Iranian Journal of Operations Research
Vol. 11, No. 1, 2020, pp. 144-156
DOI: 10.29252/iors.11.1.144

Discriminant Analysis and Data Envelopment Analysis with Specific Data and Its Application for Companies in the Iranian Stock Exchange

M. Fallah [1], F. Hosseinzadeh Lotfi * [2], M. Hosseinzadeh[3]

Using the experiences of successful and unsuccessful companies can be a criterion for predicting the situation of emerging companies. Each company can have a vector include both financial and non-financial characteristics. Accordingly, for an active or emerging company, it is possible to determine the characteristic vector and predict which group it is likely to belong to. The techniques used in this research are discriminant analysis and data envelopment analysis. Based on this technique, discriminant functions are designed to separate known sets. The main idea for finding discriminant functions is from data envelopment analysis, which makes a limit of efficiency for separating efficient units from inefficient ones. The discriminant functions of this method are used to predict the state of the company. Hyper planes are obtained as discriminant functions to separate companies. These hyper planes are based on multiple indicators. Each of these indicators can also apply in certain situations. The modeling used in this paper was used on oil companies listed on the Iran Stock Exchange. 15 indicators and criteria have been defined for each company. The data were for 2015 and 2016, and the number of oil companies was 18, of which 9 were successful and 9 were bankrupt. In this paper, with the help of data envelopment analysis and discriminant analysis, a new modeling was designed to find hyper planes for separating two sets. Modeling has been performed based on the different criteria that have existed, and each one applies in certain circumstances. In the following, the properties of the designed model are expressed and proved. The specific conditions of the criteria have become limitations that have been added to the multiplicative form of the designed model.

Keywords: *Discriminant Analysis, Data Envelopment Analysis, Efficiency, Stock Exchanges and OTC (Over the Counter).*

Manuscript was received on 02/08/2020, revised on 04/08/2020 and accepted for publication on 05/11/2020.

1. Introduction

The discriminant analysis technique is used for finding the discriminant function between two or more sets. It is assumed that the observations are divided to two or more sets and each observation has a vector for its specific characteristics. In this case, this technique looks for the discriminant functions so that they can be used for prediction of the new member to each of the sets. The indices affecting separation of the sets will be different based on the type of population.

The companies active in the oil industry are always seeking to be able to maintain their financial indices at a good or even excellent level. The financial indices are often stated in an absolute form

[1] *Department of Industrial Engineering, Central Tehran Branch, Islamic Azad University, Tehran , Iran* *mohammad.fallah43@yahoo.com*

* Corresponding author:
[2] *Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran, Email: farhad@hosseinzadeh.ir*

[3] *Department of Management , University of Bergamo, Italy· m.hosseinzadeh@unibg.it*

or as the ratio between two important items influencing the performance. It is important for each individual or investing company to be able to recognize whether there is stable profitability or success for the company in the shares they buy or in other words, the company in which they invest or in other words, if a specific company can be trusted in a special domain in a competitive market and can the shares of that company be purchased.

Also, it is important to know whether an acceptable profit can be presented for investment in an oil company, considering the competitive market between other companies. Each oil company has different factors which can affect its performance. For example, the selling price, different types of cost, net profit, earnings per share, current ratio, quick ratio, debt ratio, etc. can be mentioned which can be important factors for decision making and choosing an oil company for investment.

Whereas the data envelopment analysis (DEA) is also a technique for performance evaluation and division of the decision making units into two efficient and inefficient sets, many relations can be created between the two techniques of data envelopment analysis and discriminant analysis. Both techniques use the idea that the observations can be separated to two or more dependents and both techniques seek to find the discriminant function between the subsets.

In the area of discriminant analysis, some examples of the discriminant analysis were presented by Smith (1974). Alman (1968) discussed the discriminant analysis by the financial ratios. Press et al. (1978) presented a method for comparison of the discriminant analysis and regression. The data envelopment analysis (DEA) was invented by Charnes et al. (1978). Bajgier et al. (1982) discussed the discriminant analysis using the statistical concepts. Duata Silva et al. (1994) presented a new concept for the discriminant analysis based on mathematical programming. This technique was further developed by Charnes et al. (1985) and the modeling was performed with the help of Pareto-Koopman. Separation of two groups using mathematical programming was further developed by Freed et al. (1986). Using the concept of the Additive model, Sueyoshi (1990) discussed the discriminant analysis for the first time. A method based on mathematical programming was discussed by Ragsdale et al. (1991) for determination of the discriminant function in the discriminant analysis. The discriminant analysis consists of the concepts of the book published by Mclachlan (1992). The polynomial discriminant function was produced by Rubin (1994). Some applications of the discriminant analysis in the financial and commercial domain were presented by Eisenbeis (1997). Sinuany-stern et al. (1998) presented a method for rating using DEA/DA concept. The additive model and concept of inefficiency were introduced by Cooper et al. (1999). Subsequently, Sueyoshi (1999) introduced a new method by combining DEA/DA and concept of goal programming. The nonparametric statistical methods were presented by Hollander et al. (1999). Yanev et al (1999) introduced a combined method for classification of the problems. Sueyoshi (2001) presented the further developed DEA/DA. The classification problems were discussed by Doumpos et al. (2001). Comparison of different methods in discriminant analysis was discussed by Sueyoshi (2006). Jahanshaloo et al. (2007) presented a method based on Monte Carlo for determination of the amount of overlap and discriminant analysis. Hosseinzadeh Lotfi et al. (2008)developed a method for DEA/DA by fuzzy data. A comparison between DEA and DEA/DA was provided by Sueyoshi et al.

