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A New Optimization Model for Hierarchical Location and Districting Problem in Healthcare System Under Uncertainty

A. Rahimi[1], A. H. Azadnia[2*], M. Molani Aghdam[3], F. Harsej[4]

Health care facility systems are hierarchical as they consist of facilities at different levels such as clinics, health centers, and hospitals. Therefore, finding a proper location for the health care system can be categorized as a hierarchical location problem. Besides, partitioning a given region in a geographical area into different zones is very crucial to make sure the health services are available at their highest possible level for everyone in that region. In this study, an optimization model for the integrated problem of hierarchical location and partitioning under uncertainty in the Iranian healthcare system is proposed. The objective function of this model maximizes the total social utility of districts while workload balance and distance limitation between the zones are considered as the main constraints. Since this study involves NP-hard problems, three metaheuristic algorithms, including Genetic, Salp Swarm Algorithm (SSA), and Grey Wolf Optimizer (GWO) were developed. The numerical results suggest that the Grey Wolf Optimizer (GWO) algorithm indicates a more appropriate level of performance in almost all responses compared to the other algorithms. Therefore, the case study was solved by the Grey Wolf Optimizer (GWO). Based on the results, 10 distrcis with their zones are identified to maximize the overall utility. A sensitivity analysis also performed to show the behavior of the model. It can be stated that the findings of this study can be utilized as a useful management tool in other organizations.

Keywords*: Healthcare System; Location problem; Hierarchical Partitioning; Metaheuristic Algorithms*

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1. Introduction

Geographical partitioning refers to the classification of small demographic areas into large groups known as districting in the research literature (Lin, Chin, Fu, & Tsui, 2017). This issue has been taken into consideration in different fields such as political partitioning, commercial partitioning, and service partitioning. A novel application of this issue applies to developing strategic plans related to the healthcare system (Kalcsics & Ríos-Mercado, 2019). Each country is classified into several main regions, each region is subsequently divided into separate subdivisions (Farughi, Tavana, Mostafayi, & Santos Arteaga, 2020) aiming at improving the implementation of equipment supply operations, human resources, implementation of health-related plans, such as vaccinations, social screening plans, and also the allocation of funds for the establishment of general and specialized treatment centers. Therefore, by managing the implementation of operational plans in a hierarchical structure, it can be guaranteed that the monitoring of executive processes will be much easier. The field studies conducted suggest that the Iranian healthcare system is currently designed at three main levels as follows (Farughi, Mostafayi, & Arkat, 2018):

[1] Department of Industrial Engineering, Nour Branch, Islamic Azad University, Nour, Iran, Email: amirrahimi1368@icloud.com.
[2] Department of Business Studies, Letterkenny Institue of Technology, Ireland, Email:amir.azadnia@lyit.ie
[3] Innovation and Management Research Center, Ayatollah Amoli Branch, Islamic Azad University, Amol, Iran, Email: mohammadmolani@gmail.com.
[4] Department of Industrial Engineering, Nour Branch, Islamic Azad University, Nour, Iran, Email: harsej88@gmail.com.

Level One: It involves units in which the first and widest contact of community with the healthcare system takes place. These units are health houses and rural and urban health centers. People working in the health houses are mainly men and women health employees while general practitioners, specialists, and health technicians work in the health centers.

Level Two: It includes the health centers and hospitals that can provide a more specialized level of service. At health centers, general practitioners work with a range of healthcare professionals and diagnostic facilities, including laboratories and radiology, dentistry, and midwifery. The set of first and second-level units within the geographical scope of the county (based on political divisions) is called the healthcare network of that province.

Level Three: This level provides specialized and sub-specialized services in the medical and paramedical sectors (Farughi, Mostafayi, & Arkat, 2019).

Taking into account the structure of the healthcare system in Iran, it is necessary to study the hierarchical partitioning issue to improve the status quo, in which determining the health zones (national zones) and health networks (regional zones) should be configured in an integrated and optimized model. However, proper determination of criteria related to districting is another important issue. In other words, it is necessary to determine the criteria considered by managers to create a zone to fulfill strategic goals. In the review of related literature, some of the most important criteria for partitioning the demographic areas include balancing the population in each zone for proper distribution of workload (Darmian, Fattahi, & Keyvanshokooh, 2021). Likewise, minimization of intra-zone distances to receive a variety of healthcare services for patients is also considered as another criterion. The objective of this criterion is that the individuals in each zone travel the shortest possible distance to receive a variety of services. As another criterion, we can refer to the total inter-zone connections (Kalcsics & Ríos-Mercado, 2019). This is important because the government shall be able to meet all the medical needs of applicants in each zone. For instance, patients living in the east of the country should not need to go to medical centers in the capital for treatment of a particular disease, rather they should receive their required medication in the healthcare areas assigned to them. In this study, this case was considered as the objective function for the optimization model to finally design a healthcare system with higher productivity. It should be noted that the outcome of this study is merely the initial configuration of the healthcare system as a strategic plan and allocation of human resources and equipment at the health network level.

According to the numerical results of similar studies, solving optimization models for partitioning the demographic areas is one of the difficult problems, known in the research literature of operations as NP-hard problems (non-deterministic polynomial-time hardness) (Farughi, Mostafayi, et al., 2019). This problem cannot be solved using commercial software and requires the development of metaheuristic algorithms. Therefore, well-known metaheuristic algorithms are used to obtain the final answers in this study.

The second part of the study involves the review of the related literature to examine the research gap from a theoretical and operational perspective. The third part involves the description of the mathematical model utilized in the research and the structure of metaheuristic algorithms is then presented. In the fourth part, the studied case is described and the status quo is presented. The fifth part describes the computational results and numerical analyses, including case studies, as well as sensitivity analysis. Finally, the sixth part summarizes and presents managerial perspectives.

2. Literature Review

Districting generally involves the grouping of small geographical areas, called basic units or base areas, into larger geographical groups known as regions or zones, such that they are acceptable according to the relevant planning criteria. Common instances of base units are customer, street or postal areas. Depending on the practical context, partitioning is also called territory design, territory alignment, zone design, or sector design (Chen, Cheng, & Ye, 2019). Important domains of healthcare system partitioning are expressed in the sub-sectors of Districting of Home Health Care Services, Districting of Primary and Secondary Health Care Services, and Districting of Emergency Health Services. In today's world, the aging population is constantly increasing throughout the world, implying that the demand for common and specific healthcare services is steadily increasing (Gutiérrez-Gutiérrez & Vidal, 2015). Two notable issues of partitioning (or districting) are demand points at a tactical or strategic level and the location and planning of employees at an operational level. Partitioning involves strategic (or tactical) decision-making. This issue aims at identifying service regions that include demand scores (e.g., patients) to provide healthcare services (e.g., nurses) (Lin, Chin, Ma, & Tsui, 2018).

Planning a healthcare system in a specific region involves decisions such as locating healthcare services and identifying each base unit in territories, service delivery, capacity determination, resource allocation, staff planning, etc. Effective healthcare service planning minimizes costs, improves capacity utilization, increases patient satisfaction due to service level, improves accessibility, and ensures justice in the community in terms of access to healthcare services (Steiner, Datta, Neto , Scarpin, & Figueira, 2015).

Blais, Lapierre, and Laporte (2003) used several criteria to obtain an almost optimal partitioning plan for a home healthcare center in Ontario, Canada. To solve the problem, they utilized a multi-criteria clustering method instead of mathematical programming methods. Hertz and Lahrichi (2009) divided the workload of home healthcare providers into three parts: 1. travel load, 2. visit load, and 3. work case load.

Bennett proposed a set coverage model in which the workload of the zone is used as a constraint to create operational regions, and the costs of used operational areas were defined approximately as location costs in each area. The innovative clustering method and enhancement of local search were used to obtain and improve the initial operational areas. The partitioning model is then solved using column generation ideas and innovative local search methods. First, the linear relaxation of the partitioning model is solved. Then, the local search method is used to improve the columns to add to the set (Bennett, 2009). Cortés, Gutiérrez, Palacio, and Villegas (2018) presented a modeling approach to the partitioning problem of healthcare at home, which included a mixed-integer linear programming (MILP) and a greedy randomized adaptive search procedure (GRASP). Computational experiments performed with a set of real-life samples from a Colombian home healthcare provider showed that the metaheuristic algorithm can reduce workload imbalances by up to 57%.

