Iranian Journal of Operations Research
Vol. 6, No. 2, 2015, pp. 01-18

A Corrector-Predictor Arc-search Interior Point Algorithm for $P_*(\kappa)$-LCP Acting in a Wide Neighborhood of the Central Path

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We propose an arc-search corrector-predictor interior point method for solving $P_*(\kappa)$-linear complementarity problems. The proposed algorithm searches for the optimizers along an ellipse that is an approximation of the central path. The algorithm generates a sequence of iterates in the wide neighborhood of the central path introduced by Ai and Zhang. The algorithm does not depend on the handicap κof the problem, so that it can be used for any $P_*(\kappa)$-linear complementarity problem. Based on the ellipse approximation of the central path and the wide neighborhood, we show that the proposed algorithm has $O((1+\kappa)\sqrt{n}L)$ iteration complexity, the best-known iteration complexity obtained so far by any interior point method for solving $P_*(\kappa)$-linear complementarity problems. Some numerical results are presented to show the performance of the algorithm.

***Keywords**: Linear complementarity problem, Interior point method, Corrector-predictor algorithm, Arc search, Polynomial complexity.*

Manuscript was received on 18/05/2016 revised on 16/03/2017 and accepted for publication on 18/03/2017

1. Introduction

Interior point methods (IPMs) have provided polynomial time algorithms for solving linear optimization (LO) problems and other classes of convex optimization problems. Based on numerical experiments, the class of primal-dual path-following IPMs are considered to be the most efficient algorithms among all IPMs. Excellent practical performance of these methods is explained in part by their superlinear convergence. The Mizuno, Todd and Ye (MTY) predictor-corrector algorithm was the first algorithm for LO having both polynomial complexity and superlinear convergence [10]. Predictor-corrector algorithms operate between two neighborhoods of the central path [19, 25]. The role of the predictor step is to increase optimality while keeping the point in the outer neighborhood. It is followed by a corrector step, which brings the point back into the inner neighborhood so that the next predictor-corrector iteration can be applied. The MTY predictor-corrector algorithm was extended to the $P_*(\kappa)$-linear complementarity problems ($P_*(\kappa)$-LCPs) in 1995 by Miao [9]. His algorithm depends on κ, uses the small neighborhood of the central path, has $O((1+\kappa)\sqrt{n}L)$ as iteration complexity and is quadratically convergent for nondegenerate problems. Since the handicap of a matrix is sometimes very difficult to compute, and it is explicitly used in the construction of Miao's algorithm, so his algorithm cannot be used for general sufficient LCPs. Potra and Sheng [18] extended the MTY predictor-corrector algorithm further for sufficient complementarity problems.

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All the above algorithms operate in $\mathcal{N}_2$ neighborhoods, also known as the small neighborhoods, of the central path. In [16], Potra proposed a predictor-corrector method for monotone LCPs using the wide neighborhood $\mathcal{N}_\infty^-$ of the central path. Potra and Liu [17] extended the algorithm of [16] to sufficient LCPs. Two algorithms were analyzed in [17]. Both algorithms are of predictor-corrector type acting in between two wide neighborhoods of the central path. The radii of those neighborhoods have to satisfy an inequality that depends on the handicap κ of the problems. The first algorithm of [17] also depends on κ, while the second does not. In order to devise an algorithm that is independent of the handicap of the problem, the idea of corrector-predictor method was investigated in [3, 6, 14] .The first advantage of this approach is that only one neighborhood of the central path needs to be considered, thus avoiding the explicit relation between the radii of the neighborhoods assumed in [16, 17]. Second, the decrease of the duality gap along the predictor direction being faster if the point is closer to the central path, it makes sense to start the iteration with a corrector step. Indeed, the corrector-predictor algorithm reduces the duality gap in both the corrector and the predictor steps, and therefore it is more efficient.

The concept of the central path plays a critical role in the development of primal-dual path-following IPMs. Theoretical analysis and computational experiments [12] demonstrate that searching along the central path is the most efficient way to find optimizers. The majority of optimization algorithms search optimizers either along an arc of a power series approximation or along a straight line related to the first-order and higher-order derivatives of the central path [4, 8, 11]. Since the central path for LO appears to have sections of gentle curvature connected by sections of high curvature, intuitively ellipses can adjust center and axes parameters to approximate both gentle curvature and high curvature sections much better than straight lines which are used by most first-order and higher-order methods. Recently, Yang [20, 21, 22] devised a higher-order arc-search method. The arc-search algorithms utilized the first and second-order derivatives to construct an ellipse to approximate the central path. Yang [22] showed that arc-search along ellipse may be a better method than other one-dimensional search methods because the algorithm was proved to be polynomial with a better bound than the bounds of all existing higher-order algorithms. Ai and Zhang [1] introduced a new wide neighborhood, $\mathcal{N}_{2,\tau}^-(\alpha)$, and proposed a predictor-corrector method for solving monotone LCPs. Their algorithm has $O(\sqrt{n}L)$ iteration complexity coinciding with the same theoretical complexity as a small neighborhood algorithm. Potra [15] designed three interior point methods for solving sufficient horizontal LCPs in the wide neighborhood of the central path introduced by Ai and Zhang. Recently, Yang et al. [24] used Ai and Zhang's wide neighborhood and established a polynomial arc-search infeasible-interior-point algorithm for LO with a complexity bound of $O(n^{\frac{5}{4}}L)$. Quite recently, Pirhaji et al. [13] generalized the arc search technique proposed by Yang [21, 22] and Yang et al. [24] for LO to LCPs. Based on using the Ai-Zhang's neighborhood [1] and the Yang et al.'s new strategy [23] for obtaining the search directions, they proposed an arc search infeasible interior point algorithm for LCPs. They proved the algorithm to be well-defined and admitting the best known complexity bound,$O(n \log \varepsilon^{-1})$, for infeasible IPMs.

Motivated by Potra's algorithms [16, 17] and arc-search approximation of the central path, here we present a corrector-predictor arc-search interior point algorithm for $P_*(\kappa)$-LCP acting in the wide neighborhood $\mathcal{N}_{2,\tau}^-(\alpha)$ of the central path. The algorithm first performs a corrector step to improve

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centrality and optimality. Then, in order to enhance improvement of the optimality, the algorithm moves along an ellipsoidal curve using the predictor step. Here, we use the wide neighborhood $\mathcal{N}_{2,\tau}^{-}(\alpha)$ for any value of $\alpha \in (0,1)$, while in [1] this neighborhood was used only for $\alpha \in (0,\frac{1}{2}]$.

The rest of our work is organized as follows. In Section 2, we define $P_*(\kappa)$-LCP and review some basic concepts of IPMs for solving $P_*(\kappa)$-LCP, such as the central path and the neighborhoods of the central path. In Section 3, we describe our algorithmic scheme. Some technical lemmas are given in Section 4 and then, a complexity analysis of the algorithm is presented. Some numerical results are reported in Section 5. Section 6 gives our conclusions.

