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Average-Revenue Efficiency and Optimal Scale Sizes in Environmental Analysis

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Due to the changes of performance measures, a vital aspect for decision makers is finding optimal scale sizes of entities. Moreover, there are undesirable measures in many investigations. In the existing data envelopment analysis (DEA) approaches, optimal scale sizes (OSSs), average-cost efficiency (ACE) and average-revenue efficiency (ARE) of decision making units (DMUs) with desirable measures under strong disposability have been estimated while undesirable factors are presented in many real world examinations. Accordingly, in this research, OSSs and ARE of DMUs with undesirable outputs are addressed under managerial disposability. ARE is defined as the composite of scale and output allocative efficiencies under managerial disposability. To illustrate in detail, a two-stage DEA-based approach is rendered to estimate ARE and OSSs in the presence of undesirable outputs. A numerical example and an illustrative case are given to explain the proposed approach in this study.

Keywords: *Data envelopment analysis, average-revenue efficiency, optimal scale size, undesirable outputs, managerial disposability.*

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1. Introduction

The examination of the optimal scale size (OSS) of firms is the notable matter for decision makers in order to make plans and change the structure. In the literature, there are parametric and non-parametric approaches to address the optimal scales of activities. The non-parametric data envelopment analysis (DEA) technique, originally introduced by Charnes et al. [1], is one of popular approaches to analyze the relative efficiency, including studies dealing with scale issues.

Most productive scale size (MPSS) has been determined using DEA by Banker [2] in 1984. Podinovski [3] introduced the notion of global returns to scale in order to determine the change direction of scale related to the efficient DMUs to obtain the global maximum average productivity while the proportions of input and output are unchanged. Forsund and Hjalmarsson [4] attempted to response the following question: Does information on the optimal scale achieved from the DEA technique and also scale efficiency is significant for policy directions in DEA? Forsund and Hjalmarsson [5] estimated the scale elasticity in DEA models. Cesaroni and Giovannola [6]

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introduced an alternative definition of OSSs and presented the average-cost efficiency measure based upon the cost of the activity. Haghighatpisheh et al. [7] provided input-output allocative DEA models to describe OSSs when input and output prices are known.

On the other hand, there are different approaches to manage undesirable measures among the DEA studies. For instance, the concepts of weak, strong, natural and managerial disposability have incorporated into the DEA investigations for environmental assessment. Readers can refer to [8-10] for further information about DEA techniques to analyze the performance of DMUs with undesirable measures. Sueyoshi and Goto [10] replaced the concepts of weak and strong disposability by natural and managerial disposability. Sueyoshi et al. [11] evaluated the unified efficiency under managerial disposability and also measured the environmental efficiency. They, furthermore, estimated damages to scale and scale damages. It is clear that detecting the OSSs of entities with undesirable outputs is the important aspect for the management. Also, the definition of OSSs is more deeper when the prices of desirable and undesirable outputs are presented.

As far as we know, there is no study to address the concept of OSSs, considering undesirable outputs and the maximization of unit revenue, simultaneously. Therefore, in this study, an alternative definition of OSSs based on the maximization of the revenue and also the average-revenue efficiency (ARE) measure of DMUs are rendered under the managerial disposability. Actually, a two-stage DEA model is presented to estimate OSSs and ARE of DMUs with undesirable output. A numerical example along with a case study derived from the literature are also provided to illustrate the introduced technique.

The structure of this paper is unfolded as follows: Section 2 shows preliminaries, including the definition of managerial disposability and Hagheghatpisheh's approach [7] to deal with OSSs where measure prices are specified. The introduced approach to investigate OSSs and ARE of systems with undesirable outputs is given in Section 3. Examples are also presented in Section 4 to clarify the suggested method. Conclusions and remarks are provided in Section 5.

2. Preliminaries

Definition 1. Managerial disposability shows a DMU increases inputs to increase desirable outputs and to decrease undesirable outputs, simultaneously. This attempt to protect the environment and by technology innovation signifies managerial disposability [10].

