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Linear plus fractional multiobjective programming problem with homogeneous constraints using fuzzy approach

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We develop an algorithm for the solution of multiobjective linear plus fractional programming problem (MOL+FPP) when some of the constraints are homogeneous in nature. Using homogeneous constraints, first we construct a transformation matrix T which transforms the given problem into another MOL+FPP with fewer constraints. Then, a relationship between these two problems, ensuring that the solution of the original problem can be recovered from the solution of the transformed problem, is established. We repeat this process of transformation until all the homogeneous constraints are removed. Then, we discuss the multi objective programming part, for which fuzzy programming methodology is proposed which works for the minimization of perpendicular distances between two hyper planes (curves) at the optimal points of the objective functions. A suitable membership function is defined with the help of the supremum perpendicular distance. A compromised optimal solution is obtained as a result of the minimization of the The supremum perpendicular distance. The corresponding optimal solution to the original problem is obtained using the transformation matrix. Finally, an example is given to illustrate the proposed model.

***Key words**: Linear plus fractional functional, Transformation matrix, Distance function, Membership function.*

1. Introduction

Decision-making problems occuring in the real world are generally multi criteria decision-making (MCDM) problems. Many researchers such as Wallenius [12], Hanan [6], Feng [4], Chanas [1] and Rommelfanger [10] used and or modified the concept of decision making in fuzzy environment. They discussed different approaches to deal with the multiobjective programming problems. Also, many iterative algorithms have been studied as given by Charnes and Cooper [2], Swarup [11], Martos [9], Jain and Mangal [7] for solving fractional programming problems. Jain and Lachhwani [8] discussed a solution methodology for multiobjective linear fractional programming problem, in which the objective function in of the form $\frac{f(X)}{g(X)}$. Here, we address form which we denote by MOL+FPP.

A multiobjective linear plus fractional programming problem (MOL+FPP) seeks to optimize more than one objective functions of the form $f(X)+\frac{g(X)}{h(X)}$ [1], that is the sum of a linear and a quotient function, where $f(X)$, $g(X)$ and $h(X)$ are linear functions subject to linear

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constraints under the assumptions that the set of the feasible solutions is a convex polyhedron with a finite number of extreme points and that the denominators of the fractional part of the objective functions are nonzero on the feasible set.

We present an algorithm that performs to be better than other iterative methods as it needs computing time for the optimization process and obtains an optimum solution for a set of objective functions of such a linear plus fractional form.

The remainder of the paper is organized as follows. In Section 2, a transformation matrix T is constructed. In the Section 3, the relationship between the original problem and the transformed problem is given. In Section 4, we develop fuzzy programming model for the problem by minimizing the perpendicular distances between two hyper planes $Z_i(x) = \overline{Z_i}$, and $Z_i(x) = \underline{Z_i}$ where $\overline{Z_i}$ and $\underline{Z_i}$ are the maximum and minimum values of the function $Z_i(x)$ in the feasible region, respectively. Suitable membership functions are defined and a compromise optimal solution is obtained. In Section 5, a pseudo program is given for the construction of the transformation matrix T. An example is worked out in Section 6 to explain the model. Particular cases and conclusions are given in Section s 7 ad 8, respectively.

2. Development of transformation matrix

Consider the problem,

$$\text{Maximize } \{Z_1(X), Z_2(X), ..., Z_k(X)\} \qquad (1)$$

where, $$Z_i(X) = (L_i X + l_{0i}) + \frac{C_i X + c_{oi}}{D_i X + d_{oi}} \qquad \forall\ i = 1, 2, ..., k$$

subject to $AX = b,$

and $X \geq 0$

with $a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n = 0$, for some *i*. (2)

Here, L_i, C_i and D_i $(i = 1, 2, ..., k)$ are row vectors with n components, X and b are column vectors with n and m components, respectively, $A = (A_1, A_2, ..., A_n)$ is an m by n matrix and the c_{oi}, d_{oi} $(i = 1, 2, ..., k)$ are scalars. It is assumed that $D_i X + d_{oi} > 0$, over L , where L=$\{X : AX = b,\ X \geq 0\}$. Let $X = (x_1, x_2, ..., x_n)$ be an efficient solution of (2). If x_k and $a_{ik} > 0$, then it is obvious that there exists at least one x_l with $a_{il} < 0$. Taking this in view, we partition matrix A as $A = \{A^0, A^+, A^-\}$, where A^0 is the set of all columns of A, whenever $a_{ij} = 0$, (Let the number of such columns be r), A^+ be the set of all columns of A, whenever $a_{ij} > 0$, p in number, and finally A^- be the set of all columns of A, whenever $a_{ij} < 0$, q in number. Thus, $p + q + r = n$.