Atkinson et al. (1997), Charnes et al. (1985), Charnes et al. (1978), Chang et al. (2007),can be mentioned among the researchers who performed many studies and provided different models for finding the discriminant function.

Various practical work has also been done in the area of application of data envelopment analysis in petrochemical companies. Among them are Liang et al. (2006), Malhotra et al. (2007), Malhotra et al. (2008), Mehrani et al. (2004), and Worthington et al. (1998).

Numerous articles in the area of oil companies provided their classification or performance evaluation. Different models have been presented by data envelopment analysis with different structures.

In the second section, a background is provided for both data envelopment analysis and discriminant analysis. In the third section, a model is discussed for separation of the two groups of successful and unsuccessful oil companies with specific data. The designed model is discussed in the fourth section of this article using the actual data for 18 oil companies in Iranian stock exchange. In the end, conclusion and suggestions are provided in the fifth section.

2. Preliminaries

Assume that n is the existing decision making unit. The $j-th$ unit uses the input vector $X_j = (x_{1j},...,x_{mj})$ for production of the output vector $Y_j = (y_{1j},...,y_{sj})$ while $X_j \geq 0, X_j \neq 0, Y_j \geq 0, Y_j \neq 0,$ The CCR model for calculation of the relative efficiency of DMU_p is as follows.

The optimal value of the objective function of model 1, i.e. θ^* is called efficiency of DMU_p. It is obvious that if $\theta^* = 1$, then, DMU_p is called relatively efficient and otherwise, if $0 < \theta^* < 1$, then, DMU_p is called inefficient.

$$
\begin{aligned}
&Min \quad \theta \\
&s.t. \quad \sum_{j=1}^{n} \lambda_j X_j \leq \theta X_p \\
&\qquad\quad \sum_{j=1}^{n} \lambda_j Y_j \geq Y_p \\
&\qquad\quad \lambda \geq 0
\end{aligned}
\tag{1}
$$

The dual form of model 1 which is called a multiple model of CCR is as follows.

$$
\begin{aligned}
&Max \quad UY_p \\
&s.t. \quad UY_j - VX_j \leq 0 \quad , j = 1,...,n \\
&\qquad\qquad VX_p = 1 \\
&\qquad U \geq 0, \quad V \geq 0
\end{aligned}
\tag{2}
$$

If (V^*, U^*) is the optimal solution of model 2, then, the hyperplane H which is defined as follows is the hyperplane leaning on the production possibility set (PPS) at the projective point of DMU_p.

$$
H = \left\{ \begin{pmatrix} X \\ Y \end{pmatrix} \middle| U^*Y - V^*X = 0 \right\}
$$

By designing n models in the form of 2, different hyperplanes are obtained for each DMU_p. If (V^*,U^*) is an optimal basic feasible solution for 2, then, H is a constituent hyperplane of the production possibility set. From these constituent hyperplanes of the production possibility set, an efficient function is obtained in the form of a concave linear segment which is the discriminant function between efficient and inefficient units. Any decision making unit which is located on the function is efficient and if it is located under this function, it is inefficient. The main purpose of data envelopment analysis technique is to be able to find an efficient function. The efficiency will be different depending on acceptance of different postulates for the type of the discriminant function.

In discriminant analysis, it is assumed that n is the existing observation. Each observation has a characteristic vector $Z_i = (z_{1i},...,z_{mi})$. Then observations are divided to two groups of G1 and G2. Finding a discriminant function similar to a function with the equation $f\left(Z\right) = \alpha Z - d$ is the main purpose of the technique so that we want to find the parameters of function f in such a way that

$$\begin{aligned} &\forall Z_i \in G_1 \quad f(Z_i) \le 0 \\ &\forall Z_i \in G_2 \quad f(Z_i) > 0 \end{aligned} \tag{3}$$

In other words

$$\begin{aligned} &\forall Z_i \in G_1 \quad \alpha Z_i \le d \\ &\forall Z_i \in G_2 \quad \alpha Z_i > d \end{aligned} \tag{4}$$

Whereas it is possible that parameters α and d cannot be found in a way that relations 4 are established. Therefore, we will try to establish relations 4. Otherwise, the intended error will become minimum.

