A truthful Screening Mechanism for Improving Information Asymmetry in Initial Public Offering Transactions

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We propose a mechanism to deal with the asymmetric information which increases the phenomenon of underpricing in the Initial Public Offering (IPO) transactions. In this regard, we develop a truthful screening mechanism by which a screening agent could assess a firm that is going to be public during an IPO. A mathematical model is developed and solved to determine the incentives of these agencies so that they find it optimal to perform truthfully. We also pursue the case of cooperation of n such agents and compare it with the situation in which each agent works independently.

Keywords: *IPO Underpricing; Information Asymmetry; Screening Mechanism; Mathematical Modeling.*

1. Introduction

An Initial Public Offering (IPO) is the process of initial offering to sell a part of shares of a company to the public. There are at least three common phenomena in initial public offerings: IPO underpricing which leads to excess positive returns in the short run, strong concentration of IPO activity in certain period, and underperformance of IPO shares in the long run [1]. During an IPO, the phenomenon of underpricing occurs if the price of a security received by the issuer in a primary market is lower than the price of the same security in the secondary market.

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Many researchers have studied IPO underpricing in different countries. The first empirical evidence of underpricing in the market of IPOs dates back to a study by the U.S. Securities and Exchange Commission (SEC) in 1963. Lowry and Murphy [9] analyzed the IPO market in the United States between 1996 and 2000 and found that in about one-third of the IPOs, the executives received stock options with an exercise price equal to the IPO price rather than the market price. Borges [3] analyzed the underpricing phenomenon in Portugal. He separately examined 57 IPOs in the 'hot issue' market of 1987 and 41 IPOs in the 'hot issue' market between 1988 and 2004. Chang et. al [4] have shown that in Chinese class A-share IPO initial returns, the initial abnormal return in the secondary market was significantly positive.

According to Ljungqvist [8], theories explaining why IPO underpricing occurs can be grouped under four broad headings: Asymmetric information, institutional reasons, control considerations, and behavioral approaches. Among them, the best established group of theories is the asymmetric information based models. The major actors of an IPO transaction are the issuing firm, the bank underwriters and investors. Asymmetric information models assume that some of these parties have more information than the others. Baron [2] assumes that the bank is better informed than the issuer about demand conditions leading to a principal-agent problem in which underpricing is used to promote optimal selling effort. Perhaps winner's curse model due to Rock [11] is the best-known asymmetric information model. Rock assumes that some investors have more information about the value of the offered shares than other investors, the issuing firm, or its underwriting bank. Unlike general investors, these informed investors have the advantage to bid only for attractive shares which impose a "winner's curse" on uninformed investors. If we assume that unattractive shares are offered at prices higher than their fair values, then the average return of uninformed investors will be lower than their expectation and they will be unwilling to participate in IPO allocations, and so the IPO market will be abandoned by them. Rock assumes that the population of informed investors is insufficient to acquire even attractive shares. Thus, the issuers must offer their shares in such a price that the expected return of uninformed investors becomes non-negative. In other words, to avoid the market failure, all IPOs must be underpriced in expectation. Of course, the attractive shares will be acquired more by the informed investors while uninformed ones will have an acceptable return. Then, because of the involuntary costs of underpricing to the issuer, this agent has incentives to reduce the information asymmetry.

Here, we employ a kind of screening mechanism to deal with the informational asymmetry phenomenon. In the market of IPOs, there are financial institutions which clarify the real quality of shares by acquiring and analyzing the necessary information about them. According to Stiglitz [14], this activity is named "screening" and the agents who perform it are named "screening agents" (SAs). Revealing the results of this clarification for all engaged actors can reduce the information asymmetry across them and also the consequent underpricing. On the other hand, because the fair values of the shares are not known a priori, and revealing these values is a costly process for these agencies, they may be tempted to bypass the necessary, needed investigation.

In other words, they may behave dishonestly and declare an imprecise estimation about the quality of shares while claiming that the screening process has been thoroughly completed. We name this kind of behavior untruthfulness. To prevent this moral hazard, the practice of such agencies is usually evaluated and their truthfulness is judged. Even though this evaluation is not surely reliable, but, their incentives will be determined according to this judgment. According to Millon and Thakor [10], screening agents also may form a group of screening agents (IGA) to reduce the costs of screening by sharing their information. They developed a mathematical model to alleviate the moral hazard problem in a two-agent IGA. Here, we extended this idea to establish the conditions under which the IGA with *n*-agents is formed and the truthful behavior of every individual SA is ensured.

2. Screening Trade-off Models
2.1. The Basics of Screening Trade-off

Consider a firm that wishes to sell new shares to the public. Let Ω denote the fair value of one unit of the new shares. We assume that Ω depends on some firm variable, *d*, and some market variable, *w*, which respectively denote the characteristics of the firm and its industry market. This dependency may be explained by some function, *g(d,w)*, which is known as a priori to all. That is,

$$\Omega = g(d, w).$$

The screening agents (SA's) should analyze inside and outside of the firm and clarify a priori unknowns *d* and *w* which can be acquired by investing in information gathering. Assume that enough SA's are available to perform this process and $\overline{B}$ is the minimum expected utility for an SA to perform it. In the spirit of asset pricing, an SA may face pieces of information from outside (outsider signals) that makes *d* or *w* known without imposing any noticeable cost. These signals may be received by the SA at any time during the process of screening. Here, we focus on two special cases: Receiving the signals before starting the screening process and after complete it. If we assume receiving the signals before the process, the SA makes a decision about screening the firm and market in accordance with the information received. Otherwise, if the SA receives the signals after the process, it may decide not to screen the firm and market and hopes to receive the necessary information later. In this case, if the signal doesnot contain desired information, the behavior of the SA is not truthful. Therefore, a truthful mechanism must ensure the following issues:

1. Under the first assumption, the SA should clarify *d* and *w* merely if the signals do not provide enough information about them.

2. Under the second assumption, the SA must clarify both *d* and *w* to ensure clarification of shares.

Let *r* and *s* be the decision variables of an SA. The SA can acquire the necessary information about the firm and its market by choosing *r* and *s* from the compact sets of possible behaviors, *R* and *S*. Thus, *R* and *S* are the sets of possible decisions which can be taken by the SA about clarifying the firm and its market. Considering the

compactness of *R* and *S*, we take $R \equiv [0,1]$ and $S \equiv [0,1]$. With these considerations, $r = s = 1$ represents complete screenings of firm and market while *r* and $s < 1$ indicate imprecise screenings. Let ϕ be a compensation function by which some incentives is assigned to the SA in accordance with its behavior and Φ be the support of ϕ.

