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A two-stage multi- parametric model for solving Animal Diet Formulation with Floating Price based on a fuzzy flexible linear programming model

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New concepts of $\bar{\alpha}$-feasibility and $\bar{\alpha}$-efficiency of solutions for fuzzy mathematical programming problems are used, where $\bar{\alpha}$ is a vector of distinct satisfaction degrees. Recently, a special kind of fuzzy mathematical programming entitled Fuzzy Flexible Linear programming (FFLP) is attracted many interests. Using the mentioned concepts, we propose a two-phase approach to solve FFLP. In the first phase, the original FFLP problem converts it to a Multi-Parametric Linear Programing (MPLP) problem, and then in phase II using the convenient optimal solution with the higher feasibility degree is concluded. Using this concept, we have solved the problem of the animal diet. In the process of milk production, the highest cost relates to animal feed. Based on reports provided by the experts, around seventy percent of dairy livestock costs included feed costs. In order to minimize the total price of livestock feed, according to the limits of feed sources in each region or season, and also the transportation and maintenance costs and ultimately milk price reduction, optimization of the livestock nutrition program is an essential issue. Because of the uncertainty and lack of precision in the optimal food ration done with existing methods based on linear programming, there is a need to use appropriate methods to meet this purpose. Therefore, in this study formulation of completely mixed nutrient diets of dairy cows is done by using a fuzzy linear programming in early lactation. Application of fuzzy optimization method and floating price make it possible to formulate and change the completely mixed diets with adequate safety margins. Therefore, applications of fuzzy methods in feed rations of dairy cattle are recommended to optimize the diets. Obviously, it would be useful to design suitable software, which provides the possibility of using floating prices to set feed rations by the use of fuzzy optimization method.

Keywords*: Fuzzy linear programming, Feasibility and efficiency, Fuzzy flexible linear programming, Diet, Floating price*.

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1. Introduction

Linear programming is one of the most frequently applied operations research techniques as an important part of mathematical programming. In the real world situations, the decision marker might not really want to actually maximize or minimize the objective function. Rather, he or she might want to reach some aspiration levels that might not even be definable crispy. Thus he or she might want to improve the present cost situation considerably and so on. Also, the role of the constraints can be different from that in the classical one, where the violation of any single constraint by any amount renders the solution infeasible. The decision marker might accept small

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violations of constraints but might also attach different (crisp or fuzzy) degrees of importance to violations of different constraints. Fuzzy mathematical programming offers a number of ways to allow for these types of imprecisions. It is necessary to distinguish between flexibility in constraints and goals and uncertainty of the data. Flexibility is modeled by fuzzy sets and may reflect the fact that constraints or goals are linguistically formulated. Their satisfaction is a matter of tolerance and degrees or fuzziness [2]. Ramik and Rimanek [40] also dealt with and LP problem with fuzzy parameters in the constraints. Later, Verdegay [45] and Chanas [7] have shown an application of parametric programming techniques in the fuzzy LP. Livestock ration formulation models have been developed for commercial purposes as well as for its development, using various mathematical techniques for several decades. Some of them are Pearson square method, two by two matrix method, and trial and error method. Some mathematical programming techniques are being used to formulate the ration such as linear programming, multiple objective programming, goal programming, separable programming, quadratic programming, nonlinear programming, and genetic algorithm. Linear programming is the common method of least cost feed formulation and for the last fifty years, it has been used as an efficient technique in ration formulation [21]. In milk production, the main costs are related to animal feed**.** Therefore, to reduce the total price of milk, the use of diets with the lowest price is essential**.** The linear programming model can be applied to find the lowest price that will meet all the needs of lactating cows. However, the parameters of this model are often considered as definitive, which is not real and are by approximate nature**.** Meeting the real needs of food rations during the use of these models is impossible. Animal feed price is usually stated as an interval or range, and are not accurate estimates, because it has always been volatile and may vary from region to region. Transport and harvest costs and above all the existence of feed material in a region may effect on total cost of feed. So fuzzy sets and numbers are useful tools for modelling of such uncertain and imprecise problems.

Impressed by Zadeh's fuzzy set theory set theory [50], sometimes the vagueness in the coefficients may not be of a probabilistic type. In these conditions, the decision maker can model the inexactness by means of fuzzy parameters. First of all, the decision maker might not actually want to go maximize or minimize the objective function. Instead, s/he might want to get some aspiration levels that might not even be definable crisply. Thus, s/he might want to reduce the present cost situation considerable, and so on [7]. Next, the constraints might not be clear and be ambiguous of the following way. The " $\leq$ " sign might not be meant in the strictly mathematical sense, but smaller violations might well be all right. This can occur if the constraints show aspirations levels as pointed out above or if for example the constraints represent sensory. Requirements (taste, colour, smell, etc.), which cannot come adequately, be approximated by a crisp constraint. Obviously, the technological coefficients itself can have a fuzzy character either because of being fuzzy in nature or due to the fuzzy perception of them. At least the role of the constraints can be unlike classical LP, where the violation of any single constraint by any amount renders of the solution infeasible is considered. The decision maker might accept small violation of constraints but might also attach different (crisp or fuzzy) degrees of importance to violations of different constraints. Many authors considered different kinds of the fuzzy linear programming problems and suggested several approaches for solving these problems [9, 12, 19, 25, 35, 41, 42, 44].

It is necessary to differentiate between flexibility in constraints and goal and uncertainty of the data. Flexibility is modeled by fuzzy sets and may reflect the fact that constraints or goal are linguistically formulated, and their satisfaction is a matter of tolerance and degrees or fuzziness [3]. On the other hand, there is an ambiguity corresponding to an objective variability in the model parameters (Randomness), or a Lack of knowledge of the parameter values (epistemic uncertainty). Randomness originates from the random nature of events and it is about uncertainty regarding to the membership or non-membership of an element in a set. Epistemic uncertainty deals with ill-known parameters modeled by fuzzy intervals in the setting of possibility theory [10,50]. In [47], Verdegay proposed a parametric linear programming model with single parameter using $\alpha-$cuts to achieve an equivalent model for the fuzzy linear programming with flexible constraints. After that, Verdegay in [45] used the duality results to solve the original fuzzy linear programming. Werner's in [48] introduced an interactive multiple objective programming model subject to its constraint are flexible and proposed a special approach for solving multiple objective programming model based on fuzzy sets theory. In the mentioned work, the classical model is extended by integration flexible constraints. After that, Delgado et al. [12] proposed a general model for fuzzy linear programming problem. In particular, they suggested a resolution method for the mentioned problem. Campos et al. [8] considered a linear programming problem with fuzzy constraints including fuzzy coefficients in both matrix and right hand side. They dealt with an auxiliary model resulting from the embedding constraints in the main model. After that, Nasseri et al. [33] introduced an equivalent fuzzy linear model for the flexible linear programming problems and proposed a fuzzy primal Simplex algorithm to solve these problems. Recently, Attari and Nasseri in [1] introduced a concept of feasibility and efficiency of the solution for the fuzzy mathematical programming problems. The suggested algorithm needs to solve two classical associated linear programing problems to achieve an optimal flexible solution. While unfortunately this process because of solving much computational operation is not efficient and hence in the task of the sensitivity analysis on this model, their approach doesn't work appropriately. Now, we are going to improve their method and propose a new approach, which is more flexible in order to overcome the mentioned shortage. The new approach can determine the optimal solution by solving an associated auxiliary problem in just one phase. And hence, our method can obtain the flexible optimal solution with the higher satisfaction degree in comparison with the earlier approach, which was introduced by Attari et al. [1].

