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A new study of an EOQ model for deteriorating items with shortages under inflation and time discounting

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We discuss the effects of inflation and time discounting on an EOQ model for deteriorating items under stock-dependent demand and time-dependent partial backlogging. The inventory model is studied under the replenishment policy starting with no shortages. We then use MATLAB to find the optimal replenishment policies. The objective of this model is to maximize the total profit (TP) which includes the sales revenue, purchase cost, the set up cost, holding cost, shortage cost and opportunity cost due to lost sales. Analytical results are given to justify the model. Finally, numerical examples are presented to determine the developed model and the solution procedure. Sensitivity analysis of the optimal solution with respect to major parameters is carried out. We propose a solution procedure to find the solution and obtain some managerial results by using sensitivity analysis.

Keywords: *Inventory control, Inflation, Stock-dependent Demand, Deteriorating items, Time-dependent demand.*

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1. Introduction

An increase in the general price level results in a reduction in the consumption power of money. Most classical inventory models did not consider the effect of inflation. But most countries suffer from large scale inflation. It is therefore necessary to investigate how inflation influences various inventory policies. The inflationary effect on an inventory policy has been examined by several authors. The first attempt in this direction was by Buzacott [1]. Later, several researchers extended their advent in various exploring situations. Jaggi and Khanna [18] studied retailer's ordering policy for deteriorating items with inflation-induced demand under trade credit policy.
A stock-dependent consumption rate item enumerates that its demand rate depends on the stock level. Much studies have updated inventory policies by focusing on the stock-dependent consumption rate. Gupta and Vrat [12] studied an inventory model for stock dependent consumption rate.

Mandal and Phaujdar [22] proposed an inventory model for deteriorating items and stock dependent consumption rate. Su et al. [33] have investigated it. More investigations can be found in Hou [15], Khanra et al. [20], Nahmias [25], Nandhakumar and Morton [26], Padmanabhan and Vrat [28], Panda et al. [29], Roy and Chaudhuri [30], Soni and Shah [32], Yang et al [37].
Deterioration of items is a common factor. There are many productions in the real world that are subject to significant rate of deterioration. Ghare and Schrader [9] were the first proponents to establish a model from an exponentially decaying inventory. Dave and Patel [6] presented a (T, Si) policy inventory model for deteriorating items with time proportional demand.

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Sachan [31] developed a model on (T, S_i) Policy inventory model for deteriorating items with time proportional demand. Hariga [14] proposed inventory models on deteriorating items. Then, Goyal and Giri [11] generalized the study on inventory models with deteriorating items. Recent related research works are: Chang [3], Chaudhuri and Roy [4], Chern et al. [5], Dye et al. [8], He et al. [13], Jaggi et al. [19], Liao [21], Manna et al. [23], Manna and Chiang [24], Ouyang et al. [27], Tsao and Sheen [35], Wee [36].

The assumption of constant demand rate is not often applicable to many inventory items (e.g., electronic goods, fashionable clothes, tasty foods, etc.) as they experience fluctuation on the demand rate. This phenomenon has been notified by many researchers to develop deteriorating inventory models with time-varying demand pattern. Giri and Chaudhuri [10] developed heuristic models for deteriorating items with shortages and time varying demand and costs. Chakroborty et al. [2] proposed a heuristic for replenishment of deteriorating items with time-varying demand and shortages in all cycles and Teng et al. [34] established deterministic lot size inventory models with shortages and deterioration for fluctuating demand.

Here, we investigate the effects of inflation and time discounting on an EOQ model for perishable items with both stock-varying and time-varying demand and partial backlogging. The Dye and Ouyang [7] model is extended and modified in three was. First, instead of stock-dependent demand, time and stock-dependent demand is considered. Second, inflation is considered in the model. Third, we extend the results of the described model to ameliorating item see Hwang ([16], [17]).

The rest of our work is organized as follows. In Section 2, the notations and assumptions are given. In Section 3, we present the mathematical model. In Section 4, numerical examples are presented to demonstrate the model. Finally, we conclude in Section 5.

2. Notations and Assumptions

To develop the mathematical model, the following notations and assumptions are used.

2.1. Notations

K the ordering cost of inventory per order
C the purchase cost per unit
P the selling price per unit, where $P > C$
h the holding cost per unit per unit time
s the shortage cost per unit per unit time
θ the deterioration rate, a fraction of the on hand inventory
π the opportunity cost due to lost sales per unit
$I(t)$ the inventory level at time t, where $t \in [0,T]$
$R(t)$ the demand rate at time t, where $t \in [0,T]$
η the discount rate of net inflation
δ the backlogging parameter, where $0 \leq \delta \leq 1$
T the length of the replenishment cycle
t_1 the time at which the shortage starts, where $0 \leq t_1 \leq T$
TP the total relevant profit per unit time.

