

Interdiction problem as a tool to identify an effective budget allocation to quality improvement plans

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In the face of budgetary limitations in organizations, identifying critical facilities for investing in quality improvement plans could be a sensible approach. Here, hierarchical facilities with specified covering radius are considered. If disruption happens to a facility, its covering radius will be decreased. For this problem, a bi-objective mathematical formulation is proposed. Critical facilities are equivalent to the facilities which attacking them cause the most reduction in the quality of the system performance. Consequently, this problem is studied in the framework of interdiction problem. To solve the multi-objective model, the weighting-sum approach is applied. The first interdicator's objective function helps decision makers to identify the vulnerability of the system. Moreover, the second objective function may assist in minimizing the cost of applied quality improvement plans.

Keywords: bi-objective mathematical model, critical facilities, interdiction problem, weighting approach.

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1. Introduction and Overview

Nowadays some companies are obsessive about the quality of their services in order to survive. To provide a profitable service system, most companies try to maximize the reliability and availability of their facilities to more customers [1]. In the face of this fierce competition in the marketplace, interests in the implementation of quality improvement plans have increased (as a comprehensive reference book for quality improvement plans, see [2]).

Regarding the increasing attention to the level of customer satisfaction, a lot of plans for quality improvement have been proposed [3]. Due to availability of various plans and also restrictions on available budget, managers are required to decide how to invest in quality improvement plans. Furthermore, with respect to the inherent uncertainty of disruptions, risk management is one of the most crucial tasks for managers. Based on [4], "Risk-based decision-making and risk-based approaches in decision-making are the terms frequently used to indicate some systematic process that deals with uncertainties being used to formulate policy options and assess their various distributional impacts and ramifications". For example, in [5], defining alternative future scenarios is applied as a risk-based decision-making approach to deal with the uncertainty associated with future events for service facilities and it is indicated that there are two common approaches to optimize the objective function: (1) optimizing the expected performance over all future scenarios, and (2) optimizing the expected performance over the expected scenario. These two popular

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approaches have important drawback; the planner needs to estimate the probabilities which are associated with the defined scenarios. Since there are numerous disruption possibilities in natural disasters and random disruptions, defining all scenarios and estimating their probabilities is a tough task [6]. As a remedy for this drawback, here, focusing on the worst-case vulnerability reduction which obviates the need for these probabilities is used. Optimizing the worst-case scenario is potentially effective but does not necessarily ensure the best investment scheme and allocation of protective resources [7]. Therefore, to arrive at a more intelligent decision, managers should accumulate more knowledge about other related non-quantitative factors and take them into account.

Here, we consider the problem as a multi-objective one to overcome the rigid behavior of making decision based on the worst-case scenario. We take the worst-case scenario into account as a risk management method for identifying critical infrastructure (for more information on other methods of risk management, see [4]). "Critical infrastructure can be defined as those elements of infrastructure that, if lost, could pose a significant threat to needed supplies, services, and communications or a significant loss of service coverage or efficiency" [8]. For more information on the background of our work, we refer to [8]-[10].

Inspired by many real service systems, here, a non-nested hierarchical system is considered. "Hierarchical systems have multiple layers of interacting facilities. A system is classified as nested or non-nested according to the service availability at the levels of hierarchy. In a nested hierarchy, a higher-level facility provides all the services provided by a lower level facility and at least one additional service. In a non-nested hierarchy, facilities on each level offer different services." [11].

In covering models, a demand point is covered if at least one facility can serve it within a specified distance standard [9] and in the majority of hierarchal systems, it is assumed that each facility, based on its service level, can cover demands with their distances from the facility being less than a predefined value. Nevertheless, in our model a demand is considered to be satisfied if it is covered directly and/or indirectly by the hierarchical facilities that provide the customer with the necessary service levels.

Interdiction problem dates back to 1960 [12]. An intentional strike against a system is called interdiction [8]. Generally, interdiction problem can be considered as a two-player Stackelberg game between a system defender whose aim is to conserve her system performance and an interdictor who attempts to cause the most damage.