2. Preliminaries

The $P_*(\kappa)$-LCP consists of finding a pair of vectors $(x,s) \in R^{2n}$ such that

$$s = Mx + q, x^T s = 0, x, s \geq 0,$$

where $q \in R^n$ and $M \in R^{n\times n}$ is a $P_*(\kappa)$-matrix, i.e., for some nonnegative constant κ,

$$(1+4\kappa)\sum_{i\in I_+} x_i(Mx)_i + \sum_{i\in I_-} x_i(Mx)_i \geq 0, \forall x \in R^n, \tag{1}$$

where $I_+ := \{i: x_i(Mx)_i \geq 0\}$ and $I_- := \{i: x_i(Mx)_i < 0\}$ are two index sets. The smallest κ with the property (1) is called the handicap of the matrix. The class of $P_*(\kappa)$-matrices was first introduced by Kojima et al. [5], where the authors proved the existence and uniqueness of the central path for $P_*(\kappa)$-LCP and extended the primal-dual IPM for LO to $P_*(\kappa)$-LCP. Denote the set of all feasible points and strictly feasible points of $P_*(\kappa)$-LCP by

$$\mathcal{F} := \{(x,s) \in R^{2n}: s = Mx + q, (x,s) \geq 0\},$$

$$\mathcal{F}^0 := \{(x,s) \in R^{2n}: s = Mx + q, (x,s) > 0\}.$$

Moreover, we denote its solution set by

$$\mathcal{F}^* := \{(x^*,s^*) \in \mathcal{F}: (x^*)^T s^* = 0\}.$$

It is known (see [5]), under the assumption that $\mathcal{F}^0$ is nonempty, the nonlinear system

$$\begin{aligned} -Mx + s &= q, \\ xs &= \mu e, \end{aligned}$$

has a unique positive solution $\big(x(\mu), s(\mu)\big)$, for any $\mu > 0$. We call $(x(\mu), s(\mu))$ the μ-center of $P_*(\kappa)$-LCP. The set of μ-centers form the central path $\mathcal{C}$ of $P_*(\kappa)$-LCP:

$$\mathcal{C} := \{(x(\mu), s(\mu)): \mu > 0\}.$$

It has been shown that the limit of the central path (as μ goes to zero) exists and yields a solution for $P_*(\kappa)$-LCP ([5, Theorem 4.4]). Theoretical analysis and computational experiments demonstrate

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that searching along the central path is the most efficient way to find optimizers [12]. Many IPMs search optimizers along an arc of a power series approximation. However, there is no practical way to calculate the entire arc of the central path. Recently, Yang [20] suggested approximating the central path using ellipse, and developed an algorithm for LO which searches optimizers along the ellipse. The ellipse Υ in $2n$-dimensional space is defined as follows:

$$\Upsilon = \{(x(\theta), s(\theta)) : (x(\theta), s(\theta)) = \mathbf{i}\cos(\theta) + \mathbf{j}\sin(\theta) + \mathbf{k}\}, \tag{2}$$

where $\mathbf{i} \in R^{2n}$ and $\mathbf{j} \in R^{2n}$ are the axes of the ellipse, which are perpendicular to each other, and $\mathbf{k} \in R^{2n}$ is the center of ellipse.

Let $(\dot{x}, \dot{s})$ and $(\ddot{x}, \ddot{s})$ respectively denote the first and second derivatives of (x, s) and $z = (x, s) = \big(x(\theta_0), s(\theta_0)\big) \in \Upsilon$, which is close to or on the central path. Based on Yang's idea [20, 21, 22], we proceed to determine the vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ and the angle θ_0 such that the first and second derivatives of (x, s) satisfy

$$\begin{aligned} M\dot{x} - \dot{s} &= 0, \\ s\dot{x} + x\dot{s} &= xs, \end{aligned} \tag{3}$$

$$\begin{aligned} M\ddot{x} - \ddot{s} &= 0, \\ s\ddot{x} + x\ddot{s} &= -2\dot{x}\dot{s}. \end{aligned} \tag{4}$$

Let $\theta \in [0, \frac{\pi}{2}]$. It has been shown in [20] that one can avoid the calculation of the vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ in the expression for ellipse, which leads to the following lemma.

Lemma 1. ([22, cf. Theorem 3.1]) Let $\big(x(\theta), s(\theta)\big)$ be an arc defined by (2) passing through a point (x, s), and its first and second derivatives at (x, s) be $(\dot{x}, \dot{s})$ and $(\ddot{x}, \ddot{s})$, which are defined by (3) and (4). Then, an ellipsoidal approximation of the central path is given by

$$x(\theta) := x - \sin(\theta)\dot{x} + (1 - \cos(\theta))\ddot{x}, \tag{5}$$

$$s(\theta) := s - \sin(\theta)\dot{s} + (1 - \cos(\theta))\ddot{s}, \tag{6}$$

where $\theta \in [0, \frac{\pi}{2}]$.

The distance of a point $z = (x, s) \in \mathcal{F}$ to the central path can be quantified by different proximity measures. The following proximity measures have been extensively used in the literature:

$$\delta_2(z) := \| \frac{xs}{\mu} - e \|_2, \quad \delta_\infty(z) := \| \frac{xs}{\mu} - e \|_\infty, \quad \delta_\infty^-(z) := \| (\frac{xs}{\mu} - e)^- \|_\infty,$$

where $(v)^-$ denotes the negative part of the vector v, i.e., $(v)^- = -\max\{-v, 0\}$ and $\mu = \frac{x^T s}{n}$. According to the above-defined proximity measures, the neighborhoods of the central path are defined as follows:

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$$\mathcal{N}_2(\alpha) = \{z \in \mathcal{F}^0 : \delta_2(z) \le \alpha\},$$

$$\mathcal{N}_\infty(\alpha) = \{z \in \mathcal{F}^0 : \delta_\infty(z) \le \alpha\},$$

$$\mathcal{N}_\infty^-(\alpha) = \{z \in \mathcal{F}^0 : \delta_\infty^-(z) \le \alpha\} = \{z \in \mathcal{F}^0 : xs \ge (1-\alpha)\mu e\},$$

where $0 < \alpha < 1$ is a given parameter. In 2005, Ai and Zhang [1] introduced a new wide neighborhood as follows:

$$\mathcal{N}_{2,\tau}^-(\alpha) = \{(x,s) \in \mathcal{F}^0 : \| (xs - \tau\mu e)^- \|_2 \le \alpha\tau\mu\}, \tag{7}$$

where $0 < \tau < 1$. It is clear that $\| (xs - \tau\mu e)^- \|_2 = 0$, for all $(x,s) \in \mathcal{N}_\infty^-(1-\tau)$, and that for any $(x,s) \in \mathcal{N}_{2,\tau}^-(\alpha)$,

$$\| (xs - \tau\mu e)^- \|_2 \le \alpha\tau\mu \quad \text{and} \quad x_i s_i \le \tau\mu,$$

which imply

$$0 \le 1 - \frac{x_i s_i}{\tau\mu} \le \alpha, \quad \text{or equivalently} \quad x_i s_i \ge (1-\alpha)\tau\mu. \tag{8}$$

Therefore, we have

$$\mathcal{N}_\infty^-(1-\tau) \subset \mathcal{N}_{2,\tau}^-(\alpha) \subset \mathcal{N}_\infty^-(1-(1-\alpha)\tau), \qquad \forall \alpha, \tau \in (0,1). \tag{9}$$

Since $\mathcal{N}_\infty^-(1-\tau)$ is a wide neighborhood, so is $\mathcal{N}_{2,\tau}^-(\alpha)$.