Therefore, the correspondent model for finding the discriminant function with the minimum error for separation of the two sets G1 and G2 is as follows:

$$\begin{aligned} Min \quad & \sum_{Z_i \in G_1} S_j^+ + \sum_{Z_i \in G_2} S_j^- \\ s.t. \quad & \alpha Z_i - S_j^+ \le d \quad , \quad Z_i \in G_1 \\ & \alpha Z_i + S_j^- \ge d + \varepsilon \quad , \quad Z_i \in G_2 \\ & 1\alpha = 1 \\ & S^+ \ge 0 , S^- \ge 0 \end{aligned} \tag{5}$$

In model 5, assume that (d^*,α^*) is the optimal solution. In this case, the hyperplane with equation $\alpha^* Z = d^*$ is recognized as the hyperplane separating the two sets. For a new observation $\overline{Z}$, if $\alpha^* \overline{Z} \le d^*$, it is predicted that $\overline{Z} \in G_1$ and if $\alpha^* \overline{Z} > d^*$, it is predicted that $\overline{Z} \in G_2$.

3. A Model for Prediction of Performance of Oil Companies

In this section, the oil companies listed in Iranian stock exchange are studied. The indices affecting the performance of oil companies will be as follows:

Table 1. Index Performance

Item	Index	Definition	Remarks
1	Income	Grand total of sales in a period	The higher it is, the better for the company.
2	Cost	The difference of the inventory of goods at the end of the period is calculated from the total purchase and inventory of goods at the beginning of the period.	The lower it is, the better.
3	Administrative Costs	It is total costs except the direct costs of production.	The lower it is, the better.
4	Net Profit	The difference of all costs and tax is obtained from total sales.	The higher it is, the better.
5	Earnings per Share	It is obtained through dividing the shareholders' dividend by the total number of shares of a company, excluding the preference shares.	The higher it is, the it indicates a better status for the company.
6	Current Ratio	It is calculated through dividing the current asset by the current liability in each period.	The more the number is closer to 2, the liquidity is in a more favorable status.
7	Quick Ratio	Total inventory + advance payment - current asset Current Liability	The more the number is bigger than 1, the better is the company's status.
8	Debt Ratio	It is calculated through dividing the total liability by the total asset.	The more the number is smaller than 1, the better it is.
9	Inventory Turnover	It is obtained through dividing the costs of the goods sold by the average inventory.	The more the number is smaller than 1, the better is the company's status.
10	Current Assets to Total Assets Ratio	It is obtained through dividing the current asset by total assets.	The more the number is bigger than 1, the better is the company's status.
11	Proprietary Ratio	It is obtained through dividing total shareholders' equity by total assets.	The more the number is bigger than 0.5, the better it is for the company.
12	Return on Asset Ratio	It is obtained through dividing the net profit by total assets.	The more the number is bigger than 0.20, the better it is.
13	Fixed Asset Turnover Ratio	It is calculated through dividing the sales by total assets.	The bigger and closer to 1 it is, the better is the company's status.
14	Profit Margin Ratio	It is obtained through dividing the net profit by net sales.	The more it is bigger than 0.3, the better it is.
15	Debt-to-capital Ratio	It is calculated through dividing total liability to the shareholders' equity.	The more it is smaller than 0.7, it indicates the company's better status.
16	Investment rate	Net profit to Total assets	The bigger and closer to 0.2 it is, the better is the company's

Income, cost, administrative costs, net profit, earnings per share, current ratio, quick ratio, debt ratio, inventory turnover, current assets to total assets ratio (CATA), proprietary ratio, return on asset (ROA), fixed asset turnover ratio, profit margin ratio, debt-to-capital ratio.