Tayyebi and Mostafayi Darmian (2020) analyzed one of the most widely used issues in the field of operations management entitled the problem of partitioning. The objective of solving this problem is to divide a community into several regions such that each region can fully cover the healthcare services of its population to the desired extent. According to the strategic model of the healthcare system, an approach based on genetic optimization algorithms, particle swarm optimization, and differential evolution was investigated to divide population areas into ten zones. The findings indicated that the particle swarm algorithm had the highest efficiency and the differential evolution algorithm had the lowest efficiency. Sudtachat, Mayorga, Chanta, and Albert (2020) proposed a modeling method for relocation and partitioning in emergency healthcare systems. This mixed-method was performed to increase the efficiency of EMS systems. They developed the nested adaptation model (which offers a relocation policy) to provide a high limit on relocation time by dividing the whole region into smaller sub-areas (zones). Farughi, Tavana, Mostafayi, and Santos Arteaga (2019) presented a new multi-objective mathematical model for designing compact, balanced, and adjacent areas in healthcare systems. Fitness functions minimize heterogeneity and the cost of implementing monitoring programs to promote public healthcare. Expert teams carry out these programs periodically by maximizing the covered areas. The objective of the districting problem is to determine how teams are formed and assigned based on their service capacity and type of expertise. Improper team allocation and inadequate service delivery increase time and cost and harm promoting public healthcare in the area. To solve the real-scale mathematical model, two metaheuristic algorithms, including Multi-Objective Genetic Algorithm II (Non-dominated Sorting Genetic Algorithm II - NSGAII) and Multi-Objective Grey Wolf Optimizer (MOGWO), were defined, and a case study was presented to demonstrate the application and efficiency of the proposed model. To recap, the most important studies with similar concerns can be observed in the table below.

Table 1. A summary of articles with similar topics

Reference	Emergency healthcare services	Primary and Secondary Health Care Services	Home Health Care Services	Health system districting	Uni-objective	Multi-objective	Metaheuristic	Precise	Hierarchical
Darmian et al.,) (2021				✓	✓		✓		
(Nikzad, Bashiri, & Abbasi, 2021)			✓		✓			✓	
(Diglio, Peiró, Piccolo, & Saldanha-da-Gama, 2021)				✓	✓		✓		
(Diogo, Vargas, Wanke, & Correa, 2021)			✓		✓		✓		
Ghollasi, Hosseini) Nasab, Fakhrzad, & (Tayyebi, 2020				✓		✓	✓		
(Kim, 2020)				✓	✓		✓		
Han, Hu, Mao, &) (Wan, 2020				✓		✓	✓		
Liu, Erdogan, Lin,) (& Tsao, 2020				✓		✓	✓		
Sudtachat et al.,) (2020	✓				✓		✓		
Tayyebi &) Mostafayi Darmian, (2020				✓	✓		✓		
Farughi, Tavana, et) (al., 2019				✓		✓		✓	
Farughi, Mostafayi,) (et al., 2019				✓		✓	✓		
(Cortés et al., 2018)			✓		✓			✓	
Regis-Hernández,) Lanzarone, Bélanger, & Ruiz, (2018	✓				✓		✓		
Tran, Dinh, &) (Gascon, 2017				✓	✓		✓		

(Lin et al., 2017)			✓		✓		✓		
(Steiner et al., 2015)		✓				✓		✓	
Gutiérrez-Gutiérrez) (& Vidal, 2015			✓			✓		✓	
(Jia et al., 2014)		✓			✓		✓		
Datta, Figueira,) Gourtani, & Morton, (2013		✓				✓	✓		
Benzarti, Sahin, &) (Dallery, 2013			✓		✓		✓		
Mahar, Bretthauer,) (& Salzarulo, 2011		✓				✓	✓		
(Bennett, 2009)			✓		✓		✓		
Hertz & Lahrichi,) (2009			✓		✓		✓		
(Blais et al., 2003)			✓		✓		✓		
The Present Study				✓	✓		✓		✓

Table 1 shows the analysis of the relevant literature in the field. Taking into account the literature review, it can be observed that districting has not been considered hierarchically by researchers hitherto. However, according to the definition provided for the research problem, developing multilevel districting as hierarchical structures have many applications in the real world. Additionally, this study will also be one of the few investigations to design a new mathematical model with the purpose of creating very difficult mathematical constraints to ensure optimal continuity in the territories and also preparing appropriate conditions for ensuring the development of compact zones. Furthermore, utilizing metaheuristic algorithms as a common solution method is also investigated.

3. Problem Statement and the Mathematical Model

The foremost concern of this study is to design an approach based on optimization methods to configure geographical zones for districting the Iranian healthcare system in line with improving the strategic executive plan by the Ministry of Science and Research to determine healthcare poles in Iran. According to this plan, the country's demographic regions are divided into 10 main categories, namely, "health zones", each of which implements action plans approved by the Ministry of Health and Medical Education under the supervision of a central committee. In each "health zone", hospital and treatment facilities at both general and specialized levels will be created as per the demand of people in that area for various health services to minimize the number of patients referring to other centers outside the designated zone. Hereby the government budgeting for tackling the problems in each region can be facilitated in addition to providing better supervision over the health status of a zone, like the spread of certain diseases such as cholera and influenza. Thus, to improve the status quo, it is necessary to investigate the country's health system as a hierarchical districting issue in which health zones (national zones) and health networks (regional zones) are configured in an integrated way as an optimization model. In fact, decisions should be made in such a way that a main zone is primarily determined and sub-zones are later recognized in each zone. This guarantees the better performance of management affairs related to the system because by designating a specific center for each sub-zone, operational level decisions are made in the same sub-zone and the macro-level management does not get engaged in such affairs. However, the status of implementation will be informed through correspondences and reporting and the necessary decisions will be made if major correction is required. This issue is more important in hospital systems that originally have a hierarchical organizational and operational structure. In this system, each hospital has several subdivisions that are responsible for providing services.

The proposed formulation in an Undirected Graph is G = (V, E) where V is a set of vertices (applicants) and E is a set of relations between the vertices. In other words, the set $V = \{v_i | i = 1,2, \dots, |V|;\}$ represents the set of vertices in the graph, where N is the number of vertices. In this set, each v_i vertex is specified with vertical and horizontal coordinates (x_i, y_i). Similarly, $E = \left\{e_{ij}; i, j = 1,2, \dots, |E|; i \neq j; e_{ij} = e_{ji}\right\}$ is a set of graph edges ($|E|$ Size), indicating the relationship between vertices v_i and v_j with $e_{ij} \in \{0,1\}$. If there is a relationship between v_i and v_j, then $e_{ij} = 1$; otherwise $e_{ij} = 0$. Each v_i vertex also has a demand equal to $\alpha_i \geq 0$.

A major issue in solving the districting problem is the difficulty of designing mathematical constraints to ensure the continuity of zones. Two path-based and flow-based approaches can be applied to establish continuity constraints; In the path-based approach, the shortest distance between two base units in a zone must be within that zone, meaning that the zone shall be almost convex. In this study, a flow-based approach was used to design the mathematical constraints to ensure the continuity of zones. This constraint ensures that when the flow begins from the center of each zone, all nodes assigned to it have a neighboring structure. A node can be assigned to a zone when there is a continuous (direct or indirect) flow between the center of the zone and that node. According to the explanations provided, the mathematical structure of the research model can be described as follows.

Sets	
V	Set of basic units
K	Set of first level zones
P	Set of second level zones
C	Set of potential points for the establishment of centers $C \subseteq V$,
A	Set of pairs of base units adjacent to the first level
B	Set of pairs of base units adjacent to the second level
Input Parameters	
α_i	Demand in node *i*
θ_i	Service delivery capacity in node *i*
L_{ij}	Distance between points *i* and *j*
T_{max}	Maximum allowable distance between points in each zone
μ	Maximum allowable workload difference between second level zones
β_i	Social utility coefficient for choosing the center *i* of first level zone
Decision Making Parameters	
X_{ip}	Binary variable which is equal to 1 if node *i* is assigned to domain *p*.
W_{ip}	Binary variable which is equal to 1 if node *i* is selected as the center of zone *p*. In other words, if (*i* = *p*)
Y_{ijp}	The rate of flow from node *i* to node *j* for zone p
U_{ik}	Binary variable equal to 1 if node *i* is assigned to first level zone *k*.