Now, assume *Von-Neumann-Morgenstern* utility function for the SA,

$$U : \Phi \rightarrow \Re,$$

where $\Re$ is the set of real numbers and *U* is a bounded concave increasing function. Then, we determine a convex increasing function $h(\cdot) = U^{-1}(\cdot)$. Moreover, each SA has a regret function *W* from *R* and *S* to $\Re$; i.e,

$$W : R \times S \rightarrow \Re.$$

Here, we assume *W* to be a symmetric function, *W*(*r, s*) =*W*(*s, r*), with *W*(*0, 0*) =0. Then, the net utility function for an SA is:

$$NU(\phi, r, s) = U(\phi) - W(r, s) \quad W(.,.) > 0.$$

Our objective is to define the compensation function ϕ such that the SA voluntarily decides to behave truthfully. In other words, every SA chooses *r=s=1* if the positive outsider signals are not received. It is obvious that ϕ cannot be based on whatever declared by the SA, because it would be tempted to misrepresent the values of *d* and *w*. Thus, we must depend ϕ only on the values of *r* and *s*. Also, because only the SA is aware of the real values of *r* and *s*, and it is inclined to choose *r=s=0* and declare any arbitrary values from the support of *d* and *w*, the screening process is faced with a moral hazard problem, [12], [13], [5], [6] and [7]. To confront this moral hazard, ϕ can be determined on the basis of a posterior evaluation function *T*. This evaluation function is:
$$T : R \times S \times V \times V^{'} \rightarrow [0 \quad 1].$$

T measures the SA efforts to clarify *d* and *w*. The sets *V* and *V'* are the state spaces of Bernoulli distributed random variables *v* and *v'* which respectively indicate the information received about firm and market via outsider signals. In other words, *V=V'={positive, negative}*, where and the *positive* value for *v* or *v'* indicates receiving desired information about firm or market, vice versa for the *negative* value.
Assume that the probability that an SA receives desired information about firm and market are γ and λ, respectively. It means that:

Prob{(*v, v'*) = (*pos*[3], *pos*)} = Prob{*v= pos, v'= pos*} = $\gamma\lambda$.

Now, we can define the compensation function which determines the incentives of the SA as:

$$\phi(T) = \begin{cases} X & if \ T = 1 \\ Y & if \ T = 0, \end{cases}$$

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3. pos represents positive

where *X* and *Y* are the corresponding amount of money given to the SA according to evaluation of its efforts. Assuming that the values *X* and *Y* result in utilities *x* and *y* we have $U(X) = x$ and $U(Y) = y$.

Consider the probability function $f_{v,v'}(r,s)$ which denotes the probability that the SA is judged to have behaved truthfully:

$$f_{v,v'}(r,s) = \text{Prob}(T = 1 | s,r,v,v').$$

Notice that a truthful behavior is acquiring precise information about shares, regardless of being gathered from a complete screening or from the random outsider signals. It is logical to assume *f* to be an increasing function of *s* and *r*.

We also define the errors of type I and II, which refer to the evaluation of SA's efforts:

Type I error (α): The probability that the SA is not believed to have behaved truthfully, whereas the situation has been clarified.

Type II error ($\beta_{r,s}$): The probability that the SA is deemed to have behaved truthfully, whereas it has not done so.

Without loss of generality[4], assume that there exist real values $0 = \theta_1 < \theta_2 < ... < \theta_n = 1$ such that for all values of (r,s) such that $\theta_i \leq r < \theta_{i+1}$ and $\theta_j \leq s < \theta_{j+1}$, $f(r,s) = \beta_{ij}$, where $i = 1, 2, ..., n-1$ and $j = 1, 2, ..., n-1$. Also, $f(\theta_n, s) = \beta_{nj}$ and $f(r, \theta_n) = \beta_{in}$. Then, *f* is a discrete function, where for every $i < \bar{i}$ and $j < \bar{j}$, $\beta_{ij} < \beta_{\bar{i}j}$ and $\beta_{ij} < \beta_{i\bar{j}}$.

Lemma 1: The optimal policy for an SA is obtained by choosing $r^* = \theta_i$ and $s^* = \theta_j$, where $i, j \in \{1, 2, ..., n\}$.

Proof: Consider $\theta_i < r^* < \theta_{i+1}$. Then,

$$U(r^*, s^*) = NU(r^*, s^*) - W(r^*, s^*) = \beta_{ij} x - (1 - \beta_{ij}) y - W(r^*, s^*) \tag{1}$$

$$U(\theta_i, s^*) = NU(\theta_i, s^*) - W(\theta_i, s^*) = \beta_{ij} x - (1 - \beta_{ij}) y - W(\theta_i, s^*) \tag{2}$$

Because *W* is increasing, $W(\theta_i, s) < W(r^*, s)$. Then, from (1) and (2) we get,

$$U(r^*, s) < U(\theta_i, s).$$

In a similar way, it can be proved that $U(r, s^*) < U(r, \theta_i)$. Then, the optimal policy for an SA is obtained by choosing $(r^*, s^*) = (\theta_i, \theta_j)$, where $i, j \in \{1, ..., n\}$.

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4. The generality of our discussion is not lost because *n* can be any big number.

For simplicity, assume $0 = \theta_1 < \theta_2 = 1$. Then, the optimal policy for the SA is obtained by choosing $r, s \in \{0, 1\}$. Also, assume $\beta_{1,0} = \beta_{0,1} = \beta_{0,0}$ and let β denote the corresponding value. Now, we can rewrite two types of errors as follows:

$$P(T = 1 | \text{Shares are clarified}) = 1 - \alpha. \quad \text{Let } \bar{\alpha} = 1 - \alpha.$$
$$P(T = 0 | \text{Shares are not clarified}) = 1 - \beta.$$

In other words,

$$P(T = 1 | r = s = 1) = P(T = 1 | r < 1, s = 1, v = pos) = P(T = 1 | r = 1, s < 1, v' = pos) =$$
$$P(T = 1 | r, s < 1, v = v' = pos) = 1 - \alpha = \bar{\alpha}, \text{ and}$$
$$P(T = 0 | r = s = 1) = P(T = 0 | r < 1, s = 1, v = pos) = P(T = 0 | r = 1, s < 1, v' = pos) =$$
$$P(T = 0 | r, s < 1, v = v' = pos) = 1 - \beta,$$

where $\alpha \in [0.5, 1)$ and $\beta \in (0, 0.5)$.

Next, we develop a convex nonlinear model to choose proper values for X and Y ensuring truthful behavior of SAs, while their total payment is minimized. This model has two linear constraints: The Incentive Compatibility (IC) constraint which ensures the truthful behavior of SAs and the Individual Rationality (IR) constraint which ensures that the expected utility for any SA to be at least $\bar{B}$, the minimum expected utility for any SA.

2.2. Models for Screening Trade-off after Receiving the Signals
Here, we restrict our attention to the case of receiving outsider signals before r and s are chosen by the SA.

2.2.1. A single screening agent
This is the case in which a single SA is employed to clarify a special share in an IPO. We assume that the signals are Bernoulli random variables with parameters being $0 \le \gamma, \lambda \le 1$. Then, we first study the case of $\gamma \ne 1, \lambda \ne 1$.

The second case is when $\gamma \ne 1, \lambda = 1$. In this case, it is assured that the market will be clarified. Assessing other situations does not lead to valuable theoretical insights and will not be discussed here.