Now, we define a transformation matrix T (n by $pq + r$) such that the i-th equation of $AT\text{w} = b$ will be automatically zero. Here, w is a column vector with $pq + r$ components. This is accomplished by defining w_{kl} for each pair (k, l) such that $A_k \in A^+$ and $A_l \in A^-$.

Now, we partition $T = (T_1, T_2)$ where T_1 consists of unit column vectors e_j corresponding to w_{kl}. A column corresponding to the w_{kl} looks like:

$$\begin{bmatrix} 0 \\ -a_{ij} \\ 0 \\ a_{ik} \\ 0 \end{bmatrix}.$$

Thus, T, which has n rows and r + pq columns, can be represented as:

$$T = (T_1, T_2) = [\langle e_j \rangle \ \forall \ j \in a_{ij} = 0; \ \langle t_{kl} \rangle \ \forall \ k \in A^+ \wedge l \in A^-],$$

that is, e_j is the j th column of the identity matrix I_n, and $t_{kl} = -a_{il}e_k + a_{ik}e_l$. (3)

A pseudo code program is also framed for the construction of transformation matrix T in Section 5.

Theorem 2.1 *In matrix* T *of order* $n \times (pq + r)$, *the* $i-th$ *equation of* $AT\mathrm{w} = b$ *will be identically zero.*

Proof: To prove this theorem, it is sufficient to show that the i-th equation of AT will have all the zero. Let A^i denote the i-th row of A. For any $j \in A^0$,it is clear that $A^i e_j = 0$, implying thereby that $A^i T_1 = 0$. Again for any (k,l), $k \in A^+$, and $l \in A^-$, we have $A^i t_{kl} = A^i(-a_{il}e_k + a_{ik}e_l)$ $= -a_{il}a_{ik} + a_{ik}a_{il} = 0$ .Thus, the left hand side of the i-th equation of $AT\mathrm{w}$ will be zero and $b_i = 0$.

3. Transformed problem

Using the transformation $X = T\mathrm{w}$, we define the following problem corresponding to problem (2):

$$\text{Maximize} \quad z_i = \left(\overline{L_i}\mathrm{w} + l_{0i}\right) + \frac{\overline{C_i}\mathrm{w} + c_{oi}}{\overline{D_i}\mathrm{w} + d_{oi}} \quad \forall \ i = 1,2,...,k \qquad (4)$$

$$\text{Subject to} \quad \overline{A}\,\mathrm{w} = b$$

$$\text{and} \quad \mathrm{w} \geq 0. \qquad (5)$$

In view of Theorem 2.1, the i-th equation in system (5) will be identically zero and hence can be removed while solving the program (4)-(5). The following theorems are now in order.

Theorem 3.1 *If X solves (2), then there exists w (X=Tw) which solves (5).*

Proof: Here, constraints and non-negativity restrictions are similar in both original and the transformed problem for the single objective to generalized problem taken by Chadha [3]. So, the proof can be seen in Chadha [3].

Theorem 3.2 *If* X^* *solves the program (1)-(2), then* w^* *(*$X^*=T\mathrm{w}^*$*) solves the program (4)-(5).*

Proof: Theorem 3.1 guarantees the existence of a feasible w^*, i.e., $\overline{A}\mathrm{w}^* = b, \mathrm{w}^* \geq 0$. Next, since X* solves the program (1)-(2), then

$$\left(L_i X^* + l_{0i}\right) + \frac{C_i X^* + c_{0i}}{D_i X^* + d_{0i}} \geq \left(L_i X + l_{0i}\right) + \frac{C_i X + c_{0i}}{D_i X + d_{0i}}, \ \forall \ X \in L, \ \ i = 1,2,...,k$$