Considering the conditions of the problem, the initial model for finding the separating hyperplane in the sets G1 and G2 is as follows (Sueyoshi (2001)).

$$\begin{aligned}
Min \quad & \sum_{Z_j \in G_1} S_{1j}^{+} + \sum_{Z_j \in G_2} S_{2j}^{-} \\
s.t. \quad & \alpha Z_j + S_{1j}^{+} - S_{2j}^{-} = d + 1 \quad , \quad Z_j \in G_1 \\
& \alpha Z_j + S_{2j}^{+} - S_{2j}^{+} = d \quad , \quad Z_j \in G_2 \\
& 1|\alpha| = 1 \\
& S_{1j}^{+} \geq 0 , S_{1j}^{-} \geq 0 \quad , \quad Z_j \in G_1 \\
& S_{2j}^{+} \geq 0 , S_{2j}^{-} \geq 0 \quad , \quad Z_j \in G_2
\end{aligned} \tag{6}$$

Consider that α_i for each i is free in sign. Suppose that $\alpha_i = \alpha_i^{+} - \alpha_i^{-}$ where $\alpha_i^{+}, \alpha_i^{-}$ are non-negative, model 6 is changed as follows:

$$\begin{aligned}
Min \quad & \sum_{Z_j \in G_1} S_{1j}^{+} + \sum_{Z_j \in G_2} S_{2j}^{-} \\
s.t. \quad & (\alpha^{+} - \alpha^{-}) Z_j + S_{1j}^{+} - S_{2j}^{-} = d + 1 \quad , \quad Z_j \in G_1 \\
& (\alpha^{+} - \alpha^{-}) Z_j + S_{2j}^{+} - S_{2j}^{+} = d \quad , \quad Z_j \in G_2 \\
& 1(\alpha^{+} + \alpha^{-}) = 1 \\
& S_{1j}^{+} \geq 0 , S_{1j}^{-} \geq 0 \quad , \quad Z_j \in G_1 \\
& S_{2j}^{+} \geq 0 , S_{2j}^{-} \geq 0 \quad , \quad Z_j \in G_2 \\
& \alpha^{+}, \alpha^{-} \geq 0
\end{aligned} \tag{7}$$

Assume that $\alpha_i^{*} = \alpha_i^{+^{*}} - \alpha_i^{-^{*}}$, $(i = 1, \ldots, m)$ and d* is the optimal solution of model 7. In this case, consider the following states in order to recognize the new member $\overline{Z}$ is in which of the sets G_1 *or* G_2 .

$$\begin{aligned}
&if \quad \alpha^{*} \overline{Z} \geq d^{*} + 1 && then \quad \overline{Z} \in G_1 \\
&if \quad d^{*} < \alpha^{*} \overline{Z} < d^{*} + 1 && then \quad \overline{Z} \in G_1 \bigcap G_2 \\
&if \quad \alpha^{*} \overline{Z} \leq d^{*} && then \quad \overline{Z} \in G_2
\end{aligned} \tag{8}$$

Now, assume $G \in G_1 \bigcap G_2$

$$R_1 = \left\{ Z_j \in G \left| \alpha^* Z_j \geq d^* + 1 \right. \right\}$$
$$R_2 = \left\{ Z_j \in G \left| d^* < \alpha^* Z_j < d^* + 1 \right. \right\}$$
$$R_3 = \left\{ Z_j \in G \left| \alpha^* Z_j \leq d^* \right. \right\}$$
$$C_1 = \left\{ Z_j \in R_1 \left| Z_j \in G_1 \right. \right\}$$
$$C_2 = \left\{ Z_j \in R_2 \left| Z_j \in G_2 \right. \right\} \quad (9)$$

In fact, R_1 is the set of all observations which are located in the half-space $\alpha^* Z \geq d^* + 1$ and R_3 is the set of all observations which are located in $\alpha^* Z \leq d^*$ and C_1 and C_2 are the sets of all observations from G_1 and G_2 , respectively, which are correctly located in the relevant half-spaces according to the null hypothesis. For all points that are located in the interface of $G_1 \bigcap G_2$ in algorithm 8, the following problem is solved in order to determine their membership in one of the sets.

$$\begin{aligned}
Min \quad & \sum_{Z_j \in G_1 \bigcap G_2} S_{1j}^+ + \sum_{Z_j \in G_1 \bigcap G_2} S_{2j}^- \\
s.t. \quad & (\alpha^+ - \alpha^-) Z_j \geq d + 1 \quad , & Z_j \in G_1 \\
& (\alpha^+ - \alpha^-) Z_j + S_{1j}^+ - S_{1j}^+ = d' \quad , & Z_j \in G_1 \bigcap R_2 \\
& (\alpha^+ - \alpha^-) Z_j + S_{2j}^+ - S_{2j}^+ = d' + \varepsilon \quad , & Z_j \in G_2 \bigcap R_2 \\
& (\alpha^+ - \alpha^-) Z_j \leq d \quad , & Z_j \in G_2 \\
& 1(\alpha^+ + \alpha^-) = 1 & \\
& d \leq d' \leq d + 1 & \\
& S_{1j}^+ \geq 0, S_{1j}^- \geq 0 \quad , & Z_j \in G_1 \bigcap R_2 \\
& S_{2j}^+ \geq 0, S_{2j}^- \geq 0 \quad , & Z_j \in G_2 \bigcap R_2 \\
& \alpha^+, \alpha^- \geq 0 &
\end{aligned} \quad (10)$$

Assume that $\overline{d'}$ and $\overline{\alpha'}$ are the optimal solution of model 10. In this case,

If $\overline{\alpha}\overline{Z} \geq \overline{d'}$, then $\overline{Z} \in G_1$ and if $\overline{\alpha}\overline{Z} < \overline{d'}$, then $\overline{Z} \in G_2$.