2.2.1.1. Case $\gamma \ne 1, \lambda \ne 1$
According to the IC constraint, the SA should find it optimal to acquire precise information about both firm and market. Then, if the signals clarify only the firm factor, clarification of market must be the best behavior:

$$\bar{\alpha}x + (1-\bar{\alpha})y - W(0,1) \geq \beta x + (1-\beta)y - W(0,0). \tag{3}$$

Also, clarification of the firm must be the best decision if the signals clarify only the market:

$$\bar{\alpha}x + (1-\bar{\alpha})y - W(1,0) \geq \beta x + (1-\beta)y - W(0,0). \tag{4}$$

And finally, if no positive signal is received, clarification of both items must be better than performing any other task:

$$\bar{\alpha}x + (1-\bar{\alpha})y - W(1,1) \geq \beta x + (1-\beta)y - W(0,0) \tag{5-a}$$

$$\bar{\alpha}x + (1-\bar{\alpha})y - W(1,1) \geq \beta x + (1-\beta)y - W(1,0) \tag{5-b}$$

$$\bar{\alpha}x + (1-\bar{\alpha})y - W(1,1) \geq \beta x + (1-\beta)y - W(0,1). \tag{5-c}$$

All the constraints (3) to (5) can be replaced by:

$$(x-y)(\bar{\alpha}-\beta) \geq W(1,1) - W(0,0). \tag{6}$$

According to the IR constraint, the SA must find it rational to perform the screening task. Because the utility of this process is uncertain, the expected utility should be at least equal to $\bar{B}$:

$$\bar{\alpha}x + (1-\bar{\alpha})y - [(\gamma + \lambda - 2\gamma\lambda)W(1,0) + (1-\gamma)(1-\lambda)W(1,1)] \geq \bar{B}. \tag{7}$$

Considering $h(x)=X$ and $h(y)=Y$, the model of screening trade-off is given by:

$$(PI): Min \quad \bar{\alpha}h(x) + (1-\bar{\alpha})h(y)$$

$$s.t. \quad \text{Constraints (6) and (7)}.$$

Lemma 2: Suppose w is a priori unknown. The optimal incentives which must be offered to an SA are:

$$x^* = \bar{B} + W(1,0)(\gamma + \lambda - 2\gamma\lambda) + \frac{[(1-\bar{\alpha}) + (1-\gamma)(1-\lambda)(\bar{\alpha}-\beta)]W(1,1)}{(\bar{\alpha}-\beta)}$$

$$y^* = \bar{B} + W(1,0)(\gamma + \lambda - 2\gamma\lambda) + \frac{[-\bar{\alpha} + (1-\gamma)(1-\lambda)(\bar{\alpha}-\beta)]W(1,1)}{(\bar{\alpha}-\beta)}.$$

Proof: The optimal solution of this problem must satisfy Karush-Kuhn-Tucker (KKT) first-order necessary conditions:

$$\bar{\alpha}h'(x) - \bar{\alpha}\mu_1 - (\bar{\alpha}-\beta)\mu_2 = 0 \tag{8}$$

$$(1-\bar{\alpha})h'(y) - (1-\bar{\alpha})\mu_1 + (\bar{\alpha}-\beta)\mu_2 = 0 \tag{9}$$

$$\mu_1 (\bar{\alpha}x + (1-\bar{\alpha})y - [(\gamma + \lambda - 2\gamma\lambda)W(1,0) + (1-\gamma)(1-\lambda)W(1,1)] - \bar{B}) = 0 \quad (10)$$

$$\mu_2 [(x - y)(\bar{\alpha} - \beta) - W(1,1)] = 0 \quad\quad\quad (11)$$

$$\mu_1 \geq 0 \quad\quad\quad (12)$$

$$\mu_2 \geq 0 \quad\quad\quad (13)$$

Constraints (6) and (7).

Adding (8) and (9), we get $\mu_1 = \bar{\alpha}h'(x) + (1-\bar{\alpha})h'(y)$. Since $\bar{\alpha} \in [0.5 \quad 1]$ and $h(.)$ is strictly increasing. It is concluded that $\mu_1 > 0$. Then, according to KKT conditions, we have:

$$\bar{\alpha}x + (1-\bar{\alpha})y = [(\gamma + \lambda - 2\gamma\lambda)W(1,0) + (1-\gamma)(1-\lambda)W(1,1)] + \bar{B}. \quad (14)$$

Also note that $\mu_2 = \dfrac{\bar{\alpha}(1-\bar{\alpha})(h'(x) - h'(y))}{\bar{\alpha} - \beta}$ will be positive due to the convexity of $h(.)$. Thus,

$$(x - y)(\bar{\alpha} - \beta) = W(1,1) \quad\quad\quad (15)$$

By solving (14) and (15) we get:

$$x^* = \bar{B} + W(1,0)(\gamma + \lambda - 2\gamma\lambda) + \frac{[(1-\bar{\alpha}) + (1-\gamma)(1-\lambda)(\bar{\alpha} - \beta)]W(1,1)}{(\bar{\alpha} - \beta)}$$

$$y^* = \bar{B} + W(1,0)(\gamma + \lambda - 2\gamma\lambda) + \frac{[-\bar{\alpha} + (1-\gamma)(1-\lambda)(\bar{\alpha} - \beta)]W(1,1)}{(\bar{\alpha} - \beta)}.$$

The proof is now complete. $\square$

2.2.1.2. Case $\gamma \neq 1, \lambda = 1$

In this case, the SA is assured that sufficient information about the market will be received. Then, (PI) can be simplified as follows:

$$(PII): Min \quad \bar{\alpha}h(x) + (1-\bar{\alpha})h(y)$$

s.t.

$$\bar{\alpha}x + (1-\bar{\alpha})y \geq \bar{B} + (1-\gamma)W(1,0)$$

$$(x - y)(\bar{\alpha} - \beta) \geq W(1,0).$$

Lemma 3: Suppose w is a priori known. Then, the optimal incentives which must be offered to an SA are:

$$x^{**} = \bar{B} + \frac{[(1-\bar{\alpha})+(1-\gamma)(\bar{\alpha}-\beta)]W(1,0)}{(\bar{\alpha}-\beta)}$$

$$y^{**} = \bar{B} + \frac{[-\bar{\alpha}+(1-\gamma)(\bar{\alpha}-\beta)]W(1,0)}{(\bar{\alpha}-\beta)}.$$

Proof: Similar to the proof of Lemma 2.

2.2.2. Analysis of IGA Formation

Now, suppose n identical SAs form an IGA. Each screens one distinct firm in the same market. Also, SAs perform independently, but all will share their information and finally pool their payoffs together and share it equally. Inherently, agents have the tendency to join together and make a formation because they can profit from sharing their information about the same market to be screened; i.e., if at least one of them receives some positive signal about a market, then the market factor for all offered shares in an special market is clarified. On the other hand, each agent may be tempted not to spend any time for clarifying the market and just use the information shared by others. In what follows, we develop a convex nonlinear mathematical model to determine the incentives of the SA s such that truthfulness of their optimal behavior is assured.

Because the payoffs will be shared equally, the compensation of each SA will be as follows:

$$\phi(T) = \begin{cases} X & \text{if } T=1 \text{ for all agents} \\ Z_{n-1} = \dfrac{(n-1)X+Y}{n} & \text{if } T=1 \text{ for only } (n-1) \text{ agents} \\ \dots \\ Z_i = \dfrac{iX+(n-i)Y}{n} & \text{if } T=1 \text{ for only } i \text{ agents} \\ \dots \\ Y & \text{if } T=0 \text{ for all agents.} \end{cases}$$

Also, the probabilities of receiving positive outsider signals and the cost of clarifying shares are shown in Table 1.