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2.2. Assumptions

The proposed model is explored under the same assumptions as adopted by Dye and Ouyang [7], except the ones related to the time-dependent demand, the inflation and time- discounting. The assumptions are:

1. The replenishment rate is infinite and lead time is zero.
2. The distribution of time to deterioration of the items follows an exponential distribution with parameter θ (i.e., constant rate of deterioration).
3. The unit cost and the inventory carrying cost are known.
4. The selling price per unit and the ordering cost per order are known.
5. The demand rate function *R(t)* is given by

$$R(t)=\begin{cases} f(t)+\beta I(t), & 0\le t\le t_1 \\ f(t), & t_1\le t\le T \end{cases},$$

where $f(t)$ is non-negative and a continuous function of t, $t \in (0,T]$ and β is a positive constant.

6. Shortages are allowed and unsatisfied demand is backlogged at the rate of $1/[1+\delta(T-t)]$. The backlogging parameter δ is a positive constant, and $t_1 \le t \le T$.
7. There is no repair or replacement of the deteriorated items during the production cycle.

3. Model Formulation

A typical behavior of the inventory in a cycle is depicted in Figure 1.

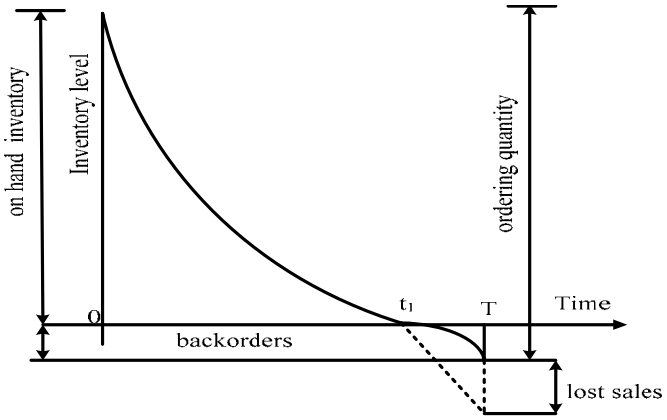


Figure 1. Graphical representation of inventory system

The reduction of the inventory occurs due to the combined effects of the demand and deterioration in the interval $[0, t_1)$ and demand backlogged in the interval $[T, t)$. The instantaneous states of the inventory level $I(t)$ at time t $(0 \le t \le t_1)$ can be described by the following differential equation:

$$\frac{dI(t)}{dt}=-f(t)-\beta I(t)-\theta I(t),\quad 0\le t\le t_1 \qquad (1)$$

with the boundary condition $I(t_1)=0$. Solving equation (1), we get the inventory level as:

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$$I(t) = \int_{t}^{t_1} e^{(\theta+\beta)(u-t)} f(u)du,\ 0 \le t \le t_1. \tag{2}$$

The instantaneous states of the inventory level $I(t)$ at time t ($t_1 \le t \le T$) can be described by the following differential equation:

$$\frac{dI(t)}{dt} = \frac{-f(t)}{[1+\delta(T-t)]},\ t_1 \le t \le T \tag{3}$$

with the boundary condition $I(t_1)=0$. Solving equation (3), we get the inventory level as:

$$I(t) = -\int_{t}^{t_1} \frac{f(u)}{[1+\delta(T-u)]}du, \quad t_1 \le t \le T. \tag{4}$$

The present worth of the profit per unit time of our model consists of the following elements:

- present worth of setup cost per cycle K
- present worth of holding cost per cycle (HC)
- present worth of shortage cost per cycle (SC)
- present worth of opportunity cost due to lost sales per cycle (OC)
- present worth of purchase cost per cycle (PC_1, PC_2)
- present worth of the sales revenue per cycle (SR).

The present worth of the purchase cost during the period [0, t_1] and [t_1, T] is given respectively by

and $$PC_1 = \int_{0}^{t_1} Pe^{-\eta t} f(t) e^{(\theta+\beta)t}\, dt \tag{5}$$

$$PC_2 = \int_{t_1}^{T} P e^{-\eta T} \frac{f(t)}{[1+\delta(T-t)]}\, dt. \tag{6}$$

The present worth of the holding cost for carrying inventory over the period [0, t_1] is given by

$$HC = \int_{0}^{t_1} h e^{-\eta t} I(t) dt$$

$$= \int_{0}^{t_1} h e^{-\eta t} \left[\int_{t}^{t_1} e^{(\theta+\beta)(u-t)} f(u)du \right] dt. \tag{7}$$