3. Arc-search Corrector-Predictor Algorithm

Here, we describe an arc-search corrector-predictor method which follows approximately the ellipsoidal central path defined by (2). Let $(x,s) = (x(\theta_0), s(\theta_0)) \in \mathcal{N}_{2,\tau}^-(\alpha)$ be given. We first perform a corrector step in order to improve centrality and optimality of (x,s). By modifying (3) we define the first and second derivatives at $(x,s) \in \Upsilon$ in the corrector step to satisfy

$$\begin{aligned} M\dot{x} - \dot{s} &= 0, \\ s\dot{x} + x\dot{s} &= -[(\tau\mu e - xs)^- + \sqrt{n}(\tau\mu e - xs)^+], \end{aligned} \tag{10}$$

$$\begin{aligned} M\ddot{x} - \ddot{s} &= 0, \\ s\ddot{x} + x\ddot{s} &= -2\dot{x}\dot{s}. \end{aligned} \tag{11}$$

By solving systems (10), (11), we consider the point $(x(\theta), s(\theta))$ as defined in (5) and (6). Then, we compute $\sin(\hat{\theta})$ to obtain the point $(\bar{x}, \bar{s}) := (x(\hat{\theta}), s(\hat{\theta})) \in \mathcal{N}_{2,\tau}^-(\bar{\alpha})$, with $\bar{\alpha} < \alpha$ and $\bar{\mu} \le \mu$.

In the predictor step, we improve optimality by moving along an ellipsoidal curve defined by

$$\begin{aligned} M\dot{\bar{x}} - \dot{\bar{s}} &= 0, \\ \bar{x}\dot{\bar{s}} + \bar{s}\dot{\bar{x}} &= \bar{x}\bar{s}, \end{aligned} \tag{12}$$

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$$\begin{aligned} M\ddot{\bar{x}} - \ddot{\bar{s}} &= 0, \\ \bar{x}\ddot{\bar{s}} + \bar{s}\ddot{\bar{x}} &= -2\dot{\bar{x}}\dot{\bar{s}}. \end{aligned} \tag{13}$$

Then, by calculating step length $\sin(\hat{\xi})$ and using (5) and (6), we get $(\bar{x}(\hat{\xi}), \bar{s}(\hat{\xi}))$. As an immediate consequence, we have

$$(x^+, s^+) := \left(\bar{x}\left(\hat{\xi}\right), \bar{s}\left(\hat{\xi}\right)\right) \in \mathcal{N}_{2,\tau}^-(\alpha), \qquad \mu(z^+) < \bar{\mu},$$

and this implies that a new corrector step can be started. Therefore, a formal description of the algorithm is given in Fig. 1.

Algorithm 1: A corrector-predictor algorithm with arc-search

Input:

accuracy parameter$\varepsilon > 0$;

neighborhood parameterα, $0 < \alpha < 1$;

centering parameterτ, $0 < \tau \leq \frac{1}{4}$;

aninitial point$(x^0, s^0) \in \mathcal{N}_{2,\tau}^-(\alpha)$, $\mu_0 = (x^0)^T s^0/n$;

set$k := 0$.

begin

while$\mu_k > \varepsilon$ **do**

(corrector step)

Compute the directions $(\dot{x}^k, \dot{s}^k)$ and $\left(\ddot{x}^k, \ddot{s}^k\right)$ by solving (10) and (11).

Compute the largest positive $\sin\left(\hat{\theta}_k\right)$ such that the relations

$$(x(\theta), s(\theta)) \in \mathcal{N}_{2,\tau}^-(\bar{\alpha}),$$

$$\mu(\theta) \leq \mu_k,$$

hold for every $\sin(\theta) \in [0, \sin(\hat{\theta}_k)]$ with $\bar{\alpha} < \alpha$.

Compute $(\bar{x}, \bar{s}) = \left(x\left(\hat{\theta}_k\right), s\left(\hat{\theta}_k\right)\right)$ by (5) and (6).

Set $(\bar{x}^k, \bar{s}^k) \leftarrow (\bar{x}, \bar{s}), \bar{\mu}_k \leftarrow (\bar{x})^T \bar{s}/n$.

(predictor step)

Compute the directions $(\dot{\bar{x}}^k, \dot{\bar{s}}^k)$ and $\left(\ddot{\bar{x}}^k, \ddot{\bar{s}}^k\right)$ by solving (12) and (13).

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Compute the largest positive $\sin(\hat{\xi}_k)$ such that the relations

$$(\bar{x}(\xi), \bar{s}(\xi)) \in \mathcal{N}_{2,\tau}^{-}(\alpha),$$

$$\bar{\mu}(\xi) \leq \bar{\mu}_k,$$

hold for every $\sin(\xi) \in [0,\ \sin(\hat{\xi}_k)]$.

Compute $(x^+,\ s^+) = \left(\bar{x}(\hat{\xi}_k),\ \bar{s}(\hat{\xi}_k)\right)$ by using (5) and (6).

Set $(x^{k+1},\ s^{k+1}) \leftarrow (x^+,\ s^+), \quad \mu^{k+1} \leftarrow (x^+)^T s^+/n.$

Set $k \leftarrow k + 1$.

end while

end

Fig. 1. Algorithm 1

Before proceeding to the analysis of the algorithm, we recall two technical lemmas that are widely used in subsequent sections. For ease of notation, we shall adopt the following conventions:

$$D := X^{-1/2}S^{1/2}, \quad \| (u,v) \|_z^2 := \| Du \|_2^2 + \| D^{-1}v \|_2^2, \quad \tilde{a} := (xs)^{-1/2}a.$$

Lemma 2. [3, cf. Lemma 2] If *LCP* is $P_*(k)$, then for any $(x,s) \in R_{++}^{2n}$ and any $a \in R^n$ the linear system

$$Mu - v = 0,$$
$$su + xv = a.$$

has a unique solution (u,v) , for which the following estimates hold:

$$\| uv \|_2 \leq \left(\frac{1}{\sqrt{8}} + \kappa\right) \| \tilde{a} \|_2^2 \leq \frac{1}{2}(1 + 2\kappa) \| \tilde{a} \|_2^2, \qquad \| (u,v) \|_z^2 \leq (1 + 2\kappa) \| \tilde{a} \|_2^2.$$

Lemma 3.[24, cf. Lemma 4] Let $(x,s) \in \mathcal{N}_{2,\tau}^{-}(\alpha)$. Then,

$$\| (xs)^{-1/2}((\tau\mu e - xs)^- + \sqrt{n}(\tau\mu e - xs)^+) \|_2^2 \leq (1 + \frac{\alpha^2\tau}{1-\alpha})n\mu.$$

4. Analysis of the Algorithm

4.1. Analysis of the Corrector Step

Here, we analyze the corrector step such that its two requirements are ensured.