$$Max\ Z = \sum_{i \in C} \left(\beta_i \sum_{k \in K} U_{ik} \right)$$ ١

s.t

$$\sum_{i \in V} (\theta_i - \alpha_i) X_{ip} - \sum_{i \in V} (\theta_i - \alpha_i) X_{ip'} \leq \mu \sum_{i \in V} \alpha_i \qquad p \neq p' \in P$$ ٢

$$L_{ij} \leq T_{max} + |M| \left(2 - X_{ip} + X_{jp} \right) \qquad i, j \in V$$ ٣

$$\sum_{p \in P} X_{ip} = 1 \qquad i \in V$$ ٤

$$\sum_{i \in V} W_{ip} = 1 \qquad p \in P$$ ٥

$$\sum_{j:(i,j) \in B} Y_{ijp} - \sum_{j:(i,j) \in B} Y_{jip} = X_{ip} - |V|\, W_{ip} \qquad p \in P, i \in C$$ ٦

$$\sum_{j:(i,j) \in B} Y_{ijp} \leq (|V| - 1) X_{ip} \qquad p \in P, i \in V$$ ٧

$$\sum_{k \in K} W'_{ik} = 1 \qquad i \in C$$ ٨

$$\sum_{i \in C} U_{ik} = 1 \qquad k \in K$$ ٩

$$\sum_{j:(i,j) \in A} Y'_{ijk} - \sum_{j:(i,j) \in A} Y'_{jik} \geq W'_{ik} - |V|\, U_{ik} - |V| \sum_{p \in P} W_{ip} \qquad k \in K, i \in C$$ ١٠

$$\sum_{j:(i,j) \in A} Y'_{ijk} \leq (|V| - 1) W'_{ik} + |V| \sum_{p \in P} W_{ip} \qquad k \in K, i \in C$$ ١١

$$U_{ik} \leq |V| \sum_{p \in P} W_{ip} \qquad k \in K, i \in C$$ ١٢

$$W_{ip}, W'_{ik}, X_{ip}, U_{ik} \in \{0,1\} \qquad p \in P, k \in K, i \in V$$
$$Y_{ijp}, Y'_{ijk} \geq 0 \qquad p \in P, (i,j) \in B \qquad k \in K, (i,j) \in A$$ ١٣

The problem's objective function deals with the maximization of the utility of choosing the center of the first-level zone. This objective function is presented because the community requesting healthcare services always tends to have basic services and management of different sectors under the supervision of a well-known center in the country. For instance, the regional population in southeastern Iran tends to be covered by the Zahedan supervision system. However, it might be desirable to choose Kerman in terms of workload balance (of course, Kerman is currently selected based on the information provided by the Health Organization). Constraint (2) ensures that the population located in each second-level zone does not exceed the predetermined value. Constraint (3) guarantees that the geographical distance between points located in each second level zone does not exceed the predetermined value. Constraint (4) ensures that each population point is allocated to only one first-level zone. Constraint (5) guarantees that only one point is recognized as the center of any first-level zone. The set of constraints (6) and (7) ensures continuity in each zone. These constraints generate justified responses by assigning at least one of the points adjacent to each center must first be assigned to it. Then, according to adjacency structure of other parts in the network, only nodes can be assigned to a center that are connected to at least one of the nodes adjacent to that center. This new node is then added to set of nodes adjacent to the center and can act as a communication channel on the path between points so that other points can join the center. This ensures that the structure of all zones is created continuously. Constraints (8) to (11) exactly guarantee the necessary settings to create appropriate zones in the first level. Constraints (12) and (13) also indicate the range of quantification of decision variables.

3.1 The mathematical model under uncertainty

It is very important to consider that solving the research problem under uncertainty of input parameters (Rouzpeykar et al, 2020). This is important because it is always difficult to estimate the appropriate rate of input parameters in real-world conditions and is considered as a challenge by managers of health organizations. According to experts, determining the appropriate rate of demand is a difficult task and it is not possible to determine the specific rate of demand using expert approaches. Therefore, it is necessary to use scientific approaches to create appropriate control over the numerical results obtained under uncertainty. Numerous methods have been proposed to control the level of uncertainty hitherto, one of the most effective of which is utilizing a solid planning approach. In this study, the uncertainty in demand is considered as a time interval and the model presented below is used to address it.

In this approach, it is assumed that the demand parameter is in the range $[\bar{\alpha}_i - \hat{\alpha}_i, \bar{\alpha}_i + \hat{\alpha}_i]$. It is necessary to make changes in the original model to use the approach proposed by Bertsimas and Sim (2004). In this way, the following constraints replace constraint (2) of the original model.

$$\sum_{i \in V} \Delta_i X_{ip} \leq F_p \qquad p \in P \qquad ۱۴$$

$$F_p - F_{p'} \leq \mu \qquad p \neq p' \qquad ۱۵$$

Where F_p is a positive variable, calculating the population assigned to each zone. Likewise, $\Delta_i = \delta_i - \alpha_i$. Clearly, there is uncertainty in the parameter Δ_i which be defined in $[\bar{\Delta}_i - \hat{\Delta}_i, \bar{\Delta}_i + \hat{\Delta}_i]$ range. Therefore, constraint structure (14) changes according to the theory presented in the study by Bertsimas and Sim (2004) as follows:

$$\sum_{i \in V} \left(\bar{\Delta}_i X_{ip} + \overbrace{\max_{\substack{S: S \subseteq V, |S| \leq \Gamma \\ (i_t) \in V/S}} \left(\hat{\Delta}_i X_{ip} + (\Gamma - \lfloor \Gamma \rfloor)\, \hat{\Delta}_{i_t} X_{i_t p} \right)}^{\theta} \right) \leq F_p \qquad p \in P \qquad ۱۶$$

Where Γ is the parameter regulating the level of uncertainty and takes a value in $[0, |V|]$ range. If $\Gamma = 0$, then no change is allowed and it is equivalency problem or certainty status. Also, if the Γ is an integer, the maximum value of statement (16) will be equal to $\max_{\substack{S: S \subseteq V, |S| \leq \Gamma \\ (i_t) \in V/S}} \left(\hat{\Delta}_i x_{ip} \right)$ But if $\Gamma = \lfloor J \rfloor$J |, then we will observe equivalency problem with the worst status and will be similar to Soyster Method. As it is clear, part θ of equation (3-16) is nonlinear and cannot be solved optimally globally. Therefore, it is necessary to develop its linear structure using appropriate transformations. For this purpose, the variable K_{ip} is defined by the following conditions.

$\sum_{i\in V} K_{ip} \le \Gamma$	$p \in P$	۱۷
$0 \le K_{ip} \le 1$	$i \in V, p \neq p'$	۱۸

Hence, part θ of Equation (16) can be considered equivalent to the following model.

$\max_{p\in P}\left(\hat{\Delta}_i x_{ip} K_{ip}\right)$		۱۹
$s.t$		
$\sum_{i\in V} K_{ip} \le \Gamma$	$p \in P$	۲۰
$0 \le K_{ip} \le 1$	$i \in V, p \neq p'$	۲۱

The optimal solution for this model must have $\lfloor \Gamma \rfloor$variable, $K_{ip} = 1$ and a $K_{ip} = \Gamma - \lfloor \Gamma \rfloor$ which is equivalent to the optimal solution of part θ of equation (16). Using a strong duality for the given values $\left(X_{ip}\right)_{i=1,\dots,V\ ,p=1,\dots,P}$, the duality of the above model is formulated as follows.

$\min_{p\in P}\left(\Gamma\ U_p + \sum_{i\in P} U_{ip}\right)$		۲۱
$s.t$		
$U_p + U_{ip} - \hat{\Delta}_i X_{ip} \ge 0,$	$i \in V, p \neq p'$	۲۲
$U_{ip} \ge 0,$	$p \in P$	۲۳
$U_p \ge 0,$	$i \in V, p \neq p'$	۲۴

By placing the above model in constraint (16), the linear structure of the problem can be obtained.

$\sum_{i\in V}\left(\bar{\Delta}_i x_{ip} + \Gamma\ U_p + \sum_{i\in P} U_{ip}\right) \le F_p$	$p \in P$	۲۵
$U_p + U_{ip} - \hat{\Delta}_i X_{ip} \ge 0,$	$i \in V, p \neq p'$	۲۶
$U_{ip} \ge 0,$	$p \in P$	۲۷
$U_p \ge 0,$	$i \in V, p \neq p'$	۲۸

Finally, the robust counterpart of the research model is as follows.

$Max\ Z = \sum_{i \in C} \left(\beta_i \sum_{k \in K} U_{ik} \right)$ $s.t.$ $\sum_{i \in V} \left(\bar{\Delta}_i x_{ip} + \Gamma\ U_p + \sum_{i \in P} U_{ip} \right) \leq F_p$ $F_p - F_{p'} \leq \mu$ (3-3) – (3-13) $U_p + U_{ip} - \hat{\Delta}_i X_{ip} \geq 0,$ $U_p \geq 0$ $U_{ip} \geq 0$	۲۹

By changing the value of Γ in the defined interval, the behavior of the model under different conditions of uncertainty can be investigated.

4. Solution Method

Considering that districting is an NP-hard problem (Farughi, Tavana, et al., 2019), metaheuristic algorithms are used to solve medium and large numerical samples. According to a review of the related literature, using population-based algorithms derived from natural structures is more applicable than other algorithms for solving the districting problem (Farughi, Mostafayi, et al., 2019). The most well-known algorithm in this regard is the genetic algorithm, which is highly efficient in solving districting problems (Bacao, Lobo, & Painho, 2005). Therefore, this algorithm is also used in this research.

However, the important issue in evaluating the performance of the proposed model and solution algorithm is a proper generation of different numerical instances and their solution. Therefore, in this study, a systematic structure is presented to produce different numerical instances. The noteworthy point in these instances is the coordinates of population points and the Euclidean distance between population points, as well as the adjacency matrix between population points, which must be precisely defined. The communication between the points should be as a connected graph network and the distance between the points is then calculated using the Euclidean distance. For this purpose, for instance, a two-dimensional environment is first considered. For all the produced instances, the coordinates of base units are considered between 0-500.