Identifying critical facilities and offering protective strategies are not the only cases of terrorist attacks. Even in a safe situation with no possibility of an intentional attack, to strengthen the system sustainability and reliability in the face of natural catastrophes and random disruptions, being aware of the vulnerability of the system components and fortifying the critical components are absolutely vital [13]. Here, interdiction problem is employed as a framework to recognize the vulnerability of the system components. In this problem, critical facilities are equivalent to the facilities cause the most reduction in the quality of the system performance, if attacked by an imaginary interdictor. Decision makers interpret the result of this interdiction problem as a measure of importance for system components. Partial interdiction is used versus full interdiction in order to provide a better image for the importance of facilities. In full interdiction, the interdicted facility loses its total capability to serve customers while in partial interdiction the interdicted facility does not necessarily end up with a total reduction of its functionality [10]. The higher level a facility is attacked, the more crucial role it plays in the quality of the services; therefore, it attracts more budgets for investment in its quality improvement plans.

The rest of our work is organized as follows. Section 2 provides a reasonably comprehensive problem description. A multi-objective mathematical formulation is given in Section 3. We present an example with a computational result analysis to illustrate the applicability of our work in Section 4. Finally, Section 5 gives a brief summary of our findings and recommendations for further research.

2. Problem Description

In this section, a detailed description of the problem is given. A non-nested hierarchical service system is studied in order to ameliorate its performance in the wake of the worst-case scenario by means of the implementation of quality improvement plans.

Each facility in the system can serve the demand points with their distances from the facility being less than a specified value. By experience, it is known that each customer's specific percentages of demands require particular service levels. Moreover, as a result of the hierarchical nature of the system, distinct percentages of demands of each customer are required to be covered directly and/or indirectly for multi-stage services.

To illustrate multi-stage services, assume $\varphi_{hh'}$ is a percentage of demands of each demand point that is required to be served by hh' multi-stage service strategy: First, a customer should obtain her direct service demand from a facility at level h with distance from the customer being in its radius of coverage. The facility may also refer the customer to a facility at level h' with distance between the two facilities being less than the radius of coverage of the facility at level h' . For clarification, see Fig. 1. Note that each dotted circle with its corresponding centroid facility illustrates the coverage area of a facility. Demand point 14 is in the coverage area of facility 3 and facility 6, respectively at levels 2 and 3, so the direct demand for the service level is satisfied. However, the multi-stage demand for level $hh' = 23$ and also for level $hh' = 32$ are not satisfied, because these facilities are not located in each other's coverage area.