In this case, the algorithm for determination of membership of the new member $\overline{Z}$ is as follows:

Step 1 : solve the model (7) and find α^*, d^*

Step 2 : $if \quad \alpha^* \overline{Z} \geq d^* + 1 \quad then \quad \overline{Z} \in G_1$

Step 3 : $if \quad \alpha^{*}\overline{Z} \le d^{*} \quad then \quad \overline{Z} \in G_2$

Step 4 : solve the model (10) and find $\overline{\alpha}, \overline{d^{'}}$

Step 5 : $if \quad \overline{\alpha}\overline{Z} \ge \overline{d^{'}} \quad then \quad \overline{Z} \in G_1$

Step 6 : $if \quad \overline{\alpha}\overline{Z} < \overline{d^{'}} \quad then \quad \overline{Z} \in G_2$

Now, assume that we want to write the dual form of model 7. In this case, by introduction of the dual variables corresponding to the constraints of 7, we will have:

$$\begin{aligned}
Max \quad & \theta + \sum_{Z_j \in G_1} W_j \\
s.t. \quad & \sum_{Z_j \in G_1 \cup G_2} W_j Z_j + \theta \le 0 \quad , \\
& -\sum_{Z_j \in G_1 \cup G_2} W_j Z_j + \theta \le 0 \quad , \\
& \sum_{Z_j \in G_1 \cup G_2} W_j Z_j = 0 \\
& 0 \le W_j \le 1 \quad , \quad Z_j \in G_1 \\
& -1 \le W_j \le -0 \quad , \quad Z_j \in G_2
\end{aligned} \tag{11}$$

Based on table 1, there are 16 indices affecting determination of the discriminant function. Some of these indices have a standard limit which are required to be observed in modeling as far as possible. Consequently, according to table 1, model 11 is amended and suggested as follows:

$$
\begin{aligned}
Max \quad & \theta + \sum_{Z_j \in G_1} W_j - \sum_{i=6,7,10,11,12,13,14,16} (\delta_i^+ - \delta_i^-) + \sum_{i=8,9,15} (\delta_i^+ - \delta_i^-) \\
s.t. \quad & \sum_{Z_j \in G_1 \cup G_2} W_j z_{ij} + \theta \le 0 \quad , \quad i = 1,\dots,16 \\
& -\sum_{Z_j \in G_1 \cup G_2} W_j z_{ij} + \theta \le 0 \quad , \quad i = 1,\dots,16 \\
& \sum_{Z_j \in G_1 \cup G_2} W_j Z_j = 0 \\
& 0 \le W_j \le 1 \quad , \quad Z_j \in G_1 \\
& -1 \le W_j \le -0 \quad , \quad Z_j \in G_2 \\
& \sum_{Z_j \in G_1 \cup G_2} W_j z_{ij} + \delta_i^+ - \delta_i^- = 1 \quad , \quad i = 6,7,8,9,10,13 \\
& \sum_{Z_j \in G_1 \cup G_2} W_j z_{ij} + \delta_i^+ - \delta_i^- = 0.5 \quad , \quad i = 11 \\
& \sum_{Z_j \in G_1 \cup G_2} W_j z_{ij} + \delta_i^+ - \delta_i^- = 0.2 \quad , \quad i = 12,16 \\
& \sum_{Z_j \in G_1 \cup G_2} W_j z_{ij} + \delta_i^+ - \delta_i^- = 0.3 \quad , \quad i = 14 \\
& \sum_{Z_j \in G_1 \cup G_2} W_j z_{ij} + \delta_i^+ - \delta_i^- = 0.7 \quad , \quad i = 15 \\
& \delta_i^+, \delta_i^- \ge 0 \quad , \quad i = 1,\dots,16.
\end{aligned}
\tag{12}
$$

Now, we conclude that the dual form of model 12 is as follows.