Nasseri proposed a method for solving fuzzy linear programming problems by solving the classical linear programming [30]. Ebrahimnejad and Nasseri used the complementary slackness theorem to solve fuzzy linear programming problem with fuzzy parameters without the need of a simplex tableau [13]. Ebrahimnejad et al. proposed a new primal-dual algorithm for solving linear programming problems with fuzzy variables by using duality results [14]. Nasseri and Ebrahimnejad proposed a fuzzy primal simplex algorithm for solving the flexible linear programming problem [33]. Ebrahimnejad et al developed the bounded simplex method for solving a special kind of linear programming with fuzzy cost coefficients, in which the decision variables are restricted to lie within lower and upper bounds [17]. Many authors considered various types of the FLP problems and proposed several approaches for solving these problems [13-17, 20, 24, 25, 30, 33-35]. The diet problem was one of the first optimization problems studied in the 1930s and 1940s [28]. Linear programming techniques have been extensively used for animal diet formulation for more than last fifty years. To overcome the drawback of linear approximation of the objective function for diet formulation, a mathematical model based on nonlinear programming technique is proposed to measure animal performance in terms of milk yield and weight gain [43]. Some biological optimization problems imply finding the best compromise among several conflicting demands in a fuzzy situation. For example, experimental results show that a micro-organism may reflect the resilience phenomenon after stressful environmental changes and genetic modification [47]. Fuzzy linear programming is considered as an appropriate method for solving the problems of dairy cows' diets when feeding prices used in completely mixed diets are expressed as fuzzy numbers. In this case, all the numerical coefficients that are expressed as approximate and imprecise can be stated in terms such as approximately, about or in range. On the other hand, in completely mixed diet formulation, if feeding prices are expressed in the interval, their meaning may not provide good information about that interval; while if the membership functions of these intervals were available, decisions would have been made on the basis of information that is more complete. In this context because of flexibility in choosing the coefficients, fuzzy linear programming can help more effectively to decide about the appropriate food formulation. Since it is helpful for better modelling of the inherent uncertainty that the user faced about the feed rations data base. Linear

programming, fuzzy linear programming and mathematical techniques, techniques have been extensively used for diet formulation [2,22,27,29,39]. Cadenzas et al. presented the application of fuzzy optimization to diet problems in Argentinean farms and also developed software which has several capabilities such as the suggestion of different diets with satisfied diet requirements, price and the amount of constraint satisfaction [4]. Castrodeza et al. gave a multi-criteria fractional model for feed formulation with economic, nutritional and environmental criteria. Together with the search for the lowest possible cost, they introduced some other aspects such as maximizing diet efficiency and minimizing any excess that may lead to unacceptable damage to the environment [6]. Pomar et al developed multi-objective optimization model based on the traditional least-cost formulation program to reduce both feed cost and total phosphorus content in pig feeds [39]. Niemi et al. used stochastic dynamic programming to determine the value of precision feeding technologies for grow-finish swine [37]. Darvishi et al used fuzzy optimization in diet formulation and using a fuzzy model in comparison to linear programming models, feed costs was reduced to about 8 percentages. The result of this experiment guarantees the formulation of ration using fuzzy models can be used to reduce feed cost and obtain different ration that they may met dairy cow Iranian Journal of Optimization, Vol 8, Issue 2, spring 2016 1037 nutrient requirements over different situations [32]. Mamat et al concentrates on the human diet problem using fuzzy linear programming approach. This research aims to suggest people have healthy food with the lowest cost as possible [26]. In order to develop the decision making approach of Operations Research (OR) in the other subjects, Fuzzy and Stochastic approaches are used to describe and treat imprecise and uncertain elements present in a real decision problem. In fuzzy programming problems the constraints and goals are viewed as fuzzy sets and it is assumed that their membership functions are known. In this paper, Fuzzy Flexible Linear Programming (FFLP) problem with fuzzy cost coefficients was used for the completely mixed rations of lactating cows in early lactation intake (from birth to 70 days postpartum), weighing between 600 to 700 kg. In order to determine the fluctuation of food prices, fuzzy theory approach was employed, where the prices of food were assumed as fuzzy numbers.
The rest of the paper is organized as follows. In Section 2, we demonstrate some preliminaries of fuzzy and fundamental definitions. In particular, a certain linear ranking function for ordering trapezoidal fuzzy numbers is emphasized. In Section 3, first based on the Fuzzy flexible linear programming problem with fuzzy cost coefficients is introduced and then a new two-phase approach for solving the associated problem is suggested, an algorithm is proposed for the suggested approach. in Section 4, we shall illustrate our approach by solving a case study on Animal Diet Formulation with Floating price. we will allocate Section 5 to conclusions.

2. Preliminaries and fundamental definitions

In this section, some basic concepts of fuzzy sets theory and concept of feasible solution to the fuzzy programming problem is given.

2.1. FFLP problem with linear membership function

Consider a decision maker faced with a linear programming problem in which s/he can endure violation in completing the constraints, that is, s/he allows the constraints to be held as well as possible. For each constraints, in the constraints set this assumption can be denoted by $a_i x \preceq \tilde{b}_i$, $i = 1,\ldots,m$ and for every, modeled by the use of a membership function

$$\mu_i(x)=\begin{cases}1, & a_i x \le b_i\\ f_i(a_i x), & b_i \le a_i x \le b_i+p_i\\ 0, & a_i x \ge b_i+p_i\end{cases}$$

(1)

Definition 2.1. A Fuzzy Flexible Linear Programming (FFLP) problem is defined as follows:

$$\begin{aligned}&\max \tilde{z}=\tilde{c}x\\ &s.t. Ax \preceq \tilde{b} \qquad (2)\\ &x \ge 0\end{aligned}$$

where $f_i(0)$ is strictly decreasing and continuous for $a_i x$, $f_i(b_i)=1$ and $f_i(b_i+p_i)=0$.

This membership function expresses that the decision maker tolerates violation in the accomplishment of the constraints i up the value b_i+d_i. The function $\mu_i(x)$ gives the degree of satisfaction of the i-th constrains for $x\in\mathbb{R}^n$, but this value is obtained by means of the function f_i which is defined over $\mathbb{R}$. Based on the above assumption the associated FFLP Problem can be presented as:

$$\begin{aligned}&\max \tilde{z}=\tilde{c}x\\ &s.t.\ a_i x \le b_i+p_i(1-\alpha_i)\\ &x\ge 0, \alpha_i \ge \alpha_i^D,\ 0\le\alpha_i\le 1,\ i=1,\dots,m.\end{aligned}$$

(3)

We name the above problem as Multi-Parametric Linear Programming (MPLP) problem [1,5,18].

