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A Robust Modeling of Inventory Routing in Collaborative Reverse Supply Chains

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We propose a robust model for optimizing collaborative reverse supply chains. The primary idea is to develop a collaborative framework that can achieve the best solutions in an uncertain environment. Firstly, we model the exact problem in the form of a mixed integer nonlinear program. To consider uncertainty, robust optimization is employed that searches for an optimal solution with nearly all possible deviations in mind. In order to allow the decision maker to vary the protection level, we use the "budget of uncertainty" approach. To solve the NP-hard problem, we suggest a hybrid heuristic algorithm combining dynamic programming, ant colony optimization and tabu search. To assess the performance of the algorithm, two validity tests are made, first by comparing with the previously solved problems and next by solving a sample problem with more than 900 combinations of parameters and comparing the results with the nominal case. In conclusion, the results of different combinations and prices of robustness are compared and some directions for future research are suggested.

Keywords: *Reverse supply chain, Collaboration, Robust optimization, Ant colony optimization (ACO), Dynamic programming, Tabu search.*

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1. Introduction

European working group on reverse logistics (RevLog) defines reverse supply chain as "the process of planning, implementing and controlling flows of raw materials, in process inventory, and finished goods, from a manufacturing, distribution or use point to a point of recovery or point of proper disposal." In today's highly competitive business, organizations have begun to realize the importance of collaboration between supply chain members to ensure efficiency and responsiveness of supply chain. A global survey about supply chain management in 2004 and 2007 defines collaboration as the key to success by SCMR and CSC ([26] and [27]). Though many models and techniques are identified for collaboration in forward supply chains, the subject is much newer in reverse chain. On the other hand, some reverse supply chain parameters such as return demand, consumption, quality level, and disposition option are largely unpredictable and affected by high levels of uncertainty. Our work here is an expansion of the previously proposed collaborative model of reverse supply chains, with a robust inventory routing problem (RIRP). Robust optimization as an emerging paradigm deals with uncertainty and gives a solution capable of withstanding variants in inputs as demonstrated by Coelho [7]. Our proposed collaborative model works in 3-tier reverse supply chains, consisting of

- return generators (G): the centers that produce some returns (e.g., in the form of end-of-life, end-of-use or guarantee) that need to be recovered at other places;
- recovery centers (R): the centers that recover returns received from Gs in the form of reuse, repair, remanufacture, refurbish, recycle or disposal. The type of recovery depends on the product type and its phase in the life cycle. To be general, the term "recovery" is used;.

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- collection center (CC): the center collects and holds returns from Gs and sends them to Rs.

In the non collaborative situation, when the inventory level of one R reaches its order point, it will order returns from Gs. But in a simple collaborative case, CC works as an intermediate warehouse that can keep the returns and transfer them through the chain from Gs to Rs. However, in the proposed model, reverse supply chains share their inventory data to decide collaboratively about transferring returns from Gs to Rs. This type of collaboration as a variant of VMI[3] in forward supply chains contains three layers sharing their inventory information. These members have their own inventory levels that define the time for picking up and delivering.

In Section 2, a review of the literature on reverse supply chains and collaborative models is presented. In Section 3 the problem is described, and the assumptions, symbols and mathematical models are defined. The first model is for the nominal case (deterministic) and the second one is a robust model. The proposed hybrid heuristic algorithm (HHA) to solve the robust model is described in Section 4. Finally, to validate HHA, some tests are performed in Section 5. Concluding remarks and ideas for further research are provided in Section 6.

2. Literature Review

For a sustainable growth in the highly competitive business world, organizations of a supply chain need to use other members' resources and cannot work in isolation [25]. By such an approach, a collaborative supply chain will have a better performance as shown by McLarn et al. [19], Laseter et al. [17] and Audy et al. [3]. Collaboration has been defined as "a way by which all the companies in a supply chain are actively working together toward common objectives, and are characterized by sharing information, knowledge, risks and profits. Moreover, organizations routinely make decisions that require consultations with multiple participants", as stated by Hernández et al. [14]. Collaboration can take place between different layers of one supply chain or in special activities and processes of different chains (vertical and horizontal collaborations). Also, it can be defined in special processes, for example, CPFR is a collaborative planning, forecasting, and replenishment and VMI needs collaboration in inventory management.

In comparison to the forward supply chain, collaboration has been less studied in the field of reverse supply chain. Information systems (ISs), decision support systems (DSSs), communication and relationship management are collaborative tools in reverse supply chains as suggested by Pokharel et al. [24], Lambert et al. [16]. Kovács [15] explained the ethical and behavioral aspects of collaboration between supply chain members. Zhang et al. [33] explored the reasons for cooperation in reverse supply chains and opportunities arising from that.

The model of coordination between customers, retailers and manufacturers, developed by Bai [4] for ink cartridge, has been evaluated in a closed loop supply chain to maximize returns. Comparing the results of coordinated model with the previous one, it was found out that with the new system, return volume will increase and costs will decrease. In an open-loop reverse supply chain, Gou et al. [12] introduced a modified inventory policy. They found the optimal economic parameters such as inventory level, delivery level and inventory costs to minimize the whole system cost. A year later, Gou et al. [13] introduced joint inventory management and economic inventory levels for managing a central return center (CRC) and multiple local collection points (LCPs). These studies were mainly concerned with inventory management, but the vehicle routes were not considered. Furthermore, the economic inventory level was assumed to be the same for all LCPs, which may not be always the

[3] vendor managed inventory

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case in the real world. The authors suggest developing other joint inventory policies similar to VMI in forward supply chains. However, in 2006 a concept called collector managed inventory (CMI) was defined as the reverse logistics variant of VMI in a PhD thesis by le Blanc [18]. CMI had two levels for the used oil and coolants inventory were defined: can-order and must-order.

A major difference between forward and reverse supply chains is in the supply and demand side, which being largely unpredictable, introduces a high level of uncertainty in reverse supply chains. Robust optimization has emerged as a powerful methodology for uncertain environments with no information about the probability distributions that provides a solution which is able to remain feasible even if the input data is uncertain at the cost of optimality. Different approaches are defined to solve robust problems, for example, the over-conservative method of Soyster [29], the ellipsoidal uncertainty sets of Ben-Tal and Nemirovski [5] and budget of uncertainty approach of Bertsimas and Sim [6]. In the latter, some uncertain parameters are allowed to deviate from their nominal values simultaneously according to the defined budget of uncertainty to control the level of conservativeness. Thiele [32] in her PhD thesis proved that robust optimization opens a new research direction and many old problems in supply chain and revenue management can be revisited by this approach. A survey on robust optimization provides some successful applications across different domains, with supply chain being a main one. In single-station inventory control, the benefits and advantages exclude not needing to know the probability distributions, tractability and the ability to extend to problems with capacities and over networks. Also for inventory control in flexible commitments of retailer and supplier and for computing base-stock levels, robust optimization showed to be a good tool by Solyali et al [28]. They solved an IRP for one supplier that distributed a single product to multiple customers using VMI when the customers' demand were uncertain and the probability distribution was not known. By a robust optimization approach, they proposed two mixed integer programming formulations and a branch and cut algorithm.