Table 1. States of outsider signals

Cost of clarification	Probability	State of outsider signals
$W(1,\dfrac{1}{n})$	$(1-\gamma)(1-\lambda)^n$	Neither firm nor market is clarified
$W(0,\dfrac{1}{n})$	$\gamma(1-\lambda)^n$	Only the firm is clarified
$W(1,0)$	$(1-\gamma)(1-(1-\lambda)^n)$	Only the market is clarified
$W(0,0)$	$\gamma(1-(1-\lambda)^n)$	Both firm and market are clarified

2.2.2.1. Case $\gamma \neq 1, \lambda \neq 1$

Assuming the truthful behavior of all agents being assured by IC constraint, the probability that each SA receive payoffs equal to X, Z_i and Y, respectively are:

$$\overline{\alpha}^n, \binom{n}{i} \overline{\alpha}^i (1-\overline{\alpha})^{n-i} \text{ and } (1-\overline{\alpha})^n.$$

Then, the objective function for the minimization problem is:

$$Min \quad \overline{\alpha}^n h(x) + \sum_{i=1}^{n-1} \binom{n}{i} \overline{\alpha}^i (1-\overline{\alpha})^{n-i} h(z_{n-i}) + (1-\overline{\alpha})^n h(y).$$

The process may be analyzed using Game Theory. According to the Nash theorem, this process has two Nash equilibriums [10]: (1) All SA s behave untruthfully, and (2) all SA s behave truthfully. The IC constraint must force the game to second equilibrium. This aim is achieved by the following constraints which respectively ensure the truthful behavior of any SA whether other SA s behave truthfully, as (16) below, or not, as (17) below:

$$\overline{\alpha}^{n-1}(\alpha-\beta)x + \sum_{i=1}^{n-1} \overline{\alpha}^{n-i-1}(1-\overline{\alpha})^{i-1}\left[\binom{n-1}{i}(1-\overline{\alpha}) - \binom{n-1}{i-1}\overline{\alpha}\right](\overline{\alpha}-\beta)z_{n-i} - (1-\overline{\alpha})^{n-1}(\overline{\alpha}-\beta)y \geq W(1,\frac{1}{n})$$

(16)

$$\beta^{n-1}(\overline{\alpha}-\beta)x + \sum_{i=1}^{n-1} \beta^{n-i-1}(1-\beta)^{i-1}\left[\binom{n-1}{i}(1-\beta) - \binom{n-1}{i-1}\beta\right](\overline{\alpha}-\beta)z_{n-i} - (1-\overline{\beta})^{n-1}(\overline{\alpha}-\beta)y \geq W(1,\frac{1}{n})$$

(17)

And the IR constraint is:

$$\overline{\alpha}^n x + \sum_{i=1}^{n-1} \binom{n}{i} \overline{\alpha}^{n-i} (1-\overline{\alpha})^i z_{n-i} + (1-\overline{\alpha})^n y \geq \tau_1, \quad\quad (18)$$

where,

$$\tau_1 = \overline{B} + (1-\gamma)(1-\lambda)^n W(1,\frac{1}{n}) + \gamma(1-\lambda)^n W(0,\frac{1}{n}) + (1-\gamma)(1-(1-\lambda)^n) W(1,0).$$

Lemma 4: The IC constraint (17) is a redundant constraint.

Proof: we use induction. To simplify the proof, define t_i where $t_0 = y$, $t_i = z_i$, for $i = 1,...,n-1$, and $t_n = x$.

First, consider the case of a two-agent IGA:

$$h(t_1) = \frac{h(t_2) + h(t_0)}{2}.$$ Considering the convexity of $h(.)$:

$$t_1 - t_0 \geq t_2 - t_1.$$

Since $\overline{\alpha} \geq \beta$, Then,

$$\beta(t_2 - t_1) + (1 - \beta)(t_1 - t_0) \ge \overline{\alpha}(t_2 - t_1) + (1 - \overline{\alpha})(t_1 - t_0) \ ,$$

and then ,

$$(\overline{\alpha} - \beta)[\beta(t_2 - t_1) + (1 - \beta)(t_1 - t_0)] \ge (\overline{\alpha} - \beta)[\overline{\alpha}(t_2 - t_1) + (1 - \overline{\alpha})(t_1 - t_0)].$$

Now, suppose that:

$$\sum_{i=0}^{n-1} \binom{n-1}{i} \beta^{n-i-1} (1-\beta)^i (t_{n-i} - t_{n-i-1}) \ge \sum_{i=0}^{n-1} \binom{n-1}{i} \overline{\alpha}^{(n-i-1)} (1-\overline{\alpha})^i (t_{n-i} - t_{n-i-1}) . \qquad (19)$$

Since the right hand sides of (16) and (17) are equal, Then we must show that the left hand side of (17) is no less than that of (16); i.e,

$$\sum_{i=0}^{n} \binom{n}{i} \beta^{n-i} (1-\beta)^i (t_{n-i+1} - t_{n-i}) \ge \sum_{i=0}^{n} \binom{n}{i} \overline{\alpha}^{(n-i)} (1-\overline{\alpha})^i (t_{n-i+1} - t_{n-i}) . \qquad (20)$$

According to (5), the following inequalities are satisfied:

$$D_\beta = \sum_{i=0}^{n-1} \binom{n-1}{i} \beta^{n-i-1} (1-\beta)^i (t_{n-i} - t_{n-i-1}) \ge \overline{D}_\alpha = \sum_{i=0}^{n-1} \binom{n-1}{i} \overline{\alpha}^{(n-i-1)} (1-\overline{\alpha})^i (t_{n-i} - t_{n-i-1})$$

$$\overline{D}_\beta = \sum_{i=0}^{n-1} \binom{n-1}{i} \beta^{n-i-1} (1-\beta)^i (t_{n-i+1} - t_{n-i}) \ge D_\alpha = \sum_{i=0}^{n-1} \binom{n-1}{i} \overline{\alpha}^{(n-i-1)} (1-\overline{\alpha})^i (t_{n-i+1} - t_{n-i})$$

Considering the inequalities $(t_{n-i+1} - t_{n-i}) \ge (t_{n-i} - t_{n-i-1})$, $D \ge \overline{D}$ and $\alpha \ge \beta$, we have:

$$\beta(\overline{D}_\beta - \overline{D}_\alpha) \ge 0 \qquad\qquad\qquad (21)$$

$$(1 - \beta)(D_\beta - D_\alpha) \ge 0 \qquad\qquad\qquad (22)$$

$$(\alpha - \beta)(D_\alpha - \overline{D}_\alpha) \ge 0 \qquad\qquad\qquad (23)$$

(21) to (23) lead to: $\quad \beta \overline{D}_\beta - (1-\beta)D_\beta \ge \alpha \overline{D}_\alpha + (1-\alpha)D_\alpha$, which is equal to inequality (20), and the proof is complete.$\square$

Thus, the screening trade-off model is reformulated as follows:

$$(PIII): Min \quad \overline{\alpha}^n h(x) + \sum_{i=1}^{n-1} \binom{n}{i} \overline{\alpha}^i (1-\overline{\alpha})^{n-i} h(z_{n-i}) + (1-\overline{\alpha})^n h(y)$$

s.t.

   constraints (16) and (18).