Now, we are going to give the fundamental concept of feasible solution to the fuzzy linear programming problem, which is defined in (3).

Definition 2.2. The α-cut or α-level set of a fuzzy set $\tilde{a}$ is a crisp set defined by $A_\alpha=\{x\epsilon\square \mid \mu_{\tilde{a}}(x)>0\}$

Definition 2.3. Let $\bar{\alpha}=(\alpha_1,\cdots,\alpha_m)\epsilon(0,1]^m$ be a vector, and $x_{\bar{\alpha}}=\{x\epsilon\square^n \mid x\ge 0,\ \mu_i\{g_i(x)\preceq 0\}\}\ge\alpha_i \quad i=1,\cdots,m$. A vector $x\epsilon X_{\bar{\alpha}}$ is called the $\bar{\alpha}$-feasible solution to problem

Definition 2.4. Let $\preceq$ be a fuzzy extension of binary relation $\le$ and let $x=(x_1,\cdots,x_n)^T\epsilon\square^n$ be an $\bar{\alpha}$-feasible solution to (3), where $\bar{\alpha}=(\alpha_1,\cdots,\alpha_m)\,\epsilon\,(0,1)^m$ and let $Z(\tilde{c},x)$ be a fuzzy objective. The vector $x\in\square^n$ is an $\bar{\alpha}-$efficient solution to (3) with maximization of the objective function, if there is no any $x'\in X_{\bar{\alpha}}$ such that $\tilde{c}x\preceq\tilde{c}x'$.

Similarly, an $\bar{\alpha}-$efficient solution with minimization of the objective function can be defined.

Pay attention that any $\bar{\alpha}-$efficient solution to the FFLP problem is an $\bar{\alpha}-$feasible solution to the FFLP problem with some extra properties. In the following theorem, we represent the necessary and sufficient condition for an $\bar{\alpha}-$efficient solution to (3).

Theorem 2.1. Let $\bar{\alpha}=(\alpha_1,\ldots,\alpha_m)\in(0,1]^m$ and $x^*=(x_1^*,\ldots,x_n^*)^T$, $x_j^*\geq 0,\ j=1,\ldots,n$ be an $\bar{\alpha}-$feasible solution to (3). Then a vector $x^*\in\mathbb{R}^n$ is an $\bar{\alpha}-$efficient solution to Problem (3) with maximization of the objective function, if and only if x^* is an optimal to the following problem:

$$\begin{aligned} \max\quad & \tilde{z}(x)=z(\tilde{c},x) \\ s.t.\quad & a_i x\leq b_i+p_i(1-\alpha_i),\ i=1,\ldots,m, \\ & x_j\geq 0,\ \alpha_i\geq\alpha_i^D, 0\leq\alpha_i\leq 1,\ j=1,\ldots,n, \end{aligned} \tag{4}$$

where p_i is the predefined maximum tolerance.

Proof. Let $\bar{\alpha}=(\alpha_1,\ldots,\alpha_m)\in[0,1]^m$ and $x^*=(x_1^*,\ldots,x_n^*)^T$, $x_j^*\geq 0,\ j=1,\ldots,n$ be an $\bar{\alpha}-$efficient solution to Problem (7) with maximization of the objective function. By Definition 2.3 and equation (1), we have $a_i x^*\leq b_i+p_i(1-\alpha_i),\alpha_i\geq\alpha_i^D$ for $i=1,\ldots,m$. Therefore, x^* is a feasible solution to (4). Also by Definition 2.3, there is no any $x'\in X_{\bar{\alpha}}$ such that $Z(\tilde{c},x^*)<Z(\tilde{c},x')$, it means that x^* is an optimal solution to (4), and in this case x^* is obviously an $\bar{\alpha}-$feasible solution to Problem (3). Thus by Definition 2.4, the optimality of x^* implies the $\bar{\alpha}-$efficiency of x^*. ■

Proposition 2.1. Let $\bar{\alpha}=(\alpha_1,\ldots,\alpha_m)\in(0,1]^m$, then $X_{\bar{\alpha}}=\bigcap_{i=1}^{m}X_{\alpha_i}^i$, where

$$X_{\alpha_i}^i=\{x\in\mathbb{R}^n \setminus x\geq 0,\alpha_i\geq\alpha_i^D,a_i x\leq b_i+p_i(1-\alpha_i)\} \tag{5}$$

For $i=1,\ldots,m$ (namely, X_{α}^i is the α-cut of the i-th fuzzy constraint).

Proof. For any $\bar{\alpha}=(\alpha_1,\ldots,\alpha_m)\in(0,1]^m$, let $x\in X_{\bar{\alpha}}$, therefore $\alpha_i\geq\alpha_i^D$, $a_i x\leq b_i+p_i(1-\alpha_i)$. Now and from (5) we have $x\in X_{\alpha_i}^i, i=1,\ldots,m$, and therefore $x\in\bigcap_{i=1}^{m}X_{\alpha_i}^i$. On the other hand, if $x\in\bigcap_{i=1}^{m}X_{\alpha_i}^i$, we have $x\in X_{\alpha_i}^i$, for all $i=1,\ldots,m$. Therefore $\alpha_i\geq\alpha_i^D$, $a_i x\leq b_i+p_i(1-\alpha_i)$ and hence $x\in X_{\bar{\alpha}}$. This completes the proof. ■

Proposition 2.2. Let $\bar{\alpha}'=(\alpha_1',\ldots,\alpha_m')$ and $\bar{\alpha}''=(\alpha_1'',\ldots,\alpha_m'')$, where $\alpha_i'\leq\alpha_i''$ for all i, then $\bar{\alpha}''-$feasibility of x implies the $\bar{\alpha}'-$feasibility of it.

Proof. The proof is straightforward. ■

For a given $\alpha \in (0,1]$, let $x \in \mathbb{R}^n$ be a usual $\alpha-$feasible solution to (3) (a solution with the same degrees of satisfaction in all constraints). It has the meaning of $a_i x \le b_i + p_i (1-\alpha_i), \alpha_i \ge \alpha_i^D$ or equivalently $x \in X_\alpha^i$, for all $i = 1,\dots,m$.

If $\bar{\alpha} = (\alpha, \dots, \alpha) \in (0,1]^m$, then $x \in X_\alpha$ which implies that the $\alpha-$feasibility of (3) can be understood as a special case of the $\bar{\alpha}-$feasibility. Thus, the following result can be obtained.

Remark 2.1. If the problem (3) is not infeasible, then X_α is not empty.

Proof. The proof is straightforward. ■

2.2. Ranking function

On the other hand, ranking of fuzzy numbers is an important issue in the study of fuzzy set theory. Ranking procedures are also useful in various applications and one of them will be in the study of fuzzy linear programming problems. Many methods for solving fuzzy linear programming problems are based on comparison of fuzzy numbers and in particular using ranking functions [23, 25]. An effective approach for ordering the elements of $F(R)$ is to define a ranking function $\Re : F(R) \to R$ which maps each fuzzy number into the real line, where a natural order exists.