4.1. Generating the Initial Response

To generate initial responses, a string of zeros and ones equal to the number of potential zones of the same surface is generated initially for each level of districting, such that the sum of its numbers is equal to the number of zones. This ensures that the number of generated zones is equal to the number of decision-making inputs. For instance, in the hypothetical string shown in Figure 1, 3 centers have been selected out of 5 potential centers.

۱	۰	۱	۰	۱

Figure.1 String example

In the above string, centers 1, 3, and 5 were selected as zone centers. Then another string equal to the number of vertices is produced in which integers between 1 and the number of zones are generated. Each number in each house represents the zone to which that vertex is assigned. For instance, the following string in figure 2 shows the number of 10 vertices that should be allocated to the selected centers in the above string.

۱	۳	۵	۱	۵	۳	۱	۱	۳	۵

Figure.2 String example

As observed, vertices 1, 4, 7, and 8 were assigned to center number 1, vertices 2, 6, and 9 to center number 3 and vertices 3, 5, and 10 are assigned to center number 5. In this way, an initial response can be created for the first and second levels of districting. However, this allocation only shows the number of vertices assigned to each zone, and their order may change as the response generation process continues because these allocations do not guarantee the production of continuous zones. Therefore, it can be said that 4 vertices should be allocated to center 1 and three vertices shall be assigned to centers 3 and 5 each. In this study, a Breadth-First-Search (BFS) method is used to ensure the creation of zone with a continuous structure. To this end, the points assigned to each center of the above string are determined, and then, using the BFS algorithm, the number of specified vertices is assigned to each selected center. Given that the BFS algorithm visits neighboring points in each iteration, the continuous structure of zones will always be maintained (Sbihi, 2007). The same procedure is repeated for the second level of districting, in which the vertices are actually the centers of low-level zones. In this way, justified initial responses are generated. It is noteworthy that in case of violation of zones created from the maximum allowable distance and also the maximum allowable load difference with other zones, a penalty equivalent to the fifth power of zones' utility is deducted from the total value of the fitness function of that response so that the produced response has no advantage for being in the process of improvement. In fact, unjustified answers are automatically eliminated.

4.2. Generating initial response under uncertainty in metaheuristic algorithms

The objective of the Bertsimas and Sim approach is to investigate the effect of uncertainty of the problem's input parameters on the model behavior in obtaining different answers. Therefore, an array equal to the number of network points, with integer values of zero or one, is considered in the proposed algorithms. The maximum number of "ones" in this array will be equal to Γ. Hence, the model can consider all compounds from 1-element to Γ –element compounds for selection, which is based exactly on the mathematical structure of the Bertsimas method. The elements of this array in different iterations of algorithm and in the evolution process might be inclined to the direction that the worst possible case with Γ elements are measured in the numerical results, meaning that for each element whose value in the aforementioned array was equal to one, the demand deviation limit is added to its value. In fact, the demand for elements whose value in the array is equal to one is at its worst possible status. It should be noted that there is no limit to the selection of elements with a value of one, and therefore the algorithm can choose the worst possible case for Γ-element number. This ensures that the model is robust against the worst possible conditions and can therefore continue to provide robust responses against even easier conditions.

4.3. Genetic Algorithm

The genetic algorithm was invented by Holland in 1973 (Davis, 1991). This algorithm's structure is such that initially a set of solutions to the problem are randomly generated (Naghshnilchi, 2019). This set is called the primary population (or generation) and each element, which is actually a solution, is called a chromosome. In each iteration of the algorithm, a set of new solutions called offspring are generated using genetic operators on chromosomes of the current generation (parents). These operators are classified into two main groups, crossover operator and mutation operator. The new generation is chosen using the selection operator from the parents and children of the current generation. The selection operator is applied in such a way that chromosomes with a higher fitness function have a better chance of survival. In the main loop of genetic algorithm, production of new generations continues until reaching the criterion of cessation, and finally the best chromosome in the last generation is presented as the chosen solution. It should be noted that the implementation of genetic operators is a very important issue in designing genetic algorithms, which is described below.

4.3.1. Genetic Algorithm Operators

Crossover and mutation are the main operators of genetic algorithms to generate offspring (Saadat et al, 2019). Crossover is a process in which, by combining the information of two parents, one (or more) new solutions are produced as offspring. In the present study, a crossover was utilized as the main operator, and mutation was employed as the second operator. The used crossover operator is a two-point section type. The mutation operator is also applied by choosing several genes from chromosomes and changing their values. After this step, the replacement operator is applied. The purpose of replacement is to choose the right parents in each generation to be present in the next generation. In other words, the objective of the replacement strategy is to produce a new generation that is better than the current generation average in terms of fitness. In this study, the random sampling strategy without replacement is used for the replacement operation (Deng, Liu, & Zhou, 2015). The following pseudocode presents the structure of the genetic algorithm used in the study:

Input*: fitness function, max iteration, Population size, Crossover rate, Mutation rate*
Output*: the elitist*
Initialize a population randomly
Calculate the fitness of population and find elite
$t = 0$
    While $t \leq T$ ***do***
        Perform crossover using two-point crossover operator
        Perform Mutation
        Carry out the replacement strategy and evaluate
        Calculate the fitness and return elite
        $t = t + 1$
    End
Final solution ← elite
End
Return *Final Solutions*

Figure 3. Pseudocode: Genetic Algorithm

4.4. Salp Swarm Algorithm (SSA)

Salps belong to the Salpidae family and have a clear body. Their texture is very similar to jellyfish. They also move very much like jellyfish such that water in their body flows forward as a driving force. To mathematically model the salp chains, the population is first divided into two groups: leader and follower. The leader is in front of the chain while the rest of salps are considered followers. As the name implies, the leader leads the crowd and the followers follow each other (and the leader directly or indirectly).

Similar to swarm-based techniques, the position of salps is defined in an n-dimensional search space, where n is the number of variables given. The position of all salps is stored in a two-dimensional matrix called x. It is also assumed that there is a food source called F in the search space as the target of congestion. The following equation is proposed to update the leader position.

$$X_j^1 = \begin{cases} F_j + c_1\left(\left(ub_j - lb_j\right)c_2 + lb_j\right) & c_3 \geq 0 \\ F_j - c_1\left(\left(ub_j - lb_j\right)c_2 + lb_j\right) & c_3 < 0 \end{cases} \qquad ۳۰$$

X_j^1 indicates the position of the first (leader) in j dimensions. F_j is the position of food source in dimensions j, ub_j and lb_j represent the upper and lower boundaries of dimension $_j$, respectively. c_1, c_2 and c_3 are random numbers. The above equation indicates that the leader only updates his position according to the food source. The coefficient c_1 is the most important parameter in SSA because it defines the exploration and exploitation balance as follows:

$$c_1 = 2e^{-\left(\frac{4\,l}{L}\right)^2} \qquad ۳۱$$

l is the current iteration and L is the maximum number of iterations. The parameters c_2 and c_3 are uniform random numbers generated in [0,1] interval. In fact, they decide whether the next position in j^{th} dimension should be towards infinite positive or infinite negative direction and also the step size. The following equations are used to update the position of followers (Newton's Law of Motion):

$$X_j^i = \frac{1}{2}at^2 + v_0 t \qquad ۳۲$$

In the above equation, $i\ \geq 2$ X_j^i indicates the position of the ith following salp in the jth dimension. t is time, v_0 is the initial velocity and $a = \frac{v_{final}}{v_0}$, where $v = \frac{x - x_0}{t}$. Since it is time for the iteration optimization, the difference between iterations is equal to 1, and given that $v_0 = 0$, this equation can be expressed as follows:

$$X_j^i = \frac{1}{2}\left(X_j^i + X_j^{i-1}\right) \qquad ۳۳$$

Where i $i\ \geq 2$ and X_j^i represents the position of ith following salp in the jth dimension. With the above equations, the salp chains can be simulated.

Initialize the salp population $X_i(i\ =\ 1, 2, \ldots, n)$ considering ub and lb
While*(end condition is not satisfied)*
   Calculate the fitness of each search agent(salp)
   F=the best search agentUpdate c1
   for *each salp (xi)*
      if *(i==1)Update the position of the leading salp*
      else *Update the position of the follower salp*
   End
  End
Amend the salps based on the upper and lower bounds of variables
End

return F

Figure 4. Salp Swarm Algorithm (SSA)

4.5. Grey Wolf Algorithm

The best response in each iteration of algorithm is considered as α. Subsequently, the second and third better responses are called β, δ. The rest of candidate responses are assumed to be omega. In the GWO algorithm, the hunt (optimization) is directed by α, β, and δ. The ω wolves follow these three wolves.