$$
\begin{aligned}
Min \quad & \sum_{Z_j \in G_1} S_{1j}^+ + \sum_{Z_j \in G_2} S_{2j}^- + \sum_{i=6}^{16} a_i \delta_i \\
s.t. \quad & \sum_{i=1}^{16} (\alpha_i^+ - \alpha_i^-) z_{ij} + \sum_{i=1}^{16} \gamma_i z_{ij} + S_{1j}^+ - S_{1j}^- = d + 1 \quad , \quad Z_j \in G_1 \\
& \sum_{i=1}^{16} (\alpha_i^+ - \alpha_i^-) z_{ij} + \sum_{i=1}^{16} \gamma_i z_{ij} + S_{2j}^+ - S_{2j}^- = d \quad , \quad Z_j \in G_2 \\
& \sum_{i=1}^{16} (\alpha_i^+ - \alpha_i^-) = 1 \quad , \\
& \gamma_i \ge -1 \quad , \quad i = 6,7,10,11,12,13,14 \\
& \gamma_i \ge 1 \quad , \quad i = 15,8,9 \\
& \alpha_i^+, \alpha_i^- \ge 0 \quad , \quad i = 1,\dots,16 \\
& S_{1j}^+, S_{1j}^- \ge 0 \quad , \quad Z_j \in G_1 \\
& S_{2j}^+, S_{2j}^- \ge 0 \quad , \quad Z_j \in G_2
\end{aligned}
\tag{13}
$$

Where $a = (a_6, ..., a_{16}) = (1,1,1,1,1, 0.5, 0.2, 0.3, 0.7, 0.2)$.

After solving model 13 and finding its optimal solutions, the equation for its correspondent hyperplane is calculated using the following relations.

$\sum_{i=1}^{16} \alpha_i^* z_i = d^*$, $\forall_i \quad \alpha_i^* = \alpha_i^{+^*} - \alpha_i^{-^*}$

In this case, for determination of membership of a new member, we will have .

a) if $\alpha^* \bar{Z} > d^*$ then $\bar{Z} \in G1$

b) if $\alpha \bar{Z} \leq d^*$ then $\bar{Z} \in G2$

4. Numerical examples

Finding the separating hyperplane of oil companies listed in Stock Exchange and Over-the-counter Market of Islamic Republic of Iran: In this section, by using model 14 designed for the oil companies listed in the stock exchange and over-the-counter market in 2015 and 2016, the discriminant function will be found. Among the 18 oil companies studied, 9 companies have a favorable status are put in group G1 and the remaining 9 Companies do not have a good status to continue. The two groups G1 and G2 are as follows for successful and unsuccessful companies, respectively, and each of them is introduced in the next section. Tables 2 and 3 present information on the values of each of the indices during a period of two years.

Table 2. Average, Variance, Max, Min and Variation Range of Successful companies

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Average	31702080	24077133	2549361	6304667	1835.28	2.639	0.9	0.509	4.146	1.1217	0.516	0.479	0.641	0.383	0.37	0.469
Variance	1.28E+15	1.209E+15	1.3E+13	3.3E+13	1560797	6.329	0.2	0.151	9.423	1.9558	0.023	0.092	0.115	0.043	0.07	0.093
Max	4335528	2411727	219701	1751479	222	0.98	0.2	0.18	0.39	0.46	0.14	0.15	0.28	0.05	0.04	0.08
Min	1.31E+08	119356875	1.6E+07	2.2E+07	4486	10.5	1.8	1.9	12.18	5.2	0.75	1.02	1.36	0.93	1.16	1.02
Variation Range	1.27E+08	116945148	1.5E+07	2E+07	4264	9.52	1.6	1.72	11.79	4.74	0.61	0.87	1.08	0.88	1.12	0.94

Table 3. Average, Variance, Max, Min and Variation Range of Unsuccessful companies

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Average	19647624	16277220	609083.3	618513	-34.4	0.67	0.39	0.816	5.96	0.354	0.332	-0.1	0.835	0	0.3	0.01
Variance	4.491E+14	3.26E+14	4.02E+11	1E+12	1E+06	0.16	0.08	0.217	34.9	0.035	0.138	0.33	0.9012	0.33	0.031	0.1
Max	830678	746705	46251	-4E+05	-2571	0.23	0.14	0.04	1.59	0.14	-0.49	-2	0.22	-2.2	0.11	-0.8
Min	60934401	54833831	2650531	4E+06	1993	1.7	1.15	2.29	23	0.78	1.32	0.45	3.43	0.39	0.89	0.45
Variation Range	60103723	54087126	2604280	4E+06	4564	1.47	1.01	2.25	21.4	0.64	1.81	2.45	3.21	2.59	0.78	1.23

Mean, variance, maximum, minimum and range of variations of each of the indices for the two successful and unsuccessful groups have been indicated in tables 2 and 3, respectively.