Lemma 5: Suppose w is a priori unknown to all and the SAs perform independently. Then, the optimal incentives which must be offered to each SA are:

$$\bar{x}^* = \frac{(1-\bar{\alpha})\left[W(1,\frac{1}{n})-J_1\right]+(\bar{\alpha}-\beta)(\tau_1-J_2)}{\bar{\alpha}^{n-1}(\bar{\alpha}-\beta)}$$

$$\bar{y}^* = \frac{(\bar{\alpha}-\beta)(\tau_1-J_2)-\bar{\alpha}(W(1,\frac{1}{n})-J_1)}{(1-\bar{\alpha})^{n-1}(\bar{\alpha}-\beta)},$$

where,

$$J_1 = \sum_{i=1}^{n-1}\bar{\alpha}^{n-i-1}(1-\bar{\alpha})^{i-1}\left[\binom{n-1}{i}(1-\bar{\alpha})-\binom{n-1}{i-1}\bar{\alpha}\right](\bar{\alpha}-\beta)z_{n-i}$$

$$J_2 = \sum_{i=1}^{n-1}\binom{n}{i}\bar{\alpha}^{n-i}(1-\bar{\alpha})^i z_{n-i}.$$

Proof: According to KKT first-order necessary condition,

$\bar{\alpha}^n h'(x)-\bar{\alpha}^{n-1}(\bar{\alpha}-\beta)\mu_1-\bar{\alpha}^n\mu_2=0$, and $(1-\bar{\alpha})^n h'(y)+(1-\bar{\alpha})^{n-1}(\bar{\alpha}-\beta)\mu_1-(1-\bar{\alpha})^n\mu_2=0$.

These equalities yield:

$\bar{\alpha}h'(x)-(\bar{\alpha}-\beta)\mu_1-\bar{\alpha}\mu_2=0$, and $(1-\bar{\alpha})h'(y)+(\bar{\alpha}-\beta)\mu_1-(1-\bar{\alpha})\mu_2=0$.

By solving these equations, we get:

$$h'(x)=\frac{(\bar{\alpha}-\beta)}{\bar{\alpha}}\mu_1+\mu_2,$$

$$h'(y)=-\frac{(\bar{\alpha}-\beta)}{(1-\bar{\alpha})}\mu_1+\mu_2.$$

Thus, $\mu_2>0$.

Also, since $x>y$ and considering the convexity of $h(.)$, we have:

$$(\bar{\alpha}-\beta)(\frac{1}{\bar{\alpha}}+\frac{1}{1-\bar{\alpha}})\mu_1>0.$$

Thus, $\mu_1>0$, and the optimal solution can be found similar to the case in Lemma 2.$\square$

2.2.2.2. Case $\gamma\neq1, \lambda=1$

Under this assumption, the screening model can be reformulated as:

(PIV): $Min \quad \bar{\alpha}^n h(x)+\sum_{i=1}^{n-1}\binom{n}{i}\bar{\alpha}^i(1-\bar{\alpha})^{n-i}h(z_{n-i})+(1-\bar{\alpha})^n h(y)$

s.t.

$$\bar{\alpha}^{n-1}(\alpha - \beta)x + \sum_{i=1}^{n-1}\bar{\alpha}^{n-i-1}(1-\bar{\alpha})^{i-1}\left[\binom{n-1}{i}(1-\bar{\alpha}) - \binom{n-1}{i-1}\bar{\alpha}\right](\bar{\alpha}-\beta)z_{n-i}$$

$$-(1-\bar{\alpha})^{n-1}(\bar{\alpha}-\beta)y \geq W(1,0)$$

$$\bar{\alpha}^n x + \sum_{i=1}^{n-1}\binom{n}{i}\bar{\alpha}^i(1-\bar{\alpha})^{n-i}z_{n-i} + (1-\bar{\alpha})^n y \geq \bar{B} + (1-\gamma)W(1,0)\,.$$

Lemma 6: Suppose w is a priori known to all and SAs perform independently. The optimal incentives which must be offered to each SA are:

$$\bar{x}^{**} = \frac{(1-\bar{\alpha})\left[W(1,0) - J_1\right] + (\bar{\alpha}-\beta)(\bar{B} + (1-\gamma)W(1,0) - J_2)}{\bar{\alpha}^{n-1}(\bar{\alpha}-\beta)}$$

$$\bar{y}^{**} = \frac{(\bar{\alpha}-\beta)(\bar{B} + (1-\gamma)W(1,0) - J_2) - \bar{\alpha}(W(1,0) - J_1)}{(1-\bar{\alpha})^{n-1}(\bar{\alpha}-\beta)}\,.$$

Proof: Similar to the proof of Lemma 5.□

To continue, we compare the cooperation of SAs under different assumptions about γ and λ.

Theorem 1: Suppose w is a priori known to all and there are n SAs forming an IGA. The expected screening cost for contracting with every SA from this IGA is always higher than the expected screening cost it would incur by transacting with any single SA.

Proof: It must be shown that the optimal value of ($PIII$) is greater than that of (PI). The optimal value of ($PIII$) is:

$$\overline{\alpha}^n h(\overline{x}^{**}) + \sum_{i=1}^{n-1} \binom{n}{i} \overline{\alpha}^{(n-i)}(1-\overline{\alpha})^i h(\overline{z}^{**}_{n-i}) + (1-\overline{\alpha})^n h(\overline{y}^{**}) =$$

$$\overline{\alpha}(\overline{\alpha}^{(n-1)} h(\overline{x}^{**}) + \sum_{i=0}^{n-1} \binom{n-1}{i} \overline{\alpha}^{(n-i-1)}(1-\overline{\alpha})^i h(\overline{z}^{**}_{n-i})) +$$

$$(1-\overline{\alpha})(\sum_{i=1}^{n} \binom{n-1}{i-1} \overline{\alpha}^{(n-i)}(1-\overline{\alpha})^{i-1} h(\overline{z}^{**}_{n-i}) + (1-\overline{\alpha})^{n-1} h(\overline{y}^{**})) >$$

$$\overline{\alpha} h(\overline{\alpha}^{(n-1)} \overline{x}^{**} + \sum_{i=1}^{n-1} \binom{n-1}{i} \overline{\alpha}^{(n-i-1)}(1-\overline{\alpha})^i \overline{z}^{**}_{n-i}) +$$

$$(1-\overline{\alpha}) h(\sum_{i=1}^{n} \binom{n-1}{i-1} \overline{\alpha}^{(n-i)}(1-\overline{\alpha})^{i-1} \overline{z}^{**}_{n-i} + (1-\overline{\alpha})^{n-1} \overline{y}^{**})$$

$$= \overline{\alpha} h(\overline{x}^*) + (1-\overline{\alpha}) h(\overline{y}^*)$$

which is equal to the optimal value of (PI).