The present worth of the shortage cost over the period [t_1, T] is given by

$$SC = s \int_{t_1}^{T} e^{-\eta t} \left[-I(t) \right] dt$$

$$= s \int_{t_1}^{T} e^{-\eta t} \left(-\int_{t}^{t_1} \frac{f(u)}{[1+\delta(T-u)]}du \right) dt. \tag{8}$$

The present worth of the opportunity cost due to lost sales during the period [t_1, T] is given by

$$OC = \pi \int_{t_1}^{T} e^{-\eta t} f(t) \left[1 - \frac{1}{[1+\delta(T-t)]} \right] dt$$

$$= \pi\delta \int_{t_1}^{T} e^{-\eta t} \frac{f(t)(T-t)}{[1+\delta(T-t)]} dt. \tag{9}$$

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The present worth of the sales revenue over the period [0, T] is given by

$$SR = P\int_0^{t_1} e^{-\eta t}\left[f(t) + \beta\int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)du \right]dt + \mathrm{P}\int_{t_1}^{T} \frac{e^{-\eta u} f(u)}{\left[1+\delta\left(T-u\right)\right]}du. \tag{10}$$

Therefore, the present worth of the profit per unit time during the period [0, *T*] is given by

$$\begin{aligned} TP\left(t_1,T\right) =& \frac{1}{T}\left\{ P\int_0^{t_1} e^{-\eta t}\left[f(t) + \beta\int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)du \right]dt \right. \\ &+P\int_{t_1}^{T} e^{-\eta t}\frac{f(t)}{[1+\delta(T-t)]}dt \quad -K-\int_0^{t_1} h\,e^{-\eta t}\left[\int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)du\right]dt \\ &-s\int_{t_1}^{T} e^{-\eta t}\left(-\int_t^{t_1}\frac{f(u)}{\left[1+\delta\left(T-u\right)\right]}du\right)dt \quad -\pi\int_{t_1}^{T} e^{-\eta t} f\left(t\right)\left[1-\frac{1}{\left[1+\delta\left(T-t\right)\right]}\right]dt \\ &\left.-\int_0^{t_1} C f(t) e^{(\theta+\beta)t}\,dt \quad -\int_{t_1}^{T} C\,e^{-\eta T}\frac{f(t)}{[1+\delta(T-t)]}\,dt \right\}. \end{aligned} \tag{11}$$

The solutions for the optimal values of t_1 and T (say t_1* and T*) can be found by solving the following equations simultaneously:

$$\frac{\partial TP\left(t_1,T\right)}{\partial T}=0 \text{ and } \frac{\partial TP\left(t_1,T\right)}{\partial t_1}=0 \tag{12}$$

provided they satisfy the sufficiency conditions:

$$\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial {t_1}^2}\right]_{(t_1*,T*)}<0, \left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial T^2}\right]_{(t_1*,T*)}<0 \text{ and}$$

$$\left[\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial {t_1}^2}\right]\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial T^2}\right]-\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial t_1 \partial T}\right]^2\right]_{(t_1*,T*)}>0. \tag{13}$$

3.1. Analytical Results and Optimal Solution Procedure

Differentiating equation (11) partially with respect to *T* and equating it to zero, we obtain

$$\frac{\partial TP\left(t_1,T\right)}{\partial T}=0 \quad \Rightarrow$$

$$-\frac{1}{T}\ TP\left(t_1,T\right) + \frac{1}{T}\left\{ P\int_{t_1}^{T}\frac{-\delta f\left(t\right)e^{-\eta t}}{\left[1+\delta\left(T-t\right)\right]^2}dt + Pf\left(T\right)e^{-\eta T}\right.$$

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$$-s\left(\int_{t_1}^{T}\left[\int_{t_1}^{t}\frac{-\delta f(u)}{\left[1+\left(\delta(T-u)\right)\right]^2}du\right]e^{-\eta t}dt+\left[\int_{t_1}^{T}\frac{f(u)du}{\left[1+\delta(T-u)\right]}\right]e^{-\eta T}\right)-$$
$$\pi\int_{t_1}^{T}\frac{\delta f(t)}{\left[1+\delta(T-t)\right]^2}e^{-\eta t}dt-Ce^{-\eta T}\int_{t_1}^{T}\frac{-\delta f(t)}{\left[1+\delta(T-t)\right]^2}dt+ \qquad (14)$$
$$C\eta e^{-\eta T}\int_{t_1}^{T}\frac{f(t)}{\left[1+\delta(T-t)\right]}dt-Cf(T)e^{-\eta T}\Bigg\}=0.$$