The models of le Blanc [18] and Gou et al. [13] are considered for our current study about collaborative inventory management, in an uncertain world. Here provided collaboration such that CC not only has the authority to collect returns, but must send them to recovery centers. The mathematic model combines routing and inventory decisions, called inventory routing problem (IRP). The objective in IRP is to find the routes with a minimal cost and a maximal coverage such that stock-outs in the inventory of members are prevented.

3. Problem Definition

In the literature, reverse supply chains are usually formed by members responsible for collecting and transferring returns from one or more sources to one or more recovery centers. Typically, the members of one supply chain and their relationships are studied or optimized in prior studies. For instance, de Brito et al. [9] studied some different case studies for returning reusable containers and bottles to a soft drink company in Mexico, returning containers for Canada Post, remanufacturing of used scanners, printers, copiers and faxes in Canon, repair network of IBM and city waste management.

Collecting and transferring returns from their sources of production (e.g., manufacturer, retailer, wholesaler or other sources that have a mass of returns) to the recovery centers is costly and the high cost is an obstacle for implementing reverse supply chains. Therefore, decreasing the costs will motivate more organizations to implement these chains. Collaboration is introduced here as a mean of reducing the whole supply chain costs. Here, we consider parallel reverse supply chains with the same type of return and recovery to collaborate in their inventory management and routing of vehicles to reduce the whole costs of reverse supply chains. The proposed model is to reduce costs such as

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establishment of collection centers, inspection and transportation for each chain. Part (a) of Fig. 1 shows the simple collaborative reverse supply chains, with members just sharing a CC. In part (b), the proposed model is depicted, with members sharing information and deciding collaboratively about collecting and transporting returns. As shown by Moubed et al. [21], the collaborative case results in a large decline in costs of the reverse supply chain. In this model, CC is mostly used as an information center and not just a warehouse between members.

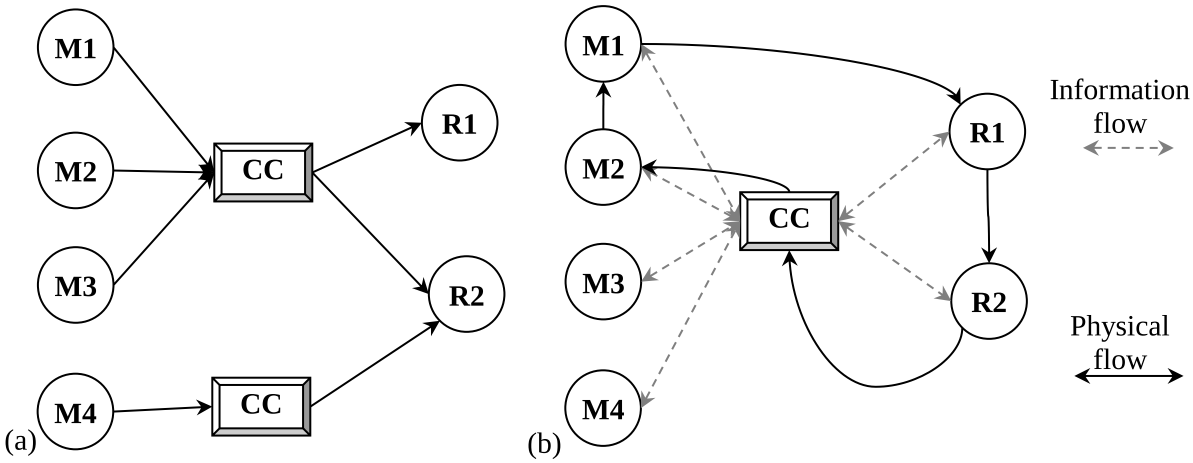

Figure 1. (a) Non-collaborative reverse supply chains and (b) the proposed collaborative model

3.1. Assumptions

The general context and our main assumptions are as follows:

- The deterministic parameters are number of Gs and Rs, different costs, lead time, inventory capacity of different centers, number of trucks and trucks' capacities.
- Uncertain parameters are return generation rates at Gs and the return consumption rates at Rs.
- Holding, stock-out and disposal costs depend on the inventory level. Transportation cost depends on the distance.
- There are multiple Gs, multiple Rs and one CC.
- The returns generated at Gs cannot be collected before reaching their CP levels.
- The inventory capacities of Gs are limited. When the inventory level of one G exceeds its MP, it cannot hold them more and will dispose them by the first disposition option (may be landfill). It is assumed that the returns more than CP are land-filled, so a penalty cost is added to the model.
- CP is equal to zero for CC, it means that whenever there is some inventory at CC, it can be picked up.
- MP is unlimited for CC, since it can hire as much warehouses as it needs.
- The inventory capacities of Rs are limited. These centers use the EOQ policy and just when reach their order point (OP), can accept returns.
- In a visit to R, the delivery quantity can be less than economic order. But to avoid several deliveries to Rs, the delivery quantity must be at least equal to the center's order point (OP).
- Partial pick up from the centers is allowed, meaning that in a visit to G, collecting all returns is not necessary. However, the loaded amount must be more than CP.
- Returns are sorted according to their disposition options at Gs.
- The model is multi-period.
- All vehicles start and finish their journies at CC.

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- At each period, just one truck can start a route to collect and deliver returns.
- Lead time is zero, meaning that returns are delivered to Rs in the same period that are collected.

3.2. Mathematical Model

To present the mathematical model, the following notations are needed:

Sets:

t	Time period (*t = 1,...,T*)
n	Number of Gs
m	Number of Rs
i, j	Nodes (*1,...,n* for Gs, *n+1* for CC and *n+2,...,n+m+1* for Rs)
k	Stops at each route (*k*=0 for start at CC, *k=1,...,KK*).

Parameters:

PG_i^t	Return production rate of G_i in period *t*
DR_i^t	Return consumption rate of R_i at period *t*
MP_i	Must pick level for G_i
CP_i	Can pick level for G_i
OP_i	Order point for R_i
EO_i	Economic order quantity for R_i
d_{ij}	Distance between *i* and *j*
C_{Gh}	Returns holding cost for Gs
C_{ch}	Returns holding cost for CC
C_{Rh}	Returns holding cost for Rs
C_m	Transportation cost per kilometer
C_l	Loading cost per unit of loaded returns
C_u	Unloading cost per unit of delivered returns
C_s	Route / truck starting cost
C_{so}	Stock-out cost for each R
C_p	Penalty cost for landfill (disposal in bad way)
TP	Truck capacity (assumed the same for all)
RTP_k^t	The remaining truck capacity at k^{th} stop of period *t*.