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Lemma 4. Let $(x,s) \in \mathcal{N}_{2,\tau}^{-}(\alpha)$. Then, the solutions of (10) and (11) satisfy the followings:

$$\| \dot{x}\dot{s} \|_2 \leq \frac{(1+2\kappa)}{2}(1+\frac{\alpha^2\tau}{1-\alpha})n\mu, \tag{14}$$

$$\| (\dot{x},\ \dot{s}) \|_z^2 \leq (1+2\kappa)\left(1+\frac{\alpha^2\tau}{1-\alpha}\right)n\mu, \tag{15}$$

$$\| \dot{x}\ddot{s} \|_2 \leq \frac{(1+2\kappa)^3(1+\frac{\alpha^2\tau}{1-\alpha})^2n^2\mu}{2(1-\alpha)\tau}, \tag{16}$$

$$\| (\dot{x},\ \ddot{s}) \|_z^2 \leq \frac{(1+2\kappa)^3(1+\frac{\alpha^2\tau}{1-\alpha})^2n^2\mu}{(1-\alpha)\tau}, \tag{17}$$

$$\| \dot{x}\ddot{s}+\dot{s}\ddot{x} \|_2 \leq \frac{(1+2\kappa)^2(1+\frac{\alpha^2\tau}{1-\alpha})^{\frac{3}{2}}n^{\frac{3}{2}}\mu}{\sqrt{(1-\alpha)\tau}}. \tag{18}$$

Proof. Applying Lemma 2 to the system (10), and using Lemma 3, we have

$$\| \dot{x}\dot{s} \|_2 \leq \frac{1+2\kappa}{2} \| (xs)^{-\frac{1}{2}}((\tau\mu e-xs)^- + \sqrt{n}(\tau\mu e-xs)^+) \|_2^2 \leq \frac{1+2\kappa}{2}(1+\frac{\alpha^2\tau}{1-\alpha})n\mu$$

and

$$\| (\dot{x},\dot{s}) \|_z^2 \leq (1+2\kappa) \| (xs)^{-\frac{1}{2}}((\tau\mu e-xs)^- + \sqrt{n}(\tau\mu e-xs)^+) \|_2^2 \leq (1+2\kappa)(1+\frac{\alpha^2\tau}{1-\alpha})n\mu.$$

These establish the inequalities (14) and (15). Similarly, applying Lemma 2 to the system (11) gives

$$\| \ddot{x}\ddot{s} \|_2 \leq \frac{1+2\kappa}{2} \| (xs)^{-\frac{1}{2}}(-2\dot{x}\dot{s}) \|_2^2 \leq 2(1+2\kappa)\frac{\| \dot{x}\dot{s} \|_2^2}{(1-\alpha)\tau\mu} \leq \frac{(1+2\kappa)^3}{2(1-\alpha)\tau}(1+\frac{\alpha^2\tau}{1-\alpha})^2n^2\mu,$$

where the second inequality follows from (8), and the last inequality is a consequence of (14). This establishes the inequality (16). Similarly, we obtain the inequality (17). In order to find an upper bound for $\| \dot{x}\ddot{s}+\dot{s}\ddot{x} \|_2$, we use the triangular inequality and the inequality $ad+bc \leq \sqrt{a^2+b^2}\sqrt{c^2+d^2}$, for all $a,b,c,d \geq 0$. In this case, we have

$$\| \dot{x}\ddot{s}+\dot{s}\ddot{x} \|_2 \leq \| D\dot{x} \|_2 \| D^{-1}\ddot{s} \|_2 + \| D\ddot{x} \|_2 \| D^{-1}\dot{s} \|_2 \leq \| (\dot{x},\dot{s}) \|_z \| (\ddot{x},\ddot{s}) \|_z.$$

Substituting the upper bounds on $\| (\dot{x},\dot{s}) \|_z$ and $\| (\ddot{x},\ddot{s}) \|_z$ into the above inequality gives the desired inequality. Therefore, the proof is complete. □

Now, for convenience, we define the following expressions:

$$x(\theta)s(\theta) = xs + [(\tau\mu e-xs)^- + \sqrt{n}(\tau\mu e-xs)^+]\sin(\theta) + d(\theta), \tag{19}$$

$$\mu(\theta) = \mu + \frac{e^T}{n}[(\tau\mu e-xs)^- + \sqrt{n}(\tau\mu e-xs)^+]\sin(\theta) + \frac{e^T d(\theta)}{n}$$

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$$= \mu + [(\tau\mu - \mu) + \frac{\sqrt{n}-1}{n} e^T(\tau\mu e - xs)^+]\sin(\theta) + \frac{e^T d(\theta)}{n}, \tag{20}$$

where

$$d(\theta) = -(1-\cos(\theta))^2 \dot{x}\dot{s} - \sin(\theta)(1-\cos(\theta))(\dot{x}\ddot{s} + \dot{s}\ddot{x}) + (1-\cos(\theta))^2 \ddot{x}\ddot{s}. \tag{21}$$

It is clear that $1-\cos(\theta) \leq \sin^2(\theta)$.

Lemma 5. Let $d(\theta)$ be defined as in (21), $0 < \alpha < 1$, and $0 < \tau \leq \frac{1}{4}$. Then, for all $\sin(\theta)$ satisfying

$$\sin(\theta) \leq \sin(\theta_0) := \frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}{2\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}(1+2\kappa)\sqrt{n}}, \quad n \geq 3, \tag{22}$$

we have

$$\| d(\theta) \|_2 \leq \frac{217}{768}\alpha\tau\sqrt{(1-\alpha)\tau}\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}\sqrt{n}\sin(\theta)\mu \leq \frac{217}{12288}(1-\alpha)\tau\mu. \tag{23}$$

Proof. Taking the 2-norm on both sides of (21) and using the triangular inequality and $1-\cos(\theta) \leq \sin^2(\theta)$, we get

$$\begin{aligned} \| d(\theta) \|_2 &= \| -(1-\cos(\theta))^2 \dot{x}\dot{s} - \sin(\theta)(1-\cos(\theta))(\dot{x}\ddot{s} + \dot{s}\ddot{x}) + (1-\cos(\theta))^2 \ddot{x}\ddot{s} \|_2 \\ &\leq \sin^4(\theta) \| \dot{x}\dot{s} \|_2 + \sin^3(\theta) \| \dot{x}\ddot{s} + \dot{s}\ddot{x} \|_2 + \sin^4(\theta) \| \ddot{x}\ddot{s} \|_2 \\ &\leq \sin(\theta)(\sin^3(\theta_0) \| \dot{x}\dot{s} \|_2 + \sin^2(\theta_0) \| \dot{x}\ddot{s} + \dot{s}\ddot{x} \|_2 + \sin^3(\theta_0) \| \ddot{x}\ddot{s} \|_2). \end{aligned} \tag{24}$$