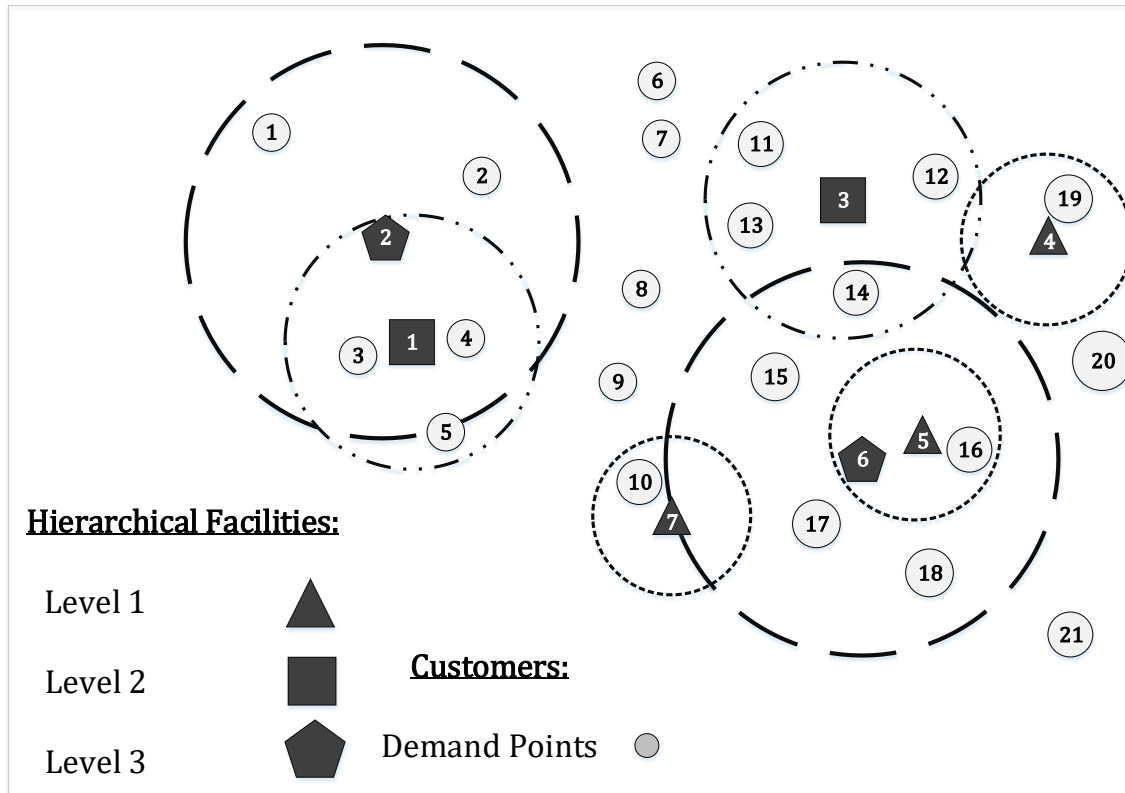


Fig. 1. An example of a hierarchical system with three levels of facilities

To sum up the notion of multi-stage demand satisfaction, we consider this kind of demand of a customer as satisfied if the customer is in the coverage radius of at least one facility at level h and the facility is in the coverage radius of at least one facility at level h' .

As previously indicated, in order to identify critical facilities, an interdiction framework with different levels of interdiction is used. In this problem, an imaginary interdictor whose aim is to cause the most damage with the least interdiction cost is considered. However, the problem is construed in a way that the quality of improvement unit is aimed to invest in most critical areas.

Decision-makers could benefit from the first interdictor's objective function to identify the vulnerability of the system. Moreover, the second objective function gives the intention to minimize the cost of applied quality improvement plans. The level of partial interdiction can aid the quality unit to draw a comparison for the role of facilities in quality indicators.

3. Problem Formulation

In this section, to have a more flexible model, a multi-objective mathematical formulation is presented.

3.1. Notations

Notations and decision variables are introduced as follows.

Indices and sets:

I	set of demand nodes
J	set of existing facilities
H	set of service levels
K	set of interdiction levels
i	index for customers ($i \in I$)
j, j'	indices for facilities ($j, j' \in J$)
h, h'	indices for service levels of facilities ($h, h' \in H$)
k	index for interdiction level ($k \in K$).

Parameters:

d_i	demand of customer i
θ_h	percentage of demand of each customer that requires a direct service at level h
$\phi_{hh'}$	percentage of demand of each customer that requires indirect service at level h and h'
σ_h	coverage radius of a facility at level h
λ_{hk}	decreased amount of the coverage radius of a facility at level h if it is interdicted at level k
α_{ij}	distance between demand point i and facility j
$\beta_{jj'}$	distance between facilities j and j'
c_k	cost of interdiction at level k
p	per unit profit of direct satisfied demand
q	per unit profit of indirect satisfied demand
f_{jh}	a binary parameter, which is equal to 1 if the service level of facility j is h , and is 0 otherwise
M	a very large positive number

Decision variables:

x_{ih}	a binary variable, which equals to 1 if demand point i is covered at service level h , and is 0 otherwise
y_{jk}	a binary variable, which equals to 1 if facility j is interdicted at interdiction level k , and is 0 otherwise
$z_{ihh'}$	a binary variable, which equals to 1 if demand point i is covered directly and indirectly at service level h and h' , and is 0 otherwise
u_{ijh}	a binary variable, which equals to 1 if facility j covers demand point i at service level h , and is 0 otherwise
$v_{jj'}$	a binary variable, which equals to 1 if facility j is in coverage radius of facility j' , and is 0 otherwise