4.5.1Encircling Prey

Gray wolves surround their prey during hunting. inTo mathematically model the encircling behavior, the following expressions are considered.

$$\vec{X}(t+1) = \overrightarrow{X_p}(t) + \vec{A}.\vec{D}, \tag{۳۴}$$

$$\vec{D} = \left|\vec{C}.\overrightarrow{X_p}(t) - \vec{X}(t)\right|, \tag{۳۵}$$

$$\vec{A} = 2\vec{a}.\vec{r}_1 - \vec{a}, \tag{۳۶}$$

$$\vec{C} = 2.\vec{r}_2, \tag{۳۷}$$

Where t indicates the current iteration. $(\vec{A})$ and $(\vec{C})$are coefficient vectors. $(\overrightarrow{X_p})$and $(\vec{X}(t))$are the hunting position vector and the wolf position vector, respectively. $(\vec{a})$ decreases linearly from 2 to 0. $(\vec{r}_1)$ and $(\vec{r}_2)$ are random vectors in [۰،۱] .

4.5.2. Hunt

There is no idea about the optimal location (prey) in a discrete search space. In order to mathematically simulate the gray wolf hunting behavior, it is assumed that alpha (the best response among the available answers), beta and delta have better information regarding the hunting location. Therefore, the 3 better results obtained are saved and other search agents (including omega) are forced to update their position according to the position of the best search agent.

$$\vec{D}_\alpha = \left|\vec{C}_1.\vec{X}_\alpha - \vec{X}\right|, \tag{۳۸}$$

$$\vec{D}_\beta = \left|\vec{C}_2.\vec{X}_\beta - \vec{X}\right|, \tag{۳۹}$$

$$\vec{D}_\delta = \left|\vec{C}_3.\vec{X}_\delta - \vec{X}\right|, \tag{۴۰}$$

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1.(\vec{D}_\alpha), \tag{۴۱}$$

$$\vec{X}_2 = \vec{X}_\beta - \vec{A}_2.(\vec{D}_\beta), \tag{۴۲}$$

$$\vec{X}_3 = \vec{X}_\delta - \vec{A}_3.(\vec{D}_\delta), \quad ۴۳$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}. \quad ۴۴$$

4.5.3. Attacking Prey

The gray wolf attacks when the prey stops, decreasing the value of $\vec{a}$. The oscillation range $(\vec{A})$ also decreases. $(\vec{A})$ is a random value in [-2a, 2a] where a decreases from 2 to 0 during iterations. In other words, the algorithm allows search agents to update their position according to the position of alpha, beta and delta.

4.5.4. Search for prey (exploration)

Grey wolves search mainly according to positions of alpha, beta and delta. They search for prey separately and attack the prey together. $(\vec{A})$is used with a random value greater than (1) or less than (-1). In other words, it helps the algorithm's exploration. $(\vec{C})$ contains random values in [0-2] and another element of exploration. This element indicates the random weights for hunting to emphasize $(C > 1)$ or trivialize $(C < 1)$the effect of hunting in determining the distance in the above equations. Vector (C) is also considered as the effect of obstacles in close hunting in nature.

Input: $\begin{cases} (n) Number\ of\ gray\ wolves\ in\ the\ pack \\ (N_{Iter})\ Number\ of\ iterations\ for\ optimization \end{cases}$

Output: $\begin{cases} (X_\alpha) Optimal\ gray\ wolf\ position \\ f(X_\alpha)\ Best\ fitness\ value \end{cases}$

Initialize a population of n gray wolves positions randomly.
Find the(α), (β) *and* (δ) *solutions based on their fitness values.*
 While *Stopping criteria not met* ***Do***
  For *each* $Wolf_i \in$ *pack do*
   Update current wolf's position according to equation (25)
  End
  I. *Update a, A, and C.*
  II. *Evaluate the positions of individual wolves.*
  III. *Update* (α), (β) *and* (δ)
 End

Figure 5. Grey Wolf Optimization Algorithm

5. Case Study

This study mainly focuses on designing an approach based on optimization methods to configure geographical zones for districting of the Iranian healthcare system to improve the executive strategic plan of the Ministry of Science and Research and also to determine healthcare capitals in Iran. According to this plan, the country's demographic regions are divided into 10 main categories, namely, "health zones", each of which implements action plans approved by the Ministry of Health and Medical Education under the supervision of a central committee. In each "health zone", hospital and treatment facilities at both general and specialized levels will be created as per the demand of people in that area for various health services to minimize the number of patients referring to other centers outside the designated zone. Hereby the government budgeting for tackling the problems in each region can be facilitated in addition to providing better supervision over the health status of a zone, like the spread of certain diseases such as cholera and influenza. For instance, the northern and southern regions of Iran have always been known to be prone to outbreaks of influenza and environmental diseases such as malaria, which, despite the great need for special facilities, have always faced a severe shortage of equipment and medicines. One of the main reasons for this problem is the lack of proper coordination between the management structure of the southern provinces of Iran as a joint committee. In fact, every southern province of Iran has separately implemented plans to combat common diseases, albeit unfortunately never successful. However, if there is a joint committee and a joint healthcare zone between these provinces, the planning of operational projects will have a more positive impact. It should be noted that management of the healthcare area is not possible in an integrated manner and each healthcare zone must be divided into smaller areas under the name of the healthcare network to create an orderly hierarchical structure to improve management. It is also necessary to determine healthcare zones and healthcare networks in an integrated manner. In other words, if the healthcare zones of the country are designated as one separate project and healthcare networks as another project, some inconsistencies might occur in the social structure of zones. For instance, the southern regions of South Khorasan province have many cultural similarities and socio-economic interactions with southern regions of Sistan and Baluchestan province such that, in some cases, patients in the northern regions of Sistan and Baluchestan province referred to South Khorasan province to receive health services, causing problems such as overcrowding and, naturally, lack of equipment and increased workload of medical staff in South Khorasan province. This is especially true for some common illnesses, such as the flu. Therefore, in determining the healthcare zones of the country, it is necessary to pay integrated attention to determining the existing healthcare networks in each region.

5.1. Computational Results

In order to evaluate the performance accuracy of the proposed optimization model, several numerical instances in small, medium and large dimensions were designed using random data and the results obtained from solving the model were examined. Considering that the districting models fall into the category of NP-hard problems, metaheuristic algorithms were used to solve the numerical instances in real-world dimensions. Since different metaheuristic algorithms have been used in the research literature, we, in this study, tried to measure the known algorithms and a number of new algorithms to strengthen the scope of the problem from a methodological point of view. It should be noted that all numerical instances are solved in a system with 3.6 GHz processing power and 16 GB of Random Access Memory in the Windows 10 operating system.

5.2. Generation of Numerical Instances

The procedure for generating random responses, which is examined to verify the performance of the proposed mathematical model and algorithms, is described in this section. In order to create a hypothetical geographical area for numerical representations, a connectivity graph $G(V, E)$is generated in which V represents the vertices of graph and E represents the communication paths between the vertices. Each vertex of the graph has two geographical characteristics x_i and y_i, indicating latitude and longitude. In this study, these two geographical features are randomly generated in a uniform interval U [10, 1200]. Then 80% of vertices of the graph are randomly considered as urban (potential) points and the other 20% as rural points. To generate population in each vertex, it is necessary to use random numbers in the uniform interval of U [500, 800] $\times 10^3$ for the population of urban areas and in the interval of U [100, 300] $\times 10^3$ for rural areas. Approximately 30% of the population is considered to determine the demand for healthcare services, which is monitored periodically. The same amount of demand varies by ± 10% for the level of service delivery in each vertex. To determine the value of maximum permitted distance parameter between points in each zone (T_{max}), the largest shortest path between the vertices of the graph is multiplied by $\left(\frac{3}{|P|}\right)$ value. It is noteworthy that the geographical space limitation of zones is considered only for the lower level of districting. Consequently, all the necessary inputs to solve the model are generated and different numerical representations in small, medium and large dimensions can be stored. The important point is that the difference between small, medium and large numerical instances is only in the number of population points and the value of parameters does not impact the dimensions of the problem. The following is a description of numerical results.

5.3. Parameter Setting

As GWO and SSA algorithms are considered intelligent algorithms, they do not have specific regulatory parameters and all their operators perform calculations based on equations with specific parameters. The only controllable parameters in these algorithms include the number of Agents and the number of Iterations of the algorithm. However, the genetic algorithm has four key operators: population number, number of iterations, mutation rate, and crossover rate. In order to improve the performance of algorithms in solving different numerical instances, it is necessary to determine the optimal levels of these parameters for each algorithm. In this study, the test design method based on Response Surface Methodology (RSM) is used. The parameters of each algorithm are considered according to the table 2.