Or equivalently,

$$\left(L_i T\mathrm{w}^* + l_{0i}\right) + \frac{C_i T\mathrm{w}^* + c_{0i}}{D_i T\mathrm{w}^* + d_{0i}} \geq \left(L_i T\mathrm{w} + l_{0i}\right) + \frac{C_i T\mathrm{w} + c_{0i}}{D_i T\mathrm{w} + d_{0i}}, \ \forall \ \mathrm{w} \in G, \ i = 1,2,...,k \ ;$$

or, $$\left(\overline{L_i}\mathrm{w}^* + l_{0i}\right) + \frac{\overline{C_i}\mathrm{w}^* + c_{0i}}{\overline{D_i}\mathrm{w}^* + d_{0i}} \geq \left(\overline{L_i}\mathrm{w} + l_{0i}\right) + \frac{\overline{C_i}\mathrm{w} + c_{0i}}{\overline{D_i}\mathrm{w} + d_{0i}},\ \forall\ \mathrm{w} \in G;$$

Thus, w^* solves the program (4)-(5).

Theorem 3.3 *If w^* solves the program (4)-(5), then there exists $X^*=Tw^*$ which solves the program (1)-(2), and the extreme values of the two objective functions are equal.*

Proof: w^* being a solution of the program (4)-(5) means that

$$\overline{A}\mathrm{w}^* = b,\ \ \mathrm{w} \geq 0;$$

or

$$AT\mathrm{w}^* = b,\ \text{or}\ \ AX^* = b, \tag{6}$$

Furthermore, $T \geq 0,\ \mathrm{w}^* \geq 0,$ imply that $X^* \geq 0$ (7)

Also, we know that,

$$\left(\overline{L_i}\mathrm{w}^* + l_{0i}\right) + \frac{\overline{C_i}\mathrm{w}^* + c_{0i}}{\overline{D_i}\mathrm{w}^* + d_{0i}} \geq \left(\overline{L_i}\mathrm{w} + l_{0i}\right) + \frac{\overline{C_i}\mathrm{w} + c_{0i}}{\overline{D_i}\mathrm{w} + d_{0i}},\ \forall\ \mathrm{w} \in G,\ i = 1,2,...,k \tag{8}$$

And,

$$\left(L_i\mathrm{X}^* + l_{0i}\right) + \frac{\mathrm{C}_i\mathrm{X}^* + c_{0i}}{\mathrm{D}_i\mathrm{X}^* + d_{0i}} \geq \left(L_i\mathrm{X} + l_{0i}\right) + \frac{\mathrm{C}_i\mathrm{X} + c_{0i}}{\mathrm{D}_i\mathrm{X} + d_{0i}},\ \forall\ \mathrm{X} \in L,\ i = 1,2,...,k \tag{9}$$

For a contradictory argument, let $\overline{X}$ and not X^* solve the program (4)-(5), which means that

$$\left(L_i\overline{\mathrm{X}} + l_{0i}\right) + \frac{\mathrm{C}_i\overline{\mathrm{X}} + c_{0i}}{\mathrm{D}_i\overline{\mathrm{X}} + d_{0i}} \geq \left(L_i\mathrm{X}^* + l_{0i}\right) + \frac{\mathrm{C}_i\mathrm{X}^* + c_{0i}}{\mathrm{D}_i\mathrm{X}^* + d_{0i}}, \qquad i = 1,2,...,k\,.$$

From Theorem 3.1 it follows that

$$\left(L_iT\overline{\mathrm{w}} + l_{0i}\right) + \frac{C_i\mathrm{T}\overline{\mathrm{w}} + c_{0i}}{D_i\mathrm{T}\overline{\mathrm{w}} + d_{0i}} > \left(L_iT\mathrm{w}^* + l_{0i}\right) + \frac{C_i\mathrm{Tw}^* + c_{0i}}{D_i\mathrm{Tw}^* + d_{0i}},\ \ \text{for any } i$$

or,

$$\left(\overline{L_i}\,\overline{\mathrm{w}} + l_{0i}\right) + \frac{\overline{C_i}\,\overline{\mathrm{w}} + c_{0i}}{\overline{D_i}\,\overline{\mathrm{w}} + d_{0i}} \geq \left(\overline{L_i}\mathrm{w}^* + l_{0i}\right) + \frac{\overline{C_i}\mathrm{w}^* + c_{0i}}{\overline{D_i}\mathrm{w}^* + d_{0i}}\,.$$