Differentiating equation (11) partially with respect to t_1 and equating it to zero, we obtain

$$\frac{\partial TP(t_1,T)}{\partial t_1}=0 \Rightarrow$$
$$\frac{1}{T}\Bigg\{P\left(\int_0^{t_1}\left(\beta e^{(\theta+\beta)(t_1-t)}f(t_1)\right)e^{-\eta t}dt+f(t_1)e^{-\eta t_1}\right)$$
$$-P\frac{f(t_1)e^{-\eta t_1}}{\left[1+\delta(T-t_1)\right]}-h\left[\int_0^{t_1}\left(f(t_1)e^{(\theta+\beta)(t_1-t)}\right)e^{-\eta t}dt\right]$$
$$-s\int_{t_1}^{T}\left(\frac{-f(t_1)}{\left[1+\delta(T-t_1)\right]}\right)e^{-\eta t}dt-\pi\left[\frac{-\delta(T-t_1)f(t_1)e^{-\eta t_1}}{\left[1+\delta(T-t_1)\right]}\right]$$
$$-Cf(t_1)e^{(\theta+\beta)t_1}-C\frac{f(t_1)e^{-\eta T}}{\left[1+\delta(T-t_1)\right]}\Bigg\}=0$$

That is,

$$f(t_1)e^{-\eta t_1}\frac{\left(P\beta-h-(\theta+\beta+\eta)C\right)}{(\theta+\beta+\eta)}\left(e^{(\theta+\beta+\eta)t_1}-1\right)=$$
$$-\frac{f(t_1)e^{-\eta T_1}}{\left(1+\delta(T-t_1)\right)}\left(\left(P-C+\pi\right)\delta(T-t_1)+\left(\frac{s}{\eta}-C\right)\left(1-e^{-\eta(T-t_1)}\right)\right)=0. \qquad (15)$$

From the above results, we have the following propositions.

***Proposition* 1.** If $\left(P\beta-h-(\theta+\beta+\eta)C\right)\geq 0$, then the optimal solution of the maximum profit, *TP* (t_1*, *T**), does not exist.

Proof. If $\left(P\beta-h-(\theta+\beta+\eta)C\right)>0$, then equation (15) holds if and only if ($T-t_1$) < 0. this is a contradiction. Similarly, if $\left(P\beta-h-(\theta+\beta+\eta)C\right)=0$, then from equation (15), we obtain $T=t_1$, which is a contradiction. Therefore, if $\left(P\beta-h-(\theta+\beta+\eta)C\right)\leq 0$, then the optimal solution of the maximum profit, *TP* (t_1*, *T**), does not exist.

***Proposition* 2.** The stationary point for the optimization problem not only exists but is also unique.

Proof. In order to prove the uniqueness of the solution, using equation (15), we take x = $T-t_1$ and let

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$$F(x) = \frac{\left(P\beta - h - (\theta+\beta+\eta)C\right)}{(\theta+\beta+\eta)}\left(e^{(\theta+\beta+\eta)t_1} - 1\right) +$$

$$\frac{1}{(1+\delta x)}\left((P - C + \pi)\delta x + \left(\frac{s}{\eta} - C\right)\left(1 - e^{-\eta x}\right)\right),$$

$$\mathrm{F'(x)} = \frac{1}{(1+\delta x)^2}\left\{(1+\delta x)(P - C + \pi)\delta + \left(\frac{s}{\eta} - C\right)\eta e^{-\eta x}\right.$$

$$\left. -\delta\left((P - C + \pi)\delta x + \left(\frac{s}{\eta} - C\right)\left(1 - e^{-\eta x}\right)\right)\right\} > 0.$$

Hence, $F(x)$ is a strictly increasing function in $x \in (0, \infty)$,

$$F(0) = \frac{\left(P\beta - h - (\theta+\beta+\eta)C\right)}{(\theta+\beta+\eta)}\left(e^{(\theta+\beta+\eta)t_1} - 1\right) < 0,$$

$$\lim_{x\to\infty} F(x) = \frac{\left(P\beta - h - (\theta+\beta+\eta)C\right)}{(\theta+\beta+\eta)}\left(e^{(\theta+\beta+\eta)t_1} - 1\right) + \frac{1}{\delta}\left((P - C + \pi)\delta\right) > 0.$$

Therefore, there exists a unique $x^* \in (0,\infty)$ such that $F(x^*) = 0$. So, from this we conclude that once we get the value of t_1, we can find a $T\ (> t_1)$ uniquely determined as a function of t_1.