By implementation of model 14 for the set of data of successful (G1) and unsuccessful (G2) oil companies, the separating hyperplane of these two sets was calculated as follows.

$$0.001 z1 - 0.001 z2 - 0.001 z3 - 0.001 z4 + 0.001 z5 - 0.15 z6 - 0.05 z7 + 0.05 z8 - 0.15 z9 - 0.05 z10 - 0.05 z11 - 0.05 z12 - 0.05 z13 - 0.05 z14 + 0.05 z15 - 0.15 z16 = 0.014$$

The above equation is the criterion of the discriminant function for determination and recognition of a new company regarding belonging to G1 or G2. This equation indicates that the indices of current ratio and inventory turnover have a higher coefficient in comparison to other indices and the quick ratio, debt ratio, Current assets to total assets ratio, proprietary ratio, return on asset ratio, fixed asset turnover ratio, profit margin ratio, debt-to-capital ratio, and return on investment ratio are in the next levels of importance.

5. Conclusion

In this article, a method was presented for discrimination of two sets with special indices by using data envelopment analysis and discriminant analysis. Discriminant analysis is used for finding a function for discrimination and separation of two or more sets.

Oil companies listed in Stock Exchange of Islamic Republic of Iran were studied in this research. The number of oil companies was 18 and they were studied during the two years 2015 and 2016. The number of indices affecting performance of companies was 16. The indices affected the performance of companies in the financial area. These indices were absolute or relative. Following the pattern of data envelopment analysis in this article, the discriminant analysis technique provided a new model for separation of the two sets of successful and unsuccessful oil companies with special indices.

In this method, a new model was presented by adding special conditions for the data and constraints for the multiple model corresponding to discriminant analysis. Among the 18 companies studied, 8 companies were successful during the period and 8 other companies were unsuccessful during the studied period. By implementation of the model designed in this article, the equation of the separating hyperplane of the two successful and unsuccessful sets was found in the end. The coefficients of indices in the equation of the hyperplane indicate that the most important indices are current ratio and inventory turnover. This research can be performed for interval, fuzzy and random data as well.

References

[1] Altman, E.I., (1968), “Financial ratios, discriminant analysis and the prediction of corporate bankruptcy”, *Journal of Finance*, 23, 589-609. DOI: 10.2307/2978933.

[2] Atkinson, A.A., Waterhouse J. H. and Wells R. B., (1997), “A stakeholder Approach to strategic performance Measurement”, Sloan Management *Review*, 25-37.

[3] Bajgier, S.M. and Hill A.V.A., (1982), “Comparison of statistical and linear programming approaches to the discriminant problem”, *Decision Sciences*, 13, 604-618. https://doi.org/10.1111/j.1540-5915.1982.tb01185.x

[4] Charnes A., and Cooper W.W. (1985), “Preface to Top in Data Envelopment Analysis”, Annals of Operational *Research*, 2, 59-70.

[5] Charnes A., Cooper W.W. and Rhodes E., (1978), “Measuring the Efficiency of Decision Making Units”, *European Journal of Operational Research*, 2, 429-444. https://doi.org/10.1016/0377-2217(78)90138-8

[6] Charnes, A., Cooper, W.W. Golany, B. Seiford, L. and Stulz, J., (1985), “Foundation of data envelopment analysis for Pareto-Koopmans efficient empirical production functions”, *Journal of Econometrics*, 30,91-107. https://doi.org/10.1016/0304-4076(85)90133-2.

[7] Cheng E.W.L., Chiang, Y. H. and Tang, B. S. (2007), ”Alternative approach to credit scoring by DEA: Evaluating borrowers with respect to PFI projects”, *Building and Environment*, 42,1752– 1760. https://doi.org/10.1016/j.buildenv.2006.02.012.

[8] Cooper,W.W., Park K.S. and Pastor, J.T., (1999) “RAM: A range adjusted measure of inefficiency for use with additive models and relations to other models and measures in DEA”, *Journal of Productivity Analysis*, 11,5-42. DOI: 10.1023/A:1007701304281.

[9] Doumpos,M., Zanakis S.H. and Zopounidis, C. (2001) “Multi-criteria preference disaggregation for classification problems with an application to global investing risk”, *Decision Sciences*, 32,333-385.

[10] Duate Silva.A.P. and Stam, A. (1994) “Second order mathematical programming formulations for discriminant analysis”, *European Journal of Operational Research*, 72,4-22. https://doi.org/10.1016/0377-2217(94)90324-7.

[11] Eisenbeis,R., (1997), “Pitfalls in the application of discriminant analysis in business, finance and economics”, *Journal of Finance*, 32 , 875-900. DOI: 10.2307/2326320.