Variables:

QG_i^t	Amount of returns generated and held at G_i in period *t*
QR_i^t	Amount of returns held at R_i in period *t*
QC^t	Amount of returns at CC in period *t*
SO_i^t	Amount of Stock-out at R_i in period *t*
x_i^t	1 if $QG_i^t \geq MP_\mathrm{i}$ and 0, otherwise
y_i^t	1 if $QG_i^t \geq CP_\mathrm{i}$ and 0, otherwise
O_i^t	1 if $QR_i^t \leq OP_\mathrm{i}$ and 0, otherwise.

Decision variables:

f^t	Number of trucks that start their route in period *t* (it is assumed that $f^t = 1$)
Z_{ij}^t	1, if truck goes from *i* to *j* in period and 0, otherwise
b_i^t	1, if returns are picked up from center *i* in period *t* and 0, otherwise (*i=1,…n+1*)
QL_{ik}^t	The amount of returns loaded at center *i* in k^{th} stop of period *t*

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a_i^t 1, if returns are delivered to center *i* in period *t* and 0, otherwise (i=n,…n+m)
QU_i^t Amount of returns delivered to center *i* in *k*th stop of period *t*.

3.2.1. The Deterministic Model

The model's objective is to collect the most possible returns from Gs and transfer them to Rs, while minimizing the total cost. Objective function (1) below consists of holding cost for returns at Gs, Rs and CC, loading and unloading, starting cost for each tour and the transportation between centers, stock-out cost for Rs and penalty for not-covering Gs that reach their MP:

$$\begin{aligned} Min\ Z = & \left(C_{Gh} \sum_{i=1}^{n} \sum_{t=1}^{T} QG_i^t + C_{Rh} \sum_{i=n+2}^{n+m+1} \sum_{t=1}^{T} QR_i^t + C_{ch} \sum_{t=1}^{T} QC^t \right) \\ & + \left(C_l \sum_{t=1}^{T} \sum_{i=1}^{n+1} \sum_{k=1}^{KK} QL_{ik}^t + C_u \sum_{t=1}^{T} \sum_{i=n+1}^{n+m+1} \sum_{k=1}^{KK} QU_{ik}^t \right) \\ & + \left(C_s \sum_{t=1}^{T} f^t + C_m \sum_{t=1}^{T} \sum_{i=1}^{n+m+1} \sum_{j=1}^{n+m+1} Z_{ij}^t \, . \, d_{ij} \right) \\ & + \left(C_{so} \sum_{t=1}^{T} \sum_{i=n+2}^{n+m+1} SO_i^t \right) \\ & + \left(C_p \sum_{t=1}^{T} \sum_{i=1}^{n} x_i^t (QG_i^t - CP_i)(1 - b_i^t) \right). \end{aligned} \tag{1}$$

Constraints are defined in four main categories:

(A) Pickup / delivery quantity constraints

These constraints limit the amount of returns that are picked up at Gs or delivered to Rs. The maximum and minimum of returns picked up at each G are shown by constraints (2) to (4). For CC, this amount cannot exceed the quantity of returns collected at that center (5). As described in the assumptions, the maximum of returns delivered to recovery centers is equal to EOQ and its minimum is equal to OP, also it cannot exceed the truckload as demonstrated in (6) to (8). The equality constraint (9) shows that sum of the returns picked up in each period is equal to sum of the returns delivered in that period:

$$QL_{ik}^t \leq QG_i^t . b_i^t, \qquad \forall t, k, i = 1, \dots, n \tag{2}$$

$$QL_{ik}^t \leq RTP_k^t, \qquad \forall t, k, i = 1, \dots, n \tag{3}$$

$$QL_{ik}^t \geq CP_i . b_i^t, \qquad \forall t, k, i = 1, \dots, n \tag{4}$$

$$QL_{n1}^t \leq QC^{t-1}, \qquad \forall t > 1 \tag{5}$$

$$QU_{ik}^t \leq EO_i, \qquad \forall t, k, i = n, \dots, n+m \tag{6}$$

$$QU_{ik}^t \leq TP - RTP_k^t, \qquad \forall t, k, i = n, \dots, n+m \tag{7}$$

$$QU_{ik}^t \geq OP_i \, . \, a_i^t, \qquad \forall t, k, i = n, \dots, n+m \tag{8}$$

$$\sum_{i=1}^{n+1} \sum_{k=1}^{KK} QL_{ik}^t = \sum_{i=n+1}^{n+m+1} \sum_{k=1}^{KK} QU_{ik}^t \,, \qquad \forall t. \tag{9}$$

(B) Pickup / delivery feasibility

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As described in assumptions, picking and delivering returns are not possible any time. At this part we will discuss these situations. If G reaches its MP, it must be picked up (10) and if reaches its CP, it can be picked up (11). Also, when R reaches its order point, returns can be delivered to that (12). Maximum number of pick up points at each period equals to the number of Gs and CC, also the maximum number of deliveries equals to the number of Rs and CC as shown in (13), (14).

$$b_i^t \geq x_i^t, \qquad \forall\, t, i = 1, \dots, n-1 \tag{10}$$

$$b_i^t \leq y_i^t, \qquad \forall\, t, i = 1, \dots, n-1 \tag{11}$$

$$a_i^t \leq O_i^t, \qquad \forall\, t, i = n+1, \dots, n+m \tag{12}$$

$$\sum_{i=1}^{n+1} b_i^t \leq n+1, \qquad \forall\, t \tag{13}$$

$$\sum_{i=n+1}^{n+m+1} a_i^t \leq m+1, \qquad \forall\, t. \tag{14}$$

(C) Transportation between centers

When there is a pick up or delivery at a center, a truck must go from (15), or to that (16). For the routes between different centers, (17) shows that the start and finish of routes must be at CC, and (18) ensures that if there is a pick up or delivery at a center, number of incoming and outgoing arcs must be equal to 2; otherwise it is zero. (19) demonstrates that if a truck goes from i to j, it cannot come back from j to i.