3.2. Bi-objective formulation

$$f_1: \text{Min} \sum_{i \in I} \sum_{h \in H} p d_i \theta_h x_{ih} + \sum_{i \in I} \sum_{h \in H} \sum_{h' \in H} q d_i \varphi_{hh'} z_{ihh'} \quad (1)$$

$$f_2: \text{Min} \sum_{k \in K} \sum_{j \in J} c_k y_{jk} \quad (2)$$

Subject to

$$\sum_{k \in K} y_{jk} = 1, \forall j \quad (3)$$

$$\sigma_h f_{jh} - \sum_{k \in K} y_{jk} \lambda_{hk} f_{jh} - \alpha_{ij} f_{jh} \leq M. u_{ijh}, \forall i, j, h \quad (4)$$

$$M. x_{ih} \geq \sum_{j \in J} u_{ijh}, \forall i, h \quad (5)$$

$$\sigma_h f_{j'h'} - \sum_{k \in K} y_{j'k} \lambda_{h'k} f_{j'h'} - \beta_{jj'} \leq M. v_{jj'}, \forall j, j', h' \quad (6)$$

$$z_{ihh'} \geq v_{jj'} f_{jh} f_{j'h'} + u_{ijh} f_{jh} - 1, \forall i, j, j', h, h' \quad (7)$$

$$x_{ih}, y_{jk}, z_{ihh'}, u_{ijh}, v_{jj'} \in \{0,1\}, \forall i, j, j', h, h', k \quad (8)$$

This model consists of two objective functions. The first objective function of the model, as shown in (1), states that the first goal of the attacker is to minimize the profit of satisfied demand of all customers. Objective function (2) shows that the second goal of the attacker is to minimize the total cost of the interdiction. Constraints (3) maintain that only one interdiction level can be chosen for each facility, including the zero level or level of no interdiction. Constraints (4) indicate that demand point i is covered at service level h by facility j if its distance from facility j is less than the coverage radius of the facility after interdiction and also the service level of facility j is h . Constraints (5) ensure that demand point i is covered at service level h if and only if its distance from at least one facility at service level h is less than the coverage radius of that facility after interdiction. Constraints (6) are similar to constraints (4), whereas in constraints (6) covering of one facility by another is considered. Constraints (7) play a significant role by making a logical connection among the covering variables of the model. These constraints guarantee indirect coverage of demand point i at service levels h and h' if customer i is covered at level h by at least one facility and that facility is covered by at least one facility at level h' . Constraints (8) are standard binary constraints on the key decision variables.

4. Solution Procedure and Computational Results

In this section, first a weighting approach as a solution procedure for small-scale and medium-scale problems is described and its advantages and disadvantages are discussed. Next, an illustrative example is given for more clarification on the practical application of the proposed approach. Finally, some estimation and relations about the required computational effort are obtained.

4.1. Weighting approach: benefits and drawbacks

To solve the multi-objective problem we use a weighting scheme (i.e., a linear combination of objective functions). Multi-objective optimization problems cope with the existence of different conflicting objectives. Since achieving a solution to optimize all the objective function simultaneously is not feasible in general, obtaining a set of solutions which are called the non-dominated frontier could be a reasonable approach [15]. The weighting method is one of the most popular multi-objective methods to obtain non-dominated (Pareto) frontier by setting different weights.

According to [14], while the weighting method is computationally tractable and potentially beneficial, it has several basic drawbacks: “First, it is often not known how much importance should be given to each objective (i.e., weighting) in advance (or during) conducting the optimization procedure. Results can be substantially different with different weights or weighting schemes. Second, the weighted-sum approach cannot identify all points in a trade-off surface of non-convex solution spaces. This is a major handicap of the weighted-sum approach since many combinatorial problems have non-convex and discontinuous solution spaces. Third, a problem of scaling among objectives can occur. It is likely that each objective takes different orders of magnitudes which can affect the mathematical procedure. Normalization can solve the problem of scaling but requires a priori setting of proper ranges along which to scale.”

The first difficulty that has been mentioned above is not a drawback of using the weighting method for our proposed problem. A reasonable weighting scheme is one that proposes the most destructive interdiction scheme (equally, the most effective quality improvement plan) with regard to interdiction budget (equally, subject to the investment program that was planned by quality unit). Therefore, to obtain a clear picture of the effect of the investment programs on the system performance, the quality unit will define some sensible weighting scenarios. The second difficulty, by contrast, is a challenging one in the proposed problem. However, with some justification, it could be ignored. By using a weighting approach, due to the discontinuous solution space there will be several non-supported solutions (i.e., the solutions that are non-dominated but cannot be obtained by using any weighting scheme). Since in this problem a high emphasis is laid on practical applications, using different weighting schemes could help a decision-maker to associate with flexible nature of the problem. Therefore, in this problem a method which needs less computational effort is valued more than a method which guarantees to obtain the whole Pareto frontier. Furthermore, the decision-maker may consider some non-quantitative indicators in addition to the result of the problem of identifying critical facilities to achieve a more sensible investment program. Finally, the third mentioned issue does not lead to any difficulties in this problem and the similarity of scaling between these two objectives is the apparent reason behind it.