Then, the proof is complete.$\square$

Theorem 2: Suppose w is a priori unknown to all and there are n SAs forming an IGA. The expected screening cost for contracting with every SA from this IGA is lower than the expected cost it would incur by transacting with any single SA if $(1-\alpha)W(1,1) \ge W(1,0.5)$.

Proof: The optimal value for the non-IGA case in (PII) is (x^*, y^*). It must be shown that if $Z_i = (iX_i + (n-i)Y_i)/2$, then, under the stated conditions, $(x^*, z_1^*, ..., z_{n-1}^*, y^*)$ is a feasible solution to (PIV).

Considering the convexity of $h(.)$,

$$\overline{\alpha}^n x^* + \sum_{i=1}^{n-1} \binom{n}{i} \overline{\alpha}^{(n-i)}(1-\overline{\alpha})^i z^*_{n-i} + (1-\overline{\alpha})^n y^* \ge \overline{\alpha}^n x^* + \sum_{i=1}^{n-1} \binom{n}{i} \overline{\alpha}^{(n-i)}(1-\overline{\alpha})^i \frac{(ix^* + (n-i)y^*)}{n} + (1-\overline{\alpha})^n y^*$$

$$= \overline{\alpha} x^* + (1-\overline{\alpha}) y^* = \tau_2$$

where,

$$\tau_2 = \overline{B} + (\gamma + \lambda - 2\gamma\lambda)W(1,0) + (1-\gamma)(1-\lambda)W(1,1).$$

Now, it must be shown that $\tau_2 \ge \tau_1$.

Knowing that τ_1 and τ_2 are functions of γ, consider $\tau_2 - \tau_1 = f(\gamma)$, where $0 \le \gamma \le 1$:

$$(\tau_2 - \tau_1)\big|_{\gamma=0} = f(\gamma)\big|_{\gamma=0} = \lambda W(1,0) + (1-\lambda)W(1,1) - (1-\lambda)^n W(1,\frac{1}{n}) - (1-(1-\lambda)^n)W(1,0) >$$

$$-(1-\lambda)W(1,0) + (1-\lambda)W(1,\frac{1}{n}) - (1-\lambda)^n W(1,\frac{1}{n}) + (1-\lambda)^n W(1,0) =$$

$$(1-\lambda)(1-(1-\lambda)^{n-1})(W(1,\frac{1}{n}) - W(1,0)) > 0.$$

Also,

$$(\tau_2 - \tau_1)\big|_{\gamma=1} = f(\gamma)\big|_{\gamma=1} = (1-\lambda)W(1,0) - (1-\lambda)^n W(0,\frac{1}{n}) >$$

$$(1-\lambda)[W(1,0) - W(\frac{1}{n},0)) > 0.$$

And because f is a linear function of γ, then,

$$\forall \gamma_{0<\gamma<1} \ \exists \zeta_{0\leq\zeta\leq1} : \tau_2 - \tau_1 = \zeta f(0) + (1-\zeta)f(1) > 0 \Rightarrow \tau_2 > \tau_1.$$

Thus far, we have shown that $(x^*, z_1^*, ..., z_{n-1}^*, y^*)$ satisfies the IR constraint of $P(IV)$.

By replacing $\bar{\alpha}$ with $(1-\bar{\alpha})$ and z_{n-i} with z_i for all $i \leq \frac{n}{2}$, we have:

$$\bar{\alpha}^{(n-1)}(\bar{\alpha} - \beta)x^* + \sum_{i=1}^{n-1} \bar{\alpha}^{(n-i-1)}(1-\bar{\alpha})^{i-1}\left[\binom{n-1}{i}(1-\bar{\alpha}) - \binom{n-1}{i-1}\bar{\alpha}\right](\bar{\alpha} - \beta)z^*_{n-i} - (1-\bar{\alpha})^{n-1}(\bar{\alpha} - \beta)y^* \geq$$

$$(1-\bar{\alpha})^{n-1}(\bar{\alpha} - \beta)(x^* - y^*) = (1-\bar{\alpha})^{n-1}W(1,1)$$

Thus, $(1-\bar{\alpha})^{n-1}W(1,1) \geq W(1,\frac{1}{n})$ is a necessary condition for satisfying $(x^*, z_1^*, ..., z_{n-1}^*, y^*)$ in IC constraint of $P(IV)$. In other words, it is a necessary condition under which the formation of an IGA is rational. $\square$

2.3. Models for Screening Trade-off before Receiving the Signals

In the case of receiving signals after the screening process, an SA may be hopeful to acquire information from random signals and choose $r = 0$ or $s = 0$. But to ensure the clarification of offered shares, the SA must find it optimal to choose $r = s = 1$.

2.3.1. A single screening agent
2.3.1.1. Case $\gamma \neq 1, \lambda \neq 1$

In Table 2, different possible policies for an SA and the expected utility from each policy are shown.

Table 2. Expected utilities under possible policies

Possible policies	Expected utility
$r = s = 1$	$\bar{\alpha}x + (1-\bar{\alpha})y - W(1,1)$
$r = 1$ and $s = 0$	$\lambda[\bar{\alpha}x + (1-\bar{\alpha})y] + (1-\lambda)[\beta x + (1-\beta)y] - W(1,0)$
$r = 0$ and $s = 1$	$\gamma[\bar{\alpha}x + (1-\bar{\alpha})y] + (1-\gamma)[\beta x + (1-\beta)y] - W(0,1)$
$r = s = 0$	$\gamma\lambda[\bar{\alpha}x + (1-\bar{\alpha})y] + (1-\gamma\lambda)[\beta x + (1-\beta)y] - W(0,0)$

Due to the IC constraints (24) to (26) below, the SA finds the first policy, $r = s = 1$, better than the others:

$$(x - y)(1 - \lambda)(\bar{\alpha} - \beta) \geq W(1,1) - W(1,0) \qquad (24)$$
$$(x - y)(1 - \gamma)(\bar{\alpha} - \beta) \geq W(1,1) - W(0,1) \qquad (25)$$
$$(x - y)(1 - \gamma\lambda)(\bar{\alpha} - \beta) \geq W(1,1) - W(0,0) \quad . \qquad (26)$$

And the IR constraint is:

$$\bar{\alpha}x + (1-\bar{\alpha})y - W(1,1) \geq \bar{B} . \qquad (27)$$

So, the screening trade-off model is formulated as follows:

$$(PV): Min \quad \bar{\alpha}h(x) + (1-\bar{\alpha})h(y)$$
$$s.t.$$

Constraints (24) to (27).