$$\sum_{j=1}^{n+m+1} Z_{ij}^t \leq a_i^t + b_i^t, \qquad \forall t, i \neq n+1 \tag{15}$$

$$\sum_{j=1}^{n+m+1} Z_{ji}^t \leq a_i^t + b_i^t, \qquad \forall t, i \neq n+1 \tag{16}$$

$$\sum_{j=1}^{n+m+1} (Z_{ij}^t + Z_{ji}^t) = 2f_i^t, \qquad \forall t, i = n+1 \tag{17}$$

$$\sum_{j=1}^{n+m+1} \left(Z_{ij}^t + Z_{ji}^t\right) = 2(a_i^t + b_i^t), \qquad \forall t, i \neq n+1 \tag{18}$$

$$Z_{ij}^t + Z_{ji}^t \leq 1, \qquad \forall\, t, i, j\ (i \neq j, i \neq n+1). \tag{19}$$

(D) Returns amounts

The returns inventory level of a G at period *t* is equal to sum of inventory level at *t-1* and produced at *t* minus the amount loaded from that point. Returns' inventory level of Rs at period t equals to its inventory level at period *t* and delivered to it at this period minus the amount used at this period. These two recursive equations will change to (20) and (21). The inventory level at CC is calculated by (22) that equals to its last period inventory plus the difference between pick up and deliveries at this period. The empty truck starts from CC at k=0 stop and moves to other centers in its route (24). The remained truck capacity at each stop is equal to its previous stop's remained capacity and the pick up at this period, minus the delivery amount at this period (23). Constraints (25) and (26) enforce integrality and non-negativity conditions on the variables.

$$QG_i^t = QG_i^0 + \sum_{tt=1}^{t} PG_i^{tt} - \sum_{tt=1}^{t} \sum_{k=1}^{KK} QL_{ik}^{tt}, \qquad \forall t, i = 1,2, \dots, n \tag{20}$$

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$$QR_i^t = QR_i^0 - \sum_{tt=1}^{t} DR_i^{tt} + \sum_{tt=1}^{t}\sum_{k=1}^{KK} QU_{ik}^{tt}, \qquad \forall t, i = n+2, \dots, n+m+1 \tag{21}$$

$$QC^t = QC^{t-1} + \sum_{i=1}^{n+1}\sum_{k=1}^{KK} QL_{ik}^t - \sum_{i=n+1}^{n+m}\sum_{k=1}^{KK} QU_{ik}^t, \quad \forall t \tag{22}$$

$$RTP_k^t = RTP_{k-1}^t + \sum_{i=n}^{n+m} QU_{ik}^t - \sum_{i=1}^{n} QL_{ik}^t, \qquad \forall t, k \geq 1 \tag{23}$$

$$RTP_0^t = TP, \qquad \forall t, k, i = 1,2, \dots, n \tag{24}$$

$$QL_{ik}^t, QU_{ik}^t, f^t, SO_i^t \geq 0 \tag{25}$$

$$b_i^t, \quad a_i^t, \quad Z_{ij}^t \in \{0, 1\}. \tag{26}$$

3.2.2. The Robust Model

The amount of returns at Gs and Rs (QG_i^t and QR_i^t) are uncertain since they depend on return generation rate ($\widetilde{PG}_i^t$) and return recovery rate ($\widetilde{DR}_i^t$) that are not deterministic. Therefore (20) and (21) that calculate QG_i^t and QR_i^t, will change to (27) and (28).

$$QG_i^t = QG_i^0 + \sum_{tt=1}^{t} \widetilde{PG}_i^{tt} - \sum_{tt=1}^{t}\sum_{k=1}^{KK} QL_{ik}^{tt} \tag{27}$$

$$QR_i^t = QR_i^0 - \sum_{tt=1}^{t} \widetilde{DR}_i^{tt} + \sum_{tt=1}^{t}\sum_{k=1}^{KK} QU_{ik}^{tt}. \tag{28}$$

Substituting these formulas in the objective function, a two-part objective function will be formed ($Z = Z_{det} + Z_{Rob}$), that we name them the deterministic and robust parts. For the robust part, the budget of uncertainty method as described by Bertsimas et al. [6] and Solyalı et al. [23] is employed. We use $\eta_{ig}^{tt} = \frac{\widehat{PG}_i^{tt} - \overline{PG}_i}{\widehat{PG}_i}$ and $\eta_{ir}^{tt} = \frac{\widehat{DR}_i^{tt} - \overline{DR}_i}{\widehat{DR}_i}$ equations to calculate the uncertain part. Thus we have $\widetilde{PG}_i^{tt} = \overline{PG}_i + \widehat{PG}_i . \eta_{ig}^{tt}$ for return generation rate at G_i and $\widetilde{D}R_i^{tt} = \overline{DR}_i + \widehat{DR}_i . \eta_{ir}^{tt}$ for return recovery rate at R_i. By these changes, the two parts of objective function will be like (29) and (30). Also equation (2) changes to (31).

$$\begin{aligned} Z_{det} = {} & C_{Gh} \sum_{t=1}^{T}\sum_{i=1}^{n}\left(QG_i^0 - \sum_{tt=1}^{t}\sum_{k=1}^{KK} QL_{ik}^{tt}\right) \\ & + C_{Rh}\sum_{t=1}^{T}\sum_{i=n+2}^{n+m+1}\left(QR_i^0 + \sum_{tt=1}^{t}\sum_{k=1}^{KK} QU_{ik}^{tt}\right) \\ & + C_{Ch}\sum_{t=1}^{T}\left(QC^0 + \sum_{tt=1}^{t}\sum_{i=1}^{n+1}\sum_{k=1}^{KK} QL_{ik}^{tt}\right. \\ & \left. - \sum_{tt=1}^{t}\sum_{i=n+1}^{n+m+1}\sum_{k=1}^{KK} QU_{ik}^{tt}\right) + C_l \sum_{t=1}^{T}\sum_{i=1}^{n+1}\sum_{k=1}^{KK} QL_{ik}^t \\ & + C_u \sum_{t=1}^{T}\sum_{i=n}^{n+m+1}\sum_{k=1}^{KK} QU_{ik}^t + C_s \sum_{t=1}^{T} f^t \\ & + C_m \sum_{t=1}^{T}\sum_{i=1}^{n+m+1}\sum_{j=1}^{n+m+1} Z_{ij}^t d_{ij} + C_{so}\sum_{i=n+2}^{n+m+1}\sum_{t=1}^{T} SO_i^t \\ & + C_p \sum_{t=1}^{T}\sum_{i=1}^{n} x_i^t \left(QG_i^0 - \sum_{tt=1}^{t}\sum_{k=1}^{KK} QL_{ik}^{tt} - CP_i\right)\left(1 - b_i^t\right) \end{aligned} \tag{29}$$