Let

$$\mathrm{G}(t_1) = -\left\{P\int_0^{t_1} e^{-\eta t}\left[f(t) + \beta\int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)\,du\right]dt\right.$$

$$+P\int_{t_1}^{T} e^{-\eta t}\frac{f(t)}{[1+\delta(T-t)]}\,dt \;-K - \int_0^{t_1} h e^{-\eta t}\left[\int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)\,du\right]dt$$

$$-s\int_{t_1}^{T} e^{-\eta t}\left(-\int_t^{t_1}\frac{f(u)}{\left[1+\delta(T-u)\right]}du\right)dt - \pi\int_{t_1}^{T} e^{-\eta t} f(t)\left[1 - \frac{1}{\left[1+\delta(T-t)\right]}\right]dt$$

$$\left. -\int_0^{t_1} C f(t) e^{(\theta+\beta)t}\,dt - \int_{t_1}^{T} C e^{-\eta T}\frac{f(t)}{[1+\delta(T-t)]}\,dt\right\}$$

$$+T\left\{P\int_{t_1}^{T}\frac{-\delta f(t) e^{-\eta t}}{\left[1+\delta(T-t)\right]^2}dt + Pf(T)e^{-\eta T}\right.$$

$$-s\left(\int_{t_1}^{T}\left[\int_{t_1}^{t}\frac{-\delta f(u)}{\left[1+(\delta(T-u))\right]^2}du\right]e^{-\eta t}dt + \left[\int_{t_1}^{T}\frac{f(u)du}{\left[1+\delta(T-u)\right]}\right]e^{-\eta T}\right) -$$

$$\pi\int_{t_1}^{T}\frac{\delta f(t)}{\left[1+\delta(T-t)\right]^2}e^{-\eta t}dt - Ce^{-\eta T}\int_{t_1}^{T}\frac{-\delta f(t)}{\left[1+\delta(T-t)\right]^2}dt +$$

$$\left. C\eta e^{-\eta T}\int_{T_1}^{T}\frac{f(t)}{\left[1+\delta(T-t)\right]}dt - Cf(T)e^{-\eta T}\right\}.$$

Then,

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$$G(0) > 0 \text{ and } \lim_{T_1 \to \bar{t_1}} G(t_1) < 0,$$

where,

$$\bar{t_1} = \frac{1}{(\theta+\beta+\eta)} \ln\left(1-\frac{(\theta+\beta+\eta)\left((P-C+\pi)\delta\right)}{\delta\left(P\beta-h-(\theta+\beta+\eta)C\right)}\right).$$

Since

$$\frac{\partial TP(t_1,T)}{\partial t_1}=0, \left(\frac{dT}{dt_1}-1\right)>0 \text{ and } \frac{\partial TP^2(t_1,T)}{\partial t_1^{\,2}}<0,$$

we have

$$\frac{dG(t_1,T)}{dt_1} \leq$$

$$T\frac{dT}{dt_1}\left(-\frac{\left(f(t_1)e^{-\eta t_1}\right.}{\left(1+\delta(T-t_1)\right)^2}(P\delta-\pi\delta)-s\delta\left(\frac{e^{-\eta T_1}-e^{-\eta T}}{\left(1+\delta(T-t_1)\right)^2}\right)\right.$$

$$-(\mathrm{s}-\mathrm{C}\eta)\mathrm{e}^{-\eta \mathrm{T}}\ln\left(1+\delta(T-t_1)\right)f(t_1)-C\mathrm{e}^{-\eta \mathrm{T}}\frac{f(t_1)}{\left(1+\delta(T-t_1)\right)^2}$$

$$\left.-C\mathrm{e}^{-\eta \mathrm{T}}(-\eta)\frac{f(t_1)}{\left(1+\delta(T-t_1)\right)}\right)+$$

$$T\left(\frac{-\delta f(t_1)e^{-\eta t_1}}{\left(1+\delta(T-t_1)\right)^2}\left((P-C+\pi)\delta(T-t_1)+\left(\frac{s}{\eta}-C\right)\left(1-e^{-\eta(T-t_1)}\right)\right)+\right.$$

$$\left.\frac{f(t_1)e^{-\eta T_1}}{\left(1+\delta(T-t_1)\right)}\left((P-C+\pi)\delta+\left(\frac{s}{\eta}-P\right)\eta e^{-\eta(T-t_1)}\right)\right)<0.$$

Therefore, there exists a unique $t_1^* \in \left(0,\bar{t_1}\right)$ such that $G(t_1^*) = 0$.