Table 2. Size of the proposed algorithms' parameters

	Low level (-1)	*Medium level (0)*	*High level (+1)*
Number of iterations	۵۰	۱۰۰	۱۵۰
Population size (X iteration size)	۱	۱,۵	۲
Crossover rate	۰,۵	۰,۷	۰,۹
Mutation rate	۰,۱	۰,۲۵	۰,۴

After carrying out the necessary tests by the RSM method through Design Expert 12 Software, the optimal levels of the proposed algorithms' parameters are presented as in table 3.

Table 3. The optimal level of parameters of the proposed algorithms

Dimensions of Problem		***Number of Iterations***	***Population size (X chromosome length)***	***Crossover rate***	***Mutation rate***
Small	*GA*	۱۰۰	۱,۳	۰,۷	۰,۲
	GWO	۱۰۰	۱,۱	–	–
	SSA	۱۰۰	۱,۶	–	–
Medium	*GA*	۱۰۰	۱,۷	۰,۷	۰,۲
	GWO	۱۰۰	۱,۸	–	–
	SSA	۱۰۰	۱,۴	–	–
Large	*GA*	۱۰۰	۲	۰,۷	۰,۳

GWO	١٠٠	١,٩	–	–
SSA	١٠٠	١,٧	–	–

5.4. Evaluating the Proposed Algorithms

Numerical analyses are presented in this section. For this purpose, numerical comparisons are made between the three genetic metaheuristic algorithms, including Genetic Algorithm, Grey Wolf Algorithm, and Salp Swarm Algorithm. It should be noted that genetic algorithms and grey wolf are among the population-based algorithms and have good performance in various problems (Karakoyun, Ozkis, & Kodaz, 2020). However, the Salp Swarm Algorithm is one of the newest congestion-based algorithms that has managed to report an acceptable result on various issues (Mirjalili et al., 2017). This algorithm offers far better performance than other congestion-based algorithms. To compare the results, 30 numerical instances are generated randomly and then the results of solving different algorithms are shown in Table 4.

Table 4. Numerical results obtained by solving numerical instances using different algorithms

		Genetic Algorithm		**Grey Wolf Algorithm**		**Salp Swarm Algorithm**	
Number of Vertices	**Number of Zones**	**Fitness Function**	**Solution Time**	**Fitness Function**	**Solution Time**	**Fitness Function**	**Solution Time**
80	10	400	148	344	160	310	181
90		510	117	480	135	456	159
100		440	130	383	145	361	156
110	12	600	129	534	128	481	150
120		660	120	568	123	523	147
130		672	119	619	129	576	135
140		612	139	527	142	485	169
150		684	116	602	136	560	143
160		696	143	592	141	545	191
170	14	728	149	648	156	590	216
180		812	161	707	190	672	220
190		826	138	744	146	700	147
200		714	174	657	174	612	188
210		672	169	572	178	521	221
220		588	156	518	164	472	159
230		798	205	695	214	640	256
240		798	182	719	190	676	264
250		630	147	567	180	539	210
260	16	816	219	719	213	669	314
270		848	211	789	238	742	261
280		912	197	830	204	764	240
290	18	774	205	682	203	628	273
300		936	208	824	218	750	250

		Genetic Algorithm		Grey Wolf Algorithm		Salp Swarm Algorithm	
Number of Vertices	**Number of Zones**	**Fitness Function**	**Solution Time**	**Fitness Function**	**Solution Time**	**Fitness Function**	**Solution Time**
310		882	187	777	206	715	242
320	20	1200	214	1020	248	918	244
330		940	198	818	221	761	257
340		920	227	792	226	729	250
350		840	244	782	268	720	297
360		820	238	730	279	657	314
370		1140	213	1038	217	976	243
380		1200	243	1128	254	1027	299
390		1000	259	870	295	792	312
400		880	217	828	248	762	285
410	22	1144	263	1030	253	948	337
420		968	264	910	278	856	322
430		1298	263	1156	263	1076	282
440		990	273	911	295	857	329
450		1144	227	1076	235	1001	320
460		1232	264	1171	290	1113	314
470		1034	234	879	283	800	264
480		1210	248	1126	277	1025	333
490		1232	229	1072	278	976	291
500		1034	259	931	272	857	336
510		902	248	848	243	806	308
520		1122	272	1010	284	960	313
530	24	960	250	845	246	769	312
540		1248	272	1186	262	1127	389
550		1296	240	1167	291	1109	293
560		1032	268	981	271	923	324
570		1224	249	1139	302	1026	334
580		1176	283	1035	309	942	277
590		1200	246	1056	286	961	259
600		1032	234	919	253	837	290
610		1296	285	1102	282	1003	322
620	25	1150	245	1001	275	911	278
630		1000	227	890	252	801	330
640		1175	258	1105	307	1017	340
650		1325	248	1259	274	1184	314
660		1350	238	1215	253	1143	295
670		1025	262	964	326	897	310

It can be observed that by increasing the dimensions of numerical instances, the solution time also increases, albeit the rate of increase is non-exponential. In other words, if the dimensions of numerical instances increase, the growth trend of the solution time also increases in proportion to the same dimensions of the instances, indicating that the case study can be solved in high dimensions. Figure 6 shows the comparison of the solving time of different algorithms.

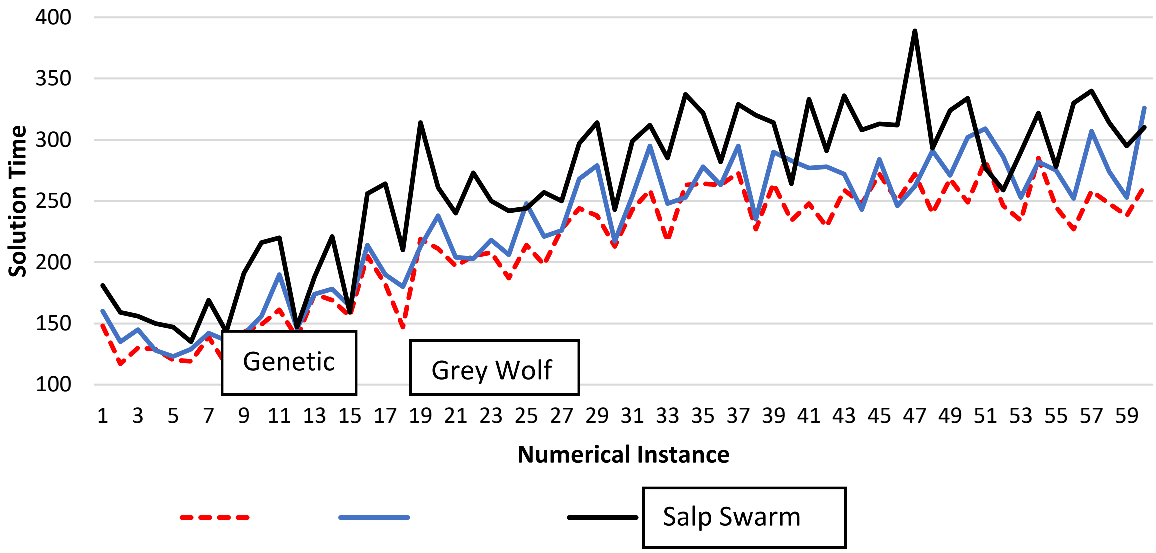

Figure 6. Comparison of Solution Time of Different Algorithms

As stated earlier, the trend of increase in the solution time has a non-exponential structure and has raised in proportion to the increase in dimensions of the problem. However, it is clear that the salp swarm algorithm had a much longer solution time because this algorithm uses more elements and consequently requires much more complex calculations than other algorithms. The grey wolf algorithm had also reported higher performance overall in terms of solution time. Comparing value of the fitness function between different algorithms depicts in figure 7.

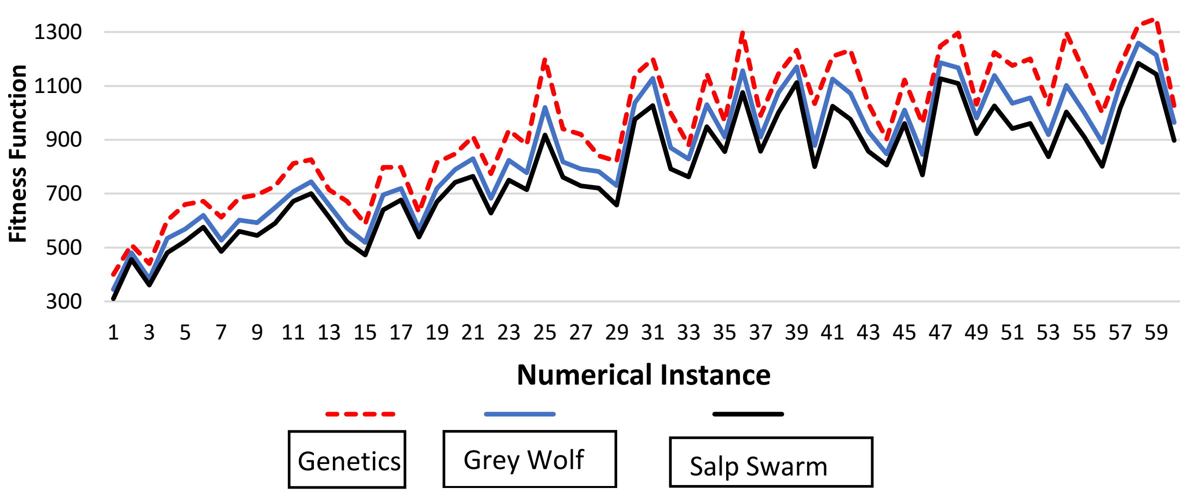

Figure 7. Comparison of fitness function of numerical instances using different algorithms

In terms of value of the fitness function, it can be stated that almost all algorithms are very similar and it cannot be claimed that a certain algorithm is better. However, a more meticulous examination reveals that the Salp Swarm Algorithm produced a much worse fitness function than the other algorithms, and the genetic algorithm was more efficient in almost all of the instances. Therefore, in this study, the genetic algorithm is chosen as the superior algorithm to solve the case study's problem.