We consider the linear ranking functions on $F(R)$ as $\Re(\tilde{a}) = c_L a^L + c_U a^U + c_\alpha \alpha + c_\beta \beta$, where $\tilde{a} = (a^L + a^U + \alpha + \beta)$ and $c_L, c_U, c_\alpha, c_\beta$ are constants, at least one of which is nonzero.

A special version of the above linear ranking function was first proposed by Yager [49] as follows: for trapezoidal numbers $\tilde{a} = (a^L + a^U + \alpha + \beta)$ we have $\Re(\tilde{a}) = \frac{a^L + a^U}{2} + \frac{1}{4}(\beta - \alpha)$.

3. Fuzzy flexible linear programming problem with fuzzy cost coefficients

Let us consider the following fuzzy mathematical programming problem,

$$
\begin{aligned}
& max f(x, \tilde{c}) \\
& s \cdot t. \; g_i(x) \preceq 0, \\
& x \ge 0, \\
& i = 1, \cdots, m,
\end{aligned}
\tag{6}
$$

where $x = (x_1, \cdots, x_n)^T$ is an n-dimensional real decision vector $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_n)$ is an n-dimensional fuzzy vector of fuzzy parameters involved in the objective function f, where $f(x, \tilde{c}) \approx \tilde{c}x$, $g_i(x) \le 0 \approx a_i^T x \le b_i$.

Generally, the model (6) is not well-defined because:

i. We cannot maximize the fuzzy quantity $f\left(x, \tilde{c}\right)$

ii. The constraints $g_i\,(x) \preceq 0,\ i = 1, \cdots, m,$ do not produce a crisp feasible set.

One appropriate approach to state a crisp optimal solution preference of alternative is comparing fuzzy quantities by means of ranking function $\Re : F\left(\mathbb{R}\right) \to \mathbb{R}$ that maps each fuzzy quantity to real line which exists a natural order (see for more detail in [7,40]).

Also, if we want to define a deterministic feasible set, an idea is to provide confidence level α_i at which it is desired that the corresponding i −th fuzzy constraint holds. Therefore, in order to remove those mentioned restrictions, the following problem will be introduced.

$$\begin{aligned} & max \sum_{j=1}^{n} \tilde{c}_j\, x_j \\ & s \cdot t \quad \mu_i\left\{g_i(x) \preceq 0\right\} \geq \alpha_i\,, \\ & x \geq 0\,, \\ & 0 < \alpha_i \leq 1\,, \\ & i = 1, \cdots, m. \end{aligned} \tag{7}$$

To motivate for a meaningful choice of membership function for each fuzzy constraints, it is argued that if $g_i(x) \leq 0$, then the i-th constraint is absolutely satisfied, where as if $g_i(x) \geq p_i$, where p_i the predefined maximum tolerance from zero, as determined by the decision marker, then the i-th constraint is absolutely violated. for $g_i(x) \epsilon\ (0, p_i)$, the membership function is monotonically decreasing. If this decrease is along a linear function, then it makes sense to choose the membership function of the i-th constraint $(i = 1, 2, \cdots, m)$ as

$$\mu_i\left\{g_i(x) \preceq 0\right\} = \begin{cases} 1\,, & g_i(x) \leq 0\,, \\ 1 - \frac{g_i(x)}{p_i}\,, & 0 \leq g_i(x) \leq p_i\,, \\ 0\,, & g_i(x) \geq p_i\,, \end{cases}$$

Note that in the objective function, the coefficient $\tilde{c}_j$ is a fuzzy number. Here, the Yager ranking functions is used. for trapezoidal numbers $\tilde{a} = \left(a^L + a^U + \alpha + \beta\right)$ we have $\Re\left(\tilde{a}\right) = \dfrac{a^L + a^U}{2} + \dfrac{1}{4}\left(\beta - \alpha\right).$

Thus, Problem (7) can then be rewritten as:

$$max\ \Re\left(f\ (x, \tilde{c})\right),$$

$$s \cdot t. \quad \mu_i\left\{g_i(x) \preceq 0\right\} \geq \alpha_i ,$$

(8)

$$x \geq 0 ,$$
$$0 < \alpha_i \leq 1 ,$$
$$i = 1 , \cdots , m$$

Also using Theorem 2.1, we have following problem:

$$max\ \Re\left(f\ (x , \tilde{c})\right)$$
$$s \cdot t. \quad g_i(x) \leq (1 - \alpha_i)p_i ,$$

(9)

$$i = 1 , \cdots , m ,$$
$$x \geq 0.$$

In Theorem 2.1, we have provided a computational method to solve fuzzy flexible linear programming problem (3). Thus, by assigning a specific $\bar{\alpha}$ by a decision maker, we may replace the α_j in the corresponding constraint of (4), and solve the resulted problem to compute the $\bar{\alpha}-$ efficient solution to the problem (3). An $\bar{\alpha}-$efficient solution to (3) has two characteristics:

i. The solution has various satisfaction degrees corresponding to each constraint.
ii. The acquired solution is optimal.

This solution permits decision maker to obtain a more flexible and more compatibility by assigning desired preferences, especially, in online optimization in more noticeable.

In Theorem 2.1, a method to FFLP problem is introduced to obtain an $\bar{\alpha}-$efficient solution. If the resulting problem (4) has only one optimal solution, then we have confirmed. So, this solution is an $\bar{\alpha}-$efficient solution to the given fuzzy problem. In the case of which Problem (4) has some multiple optimal solutions. In order to find a maximum efficient solution, i.e., in an $\bar{\alpha}'-$efficient solution with $\alpha' \geq \alpha,\ i = 1,\ldots, m,$ we apply the following two-phase approaches.

In the two-phase approach, problem (9) is solved in Phase Ⅰ, while as in Phase II, a solution is obtained and has higher satisfaction degree than the previous solution. So by using two- phase approach, we can obtain a better utilization of available resources. Also, the solution resulting by this two- phase approach is always an $\bar{\alpha}'-$efficient solution.

We will call the Problem (9) as Phase Ⅰ problem.

Let $\bar{\alpha}^0 = (\ \alpha_1{}^0 , \cdots , \alpha_m{}^0)$, and $\Re\left(x^*, f\ \left(x^*, c\right)\right)$ be the optimal solution of Phase Ⅰ with $\bar{\alpha}^0$ degree of efficiency. Set $\alpha_i{}^* = \mu_i\left\{g_i(x^*) \preceq 0\right\} \geq \alpha_i{}^0,\ i = 1, \cdots , m.$ In phase II, we solve the following problem,

$$\max \quad \textstyle\sum_{i=1}^{m} \alpha_i$$
$$s.t.\ \Re f\ \left(x , c\right) \geq \Re f\ \left(x^*, c\right)$$

(10)

$$g_i(x) \leq (1 - \alpha_i)p_i ,$$
$$\alpha_i{}^* \leq \alpha_i \leq 1, \quad i = 1 , \cdots , m$$

$$x \geq 0.$$

Now, we are a place to present the solving method. Algorithm 3.1 contains the main steps of the solving process.

3.1. Algorithm 3.1(Main steps of the proposed algorithm for FFLP problem)

Assumption1: A fuzzy mathematical model in the form of Fuzzy Flexible Linear Programming (FFLP) is given to solve. (The parameters of the model including a_i, b_i, p_i and $\tilde{c}_j$ for $j = 1,\dots,n$, and α_i^D for all $i = 1,\dots,m$ are given).