It is easily seen that the inequalities (14), (16) and (18) can be written, respectively, as

$$\| \dot{x}\dot{s} \|_2 \leq \frac{((\alpha\tau)(1-\alpha)\tau)^{\frac{3}{2}}}{16(1+2\kappa)^2\sqrt{(1+\frac{\alpha^2\tau}{1-\alpha})n}} \left(\frac{2\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}(1+2\kappa)\sqrt{n}}{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}} \right)^3 \mu,$$

$$\| \dot{x}\ddot{s} \|_2 \leq \frac{(\alpha\tau)^{\frac{3}{2}}\sqrt{(1-\alpha)\tau}\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}\sqrt{n}}{16} \left(\frac{2\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}(1+2\kappa)\sqrt{n}}{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}} \right)^3 \mu,$$

$$\| \dot{x}\ddot{s} + \dot{s}\ddot{x} \|_2 \leq \frac{(\alpha\tau)\sqrt{(1-\alpha)\tau}\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}\sqrt{n}}{4} \left(\frac{2\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}(1+2\kappa)\sqrt{n}}{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}} \right)^2 \mu.$$

Substitution of the above three bounds into (24) yields

$$\| d(\theta) \|_2 \leq \alpha\tau\sqrt{(1-\alpha)\tau}\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}\sqrt{n}\sin(\theta)\left(\frac{\sqrt{\alpha\tau}(1-\alpha)\tau}{16(1+2\kappa)^2(1+\frac{\alpha^2\tau}{1-\alpha})n} + \frac{1}{4} + \frac{\sqrt{\alpha\tau}}{16} \right)\mu$$

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$$\leq \frac{217}{768}\alpha\tau\sqrt{(1-\alpha)\tau}\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}\sqrt{n}\sin(\theta)\mu,$$

where the second inequality follows from the fact that $n \geq 3, \alpha \leq \sqrt{\alpha}$ and $\alpha(1-\alpha) \leq \frac{1}{4}$, which imply that the term in brackets is less than or equal to $\frac{217}{768}$. Since $\sin(\theta) \in [0,\ \sin(\theta_0)]$, by substituting the value of $\sin(\theta_0)$ from (22) in the last inequality for$\| d(\theta) \|_2$, we get

$$\| d(\theta) \|_2 \leq \frac{217}{12288}(1-\alpha)\tau\mu.$$

This completes the proof. □

Lemma 6. For any $(x,s) \in \mathcal{N}_{2,\tau}^{-}(\alpha)$ and $sin(\theta) \in [0,\ sin(\theta_0)]$, we have

$$\mu(\theta) \leq (1-\frac{466}{981}\sin(\theta))\mu, \tag{25}$$

$$\mu(\theta) \geq (1-(1-\tau)\sin(\theta)-\frac{316}{12653}\sin(\theta))\mu. \tag{26}$$

Proof. From the first inequality in (23), it follows that

$$\| d(\theta) \|_2 \leq \frac{217}{768}\tau^{3/2}\sqrt{\alpha(1-\alpha)+\alpha^3\tau}\sqrt{n}\sin(\theta)\mu \leq \frac{316}{12653}\sqrt{n}\sin(\theta)\mu.$$

Using (20) and the above inequality, we may verify that

$$\begin{aligned}
\mu(\theta) &= \mu + [(\tau\mu-\mu)+\tfrac{\sqrt{n}-1}{n}e^T(\tau\mu e - xs)^+]\sin(\theta) + \tfrac{e^T d(\theta)}{n} \\
&\leq \mu + (\tau-1+\alpha\tau)\sin(\theta)\mu + \tfrac{\|d(\theta)\|_2}{\sqrt{n}} \\
&\leq \mu + (\tau-1+\alpha\tau)\sin(\theta)\mu + \tfrac{316}{12653}\sin(\theta)\mu \\
&\leq (1-(1-\tau-\alpha\tau-\tfrac{316}{12653})\sin(\theta))\mu \\
&\leq (1-\tfrac{466}{981}\sin(\theta))\mu.
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
\mu(\theta) &= \mu + [(\tau\mu-\mu)+\tfrac{\sqrt{n}-1}{n}e^T(\tau\mu e - xs)^+]\sin(\theta) + \tfrac{e^T d(\theta)}{n} \\
&\geq \mu - (1-\tau)\sin(\theta)\mu - \tfrac{\|d(\theta)\|}{\sqrt{n}} \\
&\geq (1-(1-\tau)\sin(\theta)-\tfrac{316}{12653}\sin(\theta))\mu.
\end{aligned}$$

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This completes the proof. □

Lemma 7. [24, cf. Lemma 13] Let $\mu(\theta) > 0$. Then, for all $sin(\theta) \in [0,\ sin(\theta_0)]$, we have

$$\| (xs + [(\tau\mu e - xs)^- + \sqrt{n}(\tau\mu e - xs)^+]\sin(\theta) - \tau\mu(\theta)e)^- \|_2 \leq (1 - \sqrt{n}\sin(\theta))\alpha\tau\mu.$$

In the next lemma, we obtain an upper bound for the quantity $\bar{\alpha}$. Then, we guarantee that the corrector point $(\bar{x}, \bar{s})$ belongs to the wide neighborhood $\mathcal{N}_{2,\tau}^-(\bar{\alpha})$.

Lemma 8. Suppose that $(x, s) \in \mathcal{N}_{2,\tau}^-(\alpha)$ with $0 < \alpha < 1$ and $0 < \tau \leq \frac{1}{4}$, and let $\sin(\theta_0)$ be defined as in (22). Then, $\big(x(\theta),\ s(\theta)\big) \in \mathcal{N}_{2,\tau}^-(\bar{\alpha})$, for all $\sin(\theta) \in [0,\ \sin(\theta_0)]$, where

$$\bar{\alpha} = \left(1 - \frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}{8\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}(1+2\kappa)}\right)\alpha,$$

which implies that $\sin(\hat{\theta}) \geq \sin(\theta_0)$.

Proof. First, note that

$$1 - \sin(\theta) \geq 1 - \sin(\theta_0) \geq \frac{2511}{2605}.$$

Using (19), (8), (23) and the inequality $-\| z \|_2\ e \leq z \leq \| z \|_2\ e$, for all $z \in R^n$, we deduce

$$\begin{aligned} x(\theta)s(\theta) &\geq xs + [(\tau\mu e - xs) + \big(\sqrt{n} - 1\big)(\tau\mu e - xs)^+]\sin(\theta) - \| d(\theta) \|_2\ e \\ &= (1 - \sin(\theta))xs + \tau\mu\sin(\theta)e + (\sqrt{n} - 1)(\tau\mu e - xs)^+\sin(\theta) - \| d(\theta) \|_2\ e \\ &\geq \frac{2511}{2605}(1 - \alpha)\tau\mu e - \frac{217}{12288}(1 - \alpha)\tau\mu e > 0. \end{aligned}$$

Due to the above inequality and using a continuity argument, we deduce that $x(\theta) > 0$ and $s(\theta) > 0$, for all $\sin(\theta) \in [0,\ \sin(\theta_0]$. Since $-Mx(\theta) + s(\theta) = q$, it follows that $(x(\theta),\ s(\theta)) \in \mathcal{F}^0$. Thus, in order to prove $(x(\theta),\ s(\theta)) \in \mathcal{N}_{2,\tau}^-(\bar{\alpha})$, we only need to prove

$$\| (x(\theta)s(\theta) - \tau\mu(\theta)e)^- \|_2 - \bar{\alpha}\tau\mu(\theta) \leq 0.$$

