition 2. The solution $x \in R^n$ is a pareto-optimal solution to MOLP problem (1) if it is feasible and there does not exist another feasible solutionx such that $z_l(x) \leq z_l(x*)$ for all $l = 1,\ldots,k$ and $z_t(x) < z_t(x*)$ for at least one *t.*
The fuzzy-efficiency condition of the solution for model (2) can be described as Definition below.

Definition 3. A feasible solution $x*$ is a fuzzy-efficient solution to (2) if there does not exist another feasible solution x such that $\mu_l(Z_l(x*)) \leq \mu_l(Z_l(x))$ for all $l = 1,\ldots,k$ and $\mu_t(Z_t(x*)) < \mu_t(Z_t(x))$ for at least one *t.*

3. Review of two existing approaches

Here, we review two existing approaches proposed by Jim´*e*nez and Bilbao [13] and Wu *e*t al. [35] in order to solve FMOLP problem (2).
Jim´*e*nez and Bilbao [13] have been studied the pareto-optimality and fuzzy-efficiency conditions for a MOLP problem with fuzzy goals (FMOLP problem (2)), especially for the case that one of the goals is fully achieved.

Proposition 4. Any fuzzy efficient solution $x*$ to problem (2) such that $Z_l(x*) \in (g_l, g_l + t_l)$ for all l, is a pareto-optimal solution to the MOLP problem (1) (proving by reduction ad

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absurdum). However, when one goal is fully achieved, it might be possible for the solution to be fuzzy-efficient but not pareto-optimal. For example, suppose that $\mu_1\big(Z_1(x*)\big) = 1$, then it could be some feasible solution x such that $\mu_l\big(Z_l(x)\big) = \mu_l(Z_l(x*))$ for all l and $Z_1(x) < Z_1(x*)$.

According to the above proposition, they showed that, in the case of fully achievement, there is not one to one correspondence between a fuzzy-efficient solution for the FMOLP model and the pareto-optimal solution for the related MOLP model. To arrive at a pareto-optimal solution, they took advantage of the most common method of solving goal programming problems and maximized the sum of the over attainment of the goals. Also, they used the two-phase method, proposed by Guu and Wu [8], at the two first steps of their algorithm. We describe the general algorithm proposed by Jim´enez and Bilbao [13] as follows:

Phase I: After the determination of aspiration levels by DM, in order to solve FMOLP problem (2), apply the max-min operator:

$$\begin{aligned} &\max\ v \\ \text{s.t.}\ \ &1 \geq \mu_l\big(Z_l(x)\big) \geq v,\ l=1,\dots,k \\ &Ax \geq b, \\ &x \geq 0, \\ &v \in [0,1], \end{aligned} \tag{4}$$

Where $\mu_l\big(Z_l(x)\big)$ for each l is described in (3). Let $x*$ be the optimal solution resulting from Phase I.

(1) When we face with the uniqueness of the optimal solution:

(a) If for all the satisfaction levels, we have $\mu_l(Z_l(x*)) < 1$, for all l, by Definition 2 and Definition 3, it is obvious that $x*$ is fuzzy-efficient and pareto-optimal too, thus it can be considered as a preferred solution and the algorithm stops.

(b) If some satisfaction levels are completely attained, i.e. for one or more goals we have $\mu_t(Z_t(x*)) = 1$,for some t, this indicates that $x*$ is fuzzy efficient but it may not be pareto-optimal. Then go to Phase III (see Proposition 4).

(2) When we have multiple optimal solutions: the fuzzy efficiency is not guaranteed for all solutions of model (4) (because, for example, for two optimal solutions x_1 and x_2, we could have $\mu_t\big(Z_t(x_1)\big) < \mu_t\big(Z_t(x_2)\big)$ for some t), but at least one of them is fuzzy-efficient which is determined by a modified lex-maxmin algorithm (see in [38]). Then go to Phase II. *Phase II:* Solve the following model (based on additive criterion).

$$\begin{aligned} &\max \sum_{l=1}^{k} v_l \\ \text{s.t.}\ &\mu_l\big(Z_l(x)\big) \geq v_l \geq \mu_l\big(Z_l(x*)\big),\ l=1,\dots,k \\ &Ax \geq b, \\ &x \geq 0, \end{aligned} \tag{5}$$

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where the parameters of the problem are the same as defined in (4). Now, suppose that $\bar{x}$ is any optimal solution for the problem (5).