Step 1: Using the given membership function for the constraints, the main problem becomes form (7).

Step 2: Using trapezoidal numbers and Yager ranking function, Problem (7) can be written as Problem (8).

Step 3: the linear programming problem (9) is solved in phase I and first obtain the optimal value of x^* and α^*, and then the optimal value of the objective function (z^*).

Step 4: Solve Problem (10) is solved in Phase II and a solution is obtained and has higher satisfaction degree than the previous solution.

4. Case study

The objective of the diet problem is to select a family of foods that will satisfy a set of daily nutritional requirement at a minimum cost. The classical diet problem is stated as a linear optimization problem [28]. Fuzzy Flexible Diet problems (FFD) with a fuzzy objective in which, for each food $j = 1,\dots,n$ there is some vagueness on its corresponding cost which is modelled by means of a fuzzy number defined by a membership function $\mu_j \in F(R)$ such that $\mu_j : R \to [0,1]$. The problem can be formulated as follows:

$$\min \sum_{j=1}^{n} \tilde{c}_j x_j$$

$$s.t.\ \underline{\tilde{b}}_i \preceq \sum_{j=1}^{n} a_{ij} x_j \preceq \bar{\tilde{b}}_i, i = 1,\dots,m.$$

$$\underline{p}_j \leq x_j \leq \bar{p}_j, j = 1,\dots,n.$$

With regard to address a mathematical version of this problem, we need to define some variables. Let consider a set of foods $f = \{x_1, x_2, \dots, x_n\}$ a set of nutrients $N = \{A_1, A_2, \dots, A_n\}$, and be following variables:

$\tilde{c}_j$ = cost of the food j, $j = 1,\dots,n$,

x_j =amount of food j to eat, $j = 1,...,n$,

a_{ij} =amount of nutrient i in food, $j = 1,...,n$, $i = 1,...,m$,

$\underline{b}_i$ = minimum amount of nutrient i required, $i = 1,...,m$,

$\bar{b}_i$ =maximum amount of nutrient i allowed per day, $i = 1,...,m$,

$\underline{p}_j$ = minimum amount of food j desired per day, $j = 1,...,n$,

$\bar{p}_j$ = maximum amount of food j desired per day, $j = 1,...,n$.

Among several different food items used in livestock feed, only 11 foods (hay, grain, Sugar beet dried pulp, corn silage with dry matter between 32 and 38%, cottonseed meal with 41% CP, Calcium soaps, Sugar beet molasses, soybean meal with 44% CP, sunflower meal, wheat bran and Shell powder) used and for nutrients needs supply only seven nutrients (energy, protein, fat, calcium, phosphorus, NDF, NFC) were considered. Table 1 shows the chemical composition of these materials:

Table1. chemical compositions of feed

Feeds	Crude protein (g/kg)	NDF[3] (g/kg)	NFC[4] (g/kg)	Fat (g/kg)	Ca (g/kg)	P (g/kg)	NE1 (Kcal/kg)	Price[5] (Rial/kg)
Alfalfa	192	416	257	25	14.7	2.8	1190	5800 9000
Barley grain	124	208	617	22	0.6	3.9	1860	8000 9700
Sugar beet pulp	10	458	358	11	9.1	0.9	1470	8500 10500
Corn silage	88	450	387	32	2.8	2.6	1450	8250 10400

[3] Neutral Detergent Fiber (NDF)
[4] Non Fibrous Carbohydrates (NFC)
[5] The feed price obtained from Mazandaran Farming and Animal Husbandry Cooperative Union in June 2016.

Using tables of nutritional requirements of dairy cattle (NRC) [36], minimum and maximum nutritional requirements are obtained and used in fuzzy optimization problems solving. Table 2 shows the requirements and needs of lactating dairy cows in early lactation based on the minimum and maximum amount.

Cottonseed meal	449	308	157	19	2.0	11.5	1710	14500 17000
Fat supplement	0.0	0.0	0.0	845	120	0.0	5020	5000 7000
Sugar beet molasses	85	1.0	798	2.0	1.5	0.3	1840	2000 3500
Soybean meal	499	149	270	16	4.0	7.1	2130	17500 19500
Sunflower meal	284	403	222	14	4.8	10	1380	10500 12000
Wheat bran	173	425	296	43	1.3	11.8	1610	7100 8500
Oyster meal	0.0	0.0	0.0	0.0	380	0.0	0.0	20000 30000

Table 2. Nutritional requirements (kg/ dry matter) of lactating cows in early lactation, weighing between 600-700 kg

Nutrient requirements	**Unit**	**Consistent model**		
		Minimum	**Maximum**	**Equivalent**
Energy	Kcal/kg	1500	1650	
Protein	gr/kg	155	180	
Ether extract	gr/kg	30	80	
NDF	gr/kg	300	400	
NFC	gr/kg	350	420	
Calcium	gr/kg	10		
Phosphor	gr/kg	5		
Total carbohydrate	gr/kg		730	
Ratio of ca: P	gr/kg			2

Total ration	gr/kg			1
Alfalfa hay	gr/kg		250	
Barley grain	gr/kg		300	
Sugar beet pulp	gr/kg		150	
Corn silage	gr/kg		150	
Cottonseed meal	gr/kg		120	
Fat supplement	gr/kg		40	
Sugar beet molasses	gr/kg		30	
Soybean meal	gr/kg		120	
Sunflower meal	gr/kg		100	
Wheat bran	gr/kg		150	
Oyster meal	gr/kg		25	

Interval and floating prices of nutrients are gathered from the market, converted to trapezoidal fuzzy numbers.

$\tilde{c}_1 = (7000, 7500, 1200, 1500)$, $\tilde{c}_2 = (8500, 9100, 500, 600)$, $\tilde{c}_3 = (9500, 10000, 1000, 500)$,

$\tilde{c}_4 = (9900, 10000, 750, 400)$, $\tilde{c}_5 = (16000, 16500, 1500, 500)$, $\tilde{c}_6 = (5500, 6500, 500, 500)$,

$\tilde{c}_7 = (2500, 3000, 500, 500)$, $\tilde{c}_8 = (18000, 18500, 500, 1000)$, $\tilde{c}_9 = (11000, 11500, 500, 500)$,

$\tilde{c}_{10} = (7500, 8000, 400, 500)$, $\tilde{c}_{11} = (23000, 25000, 3000, 5000)$.