To this end, according to (19), (23), (26) and Lemma 7, we obtain

$$\begin{aligned} \| (x(\theta)s(\theta) - \tau\mu(\theta)e)^- \|_2 - \bar{\alpha}\tau\mu(\theta) \leq & \\ \| (xs + [(\tau\mu e - xs)^- + \sqrt{n}(\tau\mu e - xs)^+]\sin(\theta) - \tau\mu(\theta)e)^- \|_2 + \| d(\theta) \|_2 & \\ - \left(1 - \frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}{8\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}(1+2\kappa)}\right)\alpha\tau\mu(\theta) & \end{aligned}$$

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$$\leq (1-\sqrt{n}\sin(\theta))\alpha\tau\mu + \frac{217}{768}\alpha\tau\sqrt{(1-\alpha)\tau}\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}\sqrt{n}\sin(\theta)\mu$$

$$-\left(1-\frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}{8\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}(1+2\kappa)}}\right)(1-(1-\tau)\sin(\theta)-\frac{316}{12653}\sin(\theta))\alpha\tau\mu$$

$$\leq (1-\sqrt{n}\sin(\theta))\alpha\tau\mu + \frac{293}{1855}\alpha\tau\sqrt{n}\sin(\theta)\mu - \left(1-\frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}{8\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}(1+2\kappa)}}\right)\alpha\tau\mu$$

$$+((1-\tau)\sin(\theta)+\frac{316}{12653}\sin(\theta))\alpha\tau\mu$$

$$= \sqrt{n}\sin(\theta)\alpha\tau\mu\left(-1+\frac{293}{1855}+\frac{1-\tau}{\sqrt{n}}+\frac{316}{12653\sqrt{n}}\right)+\frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}{8\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}(1+2\kappa)}}\alpha\tau\mu$$

$$\leq \sqrt{n}\sin(\theta_0)\alpha\tau\mu\left(-1+\frac{293}{1855}+\frac{1-\tau}{\sqrt{n}}+\frac{316}{12653\sqrt{n}}+\frac{1}{4}\right)<0.$$

This is the desired result. □

4.2. Analysis of the Predictor Step

Now, we are ready to analyze the predictor step. Since the predictor step follows a corrector step, we take the point $(\bar{x},\ \bar{s}) := (x(\hat{\theta}),\ s(\hat{\theta}))$ obtained in the corrector step as the starting point, and try to compute the directions $(\dot{x}, \dot{\bar{s}})$ and $(\ddot{x}, \ddot{\bar{s}})$ by solving the systems (12) and (13). In this case, we get the predictor point, according to Lemma 1, as follows:

$$\bar{x}(\xi) = \bar{x} - \sin(\xi)\dot{x} + \big(1-\cos(\xi)\big)\ddot{x}, \quad \bar{s}(\xi) = \bar{s} - \sin(\xi)\dot{s} + (1-\cos(\xi))\ddot{s}.$$

Thus, after some calculations, we obtain

$$\bar{x}(\xi)\bar{s}(\xi) = (1-\sin(\xi))\bar{x}\bar{s} + \bar{d}(\xi), \tag{27}$$

$$\bar{\mu}(\xi) = (1-\sin(\xi))\bar{\mu} + \frac{e^T\bar{d}(\xi)}{n}, \tag{28}$$

where

$$\bar{d}(\xi) = -(1-\cos(\xi))^2\dot{x}\dot{s} - \sin(\xi)(1-\cos(\xi))(\dot{x}\ddot{s}+\dot{s}\ddot{x}) + (1-\cos(\xi))^2\ddot{x}\ddot{s}. \tag{29}$$

Lemma 9 Let $sin(\hat{\xi})$ be the maximum step size in the predictor step. Then, for all $\sin(\xi) \in [0, \sin(\xi_0)]$, we have $\big(\bar{x}(\xi), \bar{s}(\xi)\big) \in \mathcal{N}_{2,\tau}^{-}(\alpha)$, where

$$\sin(\xi_0) := \frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}{2(1+2\kappa)\sqrt{n}},$$

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which further implies $\sin(\hat{\xi}) \geq \sin(\xi_0)$.

Proof. According to Lemma 8, the corrector step produces a point $(\bar{x}, \bar{s}) \in \mathcal{N}_\infty^-(1-(1-\bar{\alpha})\tau) \subset \mathcal{N}_\infty^-(1-(1-\alpha)\tau)$ (see (9)). Using Lemma 2, we deduce that the directions obtained by the predictor step satisfy

$$\| (\dot{x}, \dot{s}) \|_z \leq \sqrt{(1+2\kappa)n\bar{\mu}},$$

$$\| \dot{x}\dot{s} \|_2 \leq \frac{(1+2\kappa)n\bar{\mu}}{2} = \frac{(\alpha\tau)^2((1-\alpha)\tau)^2}{32(1+2\kappa)^3 n}\left(\frac{2\sqrt{n}(1+2\kappa)}{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}\right)^4 \bar{\mu},$$

$$\| (\bar{x}\bar{s})^{-\frac{1}{2}}(-2\dot{x}\dot{s}) \|_2^2 \leq \frac{4\|\dot{x}\dot{s}\|_2^2}{(1-\alpha)\tau\bar{\mu}} \leq \frac{(1+2\kappa)^2 n^2 \bar{\mu}}{(1-\alpha)\tau},$$

$$\| (\ddot{x}, \ddot{s}) \|_z \leq \sqrt{1+2\kappa}\, \| (\bar{x}\bar{s})^{-1/2}(-2\dot{x}\dot{s}) \|_2 \leq \frac{(1+2\kappa)^{\frac{3}{2}} n\sqrt{\bar{\mu}}}{\sqrt{(1-\alpha)\tau}},$$

$$\| \ddot{x}\ddot{s} \|_2 \leq \frac{1+2\kappa}{2} \| (\bar{x}\bar{s})^{-1/2}(-2\dot{x}\dot{s}) \|_2^2 \leq \frac{(1+2\kappa)^3 n^2}{2(1-\alpha)\tau}\bar{\mu}$$

$$\leq \frac{(\alpha\tau)^2(1-\alpha)\tau}{32(1+2\kappa)}\left(\frac{2\sqrt{n}(1+2\kappa)}{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}\right)^4 \bar{\mu},$$

$$\| \dot{x}\ddot{s} + \dot{s}\ddot{x} \|_2 \leq \| (\dot{x}, \dot{s}) \|_z \| (\ddot{x}, \ddot{s}) \|_z \leq \frac{(1+2\kappa)^2 n^{\frac{3}{2}}}{\sqrt{(1-\alpha)\tau}}\bar{\mu}$$

$$= \frac{(\alpha\tau)^{3/2}(1-\alpha)\tau}{8(1+2\kappa)}\left(\frac{2\sqrt{n}(1+2\kappa)}{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}\right)^3 \bar{\mu}.$$