4.2. Illustrative example

Here, we present a computational experiment to illustrate our approach. We first provide some generic information about the solver software, platform and the parameter setting.

The model was coded C++ and the program was compiled using Microsoft Visual 2010. To solve the IP problem, we used the generic MIP solver CPLEX 12.3. The instances were tested on a 64-bit computer with an Intel Core i5 1.60 GHz processor and 4.00 GB of RAM.

To clarify the performance of the proposed model and the solution procedure, one instance is generated. In Table 1, parameters of this example are given. The size of this randomly generated instance is very small to make it possible to analyze the results by means of a number of figures and tables.

In Fig. 2, this non-nested hierarchical system is shown. Before any disruption, the coverage radiuses of the facilities cover all customers' demands except for customer 8 at direct service level 1 and indirect service level $hh' = 12$. However, customer loss is possible, due to disruption to the system components. The goal is to identify the most important components from an imaginary interdictor's point of view.

To solve this problem by the weighting method, a linear combination of the objective functions, $w_1.f_1 + w_2.f_2$, is substituted for the two objective functions. The results are examined by considering eleven different weighting schemes. This analysis helps us to draw a useful comparison among different weighting setting schemes. The results are summarized in Table 2. The numbers in the "facilities" columns are the levels of interdiction assigned to the facilities. In this problem, by using eleven weighting schemes, six non-dominated solutions were obtained.

Table 1. Parameter setting

Parameters	Values
$ I $	8
$ J $	$3 \rightarrow J_1 = 2, J_2 = 1$
$ H $	2
$ K $	$4 \rightarrow \{0, 1, 2, 3\}$
d_i	$\{138, 244, 195, 34, 163, 175, 222, 34\}$
θ_h	$\{0.6, 0.4\}$
$\varphi_{hh'}$	$\varphi_{1h'} = \{0, 0.6\}, \varphi_{2h'} = \{0.4, 0\}$
σ_h	$\{500, 900\}$
λ_{hk}	$\lambda_{1k} = \{0, 150, 200, 500\}, \lambda_{2k} = \{0, 100, 500, 900\}$
$coordination(i)$	$(-150, -300), (400, 70), (-155, 139), (400, -300), (-200, -170), (325, 260), (356, -164), (-450, 35)$
$coordination(j)$	$(136, -382), (171, 138), (-200, -281)$
c_k	$\{0, 800, 1500, 3200\}$
p	2
q	7
f_{jh}	$f_{0h} = \{1, 0\}, f_{1h} = \{1, 0\}, f_{2h} = \{0, 1\}$