The fuzzy linear programming approach is a method that will be used to solve our research problem. So, we establish the Minimize Cost Diet problem (MCD) model as follows below:

$$
\begin{aligned}
&\min \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \tilde{c}_3 x_3 + \tilde{c}_4 x_4 + \tilde{c}_5 x_5 + \tilde{c}_6 x_6 + \tilde{c}_7 x_7 + \tilde{c}_8 x_8 + \tilde{c}_9 x_9 + \tilde{c}_{10} x_{10} + \tilde{c}_{11} x_{11} \\
&s.t.\ \ 1190x_1 + 1860x_2 + 1470x_3 + 1450x_4 + 1710x_5 + 5020x_6 + 1840x_7 + 2130x_8 + 1380x_9 + 1610x_{10} \succeq 150\tilde{0}, \\
&\quad 1190x_1 + 1860x_2 + 1470x_3 + 1450x_4 + 1710x_5 + 5020x_6 + 1840x_7 + 2130x_8 + 1380x_9 + 1610x_{10} \preceq 165\tilde{0}, \\
&\quad 192x_1 + 124x_2 + 10x_3 + 88x_4 + 499x_5 + 85x_7 + 499x_8 + 284x_9 + 173x_{10} \succeq 15\tilde{5}, \\
&\quad 192x_1 + 124x_2 + 10x_3 + 88x_4 + 499x_5 + 85x_7 + 499x_8 + 284x_9 + 173x_{10} \preceq 18\tilde{0}, \\
&\quad 25x_1 + 22x_2 + 11x_3 + 32x_4 + 19x_5 + 845x_6 + 2x_7 + 16x_8 + 14x_9 + 43x_{10} \succeq 3\tilde{0}, \\
&\quad 25x_1 + 22x_2 + 11x_3 + 32x_4 + 19x_5 + 845x_6 + 2x_7 + 16x_8 + 14x_9 + 43x_{10} \preceq 8\tilde{0}, \\
&\quad 416x_1 + 208x_2 + 458x_3 + 450x_4 + 308x_5 + x_7 + 149x_8 + 403x_9 + 425x_{10} \succeq 30\tilde{0}, \\
&\quad 416x_1 + 208x_2 + 458x_3 + 450x_4 + 308x_5 + x_7 + 149x_8 + 403x_9 + 425x_{10} \preceq 40\tilde{0}, \\
&\quad 257x_1 + 617x_2 + 358x_3 + 378x_4 + 157x_5 + 798x_7 + 270x_8 + 222x_9 + 296x_{10} \succeq 35\tilde{0}, \\
&\quad 257x_1 + 617x_2 + 358x_3 + 378x_4 + 157x_5 + 798x_7 + 270x_8 + 222x_9 + 296x_{10} \preceq 43\tilde{0}, \\
&\quad 14.7x_1 + 0.6x_2 + 9.1x_3 + 2.8x_4 + 2x_5 + 120x_6 + 1.5x_7 + 4x_8 + 4.8x_9 + 1.3x_{10} + 380x_{11} \succeq 1\tilde{0}, \\
&\quad 2.8x_1 + 3.9x_2 + 0.9x_3 + 2.6x_4 + 11.5x_5 + 0.3x_7 + 7.1x_8 + 10x_9 + 11.8x_{10} \succeq \tilde{5}, \\
&\quad 673x_1 + 825x_2 + 816x_3 + 837x_4 + 465x_5 + 799x_7 + 419x_8 + 625x_9 + 721x_{10} \preceq 73\tilde{0}, \\
&\quad 9.1x_1 - 7.2x_2 + 7.3x_3 - 2.4x_4 - 21x_5 + 120x_6 + 0.9x_7 - 10.2x_8 - 15.2x_9 - 22.3x_{10} + 380x_{11} = 0, \\
&\quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} = 1, \\
&\quad x_1 \le 0.25, x_2 \le 0.30, x_3 \le 0.15, x_4 \le 0.15, x_5 \le 0.12, x_6 \le 0.04, x_7 \le 0.03, x_8 \le 0.12, \\
&\quad x_9 \le 0.10, x_{10} \le 0.15, x_{11} \le 0.025, \\
&\quad x_j \ge 0,\ j = 1, 2, \ldots, 11.
\end{aligned}
$$

In the above model the predefined maximum tolerance for the constraints is as follows: $p_1 = 15$, $p_2 = 16$, $p_3 = 1$, $p_4 = 2$, $p_5 = 0.3$, $p_6 = 0.8$, $p_7 = 3$, $p_8 = 4$, $p_9 = 3.5$, $p_{10} = 4$, $p_{11} = 0.1$, $p_{12} = 0.05$, $p_{13} = 7$.

So, using linear ranking function such as Yager's ranking function and with the following membership functions

$$
\mu_i^{\preceq}(A_i x, b_i) = \begin{cases} 1, & A_i x \le b_i, \\ 1 - (A_i x - b_i)/p_i, & b_i \le A_i x \le b_i + p_i, \\ 0, & A_i x > b_i + p_i. \end{cases}
$$

and

$$\mu_i^{\succ}\left(A_i x, b_i\right)=\begin{cases}1, & A_i x \geq b_i\,, \\ 1-\left(b_i-A_i x\right)/p_i\,, & b_i-p_i \leq A_i x \leq b_i\,, \\ 0, & A_i x \leq b_i-p_i\,.\end{cases}$$

we will obtain in Phase I the following Multi- Parametric Linear Programming Problem:

$$\begin{aligned}
\min\ & 7550x_1+8825x_2+9625x_3+9862.5x_4+16000x_5+6000x_6+2750x_7+18375x_8+11250x_9+7775x_{10}+24500x_{11} \\
s.t.\ & 1190x_1+1860x_2+1470x_3+1450x_4+1710x_5+5020x_6+1840x_7+2130x_8+1380x_9+1610x_{10} \geq 1500-15\left(1-\alpha_1\right), \\
& 1190x_1+1860x_2+1470x_3+1450x_4+1710x_5+5020x_6+1840x_7+2130x_8+1380x_9+1610x_{10} \leq 1650+16\left(1-\alpha_2\right), \\
& 192x_1+124x_2+10x_3+88x_4+499x_5+85x_7+499x_8+284x_9+173x_{10} \geq 155-1\left(1-\alpha_3\right), \\
& 192x_1+124x_2+10x_3+88x_4+499x_5+85x_7+499x_8+284x_9+173x_{10} \leq 180+2\left(1-\alpha_4\right), \\
& 25x_1+22x_2+11x_3+32x_4+19x_5+845x_6+2x_7+16x_8+14x_9+43x_{10} \geq 30-0.3\left(1-\alpha_5\right), \\
& 25x_1+22x_2+11x_3+32x_4+19x_5+845x_6+2x_7+16x_8+14x_9+43x_{10} \leq 80+0.8\left(1-\alpha_6\right), \\
& 416x_1+208x_2+458x_3+450x_4+308x_5+x_7+149x_8+403x_9+425x_{10} \geq 300-3\left(1-\alpha_7\right), \\
& 416x_1+208x_2+458x_3+450x_4+308x_5+x_7+149x_8+403x_9+425x_{10} \leq 400+4\left(1-\alpha_8\right), \\
& 257x_1+617x_2+358x_3+378x_4+157x_5+798x_7+270x_8+222x_9+296x_{10} \geq 350-3.5\left(1-\alpha_9\right), \\
& 257x_1+617x_2+358x_3+378x_4+157x_5+798x_7+270x_8+222x_9+296x_{10} \leq 430+4\left(1-\alpha_{10}\right), \\
& 14.7x_1+0.6x_2+9.1x_3+2.8x_4+2x_5+120x_6+1.5x_7+4x_8+4.8x_9+1.3x_{10}+380x_{11} \geq 10-0.1\left(1-\alpha_{11}\right), \\
& 2.8x_1+3.9x_2+0.9x_3+2.6x_4+11.5x_5+0.3x_7+7.1x_8+10x_9+11.8x_{10} \geq 5-0.05\left(1-\alpha_{12}\right), \\
& 673x_1+825x_2+816x_3+837x_4+465x_5+799x_7+419x_8+625x_9+721x_{10} \leq 730+7\left(1-\alpha_{13}\right), \\
& 9.1x_1-7.2x_2+7.3x_3-2.4x_4-21x_5+120x_6+0.9x_7-10.2x_8-15.2x_9-22.3x_{10}+380x_{11}=0, \\
& x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10}+x_{11}=1, \\
& x_1 \leq 0.25, x_2 \leq 0.30, x_3 \leq 0.15, x_4 \leq 0.15, x_5 \leq 0.12, x_6 \leq 0.04, x_7 \leq 0.03, x_8 \leq 0.12, \\
& x_9 \leq 0.10, x_{10} \leq 0.15, x_{11} \leq 0.025, \\
& x_j \geq 0, j=1,2,\ldots,11, \\
& 0<\alpha_i \leq 1, i=1,2,\ldots,13.
\end{aligned}$$