Using (29) and the above inequalities we obtain that for any $\sin(\xi) \in (0, \sin(\xi_0)]$, we have

$$\| \bar{d}(\xi) \|_2 \leq \sin^4(\xi_0) \| \dot{x}\dot{s} \|_2 + \sin^3(\xi_0) \| \dot{x}\ddot{s} + \dot{s}\ddot{x} \|_2 + \sin^4(\xi_0) \| \ddot{x}\ddot{s} \|_2$$

$$\leq \left(\frac{(\alpha\tau)^2((1-\alpha)\tau)^2}{32(1+2\kappa)^3 n} + \frac{(\alpha\tau)^{3/2}(1-\alpha)\tau}{8(1+2\kappa)} + \frac{(\alpha\tau)^2(1-\alpha)\tau}{32(1+2\kappa)}\right)\bar{\mu}$$

$$\leq \frac{217(\alpha\tau)^{3/2}(1-\alpha)\tau}{1536(1+2\kappa)}\bar{\mu} \leq \frac{217}{12288}(1-\alpha)\tau\bar{\mu}. \tag{30}$$

Due to (27), the inequality $-\| z \|_2 e \leq z \leq \| z \|_2 e$, for all $z \in R^n$, (8) and (30), we conclude that for any $\sin(\xi) \in (0, \sin(\xi_0)]$, we have

$$\bar{x}(\xi)\bar{s}(\xi) \geq (1-\sin(\xi))\bar{x}\bar{s} - \| \bar{d}(\xi) \| e$$

$$\geq (1-\sin(\xi_0))(1-\bar{\alpha})\tau\bar{\mu}e - \frac{217}{12288}(1-\alpha)\tau\bar{\mu}e.$$

Since $0 < \alpha < 1$, $n \geq 3$, $0 < \tau \leq \frac{1}{4}$, we have $\sin(\xi_0) \leq \frac{195}{5404}$, which implies

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$$1-\sin(\xi_0) > \frac{2511}{2605}.$$

Therefore,

$$\bar{x}(\xi)\bar{s}(\xi) \geq \left(\frac{2511}{2605} - \frac{217}{12288}\right)(1-\alpha)\tau\bar{\mu}e > 0.$$

Using a continuity argument, we deduce $\bar{x}(\xi) > 0,\ \bar{s}(\xi) > 0$, for all $0 < \sin(\xi) \leq \sin(\xi_0)$. Since $-M\bar{x}(\xi) + \bar{s}(\xi) = q$, it follows that $\left(\bar{x}(\xi),\ \bar{s}(\xi)\right) \in \mathcal{F}^0,$ for all $0 < \sin(\xi) \leq \sin(\xi_0)$.

According to the fact that $(\bar{x}, \bar{s}) \in \mathcal{N}_{2,\tau}^{-}(\bar{\alpha})$, using (27) and (28), we get

$$\begin{aligned} \| (\bar{x}(\xi)\bar{s}(\xi) - \tau\bar{\mu}(\xi)e)^{-} \|_2 &\leq (1-\sin(\xi)) \| (\bar{x}\bar{s} - \tau\bar{\mu}e)^{-} \|_2 + \| \bar{d}(\xi) \|_2 \\ &\leq (1-\sin(\xi))\bar{\alpha}\tau\bar{\mu} + \| \bar{d}(\xi) \|_2, \end{aligned}$$

$$\bar{\mu}(\xi) \geq (1-\sin(\xi))\bar{\mu} - \frac{\|\bar{d}(\xi)\|_2}{\sqrt{n}},$$

and by using Lemma 8 and (30), we deduce that for any $0 < sin(\xi) \leq \sin(\xi_0)$, we have

$$\begin{aligned} \| (\bar{x}(\xi)\bar{s}(\xi) - \tau\bar{\mu}(\xi)e)^{-} \|_2 - \alpha\tau\bar{\mu}(\xi) &\leq (1-\sin(\xi))(\bar{\alpha}-\alpha)\tau\bar{\mu} + \left(1 + \frac{\alpha\tau}{\sqrt{n}}\right) \| \bar{d}(\xi) \|_2 \\ &\leq -(1-\sin(\xi)) \frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau\tau}}{8\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}(1+2\kappa)}}\bar{\mu} + \frac{3}{2} \| \bar{d}(\xi) \| \\ &\leq -(1-\sin(\xi)) \frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau\tau}}{8\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}(1+2\kappa)}}\bar{\mu} + \frac{651(\alpha\tau)^{3/2}(1-\alpha)\tau}{3072(1+2\kappa)}\bar{\mu} \\ &\leq \frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau\tau}}{\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}(1+2\kappa)}} \left(\frac{-(1-\sin(\xi_0))}{8} + \frac{651\alpha\sqrt{(1-\alpha)\tau}\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}}}{3072} \right) \\ &\leq \frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau\tau}}{\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}(1+2\kappa)}} \left(\frac{-2511}{20840} + \frac{651\sqrt{\tau}\sqrt{\alpha(1-\alpha)+\alpha^3\tau}}{3072} \right) \\ &\leq -\frac{37}{812} \frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau\tau}}{\sqrt{1+\frac{\alpha^2\tau}{1-\alpha}(1+2\kappa)}} < 0. \end{aligned}$$

This completes the proof. □

Theorem 1. If *LCP* is $P_*(\kappa)$, then Algorithm 1 is well defined and produces a sequence of points $\left(x^k,\ s^k\right)$ belonging to the neighborhood $\widetilde{\mathcal{N}}_{2,\tau}^{-}(\alpha)$ such that

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$$\mu_{k+1} \leq \left(1 - \frac{445\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}{906\sqrt{n}(1+2\kappa)}\right)\mu_k, \quad k = 0,1, \ldots.$$

Proof. The first part of the theorem follows from Lemma 9. From (28) and (30) , we have

$$\begin{aligned}
\bar{\mu}(\xi_0) &\leq (1 - \sin(\xi_0))\bar{\mu} + \frac{\|\bar{d}(\xi_0)\|_2}{\sqrt{n}} \\
&\leq (1 - \sin(\xi_0))\bar{\mu} + \frac{217(\alpha\tau)^{3/2}(1-\alpha)\tau}{1536(1+2\kappa)\sqrt{n}}\bar{\mu} \\
&\leq \left(1 - \frac{\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}{2\sqrt{n}(1+2\kappa)}\left(1 - \frac{217\alpha\tau\sqrt{(1-\alpha)\tau}}{768}\right)\right)\bar{\mu} \\
&\leq \left(1 - \frac{445\sqrt{\alpha\tau}\sqrt{(1-\alpha)\tau}}{906\sqrt{n}(1+2\kappa)}\right)\bar{\mu}.
\end{aligned}$$

This completes the proof. □

Corollary 2. Under the hypothesis of Theorem 1, Algorithm 1 produces a point $(x^+, s^+) \in \mathcal{N}_{2,\tau}^-(\alpha)$ with $\mu^+ \leq \varepsilon$ in at most $O\left((1+\kappa)\sqrt{n}L\right)$ iterations, where $L = log\frac{\mu_0}{\varepsilon}$.

5. Numerical Results

Here, we present some numerical results to illustrate the performance of Algorithm 1. All of our tests were made on an Intel Core i7 Labtop with 2GB RAM under Windows XP and MATLAB (R2009a). We set $\tau = 0.001$ and $\alpha = 0.5$. We first compare the proposed corrector-predictor arc-search, Algorithm 1, with the algorithm of [7]. These two algorithms will be denoted by C-P algorithm and LSL algorithm, respectively. The comparison is carried out by testing LCPs generated as follows: $A = rand(n)$, $M = A^T A$ and $q = e - Me$. The algorithms are terminated when the relative duality gap satisfies

$$\frac{x^T s}{1+(x^0)^T s^0} < 10^{-8}.$$

Table 1 shows the average number of iterations (Iter) and the average CPU time (Time) per iteration of ten randomly generated problems with the same n. Our preliminary implementations show that our algorithm is promising.