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$$Z_{Rob} = C_{Gh}\sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{tt=1}^{t}\overline{PG}_i - C_{Rh}\sum_{t=1}^{T}\sum_{i=n+2}^{n+m+1}\sum_{tt=1}^{t}\overline{DR}_i + C_{nc}\sum_{t=1}^{T}\sum_{i=1}^{n}x_i^t\left(1-b_i^t\right)\sum_{tt=1}^{t}\overline{PG}_i + \max\Big\{C_{Gh}\sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{tt=1}^{t}\widehat{PG}_i.\eta_{ig}^{tt} + C_{Rh}\sum_{t=1}^{T}\sum_{i=n+2}^{n+m+1}\sum_{tt=1}^{t}\widehat{DR}_i.\eta_{ir}^{tt} + C_{p}\sum_{t=1}^{T}\sum_{i=1}^{n}x_i^t\left(1-b_i^t\right)\sum_{tt=1}^{t}\widehat{PG}_i.\eta_{ig}^{tt} : \sum_{tt=1}^{t}\left(\sum_{i=1}^{n}\eta_{ig}^{tt} + \sum_{i=n+2}^{n+m+1}\eta_{ir}^{tt}\right) \le \Gamma, \quad 0 \le \eta_{ig}^{tt}, \eta_{ir}^{tt} \le 1\Big\} \tag{30}$$

$$QL_{ik}^t - b_i^t\left(QG_i^0 + \sum_{tt=1}^{t}\overline{PG}_i - \sum_{tt=1}^{t}\sum_{k=1}^{KK}QL_{ik}^{tt}\right) - max\left\{b_i^t.\widehat{PG}_i\sum_{tt=1}^{t}\eta_{ig}^{tt} :\ 0 \le \eta_{ig}^{tt} \le 1, \sum_{tt=1}^{T}\sum_{i=1}^{n}\eta_{ig}^{tt} \le \Gamma\right\} \le 0. \tag{31}$$

Z_{Rob} demonstrates that by maximum changes in return generation and recovery rates, the objective function must remain feasible. It is assumed that the return generation rate changes in the $\left[\overline{PG}_i - \widehat{PG}_i, \overline{PG}_i + \widehat{PG}_i\right]$ domain and return recovery rate changes in the $\left[\overline{DR}_i - \widehat{DR}_i, \overline{DR}_i + \widehat{DR}_i\right]$ domain. Γ is a parameter to adjust the level of robustness and conservatism of the model. In our model, sum of the variations of return generation and recovery rates from their nominal value is Γ. It means that we restrict the nature so that only a subset of the parameters will change in an adverse direction, and in this case the robust solution will be feasible yet. Even in the case of more change, the robust solution will be feasible with very high probability.

4. Solution Procedure

The proposed mathematical model in deterministic case is a mixed integer nonlinear programming (MINLP) one. By adding the non-deterministic parameters to the model, it becomes much more complicated. Since IRPs are known to be NP-hard e.g. by Archetti et al. [1], it is very difficult to obtain high quality solutions in a reasonable amount of time by standard solvers and usually are solved by heuristic and meta-heuristic methods as stated by Coelho et al. [8]. We developed a hybrid heuristic algorithm (HHA), composed of dynamic programming, ant colony optimization (ACO) and Tabu search to solve it. The steps of solving the problem are as follows:

Step 1: define the states and decisions

Dynamic programming as one of the most general optimization approaches, can solve a broad class of problems, including VRP for example by Sarkis et al [25] and Pillac et al [20]. In the proposed model, steps are time periods of the study that are shown by t=1, 2, ...T and the hierarchy of decisions by l. At the first step $l = \{\}$ and at next steps the decision numbers are used to show this. For example $l = 112$ means no movement at first and second states and a tour at the third step. At each step, some states are defined and shown by S, that is a 1×(m+n+1) matrix as shown in figure 2. S_t^j is the return amount at center *j* at period *t*. Any decision changes the current state (t) to the next one and is shown by $1 \times (m+n+1)$ matrix $D_t^{ldd} = [d_1, d_2, \dots d_{n+m+1}]$. d_j shows the inventory change at center *j*,

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where the pick up is negative and deliver is positive. In D_t^{ldd}, t is the number of the state, dd is the decision number at the state and l is the number of decisions before reaching state t.

S_t^1	S_t^2	…	S_t^j	…	S_t^{n+m+1}

Figure 2. State matrix

Two alternative decisions ($dd = 1, 2$) are available at each state ($t = 1,2,\dots,T$).

Step 1-1: ($dd = 1$) No tour and movement of returns between centers ($D_t^{l1} = [0, 0, \dots, 0]$). The costs of this decision, including holding cost for all centers, stock-out and landfill penalty, is considered as TC^{l1}. Transition function (32) changes the current state to the next state ($\acute{S}^{l1}$) by this decision.

Step 1-2: ($dd = 2$) Select a route to pick up returns from CC and Gs and deliver them to Rs or CC. The next state at this decision ($\acute{S}^{l2}$) is calculated by the transition function (33):

$$\acute{S}_{tj}^{l1} = S_{(t-1)j}^{l} + \tilde{r}_{tj} \quad (32)$$
$$\acute{S}_{tj}^{l2} = S_{(t-1)j}^{l} + d_j + \tilde{r}_{tj}. \quad (33)$$

In these formulas, $\tilde{r}_{tj}$ is the production / recovery rate of center j in period t and S_{tj}^{l} stands for the state of center j in period t. For each center, $\tilde{r}_{tj}$ is created using random data generator such that for G_i, it is in $\left[\overline{PG}_i - \widehat{PG}_i, \overline{PG}_i + \widehat{PG}_i\right]$ and for R_i, it is in $\left[\overline{\mathrm{DR}}_i - \widehat{DR}_i, \overline{\mathrm{DR}}_i + \widehat{DR}_i\right]$. At each state and for each decision, $\eta_{\mathrm{ig}}^{\mathrm{tt}}$ and $\eta_{\mathrm{ir}}^{\mathrm{tt}}$ are calculated such that the constraint (34) is not violated:

$$\sum_{tt=1}^{t}\left(\sum_{i=1}^{n-1}\eta_{ig}^{tt} + \sum_{i=n+1}^{n+m}\eta_{ir}^{tt}\right) \le \Gamma. \quad (34)$$

Step 2: Finding the best routes for second decision

For the second decision ($dd = 2$), different routes can be selected, and each one impacts the next iteration's, decisions and costs. The proposed HHA that combines ACO and tabu search with the following steps employed to find the best route at the decision. ACO is a meta-heuristic technique used for problems such as VRP and IRP in previous studies, for example, by Pellonpera [23], Tatsis et al. [31], Tan et al. [30], and Moin et al. [20]. The technique is based on the behavior of a group of ants in finding food. First, ants search and move in a random fashion and deposit some chemical substances named as pheromones. Other ants follow these substances and by traveling the same routes, the pheromone amount reinforces. In selecting a route, the ones with more pheromones will be selected by ants more probably.