***Proposition* 3.** The stationary point for the optimization problem is a global maximum.

Proof. Now, we examine the corresponding second order sufficient conditions for the optimal solution. Since

$$\frac{\partial^2 TP(t_1,T)}{\partial t_1^{\,2}}= \left(P\beta-h-(\theta+\beta+\eta)C\right)e^{(\theta+\beta+\eta)t_1}+$$

$$\frac{\delta}{\left(1+\delta(T-t_1)\right)^2}\left((P-C+\pi)\delta(T-t_1)+\left(\frac{s}{\eta}-C\right)\left(1-e^{-\eta(T-t_1)}\right)\right)+$$

$$\frac{1}{\left(1+\delta(T-t_1)\right)}\left(-(P-C+\pi)\delta+\left(\frac{s}{\eta}-C\right)\eta e^{-\eta(T-t_1)}\right)<0$$

and

$$\frac{\partial^2 TP(t_1,T)}{\partial T^2}=$$

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$$\left(-P\delta+\pi\delta\right)\left(\int_{t_1}^{T}\frac{-2\delta f\left(t\right)e^{-\eta t}dt}{\left(1+\delta\left(T-t\right)\right)^{3}}+f\left(T\right)e^{-\eta T}\right)+P\left(\eta f(T)e^{-\eta T}+f'(T)e^{-\eta T}\right)$$

$$+\mathrm{s}\,\delta\left\{\int_{t_1}^{T}\frac{\left(\eta e^{-\eta T}\right)}{\left(1+\delta\left(T-t\right)\right)^{2}}dt-2\delta\int_{t_1}^{T}\frac{\left(e^{-\eta t}-e^{-\eta T}\right)}{\left(1+\delta\left(T-t\right)\right)^{3}}dt\right\}$$

$$-\left(\mathrm{s}-\mathrm{C}\,\eta\right)\mathrm{e}^{-\eta T}\left(\int_{t_1}^{T}\frac{-\delta f\left(t\right)}{\left(1+\delta\left(T-t\right)\right)^{2}}dt+f(T)\right)+\eta\left(\mathrm{s}-\mathrm{C}\,\eta\right)\mathrm{e}^{-\eta T}\left(\int_{t_1}^{T}\frac{f\left(t\right)}{\left(1+\delta\left(T-t\right)\right)}dt\right)+$$

$$\mathrm{C}\mathrm{e}^{-\eta T}\;\delta\left(\int_{t_1}^{T}\frac{-2\delta f\left(t\right)dt}{\left(1+\delta\left(T-t\right)\right)^{3}}+f(T)\right)+\mathrm{C}\,\eta\,\mathrm{e}^{-\eta T}\;\delta\;\int_{t_1}^{T}\frac{f\left(t\right)dt}{\left(1+\delta\left(T-t\right)\right)^{2}}$$

$$+\left(P-C\right)\left(f'(T)e^{-\eta T}+\eta f\left(T\right)e^{-\eta T}\right)$$

$$\leq-\left(P\delta-\pi\delta\right)\frac{\left(f(t_1)e^{-\eta t_1}\right.}{\left(1+\delta\left(T-t_1\right)\right)^{2}}-\mathrm{s}\,\delta\left(\frac{e^{-\eta t_1}-e^{-\eta T}}{\left(1+\delta\left(T-t_1\right)\right)^{2}}\right)$$

$$-(\mathrm{s}-\eta\,\mathrm{C})\mathrm{e}^{-\eta T}\;\frac{f(t_1)}{\left(1+\delta\left(T-t_1\right)\right)}-\mathrm{C}\,\mathrm{e}^{-\eta T}\;\frac{f(t_1)}{\left(1+\delta\left(T-t_1\right)\right)^{2}}-\mathrm{C}\,\eta\,\mathrm{e}^{-\eta T}\;\frac{f(t_1)}{\left(1+\delta\left(T-t_1\right)\right)}<0 \qquad (17)$$

$$\frac{\partial^2 TP\left(t_1,T\right)}{\partial T\partial t_1}=\frac{\partial^2 TP\left(t_1,T\right)}{\partial t_1\partial T}=$$

$$\frac{-\delta f(t_1)e^{-\eta t_1}}{\left(1+\delta\left(T-t_1\right)\right)^{2}}\left(\left(P-C+\pi\right)\delta\left(T-t_1\right)+\left(\frac{s}{\eta}-P\right)\left(1-e^{-\eta\left(T-t_1\right)}\right)\right)$$

$$+\frac{f(t_1)e^{-\eta t_1}}{\left(1+\delta\left(T-t_1\right)\right)}\left(\left(P-C+\pi\right)\delta+\left(\frac{s}{\eta}-C\right)\eta e^{-\eta\left(T-t_1\right)}\right) \qquad (18)$$

$$\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial {t_1}^2}\right]_{(t_1*,T*)}<0,\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial T^2}\right]_{(t_1*,T*)}<0.$$

From the above, we obtain

$$\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial T^2}\right]_{(t_1*,T*)}>\left|\frac{\partial^2 TP\left(t_1,T\right)}{\partial T\partial t_1}\right|_{(t_1*,T*)}$$

$$\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial {t_1}^2}\right]_{(t_1*,T*)}>\left|\frac{\partial^2 TP\left(t_1,T\right)}{\partial T\partial t_1}\right|_{(t_1*,T*)}$$

and

$$\left|\mathrm{H}\right|=\left[\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial {t_1}^2}\right]\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial T^2}\right]-\left[\frac{\partial^2 TP\left(t_1,T\right)}{\partial t_1\partial T}\right]^2\right]_{(t_1*,T*)}>0\,.$$

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Hence, the Hessian matrix at (t_1^*, T^*) is negative definite. So, the stationary point for our optimization problem is a local maximum. Since the maximal point is unique, we conclude that the optimal point is a global maximum.