(ʹa) If all the satisfaction levels are strictly less than 1, then$\bar{x}$ has both properties fuzzy-efficiency and pareto-optimality, thus it can be considered as a preferred solution and the algorithm stops.

(b)ʹ If some satisfaction levels are equal to 1, i.e. one or more goals are completely attained, then$\bar{x}$ is fuzzy-efficient but it may not be Pareto optimal (see Phase II, part (b)). Then go to Phase III.

Phase III: Solve the following model (based on the Goal Programming approach)

$$\begin{aligned} & max \sum_{e=1}^{p} f_e \\ \text{s.t.} \quad & Z_e(x) + f_e = Z_e(\bar{x}), \qquad e = 1,\ldots,p, \\ & \mu_d\big(Z_d(x)\big) = \mu_d\big(Z_d(\bar{x})\big), \ d = p+1,\ldots,k, \\ & \text{Ax} \geq \text{b}, \ x \geq 0, \ f_e \geq 0 \end{aligned} \tag{6}$$

where subscripts e is related to goals such that $\mu_e\big(Z_e(\bar{x})\big) = 1$ and subscripts d are related to goals such that $\mu_d\big(Z_d(\bar{x})\big) < 1$. In this stage, the solution has both properties fuzzy-efficiency and pareto-optimality (see Lemma 5), thus it can be considered as a preferred solution and the algorithm stops. Now, any optimal solution $\hat{x}$ to model (6) is a pareto-optimal solution to the MOLP problem resulting from Phase I.

Lemma 5. For any optimal solution $x*$ of model (6), $x*$ is a fuzzy-efficient solution to model (2) and a pareto-optimal solution to model (1).

Proof. See in [13]. ■

On the other hand, Wu *e*t al. [35] showed that membership functions defined in (3) cannot exceed the constraint $\mu_l\big(Z_l(x)\big) \leq 1$, for all l even though their values can be larger than 1 to (2) and therefore some goals cannot achieve the lowest value. They proposed a new simplified two-phase approach with the change of membership functions for the fuzzy goals to find a pareto-optimal solution for (2) so that it did not rely on the results of (5).

Their proposed membership function for all l is as follows:

$$\mu_l(Z_l(x)) = \begin{cases} \dfrac{(g_l + d_l) - Z_l(x)}{d_l} & \text{if } Z_l(x) \leq g_l + d_l, \\ 0 & \text{if } Z_l(x) \geq g_l + d_l, \end{cases} \tag{7}$$

According to the new membership function, they presented their modified two-phase method:

Phase I: Apply the max-min operator:

$$\begin{aligned} & \max \ v \\ s.t. \quad & \mu_l\big(Z_l(x)\big) \geq v, \ l = 1,\ldots,k, \\ & \text{Ax} \geq \text{b}, \\ & x \geq 0, \end{aligned} \tag{8}$$

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Where $\mu_l(Z_l(x))$ for each l is described in (7).

Phase II: Solve the following model

$$\begin{aligned} &\max \sum_{l=1}^{k} p_l \\ \text{s.t.}\quad &\mu_l(Z_l(x)) - p_l \geq v^*,\ \ l = 1,\ldots,k, \\ &Ax \geq b, \\ &x \geq 0,\ \ p_l \geq 0, \end{aligned} \tag{9}$$

where v^* is the optimal value obtained from (8).

They proved that the optimal solutions obtained by (9) are pareto-optimal. They also showed that the value of p_l for all l provides the potential information for $Z_l(x)$ of model (2), so that if for each l its value is 0, then no more efficient solutions exist in the Phase I, if for some l there exists $p_l > 0$, then the solution obtained from Phase II is more efficient than that of Phase I and by solving (9), if there exists $\rho_l + \nu *> 1$, then the value of objective function $Z_l(x)$ is overestimated by decision maker and the overestimated value achieves to $(\rho_l + \nu * - 1) \times d_l$.