5.5. Analysis of Results Under Uncertainty

To investigate the behavior of the model in solving numerical instances under uncertainty of input parameter, 10 numerical instances with random numerical parameters and robustness parameter values of different states are presented so that more detailed analyses can be performed. Table 5 presents the solution time and the value of the fitness function.

Table 5. Solution time and value

Problem instances	Problem characteristics		$\Gamma = 0$		$\Gamma = 0.2 \times \|V\|$		$\Gamma = 0.5 \times \|V\|$		$\Gamma = 0.8 \times \|V\|$		$\Gamma = \|V\|$	
	$\|V\|$	$\|P\|$	objective value	CPU time	objective value	CPU time	objective value	CPU time	objective value	CPU time	objective value	CPU time
P1	10	2	185	27	211	28	354	28	354	29	517	26
P2	12	2	119	39	194	36	194	40	361	45	361	38
P3	12	2	721	35	819	38	1274	36	1743	38	2501	35
P4	15	3	453	52	512	56	512	53	1488	54	1619	46
P5	20	4	614	61	614	64	1266	62	1413	68	1413	57
P6	50	2	622	194	817	206	1105	195	2019	181	2874	181
P7	50	3	785	214	940	235	1007	215	3281	204	3281	244
P8	50	4	903	483	994	468	1267	484	1341	493	1438	556
P9	50	4	1391	441	1047	477	1409	442	3127	415	3442	459
P10	80	4	2031	927	1246	973	3058	928	4812	872	7011	1047

As it can be observed, the value of fitness function under certain conditions has a certain level of difference with the uncertainty. Solving the problem under uncertainty makes the problem space larger and therefore the final response under uncertainty is not better than the final answer under certainty. This indicates that great care must be taken in determining the exact level of demand parameter to reduce such problems.

5.6. Solving the case study problem

Taking into account that problem analysis indicates the proper performance of the proposed model and algorithms in solving different numerical expressions, the case study, which was districting of Iran, was analyzed hierarchically. For this purpose, the numerical information in Appendices (1) and (2) will be used. Likewise, in order to calculate the utility of determining high-level centers, the data available on the website of the Statistics Center of Iran as well as the field data provided by experts of the University of Medical Sciences are used.

5.6.1. Determining the value of the social utility coefficient parameter

Hedonic models are widely used in determining parameters that are not dependent on time series (Sopranzetti, 2015). By separating the demand to independent variables affecting it, these models try to produce a model with a specific error level. For this purpose, a certain number of observations are extracted and then by solving the model, the coefficients of the independent variables affecting the demand are determined. In this study, this model is used to estimate the degree of social utility for receiving healthcare system services in Iran. Various transformation structures are presented for hedonic models including linear, semi-log and Box-Cox. In this study, the Box-Cox structure is used due to its more complete structure. In order to use this model, it is necessary to extract a certain number of observations and provide the necessary information for each of the independent variables affecting the determination of final demand. It is noteworthy that due to the hierarchical nature of health system under study, a separate model is designed for each level.

5.6.2. Mathematical Structure of Box-Cox Method

This method was proposed by Halvorsen and Pollakowski (1981) in order to create higher complexity in determining the coefficients of independent variables. In general, the transformation function of the Box-Cox model is as follows.

$$X^{\lambda} = \begin{cases} \dfrac{X^{\lambda}-1}{\lambda}, & \lambda \neq 0, X > 0 \\ \ln X, & \lambda = 0 \end{cases} \qquad ۴۵$$

Therefore, the mathematical structure of Box-Cox Model is as follows:

$$D = X^{\lambda}\beta + \varepsilon \qquad ۴۶$$

Where X is the vector of variables, β is the coefficient of variables, λ is Box-Cox transformation parameter, and $\varepsilon \sim N(0, \sigma^2)$ is the value of the computational error. However, determining the appropriate rate of λ and β can have a great impact on reducing the error and leading to a good estimation of the demand level. To determine the appropriate value of these parameters, the following logarithmic probability function can be used (Sopranzetti, 2015).

$$\ln L(\beta, \sigma^2, \lambda | y) = -\frac{1}{2\sigma^2}(z - X\beta)'(z - X\beta) - \frac{n}{2}\ln(2\pi\sigma^2) + (\lambda - 1)\sum_{i=1}^{n}\ln(y_i) \qquad ۴۷$$

Where y_i is the actual demand value in the i^{th} observation and z is also calculated based on the normal function. N also indicates the number of instances. Using the logarithmic likelihood function, the values β and λ can be determined based on the Maximum Likelihood Estimation MLE. It should be noted that the output of the estimation model is a percentage of social utility for choosing the center of the zone at a high level. After solving the proposed model using the BoxCox function, figure 8 shows the maximum value of the likelihood function for different values of the λ parameter is as follows.

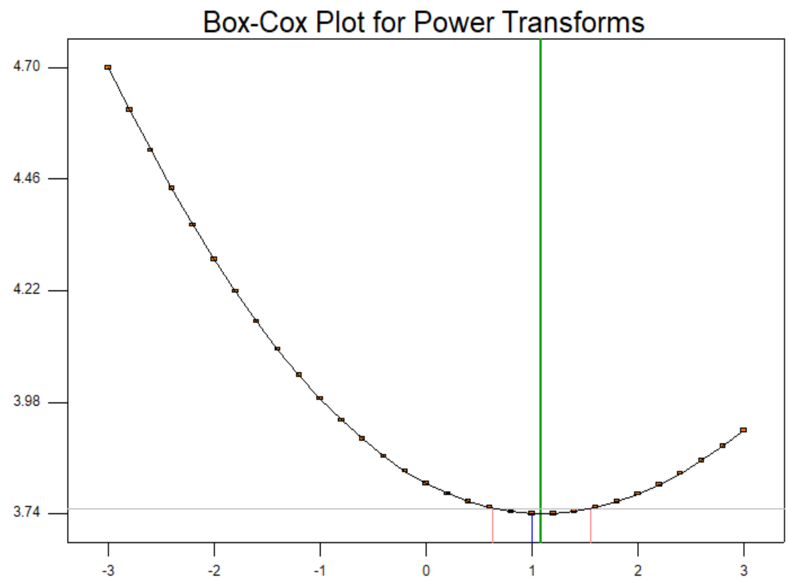

Figure 8. Graph showing the maximum value of likelihood function for different values of λ parameter for the first level

As it can be observed, the value of the transformation parameter is assumed to be 3.74. Accordingly, the utility rate for the case study can be determined. It should be noted that the data provided to determine the value of this parameter used in the hedonic model are attached. After implementing the utility estimation model for each potential point, the value of the utility coefficient is considered as shown in table 6.

Table 6. Utility coefficient of provincial centers as potential points

Potential Centers	Utility Coefficient	Potential Centers	Utility Coefficient
East Azarbaijan	14	Sistan and Baluchestan	16
Western Azerbaijan	14	Fars	15
Ardebil	6	Qazvin	2
Esfahan	8	Ghom	2
Alborz	1	Kurdistan	8
Ilam	1	Kerman	12
Bushehr	1	Kermanshah	9
Tehran	15	Kohgiluyeh and Boyer-Ahmad	3
Chaharmahal and Bakhtiari	3	Golestan	8
South Khorasan	3	Gilan	13
Khorasan Razavi	21	Lorestan	8
North Khorasan	5	Mazandaran	18
Khuzestan	11	Markazi	4
Zanjan	5	Hormozgan	9
Semnan	1	Hamedan	9
		Yazd	16

In the solution of the problem of this case study, from among of 31 potential points, 30 points need to be designated as low-level zone centers and 10 centers as healthcare capitals of the country. In fact, the total number of countable modes for chosen centers can be obtained using the equation $\left(\binom{31}{30} \times \binom{31}{10} = 31 \times 44352165\right)$, which is a very high figure. However, the proposed algorithms can drastically reduce the number of calculations and report acceptable results by applying their special operators as shown in figure 9. Therefore, the results of the research problem solved by the genetic algorithm are described as follows.