Some $\bar{\alpha}$-efficient solution with satisfaction degrees which decision maker's desire can be found in the following table.

Table 3. Some typical α -feasibility solutions (phase Ⅰ)

a	b	c	d	e	f
$\bar{\alpha}$	$(0.5,\ldots,0.5)$	$(0.5,\ldots,0.5,0.3)$	$(0.7,0.5,\ldots,0.5)$	$(0.5,0.7,0.5,\ldots,0.5)$	$(0.5,0.7,\ldots,0.7)$

$c^T x$	8609.002	8609.002	8609.002	8623.517	8634.006
x_1	0.2500000	0.2500000	0.2500000	0.2500000	0.2500000
x_2	0.2434476	0.2434476	0.2434476	0.2376289	0.2356219
x_3	0.2514197×10^{-1}	0.2514197×10^{-1}	0.2514197×10^{-1}	0.2996718×10^{-1}	0.3038981×10^{-1}
x_4	0.1500000	0.1500000	0.1500000	0.1500000	0.1500000
x_5	0.1194604×10^{-1}	0.1194604×10^{-1}	0.1194604×10^{-1}	0.1329528×10^{-1}	0.1475858×10^{-1}
x_6	0.3946235×10^{-1}	0.3946235×10^{-1}	0.3946235×10^{-1}	0.3910828×10^{-1}	0.3922969×10^{-1}
x_7	0.3000000×10^{-1}	0.3000000×10^{-1}	0.3000000×10^{-1}	0.3000000×10^{-1}	0.3000000×10^{-1}
x_8	0.000000	0.000000	0.000000	0.000000	0.000000
x_9	0.1000000	0.1000000	0.1000000	0.1000000	0.1000000
x_{10}	0.1500000	0.1500000	0.1500000	0.1500000	0.1500000
x_{11}	0.2014579×10^{-5}	0.2014579×10^{-5}	0.2014579×10^{-5}	0.3610787×10^{-6}	0.000000
α_1	0.5	0.5	0.7	0.5	0.5
α_2	0.5	0.5	0.5	0.7	0.7
α_3	0.5	0.5	0.5	0.5	0.7
α_4	0.5	0.5	0.5	0.5	0.7
α_5	0.5	0.5	0.5	0.5	0.7
α_6	0.5	0.5	0.5	0.5	0.7
α_7	0.5	0.5	0.5	0.5	0.7
α_8	0.5	0.5	0.5	0.5	0.7
α_9	0.5	0.5	0.5	0.5	0.7
α_{10}	0.5	0.5	0.5	0.5	0.7
α_{11}	0.5	0.5	0.5	0.5	0.7
α_{12}	0.5	0.5	0.5	0.5	0.7
α_{13}	0.5	0.3	0.5	0.5	0.7

If all of the satisfaction degrees are equal, then the $\bar{\alpha}$-feasibility and $\bar{\alpha}$-efficiency reduce to classic α- feasibility and α-optimality (see Table 3, column b). Let

$$x^* = (0.2500000, 0.2434476, 0.2514197\times10^{-1}, 0.1500000, 0.1194604\times10^{-1}, 0.3946235\times10^{-1},$$

$$0.3000000\times10^{-1}, 0.000000, 0.1000000, 0.1500000, 0.2014579\times10^{-5}).$$

be $(0.7, 0.5, 0.5, \dots, 0.5)$ -efficient solution with $c^T x^* = 8609.002$ as an optimal objective value (see Table 3, column d). Therefore, in Phase II, we need to solve the following linear programming