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Table 1

n	C-P algorithm		LSL algorithm	
	Iter.	Time	Iter	Time
100	4.1	0.0305	7.5	0.0081
300	4.4	0.1671	7.9	0.0831
700	4.7	1.4341	8.4	0.7868
900	4.7	2.4096	8.4	1.5269
1000	4.6	3.1464	8.3	1.9732

We also compare our algorithm with the proposed algorithm of [2]. To this end, we consider the following LCP:

$$M = \begin{pmatrix} 1 & 2 & 2 & \cdots & 2 \\ 0 & 1 & 2 & \cdots & 2 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & & 1 \end{pmatrix}, \qquad q = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}.$$

Without loss of generality, we chose $x^0 = s^0 = e$ as the starting point. We set $\tau = 0.5$ and $\varepsilon = 10^{-4}$. The number of iterations are given in Table 2.

Table 2

n	C-P algorithm	algorithm in [2]
	Iter.	Iter
10	13	46
20	14	58
30	14	67

Tables 1 and 2 show that Algorithm 1 needed smaller number of iterations.

6. Conclusions

We presented an arc-search corrector-predictor interior point algorithm for solving $P_*(\kappa)$-LCPs acting in the wide neighborhood of the central path. The proposed algorithm searches the optimizers along the ellipses that approximate the central path. Using Ai and Zhang's directions, the corrector step increased both centrality and optimality and the predictor step further improved optimality. Our algorithm did not explicitly use the handicap of the problem, and it could solve any $P_*(\kappa)$-LCP requiring at most $O((1+\kappa)\sqrt{n}L)$ iterations. The bound coincides with the currently best known theoretical bound obtained so far by any interior point method for solving $P_*(\kappa)$-LCPs. Our numerical experiments show the algorithm to be promising.

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References

[1] Ai, W. and Zhang, S. (2005), An $O(\sqrt{n}L)$ iteration primal-dual path-following method, based on wide neighborhoods and large updates, for monotone LCP,*SAIM J. Optim.*, 16(2), 400-417.

[2] Asadi, S. and Mansouri, H. (2013), Polynomial interior-point algorithm for $P_*(\kappa)$ horizontal linear complementarity problems, *Numer. Algorithms*, 63, 385-398.

[3] Gurtuna, F., Petra, C., Potra, F.A., Shevchenko, O. and Vancea, A. (2011), Corrector-predictor methods for sufficient linear complementarity problems, *Comput. Optim. Appl.*, 48, 453-485.

[4] Jansen, B., Roos, C., Terlaky, T. and Ye, Y. (1996), Improved complexity using higher-order correctors for primal-dual Dikin affine scaling, *Math. Program.*, 76, 117-130.

[5] Kojima, M., Megiddo, N., Noma, T. and Yoshise, A. (1991), A Unified Approach to Interior Point Algorithms for Linear Complementarity Problems, Lecture Notes in Comput. Sci. 538, Springer-Verlag, Berlin.

[6] Liu, X. and Potra, F.A. (2006), Corrector-predictor methods for sufficient linear complementarity problems in a wide neighborhood of the central path, *SIAM J. Optim.*, 17(3), 871-890.

[7] Liu, C., Shang, Y. and Liu, H. (2016), An $O(\sqrt{n}L)$ iteration Mehrotra-type predictor-corrector algorithm for monotone linear complementarity problem, *Optim. Lett.*, 10, 619-634.

[8] Mehrotra., S. (1992), On the implementation of a primal-dual interior point method, *SIAM J. Optim.*, 2(4), 575-601.

[9] Miao, J. (1995), A quadratically convergent $O((\kappa+1)\sqrt{n}L)$ iteration algorithm for the $P_*(\kappa)$-matrix linear complementarity problem, *Math. Program.*, 69, 355-368.

[10] Mizuno, S., Todd, M.J. and Ye, Y. (1993), On adaptive-step primal-dual interior point algorithms for linear programming, *Math. Oper. Res.*, 18(4), 964-981.

[11] Monteiro, R.D., Adler, I. and Resende, M.G. (1990), A polynomial-time primal-dual affine scaling algorithm for linear and convex quadratic programming and its power series extension, *Math. Oper. Res.*, 15(2), 191-214.

[12] Nesterov, Y. (1996), Long-step strategies in interior-point primal-dual methods, *Math. Program.*, 76, 47-94.

[13] Pirhaji, M., Zangiabadi, M. and Mansouri, H. (2017), An l_2-neighborhood infeasible interior-point algorithm for linear complementarity problems, *4OR-Q J. Oper. Res.*, 15 (2), 111-131.

[14] Potra, F.A. (2008), Corrector-predictor methods for monotone linear complementarity problems in a wide neighborhood of the central path, *Math. Program.*, 111, 243-272.

[15] Potra, F. A. (2014), Interior point methods for sufficient horizontal LCP in a wide neighborhood of the central path with best known iteration complexity, *SIAM J. Optim.*, 24(1), 1-28.

[16] Potra, F.A. (2014), A superlinearly convergent predictor-corrector method for degenerate LCP in a wide neighborhood of the central path with $O(\sqrt{n}L)$-iteration complexity, *Math. Program.*, 100(2), 317-337.

[17] Potra, F.A. and Liu, X. (2005), Predictor-corrector methods for sufficient linear complementarity problems in a wide neighborhood of the central path, *Optim. Methods Softw.*, 20(1), 145-168.

[18] Potra, F.A. and Sheng, R. (1997), A large-step infeasible-interior-point method for the P_*-matrix LCP, *SIAM J.Optim.*, 7(2), 318-335.

[19] Wright, S. J. (1997), Primal-Dual Interior-Point Methods, SIAM Publications, Philadephia.

[Downloaded from iors.ir on 2026-11-09]

[20] Yang, Y. (2009), Arc-search path-following interior-point algorithms for linear programming, *Optimization online*

[21] Yang, Y. (2011), A polynomial arc-search interior-point algorithm for convex quadratic programming, *Eur. J. Oper. Res.*, 215, 25-38.

[22] Yang, Y. (2013), A polynomial arc-search interior-point algorithm for linear programming, *J. Optim. Theory Appl.*, 158, 859-873.

[23] Yang, X., Liu, H. and Zhang, Y. (2015), A new strategy in the complexity analysis of an infeasible-interior-point method for symmetric cone programming, *J. Optim. Theory Appl.*, 166 (2), 572-587.

[24] Yang, X., Zhang, Y. and Liu, H. (2016), A wide neighborhood infeasible-interior-point method with arc-search for linear programming, *J. Appl. Math. Comput.*, 51 (1), 209-225.

[25] Ye, Y. (1997), Interior Point Algorithms : Theory and Analysis, Wiley-Interscience Series in Discrete Mathematics and Optimization, John Wiley and Sons.

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