This violates (8) and the contradiction proves the result. Finally, let Z_i^* and z_i^* be the optimal values of the objective functions of (1) and (4) at X* and w*, respectively. This means

$$\begin{aligned} Z_i^* &= \left(L_i\mathrm{X}^* + l_{0i}\right) + \frac{C_iX^* + c_{0i}}{D_iX^* + d_{0i}} \\ &= \left(L_i\mathrm{Tw}^* + l_{0i}\right) + \frac{C_i\mathrm{Tw}^* + c_{0i}}{D_i\mathrm{Tw}^* + d_{0i}} \\ &= \left(\overline{L_i}\mathrm{w}^* + l_{0i}\right) + \frac{\overline{C_i}\mathrm{w}^* + c_{0i}}{\overline{D_i}\mathrm{w}^* + d_{0i}} = z_i^* \quad \forall\ i = 1,2,..,k\,. \end{aligned} \tag{10}$$

The result then follows from (5), (6), (8) and (9).

4. Solution of linear plus fractional multiobjective programming problem

Using the transformation $X = Tw$, the problem (1)-(2) reduces to:

$$\text{Maximize } \left\{z_1(\mathrm{w}),\ z_2(\mathrm{w}),...,z_k(\mathrm{w})\right\}$$

where, $z_i(\mathrm{w}) = \left(\overline{L_i}\mathrm{w} + l_{0i}\right) + \dfrac{\overline{C_i}\mathrm{w} + c_{oi}}{\overline{D_i}\mathrm{w} + d_{oi}}$ $\forall\ i = 1, 2, \ldots, k$

subject to $\overline{A}\mathrm{w} = b$

and $\mathrm{w} \geq 0$. (11)

Problem (11) does not contain homogeneous constraints. This MOFLPP can be reduced to a fuzzy programming problem. This model is developed to minimize the perpendicular distances between two parallel hyperplanes $z_i(\mathrm{w}) = \overline{z_i}$ and $z_i(\mathrm{w}) = \underline{z_i}$, where $\overline{z_i}$ and $\underline{z_i}$ are the maximum and minimum values of the objective function $z_i(\mathrm{w})$. We define the distance function d with unit weight as:

$$d_i(\mathrm{w}) = \left|\overline{z_i} - z_i(\mathrm{w})\right| \qquad \forall\ i = 1, 2, \ldots, k .$$

This distance depends upon w. At $\mathrm{w} = \overline{\mathrm{w}}$ (ideal point in w-space) as, given by Gupta and Chakraborty [5], $d = 0$ and at $\mathrm{w} = \underline{\mathrm{w}}$ (nadir point in w-space), $z_i(\mathrm{w}) = \underline{z_i}$, and we get the maximum value of $d_i(\mathrm{w})$ as:

$$\overline{d_i} = \left|\overline{z_i} - \underline{z_i}\right| \quad \forall\ i = 1, 2, \ldots, k . \tag{12}$$

Treating this criterion to be of equal importance, the vector maximum problem (11) may be modeled as follows:

Find an action $\mathrm{w} \in G$, which minimizes

$$\text{Max } \left\{\left|\overline{z_i} - z_i(\mathrm{w})\right|,\ i = 1, 2, \ldots, k\right\},$$

where, $G = \left\{\mathrm{w} : \overline{A}\mathrm{w} = b,\ \mathrm{w} \geq 0\right\}$ ‘ (13)

We define the membership $\mu_i\left(d_i\left(\mathrm{w}\right)\right)$ as follows:

$$\mu_i\left(d_i\left(\mathrm{w}\right)\right) = \begin{cases} 0 & \text{if } d_i\left(\mathrm{w}\right) \geq p \\ \dfrac{p - d_i\left(\mathrm{w}\right)}{p} & \text{if } 0 < d_i\left(\mathrm{w}\right) < p \\ 1 & \text{if } d_i\left(\mathrm{w}\right) \leq 0 \end{cases}$$

where, $p = \sup\left\{\overline{d_i}\right\} \quad \forall \quad i = 1, 2, 3, \ldots, k$.