Lemma 7: Suppose w is a priori unknown to all and the SAs perform independently. The optimal incentives which must be offered to an SA are:

$$x^* = \bar{B} + W(1,1) + (1-\bar{\alpha})\xi$$
$$y^* = \bar{B} + W(1,0.5) - \bar{\alpha}\xi.$$

If $\quad \dfrac{W(1,0)}{W(1,1)} \geq \max\{\dfrac{\gamma(1-\lambda)}{1-\gamma\lambda}, \dfrac{\lambda(1-\gamma)}{1-\gamma\lambda}\}$, then،

$$\xi = \frac{W(1,1)}{(\alpha - \beta)(1 - \gamma\lambda)}.$$

Otherwise, if $\lambda \geq \gamma$, then ,

$$\xi = \frac{W(1,1) - W(1,0)}{(\alpha - \beta)(1 - \lambda)} \quad ,$$

and if $\lambda < \gamma$ then ,

$$\xi = \frac{W(1,1) - W(0,1)}{(\alpha - \beta)(1 - \gamma)}.$$

Proof: The optimal solution must satisfy the KKT first-order necessary conditions, given by:

$$\bar{\alpha} h'(x) = (1-\lambda)(\bar{\alpha} - \beta)\mu_1 + (1-\gamma)(\bar{\alpha} - \beta)\mu_2 + (1-\gamma\lambda)(\bar{\alpha} - \beta)\mu_3 + \bar{\alpha}\mu_4 \qquad (28)$$

$$(1-\bar{\alpha}) h'(y) = -(1-\lambda)(\bar{\alpha} - \beta)\mu_1 - (1-\gamma)(\bar{\alpha} - \beta)\mu_2 - (1-\gamma\lambda)(\bar{\alpha} - \beta)\mu_3 + (1-\bar{\alpha})\mu_4 \quad (29)$$

From (28) and (29), it is concluded that:

$$\bar{\alpha} h'(x) + (1-\bar{\alpha}) h'(y) = \mu_4 \quad \text{and thus, } \mu_4 > 0.$$

Also, since

$$(1-\lambda)\mu_1 + (1-\gamma)\mu_2 + (1-\gamma\lambda)\mu_3 = \frac{\bar{\alpha}(1-\bar{\alpha})(h'(x) - h'(y))}{\bar{\alpha} - \beta} > 0, \qquad (30)$$

at least one of the Lagrangian multipliers μ_1, μ_2 or μ_3 must be positive, which can be specified based on the conditions of the lemma. $\square$

2.3.1.2. Case $\gamma \neq 1, \lambda = 1$

In this case, the SA will not clarify the market because it is assured that sufficient information will be received. The corresponding, the model is formulated as follows:

$$(PVI): Min \quad \bar{\alpha}h(x) + (1-\bar{\alpha})h(y)$$

s.t.

$$(x - y)(1 - \gamma)(\bar{\alpha} - \beta) \geq W(1,0)$$

$$\overline{\alpha}x + (1 - \overline{\alpha})y \geq \overline{B} + W(1,0).$$

Lemma 8: Suppose w is a priori known. Then, the optimal incentives which must be offered to an SA are:

$$x^{**} = \overline{B} + \frac{[(1-\overline{\alpha}) + (1-\gamma)(\overline{\alpha} - \beta)]W(1,0)}{(1-\gamma)(\overline{\alpha} - \beta)}$$

$$y^{**} = \overline{B} + \frac{[-\overline{\alpha} + (1-\gamma)(\overline{\alpha} - \beta)]W(1,0)}{(1-\gamma)(\overline{\alpha} - \beta)}.$$

Proof: Similar to the proof of Lemma 7. $\square$

2.3.2. Analysis of IGA formation

As before, there is a trade-off between reducing the cost of information acquiring and enhancing the dilemma of moral hazard.

In Table 3, the probabilities thot shares to be clarified by positive outsider signals are shown.

Table 3. States of outsider Signals

Cost of clarification	Probability of shares to be clarified	Possible policies
$W(1,\frac{1}{n})$	1	$r = s = 1$
$W(1,0)$	$1 - (1-\lambda)^n$	$r = 1$ and $s = 0$
$W(0,\frac{1}{n})$	γ	$r = 0$ and $s = 1$
$W(0,0)$	$\gamma(1 - (1-\lambda)^n)$	$r = s = 0$

2.3.2.1. Case $\gamma \neq 1, \lambda \neq 1$

According to the IC constraint, each SA must find the first policy as the best behavior. To assure this, choosing $r = s = 1$ must be preferred to other policies whether all other SAs behave truthfully, (31) - (33), or not, (34) - (36):

$$(1-\lambda)[\overline{\alpha}^{(n-1)}(\overline{\alpha} - \beta)x + \sum_{i=1}^{n-1} \overline{\alpha}^{(n-i-1)}(1-\overline{\alpha})^{i-1}\left[\binom{n-1}{i}(1-\overline{\alpha}) - \binom{n-1}{i-1}\overline{\alpha}\right](\overline{\alpha} - \beta)z_{n-i} \quad (31)$$

$$-(1-\overline{\alpha})^{n-1}(\overline{\alpha} - \beta)y] \geq W(1,\frac{1}{n}) - W(1,0)$$

$$(1-\gamma)[\bar{\alpha}^{(n-1)}(\bar{\alpha}-\beta)x + \sum_{i=1}^{n-1}\bar{\alpha}^{(n-i-1)}(1-\bar{\alpha})^{i-1}\left[\binom{n-1}{i}(1-\bar{\alpha})-\binom{n-1}{i-1}\bar{\alpha}\right](\bar{\alpha}-\beta)z_{n-i}$$

$$-(1-\bar{\alpha})^{n-1}(\bar{\alpha}-\beta)y] \geq W(1,\frac{1}{n})-W(0,\frac{1}{n}) \tag{32}$$

$$(1-\gamma\lambda)[\bar{\alpha}^{(n-1)}(\bar{\alpha}-\beta)x + \sum_{i=1}^{n-1}\bar{\alpha}^{(n-i-1)}(1-\bar{\alpha})^{i-1}\left[\binom{n-1}{i}(1-\bar{\alpha})-\binom{n-1}{i-1}\bar{\alpha}\right](\bar{\alpha}-\beta)z_{n-i}$$

$$-(1-\bar{\alpha})^{n-1}(\bar{\alpha}-\beta)y] \geq W(1,\frac{1}{n}) \tag{33}$$

$$(1-\lambda)[\beta^{n-1}(\bar{\alpha}-\beta)x + \sum_{i=1}^{n-1}\beta^{n-i-1}(1-\beta)^{i-1}\left[\binom{n-1}{i}(1-\beta)-\binom{n-1}{i-1}\beta\right](\bar{\alpha}-\beta)z_{n-i}$$

$$-(1-\bar{\beta})^{n-1}(\bar{\alpha}-\beta)y] \geq W(1,\frac{1}{n})-W(1,0) \tag{34}$$

$$(1-\gamma)[\beta^{n-1}(\bar{\alpha}-\beta)x + \sum_{i=1}^{n-1}\beta^{n-i-1}(1-\beta)^{i-1}\left[\binom{n-1}{i}(1-\beta)-\binom{n-1}{i-1}\beta\right](\bar{\alpha}-\beta)z_{n-i}$$

$$-(1-\bar{\beta})^{n-1}(\bar{\alpha}-\beta)y] \geq W(1,\frac{1}{n})-W(0,\frac{1}{n}) \tag{35}$$

$$(1-\gamma\lambda)[\beta^{n-1}(\bar{\alpha}-\beta)x + \sum_{i=1}^{n-1}\beta^{n-i-1}(1-\beta)^{i-1}\left[\binom{n-1}{i}(1-\beta)-\binom{n-1}{i-1}\beta\right](\bar{\alpha}-\beta)z_{n-i}$$

$$-(1-\bar{\beta})^{n-1}(\bar{\alpha}-\beta)y] \geq W(1,\frac{1}{n}) \tag{36}$$

And, the IR constraint is:

$$\bar{\alpha}^n x + \sum_{i=1}^{n-1}\binom{n}{i}\bar{\alpha}^{n-i}(1-\bar{\alpha})^i z_{n-i} + (1-\bar{\alpha})^n y \geq \bar{B} + W(1,\frac{1}{n}) \tag{37}$$

Lemma 9: The IC constraints (34) to (36) are redundant.
Proof: Similar to the proof of Lemma 4. □
Here the model can be written as follows:

$$(PVII): Min \quad \bar{\alpha}^n h(x) + \sum_{i=1}^{n-1}\binom{n}{i}\bar{\alpha}^i(1-\bar{\alpha})^{n-i}h(z_{n-i}) + (1-\bar{\alpha})^n h(y)$$

s.t.
Constraints (31) - (33) and (37).