3.2. Solution Procedure

***Step* 1.** Solve equations (14) and (15), find T and t_1 values.

***Step* 2.** Using equation (11), the find TP value.

4. Numerical Examples and Sensitivity Analysis

In this section, numerical examples are given to illustrate the proposed model and its solution procedure. Example 1 below considers the exponentially increasing case and the solution procedure. Example 1 considers an exponentially increasing demand function: $f(t) = ae^{bt}$ ($a > 0$, $b > 0$). Example 2 considers a constant demand function: $f(t) = a$ ($a > 0$). Sensitivity analysis for β, δ, η are also reported for the two types of functions mentioned above.

Example 1: Let $s = 3$, $K = 250$, $\theta = 0.05$, $f(t) = ae^{bt}$, $h = 1.75$, $P = 15$, $C = 5$, $\pi = 5$, $\beta = 0.2$, $\delta = 5$, $a = 600$, $\eta = 0.14$, and $b = 3$ in appropriate units. Numerical values are shown in Table 1.

Example 2: Let $s = 3$, $K = 250$, $\theta = 0.05$, $f(t) = ae^{bt}$, $h = 1.75$, $P = 15$, $C = 5$, $\pi = 5$, $\beta = 0.2$, $\delta = 5$, $a = 600$, $\eta = 0.14$, and $b = 0$ in appropriate units. Numerical values are shown in Table 2.

Table 1. Optimal replenishment schedule for f(t) is a nonlinear function

η	T	T_1	T_1/T	TP
0.01	0.50	0.10	0.20	12095.75
0.09	0.50	0.10	0.20	11716.01
0.14	0.50	0.10	0.20	11483.75
0.16	0.50	0.10	0.20	11391.92
0.18	0.50	0.10	0.20	11300.69
0.20	0.50	0.10	0.20	11210.07

Table 2. Optimal replenishment schedule for $f(t)$ as a constant

η	T	T_1	T_1/T	TP
0.01	0.50	0.40	0.80	5135.06
0.09	0.50	0.40	0.80	4991.41
0.14	0.50	0.40	0.80	4903.35
0.16	0.50	0.40	0.80	4868.49
0.18	0.50	0.40	0.80	4833.84
0.20	0.50	0.40	0.80	4799.39

The numerical values displayed in Table 1 and Table 2 indicate that $f(t)$ is an exponentially increasing demand function of time, and the total profit is greater than that of $f(t)$ as a constant.

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Table 3. Sensitivity analysis for $f(t)$ as an exponentially increasing function

δ	β	η	T	$T1$	$T1/T$	TP
5	0.200	0.01	0.500	0.100	0.200	12095.75
5	0.001	0.01	0.800	0.100	0.125	25512.07
1	0.200	0.01	0.300	0.100	0.333	29685.99

Table 4. Sensitivity analysis for $f(t)$ as a constant

δ	β	η	T	$T1$	$T1/T$	TP
0.200	5	0.01	0.900	0.800	0.900	5064.39
0.001	5	0.01	0.500	0.400	0.800	5135.06
0.200	1	0.01	0.900	0.800	0.900	5632.95

Next, we discuss some practical applications of our model.

4.1. Managerial Implications

Based on our numerical results, we observe the following managerial concern:

- Those who are having high backlogging parameter value suffer more than those having less backlogging parameter value.
- Models having constant demands have less profit than those having time dependent demands.
- When η increases TP decreases (See Appendix 1 and 2).
- More concentration on stock turns to reduce the profit.

4.2. Special Cases

- By taking $\beta = 0$, the described model reduces to an economic order quantity (EOQ) model for deteriorating items with time dependent demand function under partial backlogging and inflation.
- By taking $\beta = 0$, $f(t)$ = a constant, the described model reduces to an EOQ model for deteriorating items with constant demand function under partial backlogging and inflation.
- By taking f(t) = a constant, the described model reduces to an EOQ model for deteriorating items with stock-dependent demand function under partial backlogging and inflation.
- By taking $\eta = 0$, the described model reduces to an EOQ model for deteriorating items with time, stock-dependent demand function under partial backlogging**.**
- By taking $\theta = -\theta$, the described model reduces to an EOQ model for ameliorating items with time, stock dependent demand function under partial backlogging and inflation.