If λ be the minimum of all $\mu_i\left(d_i\left(\mathrm{w}\right)\right)$, then ,

$$d_i\left(\mathrm{w}\right) \leq -p\lambda + p \text{, i.e. } \overline{z_i} - z_i(\mathrm{w}) \leq -p\lambda + p .$$

Now, the problem reduces to:

$$\text{Max}\lambda$$

subject to $-z_i(\mathrm{w}) + p.\lambda \leq p - \overline{z_i}$, $\forall\ i = 1, 2, \ldots, k$

$\overline{A}\mathrm{w} = b$

and $\lambda,\ \mathrm{w} \geq 0$ (14)

which can be solved by non-linear programming techniques.

5. Pseudo code program for transformation matrix

The matrix *T* can be obtained using a program in C, C++ or any other programming language. For construction of this program, a pseudo code program of the procedure is as follows.

```
// m is the number. of rows of matrix, n is the number. of columns.
// A [1: m] [1: n] is the two dimensional matrix with m rows and n columns.
// P is the number of possible elements of matrix A in the i-th row.
// Q is the number of negative elements of matrix A in the i-th row.
// R is the total number. of columns in the transformation matrix.
// ej is the j-the column of the identity matrix
{
FOR I: = 1 to m do
FOR J: = 1 to n do
READ A [1: m][1:n];
READ I;
 // FOR Ax=b where b=0:
FOR    J: = 1 to n do
{
IF (A [I][J]=0) then
{
T[R] = ej;
R=R+1;
}
}
FOR J: =1 to n do
{
IF (A [I] [J] <0) then
{
P: =P+1;
X [P]: = J;
}
IF (A [I] [J] >0) then
{
Q: =Q+1;
Y [Q] = J;
}
}
FOR K: =1 to P do
{
FOR L: =1 to Q  do
{
T[R] =- a [I] [Y [L]]* e [K] + a [I] [Y [K]]* eY[L];
R: = R+1:
}
}
WRITE T [1: R]; // output transformation matrix.
}
```

6. Numerical example

Example. The following example explains the algorithm.

$$\text{Maximize } \{Z_1(X), Z_2(X)\}$$

where, $Z_1(X) = 2x_1 + \dfrac{2x_1 + 6x_2}{x_1 + x_2 + 1}$, and $Z_2(X) = 2x_2 + \dfrac{x_1 + x_2}{x_1 - x_2 + 1}$

subject to, $x_1 + x_2 + x_3 = 4$

$$3x_1 + x_2 - x_4 = 6$$

$$x_1 - x_2 + 0.x_3 + 0.x_4 \quad = 0$$

and $\quad x_1, x_2, x_3, x_4 \geq 0$.

The solution procedure using our proposed methodology can be well explained stepwise, as given in tabular form Table 1.

Table 1. Solution of example 1

Step 1.	The equivalent problem without homogeneous constraints is given by $\text{Maximize}\{z_1(\text{w}), z_2(\text{w})\}$ where $\quad z_1 = \left(\overline{L_1}\text{w} + l_{01}\right) + \dfrac{\overline{C_1}\text{w}}{\overline{\text{D}_1}\text{w}+1} = \left(LT\text{w} + l_{01}\right) + \dfrac{CT\text{w}}{DT\text{w}+1} = 2w_3 + \dfrac{8w_3}{2w_3+1}$ and $\quad z_2 = 4w_3$ subject to $\quad \overline{A}\text{w} = b$ or $AT\text{w} = b,$ and $\quad \text{w} \geq 0$
Step 2.	Using the given pseudo code and consequently constructed C-program, we obtain: $T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
Step 3.	Using transformation matrix T, our problem becomes: $\text{Maximize}\left\{2w_3 + \dfrac{8w_3}{2w_3+1},\ 4w_3\right\}$ subject to $\quad w_1 + 2w_3 = 4$ $w_2 + 4w_3 = 6$ and $\quad w_1,\ w_2,\ w_3 \geq 0$ over region $G = \left\{\text{w} : \overline{A}\text{w} = b,\ \text{w} \geq 0\right\}$.
Step 4.	Here, we find $\overline{z_1} = 6 \quad \underline{z_1} = 0$, $\overline{z_2} = 6 \quad \underline{z_2} = 0$ and $p = \sup\left\{\overline{d_i}\right\} = 6$.
Step 5.	Then, the given problem reduces into vector maximum problem, $\text{Max. } \lambda$ subject to $\quad 6\lambda \leq 2w_3 + \dfrac{8w_3}{(2w_3+1)}$ $6\lambda \leq 4w_3$ $w_1 + 2w_3 = 4$ $-w_2 + 4w_3 = 6$ and $\quad w_1,\ w_2,\ w_3,\ \lambda \geq 0$
Step 6.	Solving this nonlinear programming problem, the compromise optimal solution of transformed problem is obtained as:

$$w_1 = 0,\ w_2 = 2,\ w_3 = 2,\ \lambda = 16/15\,.$$

The solution of the original problem (X* = Tw) is given as:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix},$$

That is, the compromise optimal solution of (2.1) is:

$$x_1 = 2,\ x_2 = 2,\quad x_3 = 0,\ \ x_4 = 2 \ \text{ and } \ Z_1^* = \frac{36}{5},\quad Z^*_2 = 8\,.$$

7. Particular cases

1. If we take $L_i = 0,\ \forall\ i = 1, 2, ..., k,$ and take only a single objective function with homogeneous constraints, then our problem is reduced to LFPP. This discussion is also given by Chadha [3].

2. If we take $D_i = 0,\ \forall\ i = 1, 2, ..., k$, then $\overline{D_i}$ = 0, and it our problem is reduced to a MOLPP. This discussion is also given by Gupta and Chakraborty [5], by defining the distance function $d_i(\mathrm{w})$ with the weight $\dfrac{1}{\left\{\sum c_{ij}^2\right\}^{1/2}}$.

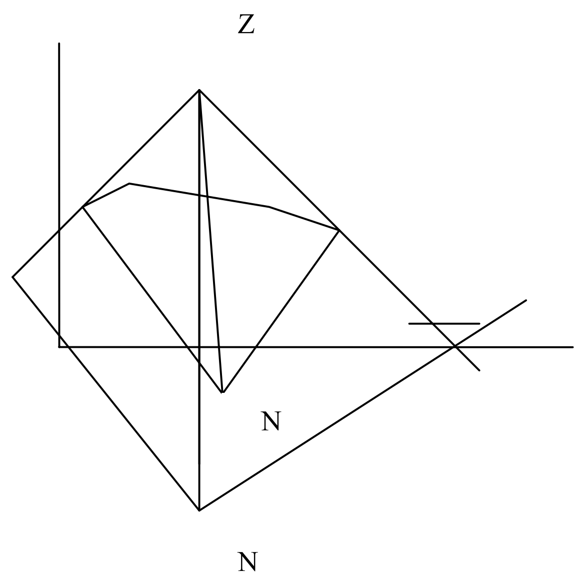

Figure 1. Selection of ideal point

8. Conclusions

When an MOFPP has more than one homogeneous constraints, then we can extend our approach for its solution. In general, T(s) is determined only when AT(1), AT(2),…,AT(s-1), have been computed. This algorithm reduces the number of constraints.

Compromised solution depends on the choice of nadir point (lowest justifiable value) of the objective function. When the justifiable value changes, the compromised solution also changes. As seen in Figure, if Z is the ideal point and N and N′ we the two different minimum aspiration levels then their compromised solution are P and P′, respectively, because NZ and N′Z are the direction in which the decision parameter λ is increased. In our methodology, to find minimum aspiration level, we used minimum value of each objective function. This point is the ideal point for the vector minimization problem of the same objective functions with the same constraints, which generally lies outside the feasible region. Knowing the nadir point (worst point), and zenith point (ideal point) we can find the direction of the decision parameter λ in which λ is increased. Considering the region by taking the lowest justifiable value each objective function gets equal importance in the optimization process.

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