To help the ants in selecting the best routes, a tabu search method is developed here. This technique's performance at VRP has been verified in previous studies, for example, by Fakhrzad et al. [10]. Tabu list is a set of rules to ban some solutions from appearing in the solutions. According to the memory structure, the tabu lists are categorized into short term and long-term as stated by Glover [11]. In the current model, to avoid trucks (ants) from selecting the previously passed centers, these centers will be placed in the long-term tabu list ($Tabu_{\mathrm{tt}}$), that lasts until an ant's tour finishes. Short-term tabu includes the centers from which truck cannot go and lasts only one iteration. For

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example, when a truck does not have any remaining capacity, all Gs are in the short-term tabu list. If truck delivers a part of its returns, at the next iteration some Gs can be removed from this list.

For each iteration, the objective function is calculated as the total cost of current iteration and the previous ones before reaching this situation. The steps of finding and selecting the best route for second decision are as follows:

Step 2-1: A truck starts its tour from CC and can pick up some returns from that center. If sum of the loadings at Gs is more than the sum of deliveries at Rs, the amount of pick up at CC will be zero. Otherwise, it is a random number less than $M = min\,\{I_{CC}, TP\}$ by a distribution function like (35). I_{CC} is the return's inventory level at CC:

$$P(i) = \begin{cases} \dfrac{0.5i}{\sum_{i=1}^{M} i}, & i > 0 \\ \\ 0.5\ , & i = 0. \end{cases} \tag{35}$$

Step 2-2: At the start of a route, Gs with less inventory than CP and Rs with more inventory than OP, are considered as long term tabu. At the next iterations, the centers according to their loading and remaining capacities are added to the short term tabu list. Sum of these lists are considered as $Tabu_{tt}$.

Step 2-3: From each center i, the pheromone amount for arc ij is calculated according to the type of j. For G as the next point, the pheromone amount for the first ant is calculated by (36) and for R it is calculated by (37). In these formulas, α is the coefficient for the importance of landfill penalty at Gs and β is the coefficient for the importance of stock-out at Rs. α and β are calculated by (38) and give priorities to more important routes. L_j stands for the pickup/delivery amount at center j and d_{ij} is the distance between centers i and j and 1 is added to avoid zero in the denominator:

$$\tau_{ij}(tt) = \frac{\alpha.x_j + y_j}{\left(RTP - L_j + 1\right).d_{ij}} \tag{36}$$

$$\tau_{ij}(tt) = \frac{\beta.SO_j + O_j}{\left(TP - RTP - L_j + 1\right).d_{ij}} \tag{37}$$

$$\alpha = \frac{C_{pn}}{C_l + C_u + C_m}, \quad \beta = \frac{C_{so}}{C_l + C_u + C_m}. \tag{38}$$

Step 2-4: Each ant chooses its route and the pheromone amount of the route changes by its decision. Equation (39) is used to calculate the pheromone amount of arc ij for (*k+1*)th ant. In this formula, $1 - \rho$ stands for evaporation rate and $\Delta\tau_{ij}^{k}$ is the pheromone amount of arc ij made by *k*th ant. The $\Delta\tau_{ij}^{k}$ calculation here is different from ACO algorithms. We used Equation (40), that is derived from the ant number method and Q is a constant number:

$$\tau_{ij}(tt + 1) = \rho.\tau_{ij}(tt) + \Delta\tau_{ij}^{k} \tag{39}$$

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$$\Delta\tau_{ij}^{k} = \begin{cases} \frac{Q}{(RTP - L_j + 1).d_{ij}}, & truck\ goes\ from\ i\ to\ j \in G \\ \frac{Q}{(TP - RTP - L_j + 1).d_{ij}}, & truck\ goes\ from\ i\ to\ j \in R \\ \frac{Q}{(TP - RTP + 1).d_{ij}}, & truck\ goes\ from\ i\ to\ j = CC \\ 0\ , & otherwise\ . \end{cases} \quad (40)$$

Step 2-5: When a truck moves into another center, its content will be updated. The next center after *i* (*j*) will be determined according to the amount of pheromones remained at each arc based on (41). *b* is a constant that stands for the importance ratio between distance and pheromone amount. To show the equal importance of these factors, *b* is considered to be 1 in our model. q is a random number ($0 \leq q \leq 1$) and q_0 is a constant that helps in finding a random route with a distribution function (P_{j_i}) as shown in (42). When all centers are in tabu lists, the next center is CC:

$$j = \begin{cases} max\left\{\frac{\tau_{iu}}{{d_{iu}}^{b}}\right\}, & u \notin Tabu_{tt}\ if\ q \leq q_0 \\ random\ center \notin Tabu_{tt}, & otherwise \end{cases} \quad (41)$$

$$P_{j_i} = \begin{cases} \frac{\frac{\tau_{ij}}{{d_{ij}}^{b}}}{\sum_{u \notin Tabu_k}(\frac{\tau_{iu}}{{d_{iu}}^{b}})}, & u \notin Tabu_k \\ 0\ , & otherwise. \end{cases} \quad (42)$$

Step 2-6: Update the tabu lists according to the truck load, its remaining capacity and centers' inventory level.

Step 2-7: Put *j* as the current center. If this is CC, then the route is finished and go to the next step. Otherwise, go to Step 2-3.

Step 2-8: After completing a route by an ant, update the pheromone amount of arcs by equations (39) and (40). Also, short-term and long-term tabu lists are deleted and the objective function (TC_k), that is the total cost for the route by *k*th ant is calculated. For the first ant, TC_1 is considered as TC^{best} and the route as R^{best}. For the next ants ($K > 1$), if the total cost is less than TC^{best}, it will replace that and the route will replace R^{best}.

Step 2-9: R^{best} reached after final ant is selected as the tour in Step 2 and its cost is equal to TC^{l2}.

ACO here has 10 ants, the evaporation rate is 0.5, $Q = 10$ and $q_0 = 0.9$. These parameter values are suggested by Moubed and Zare Mehrjerdi [22].

Step 3: Finding the best solution

For each part of the decision tree, two fitness functions are calculated by summing the iteration's total cost and the previous states before arriving the state as shown in (43). In this formula, FF_t^{ldd} shows the fitness function for period *t*, by the decision *dd*, when we have decision numbers *l* before arriving the point:

$$FF_t^{ldd} = TC^{ldd} + FF_{t-1}^{l}, \qquad dd = 1,2. \quad (43)$$

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For each state and its decisions, $\acute{S}_{tj}^{ldd}$ replaces $S_{(t-1)j}^{l}$ and the Steps 1 to 3 will repeat until reaching the last period (T). Finally, 2^T costs are calculated. The minimum of these costs is considered as the solution and the decisions to arrive the state (ldd) give the best solution.