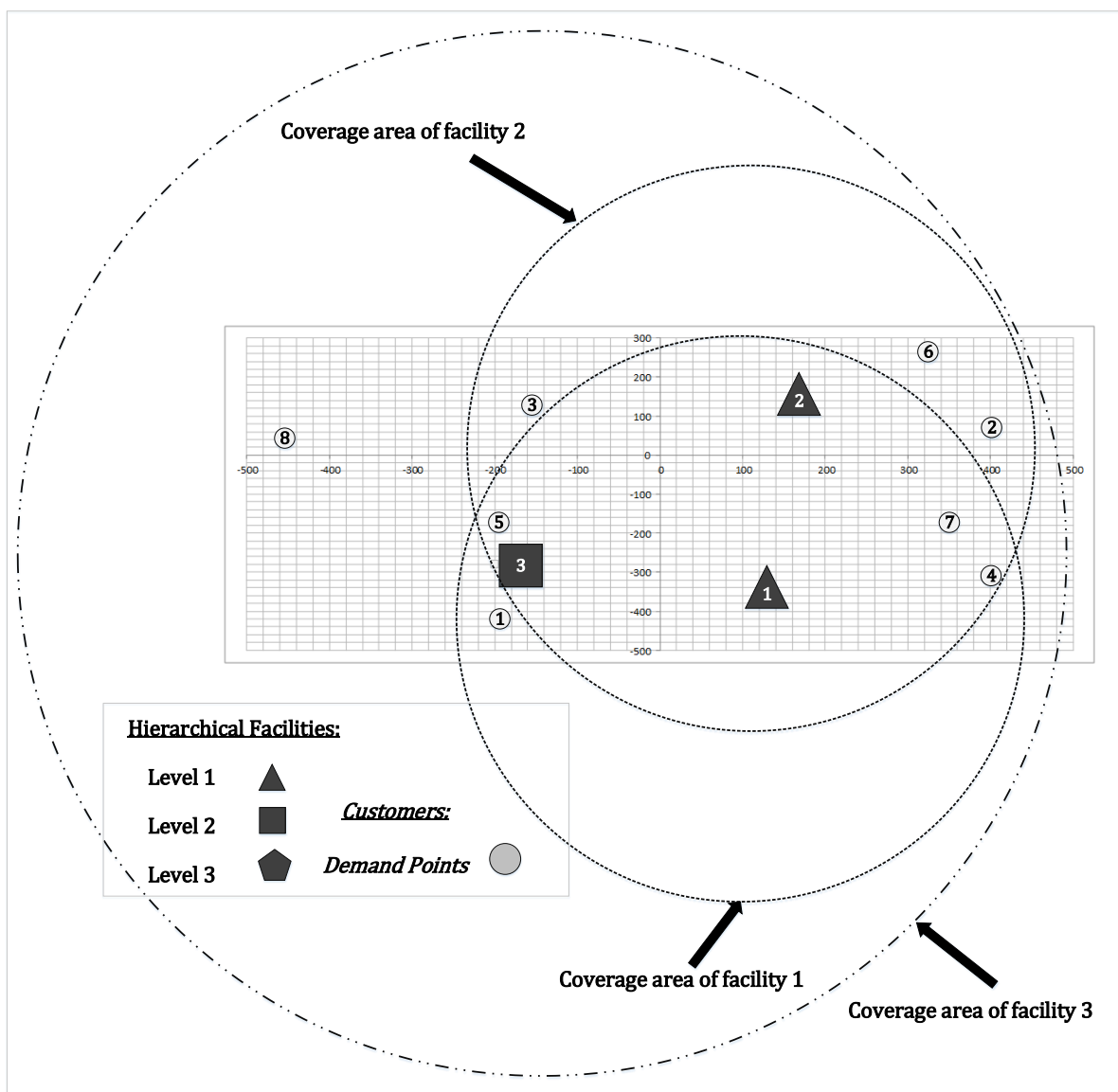


Fig. 2. The hierarchical system of the example

Table 2. Pareto solutions for the example

Scheme	weights	facilities			objective function			CPU time (S)
	(w_1, w_2)	1	2	3	f_1	f_2	$w_1 \cdot f_1 + w_2 \cdot f_2$	0/078
1	(0,1)	0	0	0	8795	0	0	0/074
2	(0/1,0/9)	0	0	0	8795	0	879	0/102
3	(0/2,0/8)	1	0	0	5421	800	1724	0/071
4	(0/3,0/7)	0	0	2	3685	1500	2155	0/097
5	(0/4,0/6)	0	0	3	502	3200	2121	0/090
6	(0/5,0/5)	0	0	3	502	3200	1851	0/089
7	(0/6,0/4)	0	0	3	502	3200	1581	0/098
8	(0/7,0/3)	0	0	3	502	3200	1311	0/102
9	(0/8,0/3)	2	3	3	40	7900	1612	0/089
10	(0/9,0/1)	3	3	3	0	9600	960	0/079
11	(1,0)	3	3	3	0	9600	0	0/078

To validate the results, a validation approach due to Pishvae et al. [16] was applied to this example.

Consider the original model:

$$\text{Minimize } f_1(x) \quad (9)$$

$$\text{Minimize } f_2(x) \quad (10)$$

$$\text{Subject to } Ax \leq b \quad (11)$$

$$x \in \{0,1\} \quad (12)$$

For validation, to verify that the solutions obtained by the weighting method are non-dominated solutions, first the original models for each objective function in the absence of the other one (i.e., a model consisting of (9), (11) and (12) and a model consisting of (10), (11) and (12)) was solved optimally by the CPLEX solver. Through solving these two models, the two extreme points of the Pareto frontier was validated (i.e., the solutions obtained by using the scheme 1, 2, 10 and 11 in Table 2).