$$\max \sum_{i=1}^{13} \alpha_i$$

$$\begin{aligned}
s.t.\quad & 7550x_1 + 8825x_2 + 9625x_3 + 9862.5x_4 + 16000x_5 + 6000x_6 + 2750x_7 + 18375x_8 + 11250x_9 + 7775x_{10} + 24500x_{11} \ge 8609.002,\\
& 1190x_1 + 1860x_2 + 1470x_3 + 1450x_4 + 1710x_5 + 5020x_6 + 1840x_7 + 2130x_8 + 1380x_9 + 1610x_{10} \ge 1500 - 15(1-\alpha_1),\\
& 1190x_1 + 1860x_2 + 1470x_3 + 1450x_4 + 1710x_5 + 5020x_6 + 1840x_7 + 2130x_8 + 1380x_9 + 1610x_{10} \le 1650 + 16(1-\alpha_2),\\
& 192x_1 + 124x_2 + 10x_3 + 88x_4 + 499x_5 + 85x_7 + 499x_8 + 284x_9 + 173x_{10} \ge 155 - 1(1-\alpha_3),\\
& 192x_1 + 124x_2 + 10x_3 + 88x_4 + 499x_5 + 85x_7 + 499x_8 + 284x_9 + 173x_{10} \le 180 + 2(1-\alpha_4),\\
& 25x_1 + 22x_2 + 11x_3 + 32x_4 + 19x_5 + 845x_6 + 2x_7 + 16x_8 + 14x_9 + 43x_{10} \ge 30 - 0.3(1-\alpha_5),\\
& 25x_1 + 22x_2 + 11x_3 + 32x_4 + 19x_5 + 845x_6 + 2x_7 + 16x_8 + 14x_9 + 43x_{10} \le 80 + 0.8(1-\alpha_6),\\
& 416x_1 + 208x_2 + 458x_3 + 450x_4 + 308x_5 + x_7 + 149x_8 + 403x_9 + 425x_{10} \ge 300 - 3(1-\alpha_7),\\
& 416x_1 + 208x_2 + 458x_3 + 450x_4 + 308x_5 + x_7 + 149x_8 + 403x_9 + 425x_{10} \le 400 + 4(1-\alpha_8),\\
& 257x_1 + 617x_2 + 358x_3 + 378x_4 + 157x_5 + 798x_7 + 270x_8 + 222x_9 + 296x_{10} \ge 350 - 3.5(1-\alpha_9),\\
& 257x_1 + 617x_2 + 358x_3 + 378x_4 + 157x_5 + 798x_7 + 270x_8 + 222x_9 + 296x_{10} \le 430 + 4(1-\alpha_{10}),\\
& 14.7x_1 + 0.6x_2 + 9.1x_3 + 2.8x_4 + 2x_5 + 120x_6 + 1.5x_7 + 4x_8 + 4.8x_9 + 1.3x_{10} + 380x_{11} \ge 10 - 0.1(1-\alpha_{11}),\\
& 2.8x_1 + 3.9x_2 + 0.9x_3 + 2.6x_4 + 11.5x_5 + 0.3x_7 + 7.1x_8 + 10x_9 + 11.8x_{10} \ge 5 - 0.05(1-\alpha_{12}),\\
& 673x_1 + 825x_2 + 816x_3 + 837x_4 + 465x_5 + 799x_7 + 419x_8 + 625x_9 + 721x_{10} \le 730 + 7(1-\alpha_{13}),\\
& 9.1x_1 - 7.2x_2 + 7.3x_3 - 2.4x_4 - 21x_5 + 120x_6 + 0.9x_7 - 10.2x_8 - 15.2x_9 - 22.3x_{10} + 380x_{11} = 0,\\
& x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} = 1,\\
& x_1 \le 0.25, x_2 \le 0.30, x_3 \le 0.15, x_4 \le 0.15, x_5 \le 0.12, x_6 \le 0.04, x_7 \le 0.03, x_8 \le 0.12,\\
& x_9 \le 0.10, x_{10} \le 0.15, x_{11} \le 0.025,\\
& 0.7 \le \alpha_1 \le 1,\quad 0.5 \le \alpha_4 \le 1,\quad 0.5 \le \alpha_7 \le 1,\quad 0.5 \le \alpha_{10} \le 1,\quad 0.5 \le \alpha_{13} \le 1,\\
& 0.5 \le \alpha_2 \le 1,\quad 0.5 \le \alpha_5 \le 1,\quad 0.5 \le \alpha_8 \le 1,\quad 0.5 \le \alpha_{11} \le 1,\\
& 0.5 \le \alpha_3 \le 1,\quad 0.5 \le \alpha_6 \le 1,\quad 0.5 \le \alpha_9 \le 1,\quad 0.5 \le \alpha_{12} \le 1,\\
& x_j \ge 0, j = 1, 2, \dots, 11,\\
& 0 < \alpha_i \le 1, i = 1, 2, \dots, 13.
\end{aligned}$$

Table 4. Comparison of the solutions of the first and second stage problems

Optimal solution Phase I of column (d)	Optimal solution phase II
$(0.7, 0.5, 0.5, ..., 0.5)$	$(0.7, 0.5, 0.5, ..., 0.5)$
8609.002	8609.002
0.2500000	0.2500000
0.2434476	0.2038018
0.2514197×10^{-1}	0.9254536×10^{-1}
0.1500000	0.1500000
0.1194604×10^{-1}	0.3495168×10^{-1}
0.3946235×10^{-1}	0.3520736×10^{-1}
0.3000000×10^{-1}	0.000000
0.000000	0.000000
0.1000000	0.8349377×10^{-1}
0.1500000	0.1500000
0.2014579×10^{-5}	0.000000
0.7	1
0.5	1
0.5	1
0.5	1
0.5	1
0.5	1
0.5	1
0.5	1
0.5	1
0.5	1
0.5	1

0.5	1
0.5	1

An optimal solution to the above problem is $x^{**} = (0.2500000, 0.2038018, 0.9254536\times10^{-1},$
$0.1500000, 0.3495168\times10^{-1}, 0.3520736\times10^{-1}, 0.000000, 0.000000, 0.8349377\times10^{-1}, 0.1500000, 0.000000).$

Also $c^T x^{**} = c^T x^* = 8609.002$. We have $\mu_2(A_2 x^{**}, b_2) = \mu_3(A_3 x^{**}, b_3) = \mu_4(A_4 x^{**}, b_4) = \mu_5(A_5 x^{**}, b_5) = \mu_6(A_6 x^{**}, b_6) = \mu_7(A_7 x^{**}, b_7) = \mu_8(A_8 x^{**}, b_8) = \mu_9(A_9 x^{**}, b_9) = \mu_{10}(A_{10} x^{**}, b_{10}) = \mu_{11}(A_{11} x^{**}, b_{11}) = \mu_{12}(A_{12} x^{**}, b_{12}) = \mu_{13}(A_{13} x^{**}, b_{13}) = 0.5$ and $\mu_1(A_1 x^{**}, b_1) = 1$.

Thus, using the two- phase approach, we can get an optimal solution x^{**}which not only achieves the optimal objective value but also gives a higher membership value in μ_1.

We saw that after improving the achieved an $\alpha-$feasibility solution in Phase II, the Convenient optimal solution with the higher feasibility degree is concluded. So, the above illustrative example showed that the proposed approach suitably helps the decision maker to obtain an $\alpha-$efficiency optimal solution.

5. Conclusion

In this study, a two- phase approach for solving fuzzy flexible linear programming as one of the comfortable model which is formulated in some real situations proposed. The method based on extending $\alpha-$feasibility solution to $\alpha-$efficiency solution is established. In the illustrative example, we saw that the defined method in Phase II suitably can improve the satisfaction degree of the solution based on the new proposed concept. One of the pillars in aquaculture farming industries is formulation of food for the animals, which is also known as feed mix or diet formulation. However, the feed component in the aquaculture industry incurs the most expensive operational cost and has drawn many studies regarding diet formulation. The lack of studies involving modelling approaches had motivated to embark on diet formulation, which searches for the best combination of feed ingredients while satisfying nutritional requirements at a minimum cost. The result of model solving is given in Table 3, Some $\bar{\alpha}$-efficient solution with satisfaction degrees which decision maker's desire and The result of model solving is given in Table 4 We saw that after improving the achieved an $\alpha-$feasibility solution in Phase II, the Convenient optimal solution with the higher feasibility degree is concluded. So, the above illustrative example showed that the proposed approach suitably helps the decision maker to obtain an $\alpha-$efficiency optimal solution. However, that the optimal value of the objective function has decreased significantly [32]. However, in sale market, prices are constantly faced changing and fluctuating, and therefore diet planners forcibly change diets according to price volatility. However, by considering safety margin for food price fluctuations, it can be possible to plan with more flexibility and confidence. Adjust the feed ration by the linear programming model is done with actual data. Although it is irrational to assume that there is often complete and accurate information about the data and needs and food prices used in the problem. Therefore, fuzzy optimization method with floating price has been recommended to more accurate formulation of nutrient needs and feed amounts. According to the concepts of fuzzy sets and numbers used in this method, diet formulation will be more profitable and realistic. By this method, it can be possible to formulate cheaper and more suitable rations. Although this method is used in the dairy food ration formulation, it can also be applied to other kinds of diet formulation. Also providing software where fuzzy optimization method is used in animal feed rations, formulation will be helpful for the work of writers and researchers.

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