[12] Freed,N and Glover, F. (1986), “Evaluating alternative linear programming models to solve the two-group discriminant problem”, *Decision Sciences*, 17,151-162. DOI: 10.1111/j.1540-5915.1986.tb00218.

[13] Hean Tat K., Chu, S. and Xu, J., (2006), “Efficiency, effectiveness and productivity of marketing in services”, *European Journal of Operational Research*, 170, 1, 265-276. https://doi.org/10.1016/j.ejor.2004.04.050.

[14] Hollander , M. and Wolfe, D.A. (1999), “Nonparametric Statistical Methods”, New York,*John Wiley*.

[15] Hosseinzade Lotfi F. and Mansouri, B. (2008), “The Extended Data Envelopment Analysis Discriminant Analysis Approach of Fuzzy Models”, Applied Mathematical Sciences, 2, 30, 1465-1477.

[16] Jahanshahloo G.R., Hosseinzadeh Lotfi, F. Balf F.R. and Rezai, H.Z. (2007), ”Discriminant Analysis of Interval Data using Monte Carlo Method in assessment of overlap”, *Applied Mathematics Computation* , 191, 2, PP.521-532. DOI: 10.1016/j.amc.2007.02.113.

[17] Liang G.S., Liu, C. F., Lin, W. C. and Yeh, C. H., (2006), “A data envelopment analysis of shipping industry bond ratings Taking”, *Journal of Science and Engineering*, 9(4), 403–408. DOI:10.6180/jase.2006.9.4.12.

[18] Malhotra R., Malhotra, D. K. and Russel, P. (2007), “Using data envelopment analysis to rate bonds”, *Proceedings of the Northeast Business & Economics Association*, 4, 420–423.

[19] Malhotra D.K., and Malhotra, R. (2008), “Analyzing Financial Statements Using Data Envelopment Analysis,” *Commercial Lending Review*, September–October: 25–31.

[20] McLachlan,G,J., (1992), “Discriminant Analysis and Statistical Pattern Recognition”, Wiley,New York.

[21] Press,S.J. and Wilson, S. (1978), “Choosing between logistic regression and discriminant analysis”, *Journal of American Statistical Association*, 73, 699-705. DOI: 10.2307/2286261.

[22] Ragsdale,C.T. and Stam, A. (1991), “Mathematical programming formulations for the discriminant problem : An old dog does new tricks”, *Decision Sciences*, 22, 296-307. DOI: 10.1111/j.1540-5915.1991.tb00348.x.

[23] Rubin,P.A., (1994), “A comment regarding polynomial discriminant functions”, *European Journal of Operational Research*, 74,29-31. https://doi.org/10.1016/0377-2217(94)90326-3.

[24] Sinuany-stern,Z. and Friedman, L. (1998), “DEA and the discriminant analysis of ratios for ranking units”, *European Journal of Operational Research*, 111,470-478. https://doi.org/10.1016/S0377-2217(97)00313-5.

[25] Smith,C.A.B., (1947), “Some examples of discrimination”, Annals of Eugenics, 13, 272-282. https://doi.org/10.1111/j.1469-1809.1946.tb02368.x.

[26] Sueyoshi,T., (1990), “Special algorithm for an additive model in data envelopment analysis”, *Journal of the Operational Research Society*, 41,249-257. https://doi.org/10.1057/jors.1990.41.

[27] Sueyoshi,T., (1999), “DEA-disceriminant analysis in the view of goal programming”, *European Journal of Operational Research*, 115,564-582. https://doi.org/10.1016/S0377-2217(98)00014-9.

[28] Sueyoshi,T., (2001), “Extended DEA-discriminant analysis”, European Journal of Operational Research, 131,324-351. https://doi.org/10.1016/S0377-2217(00)00054-0

[29] Sueyoshi,T., (2006), “DEA-discriminant analysis :Methodological comparison among eight discriminant analysis approaches”, *European Journal of Operational Research*, 169,47-272. https://doi.org/10.1016/j.ejor.2004.05.025

[30] Sueyoshi,T., Goto,M., (2009), “Methodological comparison between DEA(data envelopment analysis) and DEA-DA (discriminant analysis) from the perspective of bankruptcy assessment”, *European Journal of Operational Research.* 199(2):561-575. DOI: 10.1016/j.ejor.2008.11.030

[31] Worthington A. C. (1998), “The Application of Mathematical Programming Techniques to Financial Statement Analysis: Australian Gold Production and Exploration”, *Australian Journal of Management*, 23 (1), 97-113. https://doi.org/10.1177/031289629802300106

[32] Yanev,N. and Balev, S.A. (1999), “Combinatorial approach to the classification problem”, *European Journal of Operational Research*, 115, 339-350. https://doi.org/10.1016/S0377-2217(98)00229-X.