5. Testing Validity of Algorithm

Testing validity of the solution procedure is done in two steps. At the first step, we solve the non-robust instances defined by Leonardo Coelho at the site[4] by our algorithm. The only change in the procedure to define a deterministic solution is in the transition function (33), changing $\tilde{r}_{tj}$ to r_{tj}. Also, the type of instances defined by Coelho is different from our problem. To have comparable results, we remove CC from our model and define return generators as suppliers and recovery centers as customers. Further, the instances defined by Coelho do not have MP, CP, OP, EOQ, stock-out cost and penalty cost. We used truck capacity as MP, one (each quantity more than zero) as CP, the consumption rate as OP and maximum inventory capacity of Rs as EOQ. Also, for penalty costs, we consider it to be equal to zero and for stock-out cost, it is considered to be two times the largest value of holding cost. The solution was coded in Matlab 2013 and result of solving these instances with the proposed solution was compared with available solved procedures in table 1.

Table 1. Comparison between suggested HHA and available solutions for different instances

Code	Archetty et al. [1]		Coelho [7]		Archetty et al. [2]		*Our solution*	
	*Z**	*time*	*Z**	*Time*	*Z**	*Time*	*Z**	*time*
abs1n10h6	4141	69	4141	15	2167	0	1360	0.9
abs1n20h6	6114	475	6114	18	2793	12	4000	1.6
abs1n30h6	8052	1930	8052	1930	3918	84	1441	0.27
abs2n20h6	5957	561	5957	42	2800	6	3372	1.5
abs3n20h6	6784	429	6784	27	3101	8	3516	1.3
abs4n20h6	7309	508	7309	76	3239	4	4679	1.2
abs1n10h3	1743	10	1743	10	4499	7	497	0.08
abs1n20h3	2267	43	2267	43	6490	1830	921	0.3
abs1n30h3	3427	221	3427	221	8319	4541	1441	0.3
abs2n20h3	2497	72	2497	72	6082	282	832	0.32
abs3n20h3	2590	60	2590	60	6950	104	797	0.25
abs4n20h3	3122	90	3122	90	7432	255	1020	0.22

The comparison shows a significant improvement over the objective function and time to solve by the proposed HHA. Since some changes were made in our model to be comparable, for some instances the difference is very large. At the next step, to validate the proposed HHA in the robust case, we use a numerical example by its nominal case (Γ=0) and compare the results with the deterministic solution. The characteristics of different centers, vehicles and costs are demonstrated in tables 2 and 3.

[4] http://www.leandro-coelho.com/instances/inventory-routing.

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Table 2. Costs and truck capacity

Stock-out cost	10000
Holding cost for Gs, Rs and CC	500
Loading/ Unloading cost	300
Transport cost for empty truck per kilometer	0.5
Transport cost per kilo load per kilometer	1
Truck Capacity	500
Penalty cost	10000

Table 3. Specifications of different centers in the study

Center	$\overline{PG}_i$	$\widehat{PG}_i$	CP	MP	Initial inventory
G1	40	4	100	200	500
G2	50	5	250	380	250
G3	40	3	300	450	400
CC	--	--	1	∞	10
Center	$\overline{\mathrm{DR}}_i$	$\widehat{DR}_i$	OP	EOQ	Initial inventory
R1	40	4	100	300	20
R2	50	5	150	400	-20
R3	70	7	50	300	10

We generated instances to investigate the robust model and its sensitivity by changing Γ, truck capacity and variations in return generation and consumption ($\widehat{PG}_i$ and $\widehat{DR}_i$). The instances were coded like "Rob-AA-B-C" there AA standing for Γ, B showing the truck capacity change (I for the initial capacity, L for 50% and H for 150% of the initial value) and C demonstrating the percent of change in $\widehat{PG}_i$ and $\widehat{DR}_i$ (I for initial value, L for 50% and H for 200% of the initial value). Thus, the "Rob-00-" code stands for the nominal case of the problem. Therefore, we have 900 RIRP instances besides the 9 instances for the nominal case. Total cost and time to solve for the nominal problem with 9 different combinations are shown in tables 4 and 5.

Table 4. Comparison between total cost for the robust problem with Γ=0 with the deterministic case

Capacity level	Robust problem			Deterministic problem*
	$\widehat{PG}_i$ and $\widehat{DR}_i$ level			
	L	I	H	
L	40215556	40215556	40215556	40473294
I	37757000	37757000	37757000	37757000
H	37438878	37438878	37438878	37776134

* for the deterministic problem, we don't have $\widehat{PG}_i$ and $\widehat{DR}_i$, since they don't change.

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Table 5. Time to solve for the robust problem with Γ=0 and the deterministic case

Capacity level	Robust problem			Deterministic problem*
	$\widehat{PG}_i$ and $\widehat{DR}_i$ level			
	L	I	H	
L	0.28	0.33	0.27	0.27
I	2.56	1.85	1.8	1.59
H	13.9	15.3	14.19	13.5

* for the deterministic problem, we don't have $\widehat{PG}_i$ and $\widehat{DR}_i$, since they don't change.

To evaluate the algorithm and the impact of different combinations, each code was run at least 50 times. The results of changing Γ for the main problem are displayed in table 6. The percent of deviation for average total cost from optimal solution (*e%*) is calculated by (44) below. This number represents the percent of possible cost increase in different runs:

$$e\% = \frac{\overline{TC} - min_{TC}}{min_{TC}}. \qquad (44)$$

The obtained results show that for $\Gamma = 0$ (nominal case) the average time for solving the problem is minimal. Comparing total costs reached at each combination with the nominal case demonstrates that by changing Γ, the obtained total cost is close to the one for the nominal case. The change in total cost for different Γs is just about 3.3% of the minimal cost. The maximum cost of robustness (Rob_Cost) in the main problem is calculated by (45) below from table 6 and is equal to 0.02%, meaning that at the worst case, we have just about 0.02% increase in the total cost from the nominal case:

$$Rob_{Cost} = \frac{(Maximum\ TC\ for\ \Gamma s - TC\ for\ nominal\ case)}{TC\ for\ nominal\ case} \times 100 = 0.02\%. \qquad (45)$$

Table 6. Total cost, time and e% for different Γs in the main problem

Γ	Minimum TC	$\bar{t}$	$\overline{TC}$	e%
00	37,757,000	1.85	38,246,000	1.30
01	37,660,000	6.54	38,653,000	2.64
02	37,205,214	5.76	38,439,000	3.32
03	37,365,000	6.82	38,445,000	2.89
04	37,377,541	6.74	38,246,000	2.32
05	37,683,500	5.59	38,588,000	2.40
06	37,333,484	5.94	38,415,000	2.90
07	37,426,413	5.26	38,606,000	3.15
08	37,419,658	6.94	38,498,000	2.88
09	37,524,540	6.24	38,529,000	2.68
10	37,756,000	5.78	38,547,000	2.10
11	37,725,234	5.51	38,464,000	1.96
12	37,587,011	6.76	38,547,000	2.55
13	37,397,000	6.19	38,507,000	2.97
14	37,764,532	5.39	38,660,000	2.37
15	37,582,932	6.43	38,619,000	2.76