To validate other solutions of the weighting method, first one of the objective functions was added to the constraint set with a right-hand side equal to the value of the objective function (as shown by B) in each scheme. Afterwards, the new model was solved by the CPLEX solver.

The second objective function tries to minimize the budget which is used for each selected plan. Therefore, considering the logic behind our proposed model, the second objective function is added to the constraint set. The resulting model is expressed as:

$$\text{Minimize } f_1(x) \quad (13)$$

$$\text{Subject to } Ax \leq b \quad (14)$$

$$f_2(x) \leq B \quad (15)$$

$$x \in \{0,1\} \quad (16)$$

Note that, by selecting any value for B in the range of the two extreme points of the Pareto frontier, this approach could be applied individually to find non-dominated solutions. However, the purpose of our approach was to validate the results of the weighting method. Therefore, the value of B is limited to the values of the second objective function corresponding to the schemes of Table 2.

By applying this approach, all solutions obtained by weighting method were validated. In Fig. 3, the relations between the two objective functions of the six validated non-dominated solutions were shown. The two objective functions (i.e., f_1 , profit of the organization and f_2 , Interdiction cost) are in conflict with each other and an increase in the interdiction cost causes a reduction in the profit of the organization.

The weighting method provides managers with a wide perspective on the trade-offs between the budget invested in quality improvement plans and the customer satisfaction level and profit of the organization. The quality improvement unit could identify the degree of the importance by an interpretation of the level of the interdiction.

For example, in Table 3, in order to become more familiar with the vulnerability of the system components, the frequency of the interdiction level of each facility in the five non-dominated solutions (for all schemes of Table 2 expect for scheme 1 and 2, since in these two schemes no facility is interdicted) are summarized. A number in the “interdiction levels” column is the frequency that the interdiction level is chosen for each facility in the five non-dominated solutions. Column “MOST” presents the most frequent level for each facility among the five non-dominated solutions. In the column “IM”, the average of multiplication of frequency and interdiction level for each facility is shown.

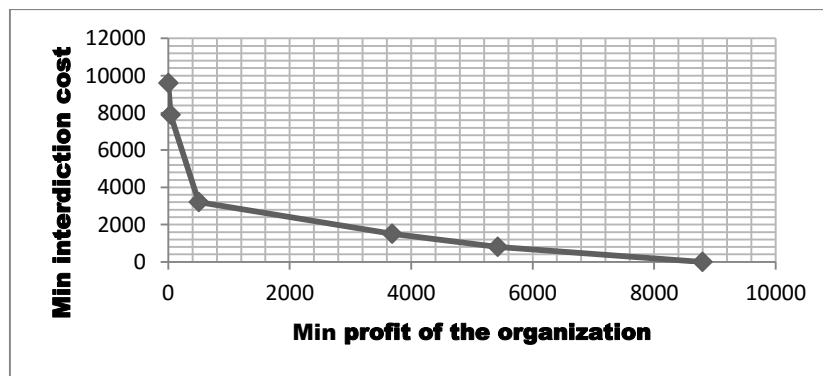


Fig. 3. Relationship between the objective functions of the non-dominated solutions

Table 3. Analysis of the Pareto set pattern for the example

facilities	interdiction levels				MOST	IM
	0	1	2	3		
1	2	1	1	1	0	1
2	3	0	0	2	0	1
3	1	0	1	3	3	1.8

By concentrating on the results, the quality improvement unit could extract some beneficial information from the Pareto set pattern. For example the columns “MOST” and “IM” in Table 3 indicate that facility 3 is the major facility to be fortified.

The more weighting schemes are produced and analyzed, the wider perspective is obtained on the role of the facilities. Moreover, decision-makers could take so many other factors, such as non-quantities indicators, collectively into account in order to choose the most appropriate quality improvement plan.

4.3. Analysis of computational effort

In Table 4, to give some estimation and relations about the required effort needed to solve medium-scale real-world problems, thirty different instances were considered. For each series, five instances were generated randomly. The average CPU time for a single weighting scheme is reported in column “Avg. time”.