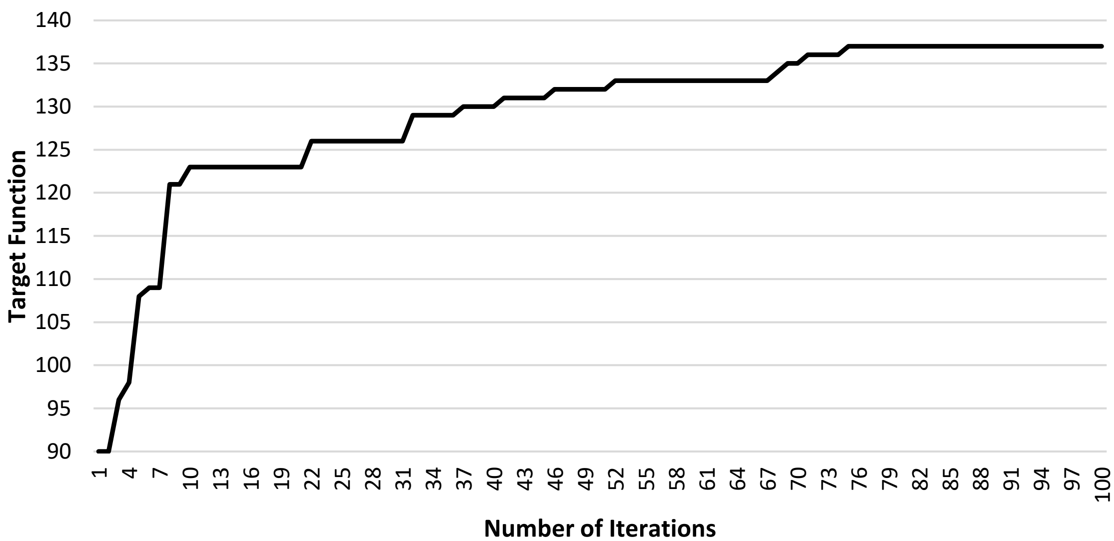

Figure 9. Convergence diagram of the case study in hierarchical mode

The algorithm initially reported a value of 90 units for the fitness function, and gradually, by applying the algorithm operators, it managed to make a relative improvement in the responses, eventually reaching the value of 137. The districting structure is as follows.

Based on the results provided in table 7 it can be observed that Iran is divided into 10 main territories and then each territory includes several other zones within it. Using these results, the healthcare system evolution plan can be developed and the subdivisions of each healthcare capital can be divided into specific zones. Accordingly, the best management decisions are applied for the country. Table 7 shows the selected centers.

Table 7. Centers chosen as health capitals

Center of the Zone	Utility Rate
East Azerbaijan	14
Tehran	15
Khorasan Razavi	21
Fars	15
Kurdistan	8
Kerman	12
Kermanshah	9

Mazandaran	18
Hormozgan	9
Yazd	16

Another very important issue is studying the performance of different algorithms in solving different random instances. The managers and decision-makers of the healthcare organizations that are the beneficiaries of this study tend to examine the performance of different solution methods for districting the areas and ultimately make the best possible management decision. For this purpose, computational studies were performed as follows.

5.6.3. Analysis of the first and second level zones

In order to further investigate the efficiency of proposed algorithm in solving the case study, the number of low-level zones was changed to 10 and 15 and the number of high-level zones was changed to 4 and 8, and the results are presented in table 8.

Table 8. districting results

Fitness Function	**Chosen Centers**	**Number of Low-Level Zones**	**Chosen Centers**	**Number of High-Level Zones**
۶۶	East Azerbaijan, Khorasan Razavi, Tehran, Fars, Sistan and Baluchestan, Yazd, Isfahan, Kermanshah, Mazandaran, Hamadan	۱۰	East Azerbaijan, Khorasan Razavi, Tehran, Sistan and Baluchestan	۴
۶۶	East Azerbaijan, Khorasan Razavi, Tehran, Fars, Sistan and Baluchestan, Yazd, Kerman, Khuzestan, Isfahan, Kermanshah, Hormozgan, Mazandaran, Hamedan, Kurdistan, Lorestan	۱۵	East Azerbaijan, Khorasan Razavi, Tehran, Sistan and Baluchestan	۴
۱۱۰	East Azerbaijan, Khorasan Razavi, Tehran, Fars, Sistan and Baluchestan, South Khorasan, Isfahan, Kermanshah, Mazandaran, Lorestan	۱۰	East Azerbaijan, Khorasan Razavi, Tehran, Fars, Sistan and Baluchestan, Kerman, Kermanshah, Isfahan	۸
۱۱۰	East Azerbaijan, Khorasan Razavi, Tehran, Fars, Sistan and Baluchestan, Hormozgan, Kerman, Gilan, Isfahan, Kermanshah, Hormozgan, Mazandaran, Hamedan, Kurdistan, Lorestan	۱۵	East Azerbaijan, Khorasan Razavi, Tehran, Fars, Sistan and Baluchestan, Kerman, Kermanshah, Isfahan	۸

As it can be observed, by changing the number of zones, the centers with highest utility were again selected. However, changes are made based on the fulfillment of restrictions of guaranteeing the maximum allowable distance in the zones as well as the maximum workload difference between the zones, but it can be stated that by making changes in zones, the answers provided are always logical.

5.7. Managerial Implications

The primary problem of this study was to provide an efficient model for dividing population areas into specific zones as healthcare networks. These healthcare networks have undergone managerial changes in recent years and are planned in a hierarchical structure. These changes became more specific after the implementation of healthcare system evolution plan in Iran, and based on a national plan, 10 healthcare capitals were determined in the country. Due to various reasons, the full implementation of this plan has encountered problems and its follow-up has been stopped. A main reason for the failure of this project can be attributed to the lack of complete and comprehensive criteria for creating health capitals. In other words, establishment of health capitals in Iran has not been properly coordinated with the healthcare networks, which are in fact the subdivisions of these capitals, creating many management problems. One of these disorders is non-establishment of a balance between workload and health networks, causing great dissatisfaction for patients as well as medical staff. Another issue is lack of appropriate criteria for the extent of space covered by each of the chosen capitals. These issues have always been considered as management challenges. In this study, using mathematical optimization models, an optimal decision-making model is designed based on the existing conditions in the real world, in which decisions about the selection of healthcare capitals and the areas covered by them as well as the creation of health networks are designed in an integrated way. Furthermore, the criteria for creating workload balance and constraints of space covered by each zone are applied as mathematical constraints to create the conditions most similar to real-world situations. The results obtained from solving the proposed model indicated that the creation of hierarchical zones is quite feasible and can meet the managerial expectations of the healthcare sector. However, valid data is needed to improve the results. At present, for various reasons, it is not possible to access the data needed to solve the model, creating a certain gap between the results and the ideal state. Therefore, as the first perspective in the development of management affairs, it is possible to propose the necessary infrastructure to monitor and obtain appropriate data to implement the model. Also, to implement the obtained results, it is better to comprehensively determine the utility coefficient of each potential zone. In this case, too, the opinion of experts in the field is needed, for which many quantitative and qualitative criteria and indicators can be determined to finally achieve the best possible result. Since the 10-zone category has not been implemented at the moment in Iran, it is not possible to make a definite comparison between the obtained results and the results of the real world. However, in any case, it can be observed that the function of the model is optimally feasible, and only reliable input data should be provided. Finally, it can be pointed out that creating operational restrictions to encourage the community to receive healthcare services from the network and the zone allocated to them can lead to the establishment of order in the workload balance. These restrictions can include reducing the insurance deductible, reducing payments of patients, and imposing discounts on health care.

6. Conclusions

The integrated problem of districting and location has been considered by academia and practitioners as one of the main important problems in the area of healthcare networks. In this study, a new mathematical model was proposed to deal with the integrated problem of location and districting in the Iranian healthcare network. The objective function of this model maximizes the total social utility of districts. The mathematical model also considered the uncertainty associated with the nature of the problem. Since the problem of this study is an NP-hard problem, metaheuristic methods were used to obtain the desired answers on a large dimension. For this purpose, three metaheuristic algorithms were used involving genetic algorithms, salp swarm, and grey wolf algorithms. The computational results indicated that the generated responses have a suitable structure. However, the genetic algorithm has a better performance level in almost all responses than other algorithms. It is therefore recommended to use these algorithms to solve large-scale numerical instances. Therefore, the case study was solved with the genetic algorithm. To develop the theoretical and operational dimensions of the present study, it is suggested that other criteria related to social development, including unemployment rate, number of elderly and sick people be considered as a measure of workload balance. Furthermore, the development of approaches based on two-level planning can lead to the coordination of decisions at the high and middle management levels.

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