3.1. Performance evaluation and comparison:

Jim´*e*nez and Bilbao [13] showed that, if at least one of the goals reaches full saturation, a fuzzy efficient solution to problem (2) is not necessarily a pareto-optimal solution. Then, in order to overcome this shortcoming, they introduced an additional step in which a model based on the conventional goal programming model is solved. This is while Wu *e*t al. [35] have shown that by modifying the membership functions, a shorter path can be made to reach a pareto-optimal solution. Another advantage of this approach is its non-dependence on the result of Guu and Wu's two-phase approach [8] (a pareto-optimal solution can be obtained directly from (9)). Also, according to the results expressed by Wu *e*t al., the value of ρ_l for all l provides more potential information for the lth objective function of model (2) such as whether the fuzzy goals of objective function are overestimated or not and the overestimated value can easily be computed if it exists.

Jim´*e*nez and Bilbao [13] used the two-phase approach of Guu and Wu [8] in the procedure of finding a fuzzy-efficient solution, although it is no restrict to use that and any other method to obtain a fuzzy-efficient solution could be used instead of that. Both Jim´*e*nez and Bilbao [13] and Wu *e*t al. [35] used the linear membership functions in their methods, but their proposed procedure can be applied to a more general form of the membership function (for details we refer to [9, 12]). In addition, in both of them, for convenience, Minimization of all the objective functions is the aim, but the algorithms can be simply extended to the case that the model includes some maximizing objectives.

In general, based on the above discussion, it can be concluded that the method proposed by Wu *e*t al. is more suitable for simplicity and efficiency than the method proposed by Jim´*e*nez and Bilbao.

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4. The new approach

It is well known that the fuzzy-efficiency and the pareto-optimality are two equally important factors in the process of finding a solution for the multi objective linear programming problem with fuzzy goals, for the former guarantees the efficiency of the solution and the latter its rationality.

On the other hand, so far, attempts, in much of the conducted research, have been made to obtain a pareto-optimal solution, which is also fuzzy-efficient, through a fuzzy-efficient solution (for instance, see the two approaches discussed in the previous section). Here, we intend to check the reverse process, especially with regard to the two proposed approaches under consideration in this paper. In point of fact, we want to examine whether in the two proposed approaches, it is possible to get a fuzzy-efficient solution, which is also pareto-optimal, through a pareto-optimal solution.

Let $x_0$be a pareto-optimal solution to (2) and therefore there does not exist another feasible solution$x$ such that $z_l(x) \le z_l(x_0)$ for all $l = 1,\dots,k$ and $z_t(x) < z_t(x_0)$ for at least one t.

Considering membership functions (3), defined by Jim´enez and Bilbao, for $z_l(x) \le z_l(x_0)$, we have:

(I) $$if\ z_l(x_0) \le g_l \quad \Rightarrow z_l(x) \le z_l(x_0) \le g_l, \\ \Rightarrow \mu_l(Z_l(x_0)) = \mu_l(Z_l(x)) = 1$$

(II) $$if\ g_l \le z_l(x_0) \le g_l + d_l \ \Rightarrow \begin{cases} g_l \le z_l(x) \le z_l(x_0) \le g_l + d_l \\ \quad or \\ z_l(x) \le g_l \le z_l(x_0) \le g_l + d_l, \end{cases}$$

$$\Rightarrow \begin{cases} 0 \le \mu_l(Z_l(x_0)) \le \mu_l(Z_l(x)) \le 1, \\ \quad or \\ 0 \le \mu_l(Z_l(x_0)) \le 1,\ \mu_l(Z_l(x)) = 1, \end{cases}$$

(III) $$if\ z_l(x_0) \le g_l + d_l \ \Rightarrow \begin{cases} z_l(x_0) \ge z_l(x) \ge g_l + d_l \\ \quad or \\ z_l(x_0) \ge g_l + d_l \ge z_l(x) \ge g_l \\ \quad or \\ z_l(x_0) \ge g_l + d_l \ge g_l \ge z_l(x). \end{cases}$$

$$\Rightarrow \begin{cases} \mu_l(Z_l(x_0)) = \mu_l(Z_l(x)) = 0 \\ \quad or \\ \mu_l(Z_l(x_0))=0,\ 0 \le \mu_l(Z_l(x)) \le 1 \\ \quad or \\ \mu_l(Z_l(x_0))=0,\ \mu_l(Z_l(x)) = 1. \end{cases}$$