A label corresponding to the series consist of four parts. For example, consider series 29 with label I-400-30-2. The first part is “I” which is used as the abbreviation for “Input”. The first number (i.e., 400) indicates the number of customer zones, the second number (i.e., 30) shows the number of facilities and the last number (i.e., 2) gives the number of hierarchies of the facilities. In all instances, four interdiction levels (i.e., $k \in \{0,1,2,3\}$) are considered. From quality improvement perspective, zero level indicates that no fortification plan is assigned to the facility and level 1, level 2 and level 3, respectively state fortifying the facility through investing small, medium and large amount of the available budget.

As shown in Table 4, the average CPU time for these thirty series is between 1.235 and 780.585 seconds. Therefore, the weighting method is able to obtain Pareto frontier in a reasonable CPU time.

Table 4. Computational effort for medium-scale samples

series	label	Avg. time	series	label	CPU time	series	label	CPU time
1	I-50-10-2	1/235	11	I-100-30-2	22/900	21	I-300-20-2	31/309
2	I-50-10-3	2/244	12	I-100-30-3	58/567	22	I-300-20-3	109/491
3	I-50-20-2	3/991	13	I-200-10-2	4/756	23	I-300-30-2	176/673
4	I-50-20-3	8/623	14	I-200-10-3	9/001	24	I-300-30-3	478/690
5	I-50-30-2	9/153	15	I-200-20-2	19/080	25	I-400-10-2	12/689
6	I-50-30-3	23/813	16	I-200-20-3	49/153	26	I-400-10-3	22/622
7	I-100-10-2	2/273	17	I-200-30-2	62/180	27	I-400-20-2	57/944
8	I-100-10-3	4/438	18	I-200-30-3	284/057	28	I-400-20-3	198/052
9	I-100-20-2	7/517	19	I-300-10-2	8/441	29	I-400-30-2	328/011
10	I-100-20-3	20/906	20	I-300-10-3	17/145	30	I-400-30-3	780/585

5. Summary and Conclusions

A new application for interdiction problem with applications mostly to military and terrorist attacks was suggested. We focused on developing a new formulation of a quality improvement problem and presenting an opportunity to achieve an appropriate quality improvement plan in an interdiction problem framework. A quality improvement problem on non-nested hierarchical facilities was examined. Each facility could cover the customer zones and other facilities located in its coverage radius. Owing to the hierarchical nature of the facilities, customer demand was required to be satisfied directly and indirectly. Identifying critical facilities having crucial roles in the level of customer satisfaction and fortification was suggested as a satisfactory way of dealing with the budgetary limitation in organizations. In order to identify critical facilities vulnerable to random disruptions and in the absence of the probability of intentional attacks, an imaginary attacker with aim to cause the least profit for the organization was considered. The level of interdiction of each facility was interpreted as the importance of that facility by the quality improvement unit. We proposed a multi-objective mathematical formulation to model the interdiction problem. Using a multi-objective formulation admitted more flexibility in our model. The proposed model may assist managers to choose more intelligent quality improvement plans and invest the budget of the organization on appropriate tasks. The application of our approach is not limited to special services. An organization offering services with coverage patterns could use the proposed model. Solving a multi-objective problem in the presence of different conflicting objectives is a challenging task. We conducted a weighting approach to solve the problem. The approach is computationally beneficial but has some inherent drawbacks. However, several compelling reasons were given to justify that the weighting method was an appropriate method for solving the proposed model. In order to clarify the connection between the quality improvement problem and the equivalent interdiction problem, we worked through an illustrative example. By using different weighting schemes, a number of Pareto solutions were produced. The decision makers could benefit from analyzing the pattern of the Pareto set to obtain a wide understanding of the importance of facilities from quality improvement perspective. To verify that the solutions obtained by the weighting method were indeed the non-dominated ones, a validation approach was used. Finally, to prove that the weighting approach was an appropriate solution procedure for small-scale to medium-scale problems, the CPU times of 150 instances in thirty different series were reported. Nowadays, organizations face serious competitions in marketplace and growing attention is attracted to customer satisfaction level and quality of the services. Therefore, identifying critical facilities may contribute to future research. Here, a covering model was developed.

For future research, other models, such as centers, median, etc. could be considered. Moreover, the weighting method for small-scale to medium-scale problems is able to obtain Pareto solutions in

a reasonable CPU time. For large-scale problems other solution approaches such as heuristic methods may be useful.

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