From the above observations it can be concluded that the most important influence of Γ is on the

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time needed to obtain solution. In other words, by increase in Γ levels, time to solve the problem will increase. At the next step, 900 combinations of the problem by different levels of Γ, truck capacity and changes in $\widehat{PG}_i$ and $\widehat{DR}_i$ are solved and the average cost and time to solve are calculated. The time to solve for $\Gamma > 0$ is shown in Table 7, with the first column being Γ and the next column showing the change in the truck capacity. The time to solve various combinations of the parameters and changes in $\widehat{PG}_i$ and $\widehat{DR}_i$ (at the second row) are shown in the three next columns of the table. The next six columns show the total cost and the amount of time change in the robust problem with respect to the nominal case, or the price of robustness. This is the price that we pay to be robust. These prices are calculated in terms of cost and time by equations (46) and (47) below. Since the total costs of different combinations are nearly the same, they are not shown in the table:

Table 7. Time to solve the combinations of problems with different Γ levels and the price of robustness

Γ	Capacity change	Time to solve for variations level of $\widehat{PG}_i$ and $\widehat{DR}_i$			Price of robustness for the variations level of $\widehat{PG}_i$ and $\widehat{DR}_i$					
					In terms of cost			In terms of time		
		L	I	H	L	I	H	L	I	H
1	L	4.01	4.05	3.94	-0.09	-0.47	-1.03	1332.14	1127.27	1357.93
1	I	6.07	14.22	14.49	1.74	2.09	1.85	137.21	668.65	705.20
1	H	6.08	15.89	19.38	5.87	5.16	5.52	15.70	3.83	36.57
2	L	4.02	4.13	3.92	-0.28	-0.53	-1.17	1335.71	1151.52	1352.00
2	I	6.55	13.32	15.62	2.26	2.04	1.50	155.68	620.00	767.94
2	H	16.1	17.57	19.28	5.00	5.73	4.42	15.84	14.86	35.85
3	L	4.01	4.01	4.40	-0.53	-0.36	-1.32	1331.61	1115.15	1528.19
3	I	6.59	14.39	13.90	1.81	1.64	2.03	157.55	677.84	672.10
3	H	13.9	14.43	16.82	5.48	6.12	5.07	0.07	-5.72	18.57
4	L	3.90	4.13	4.02	-0.47	-0.49	-1.44	1293.54	1151.52	1390.19
4	I	6.05	18.12	5.44	1.99	2.41	1.74	136.28	879.46	202.42
4	H	16.7	15.64	18.35	5.37	4.70	5.69	20.49	2.24	29.35
5	L	4.04	4.24	4.03	-0.41	-0.41	-1.08	1341.32	1184.85	1391.56
5	I	6.19	18.36	6.08	2.19	2.23	2.04	141.88	892.43	238.04
5	H	17.5	17.19	16.11	5.40	5.61	4.23	26.37	12.35	13.52
6	L	3.82	4.11	4.08	-0.49	-0.41	-0.99	1265.93	1145.45	1411.74
6	I	7.38	5.60	5.94	1.84	1.67	1.80	188.14	202.70	229.98
6	H	17.5	16.64	17.17	4.85	6.69	5.20	25.70	8.73	20.99
7	L	3.86	3.87	4.01	-0.20	-0.41	-1.30	1280.11	1072.73	1383.41
7	I	16.6	6.12	6.31	2.29	2.59	1.41	550.17	230.81	250.72
7	H	18.5	16.14	18.23	4.82	5.22	5.64	33.03	5.51	28.50
8	L	3.88	7.94	4.04	-0.30	-0.40	-1.15	1287.07	2306.06	1398.00
8	I	18.1	5.14	6.26	1.57	1.92	1.82	606.26	177.84	247.98
8	H	16.2	16.77	16.19	6.24	6.43	5.76	16.35	9.58	14.06
9	L	3.87	9.80	4.16	-0.26	-0.40	-1.53	1283.57	2869.70	1441.44
9	I	17.8	5.98	5.37	1.47	1.99	1.86	596.68	223.24	198.11
9	H	15.4	14.93	15.84	5.77	5.03	5.99	11.08	-2.44	11.61
10	L	3.91	9.35	4.14	1.18	-0.60	-1.04	1296.57	2733.33	1433.07
10	I	16.7	5.69	5.16	1.97	2.07	1.38	553.92	207.57	186.44
10	H	16.4	20.06	17.10	4.73	5.29	4.66	17.83	31.14	20.50

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$$Price\ of\ Robustness_{cost} = \frac{(Total\ Cost_{Robust} - Total\ Cost_{nominal})}{Total\ Cost_{nominal}} \times 100 \quad (46)$$

$$Price\ of\ Robustness_{time} = \frac{(Time_{Robust} - Time_{nominal})}{Time_{nominal}} \times 100. \quad (47)$$

The results given in table 7 show that the most increase in time to solve, in comparison with the nominal case, takes place in the cases that truck's capacity is at 50% of its initial value. The reason is that the time to solve at this level is very low (usually less than 10 seconds). Quite the opposite, less time is needed to solve at this level and the biggest ones are at H the capacity levels. Price of robustness in the terms of cost, is usually close to zero or negative at times, showing that the robust approach may lead to a better solution. In terms of time, the price of robustness is mostly positive and very large, especially for smaller truck capacity levels.

6. Conclusion

We introduced a robust mixed integer nonlinear programming model for collaboration in reverse supply chains. In this collaborative chain, some return generating centers and return recovery centers share their inventory information with a collection center in order to decide collaboratively about the centers to pick up and centers to deliver the returns such that the cost of the whole supply is minimized. To solve the robust model, a hybrid heuristic algorithm was proposed and its validity was tested in two steps. These tests showed a good performance of HHA in solving deterministic and robust problems. The most important influence of Γ was on the time to solve the problem that increases by increasing Γ. Since the problem was not much studied for robust situations, future researchers may be made on analytical and heuristic methods for solution. Here, we changed the truck's capacity and deviation of returns production and consumption form the average. Changing various costs can be useful to find the best ways of influencing best solutions. In our work, we only considered returns generation and consumption to be uncertain. Other parameters such as costs, routes possibility to select and trucks capacities may also be considered uncertain.

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