5. Conclusions

We discussed the effects of inflation and time discounting on an EOQ model for deteriorating items under stock-dependent selling rate and time-dependent partial backlogging. We considered demand as not only a function of stock but also fluctuating with time. For $\eta = 0$, and $f(t)$ = a, our model reduces to that of Dye and Ouyang [7]. Inflation is a main factor in the present model. Finally, the sensitivity of the solutions to changes in the values of different parameters was discussed. The proposed model can be extended in several ways. The deterministic demand

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function can be considered to have stochastic demand patterns, as a price. Also, we could generalize the model to allow for quantity discounts, and permissible delay in payments

Appendix 1

Claim. *TP* is a decreasing function of η.

Proof. Differentiate TP partially with respect to η, to obtain

$$\frac{\partial TP(t_1,T)}{\partial \eta} = \frac{1}{T}\left\{-\eta P\int_0^{t_1} e^{-\eta t}\left[f(t)+\beta\int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)du\right]dt\right.$$

$$-\eta P\int_{t_1}^{T} e^{-\eta t}\frac{f(t)}{[1+\delta(T-t)]}\,dt \quad +\eta\int_0^{t_1} h e^{-\eta t}\left[\int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)\,du\right]dt$$

$$+\eta s\int_{t_1}^{T} e^{-\eta t}\left(-\int_t^{t_1}\frac{f(u)}{\left[1+\delta(T-u)\right]}du\right)dt +\eta\pi\int_{t_1}^{T} e^{-\eta t} f(t)\left[1-\frac{1}{\left[1+\delta(T-t)\right]}\right]dt$$

$$\left.+\eta\int_0^{t_1} Cf(t)e^{(\theta+\beta)t}\,dt \quad +\eta\int_{t_1}^{T} C e^{-\eta T}\frac{f(t)}{[1+\delta(T-t)]}\,dt\right\}<0$$

Since $\dfrac{\partial TP(t_1,T)}{\partial \eta}<0$ then implies *TP* is a decreasing function of η.

Appendix 2

Claim. TP is a convex function of η.

Proof. Differentiate *TP* partially with respect to η and equate it to zero to obtain

$$\frac{\partial TP(t_1,T)}{\partial \eta} = \frac{1}{T}\left\{-\eta P\int_0^{t_1} e^{-\eta t}\left[f(t)+\beta\int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)du\right]dt\right.$$

$$-\eta P\int_{t_1}^{T} e^{-\eta t}\frac{f(t)}{[1+\delta(T-t)]}\,dt \quad +\eta\int_0^{t_1} h e^{-\eta t}\left[\int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)\,du\right]dt$$

$$+\eta s\int_{t_1}^{T} e^{-\eta t}\left(-\int_t^{t_1}\frac{f(u)}{\left[1+\delta(T-u)\right]}du\right)dt +\eta\pi\int_{t_1}^{T} e^{-\eta t} f(t)\left[1-\frac{1}{\left[1+\delta(T-t)\right]}\right]dt$$

$$\left.+\eta\int_0^{t_1} Cf(t)e^{(\theta+\beta)t}\,dt \quad +\eta\int_{t_1}^{T} C e^{-\eta T}\frac{f(t)}{[1+\delta(T-t)]}\,dt\right\}=0.$$

The second derivative of *TP* with respect to η, is:

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$$\frac{\partial^2 TP(t_1,T)}{\partial \eta^2} = \frac{1}{T}\left\{ \eta^2 P \int_0^{t_1} e^{-\eta t}\left[f(t) + \beta \int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)du \right] dt \right.$$

$$+\eta^2 P \int_{t_1}^{T} e^{-\eta t} \frac{f(t)}{[1+\delta(T-t)]}\, dt \quad -\eta^2 \int_0^{t_1} h\, e^{-\eta t}\left[\int_t^{t_1} e^{(\theta+\beta)(u-t)} f(u)du \right] dt$$

$$-\eta^2 s \int_{t_1}^{T} e^{-\eta t}\left(-\int_t^{t_1} \frac{f(u)}{\left[1+\delta(T-u)\right]} du \right) dt \quad -\eta^2 \pi \int_{t_1}^{T} e^{-\eta t} f(t)\left[1 - \frac{1}{\left[1+\delta(T-t)\right]} \right] dt$$

$$\left. -\eta^2 \int_0^{t_1} C f(t) e^{(\theta+\beta)t}\, dt \quad -\eta^2 \int_{t_1}^{T} C\, e^{-\eta T} \frac{f(t)}{[1+\delta(T-t)]}\, dt \right\} > 0.$$

This implies that *TP* is convex with respect to η.

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