Also, for $z_t(x) < z_t(x_0)$, we have

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(IV) $if\ z_t(x_0) \le g_t \Rightarrow z_t(x) < z_t(x_0) \le g_t,$
$$\Rightarrow \mu_t\big(Z_t(x_0)\big) = \mu_t\big(Z_t(x)\big) = 1,$$

V
$$if\ g_t \le z_t(x_0) \le g_t + d_t \Rightarrow \begin{cases} g_t \le z_t(x) < z_t(x_0) \le g_t + d_t \\ or \\ z_t(x) < g_t \le z_t(x_0) \le g_t + d_t, \end{cases}$$

$$\Rightarrow \begin{cases} 0 \le \mu_t\big(z_t(x_0)\big) < \mu_t\big(z_t(x)\big) \le 1 \\ or \\ 0 \le \mu_t\big(z_t(x_0)\big) \le 1,\ \mu_t\big(z_t(x)\big) = 0, \end{cases}$$

(VI)
$$if\ z_t(x_0) \ge g_t + d_t \Rightarrow \begin{cases} z_t(x_0) > z_t(x) \ge g_t + d_t \\ or \\ z_t(x_0) \ge g_t + d_t > z_t(x) \ge g_t \\ or \\ z_t(x_0) \ge g_t + d_t \ge g_t \ge z_t(x). \end{cases}$$

$$\Rightarrow \begin{cases} \mu_t\big(z_t(x_0)\big) = \mu_t\big(z_t(x)\big) = 0 \\ or \\ \mu_t\big(z_t(x_0)\big) = 0,\ 0<\mu_t\big(z_t(x)\big) \le 1 \\ or \\ \mu_t\big(z_t(x_0)\big) = 0,\ \mu_t\big(z_t(x)\big) = 1. \end{cases}$$

Gorollary 6. Based on membership functions (3), defined by Jime´nez and Bilbao, and related obtained relations (I) to (VI) and also Definition 3, it can be concluded that a pareto-optimal solution may not be fuzzy-efficient. Besides, when the value of z_l for all l at the pareto-optimal solution x_0 is less than or equal to g_l, then x_0 is not fuzzy-efficient.

Now, considering membership functions (7), defined by Wu *e*t al., for $z_l(x) \le z_l(x_0)$, we have

(A) $if\ z_l(x_0) \le g_l + d_l \Rightarrow z_l(x) \le z_l(x_0) \le g_l + d_l,$
$$\Rightarrow \mu_l\big(Z_l(x)\big) \ge \mu_l\big(Z_l(x_0)\big) \ge 0,$$

(B)
$$f\ z_l(x_0) \ge g_l + d_l \Rightarrow \begin{cases} z_l(x_0) \ge z_l(x) \ge g_l + d_l \\ or \\ z_l(x_0) \ge g_l + d_l \ge z_l(x), \end{cases}$$

$$\Rightarrow \begin{cases} \mu_l\big(Z_l(x)\big) = \mu_l\big(Z_l(x_0)\big) = 0 \\ or \\ \mu_l\big(Z_l(x_0)\big) = 0,\ \mu_l\big(Z_l(x)\big) \ge 0. \end{cases}$$

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Also, for $z_t(x) < z_t(x_0)$, we have

(C) $$if \quad z_t(x_0) \le g_t + d_t \quad \Rightarrow z_t(x) < z_t(x_0) \le g_t + d_t$$
$$\Rightarrow \mu_t(Z_t(x)) > \mu_t(Z_t(x_0)) \ge 0,$$

(D) $$if \quad z_t(x_0) \ge g_t + d_t \quad \Rightarrow \begin{cases} z_t(x_0) > z_t(x) \ge g_t + d_t \\ \quad or \\ z_t(x_0) \ge g_t + d_t > z_t(x), \end{cases}$$

$$\Rightarrow \begin{cases} \mu_t(Z_t(x_0)) = \mu_t(Z_t(x)) = 0, \\ \quad or \\ \mu_t(Z_t(x_0)) = 0, \ \mu_t(Z_t(x)) > 0. \end{cases}$$

Corollary 7. From relations (A) to (D) related to membership functions (7), defined by Wu et al., and Definition 3, it can be concluded that if the value of Z_l for all l at the pareto-optimal point x_0 is less than or equal to $g_l + d_l$, then x_0 is a pareto-optimal solution, which is also fuzzy-efficient. Though otherwise, it's not. This is due to the fact that on the right of $g_l + d_l$ the membership function μ_l is constantly equal to 0.