Lemma 10: Suppose w is a priori unknown to all and the SAs perform independently.
The optimal incentives which must be offered to each SA are:

$$\overline{\overset{*}{x}} = \frac{1}{\overline{\alpha}^{(n-1)}}[\overline{B} + W(1,\frac{1}{n}) - J_2 + \frac{(1-\overline{\alpha})(\xi - J_1)}{\overline{\alpha} - \beta}]$$

$$\overline{\overset{*}{y}} = \frac{1}{(1-\overline{\alpha})^{n-1}}[\overline{B} + W(1,\frac{1}{n}) - J_2 - \frac{\overline{\alpha}(\xi - J_1)}{\overline{\alpha} - \beta}].$$

$$\text{where } \xi = \max\{\frac{W(1,\frac{1}{n}) - W(1,0)}{(1-\lambda)}, \frac{W(1,\frac{1}{n}) - W(0,\frac{1}{n})}{(1-\gamma)}, \frac{W(1,\frac{1}{n})}{(1-\gamma\lambda)}\}.$$

Proof: Similar to the proof of Lemma 7.□

2.3.2.2. Case $\gamma \neq 1, \lambda = 1$

Under this assumption, the screening model is formulated as follows:

$$(PIX): Min \quad \overline{\alpha}^n h(x) + \sum_{i=1}^{n-1}\binom{n}{i}\overline{\alpha}^i(1-\overline{\alpha})^{n-i}h(z_{n-i}) + (1-\overline{\alpha})^n h(y)$$

$$s.t.$$

$$(1-\gamma)[\overline{\alpha}^{(n-1)}(\overline{\alpha} - \beta)x + \sum_{i=1}^{n-1}\overline{\alpha}^{(n-i-1)}(1-\overline{\alpha})^{i-1}\left[\binom{n-1}{i}(1-\overline{\alpha}) - \binom{n-1}{i-1}\overline{\alpha}\right](\overline{\alpha} - \beta)z_{n-i}$$

$$-(1-\overline{\alpha})^{n-1}(\overline{\alpha} - \beta)y] \geq W(1,0)$$

$$\overline{\alpha}^n x + \sum_{i=1}^{n-1}\binom{n}{i}\overline{\alpha}^{n-i}(1-\overline{\alpha})^i z_{n-i} + (1-\overline{\alpha})^n y \geq \overline{B} + W(1,0).$$

Lemma 11: Suppose w is a priori known to all and the SAs perform independently. The optimal incentives which must be offered to each SA are:

$$\overline{\overset{**}{x}} = \frac{1}{\overline{\alpha}^{(n-1)}}[\overline{B} + W(1,0) - J_2 + \frac{(1-\overline{\alpha})(W(1,0) - (1-\gamma)J_1)}{(1-\gamma)(\overline{\alpha} - \beta)}]$$

$$\overline{\overset{**}{y}} = \frac{1}{(1-\overline{\alpha})^{n-1}}[\overline{B} + W(1,0) - J_2 - \frac{\overline{\alpha}(W(1,0) - (1-\gamma)J_1)}{(1-\gamma)(\overline{\alpha} - \beta)}].$$

Proof: Similar to the proof of Lemma 7.□

Now we analysis the cooperation of SAs under different assumptions about γ and λ.

Theorem 3: Suppose w is a priori known to all and there are n SAs forming an IGA. The expected screening cost for contracting every SA from this IGA is always higher than the expected screening cost it would incur by transacting with any single SA.

Proof: Similar to the proof of Theorem 1, it must be shown that the optimal value of (PIX) is greater than the optimal value of (PVI). $\square$

Theorem 4: Suppose w is a priori unknown to all and there are n SAs forming an IGA. If

$$(1-\bar{\alpha})^{n-1}\max\{\frac{W(1,1)-W(1,0)}{1-\lambda},\frac{W(1,1)-W(0,1)}{1-\gamma},\frac{W(1,1)}{1-\gamma\lambda}\geq\max\{\frac{W(1,\frac{1}{n})-W(1,0)}{1-\lambda},\frac{W(1,\frac{1}{n})-W(0,\frac{1}{n})}{1-\gamma},\frac{W(1,\frac{1}{n})}{1-\gamma\lambda}\},$$

then, the expected screening cost for contracting every SA from this IGA is lower than the expected cost it would incur by transacting with any single SA.

Proof: We must show that if (x^*, y^*) is the optimal value of (PV), then, under the stated conditions, $(x^*, z_1^*, ..., z_{n-1}^*, y^*)$ is a feasible solution for ($PVII$).

Similar to proof of Theorem 2, we prove that $(x^*, z_1^*, ..., z_{n-1}^*, y^*)$ always satisfies the IR constraint of ($PVII$). Also, by replacing $\bar{\alpha}$ with $(1-\bar{\alpha})$ and z_{n-i} with z_i for all $i \leq \frac{n}{2}$, it can be shown that if

$$(1-\bar{\alpha})^{n-1}\max\{\frac{W(1,1)-W(1,0)}{1-\lambda},\frac{W(1,1)-W(0,1)}{1-\gamma},\frac{W(1,1)}{1-\gamma\lambda}\geq\max\{\frac{W(1,\frac{1}{n})-W(1,0)}{1-\lambda},\frac{W(1,\frac{1}{n})-W(0,\frac{1}{n})}{1-\gamma},\frac{W(1,\frac{1}{n})}{1-\gamma\lambda}\},$$

then, the IC constraints are also satisfied.

3. Conclusions

We have developed several mathematical models to determine the incentives of screening agencies so that they find it optimal to perform truthfully in various situations. These models have been solved analytically using the optimality conditions. We have also studied the case in which many of such agencies cooperate in a common market in order to share their information. Furthermore, the conditions have been identified under which such cooperation is more beneficial to all participating agents than the situation in which each agent works independently. It is shown that the Nash equilibrium for this situation coincides with truthful behaviors of cooperating agents. It may be possible to show in a future work that a point could be reached at which the marginal increase in the value of shared information would equal the marginal increase in the cost of moral hazard, while further growth in number of SAs cooperating in the IGA would not be advantageous at the point. Other mechanisms focusing on the behaviors of other influential agents could also be developed to improve the information asymmetry phenomenon.

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