Here we present the example, suggested by Jimenez and Bilbao [17], and solve it by our new approach to illustrate our results.

Example 8. Let the following MOLP problem be with the following vague aspiration levels: "the first objective function should be essentially less than or equal to 21", "the second should be essentially less than or equal to 8" and "the third should be essentially less than or equal to 13", the related tolerance threshold being 24, 10 and 15.

$$\begin{aligned} &\min \ z_1 = 3x_1 + 3x_2 + 3x_3 \\ &min \ z_2 = 2x_1 + x_2 + 2x_3 \\ &min \ z_3 = 4x_1 + 4x_2 + 2x_3 \\ &s.t. \ 4x_1 + 2x_2 + 4x_3 \ge 18 \\ &\qquad x_1 \ge 1, \\ &\qquad x_2 \ge 0, \\ &\quad 0 \le x_3 \le 3. \end{aligned} \tag{10}$$

Assume that the relative importance of the decision maker or the weight of the fuzzy goals is equally divided between the goals. Then, using the weighting method and with the help of Lingo software, we get the Pareto optimal solution $x^* = (x_1^*, x_2^*, x_3^*)$ where $x_1^* = 1.5$, $x_2^* = 0$, $x_3^* = 3$ and $z_1^* = 13.5$, $z_2^* = 9$, $z_3^* = 12$.

We have:

$z_1^* = 13.5 \le g_1 = 21$, $g_2 = 8 \le z_2^* = 9 \le g_2 + d_2 = 10$, $z_3^* = 12 \le g_3 = 13$,

$from(3)$: $\mu_1(Z_1(x^*)) = 1$, $\mu_2(Z_2(x^*)) = 0.5$, $\mu_3(Z_3(x^*)) = 1$,

$from(7)$: $\mu_1(Z_1(x^*)) = 3.5$, $\mu_2(Z_2(x^*)) = 0.5$, $\mu_3(Z_3(x^*)) = 1.5$,

As we can observe, with regard to $x^* = (1.5,0,3)$, for all $l \in \{1,2,3\}$, *we have* $Z_l(x^*) \le g_l + d_l$. Consequently, based on Corollary 7,x^* is a fuzzy efficient, and pareto-optimal, solution to the problem (10) and so it can be selected. But based on Corollary 6, this cannot be conclusively

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concluded. Moreover, according to the value of the obtained membership functions, we find that the goals of the first and third objective functions are over estimated by the decision maker and the number of overestimations can easily be computed (see [39]).

Corollary 9. In the above example, we use weighting method to find a pareto-optimal solution. However, any other procedure to obtain a Pareto optimal solution could be used here. Furthermore, according to the above example, it can be stated that the new approach proposed in this paper, while maintaining efficiency, is more simplified in some cases than the Wu et al.'s and Jime´nez and Bilbao's approaches, to attain a fuzzy-efficient and Pareto optimal solution for model (2).

5. Conclusion

In an MOLP problem it is improbable that all goals reach to their optimal values at the same time. In particular, if it includes fuzziness in the goals, it is practically hard for decision makers to come up with a satisfying solution. In this paper, we deal with an MOLP problem that has the additional quality of fuzzy goals. We review two existing methods to find a solution for this problem. After discussing about the disadvantages and advantages of the methods, we proposed the preferred method. Moreover, we propose a revision of the conventional point of view about the process of finding the solution of these kinds of problems and present the results regarding the two mentioned methods. Also, it can be stated that the new approach proposed in this paper, while maintaining efficiency, is more simplified in some cases than the Wu et al.'s and Jime´nez and Bilbao's approaches, to attain a fuzzy-efficient and pareto-optimal solution.

Finally, we point out that we have introduced a new approach in this paper. Though other interested researchers can develop or critique this approach.

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