The following shows a technology, containing envelopment, convex, managerial disposability, variable returns to scale (VRS), minimum extrapolation properties:

$$T_{VRS}^{m} = \{(\mathbf{x},\mathbf{y},\mathbf{b}) \mid \sum_{j=1}^{n}\lambda_j\mathbf{x}_j \geq \mathbf{x}, \sum_{j=1}^{n}\lambda_j\mathbf{y}_j \geq \mathbf{y}, \sum_{j=1}^{n}\lambda_j\mathbf{b}_j \leq \mathbf{b}, \sum_{j=1}^{n}\lambda_j = 1, \lambda_j \geq 0\} \tag{1}$$

that $\mathbf{x} \in R_{+}^{m}, \mathbf{y} \in R_{+}^{s}$ and $\mathbf{b} \in R_{+}^{p}$ indicate the vectors of input, desirable output and undesirable output, respectively. $\lambda_j \, (j = 1,...,n)$ are the intensity variables. The superscript m in T_{VRS}^{m} is used to display the technology under managerial disposability and the subscript VRS stands for the VRS property.

2.1. Haghighatpisheh et al.'s Approach

Consider there are n DMUs, $DMU_j\,(j=1,...,n)$, with the vector of input $\mathbf{x}_j \in R^m$ and the output vector $\mathbf{y}_j \in R^S$.

Definition 2. [7] Assume ρ_j is the radial scaling indicator resulted from the proportion of the input of DMU_o (with the input vector $\mathbf{x}_o$ and the output vector $\mathbf{y}_o$) and that of the reference unit and also the price vector of outputs is denoted by $q=(q_1,...,q_s)$. The average-revenue efficiency of DMU_o is defined as:

$$R_o^{(R)} = \frac{q\bar{y}}{qy_o}\rho_o \tag{2}$$

Definition 3. [7] For each DMU such as DMU_o, a production possibility $(x', y') \in T$ that maximizes $R_o^{(R)}$ is called an OSS, i.e. an OSS is a solution for the following problem:

$$\begin{aligned} &Max \;\; R_o^{(R)} \\ &s.t. \quad (x', y') \in T. \end{aligned} \tag{3}$$

T shows a set of all feasible vectors related to inputs and outputs, particularly $T=\{(x,y)\,|\,x \;\; can \;\; produce \;\; y\}$. Note that the technology T considers one type of output vectors while two types of outputs, desirable and undesirable ones, have been presented in T_{VRS}^m.

Haghighatpisheh et al. [7] provided the following two-stage approach to estimate OSSs under the variable returns to scale technology.

Stage 1:

$$RE_o^* = Max \frac{\sum_{r=1}^{s} q_r \bar{y}_r}{\sum_{r=1}^{s} q_r y_{ro}}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io},\ i = 1,\dots,m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge \bar{y}_r,\ r = 1,\dots,s, \qquad (4)$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\lambda_j \ge 0, \forall j, \bar{y}_r \ge 0, \forall r.$$

Stage 2:

$$\gamma^* = Min\ \gamma$$

$$s.t. \quad \gamma x_{io} \ge \hat{x}_i,\ i = 1,\dots,m,$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \le \hat{x}_i,\ i = 1,\dots,m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge \hat{y}_r,\ r = 1,\dots,s,$$

$$\sum_{j=1}^{n} \lambda_j = 1, \qquad (5)$$

$$RE_o^* = \frac{\sum_{r=1}^{s} q_r \hat{y}_r}{\sum_{r=1}^{s} q_r y_{ro}},$$

$$\lambda_j \ge 0, \forall j, \hat{x}_i, \hat{y}_r \ge 0, \forall i, \forall r.$$

The optimal solution $(\hat{x}_i^*, \hat{y}_r^*)$ obtained from model (5) is considered as the OSS and $R_o^{(R)} = RE_o^* \times \gamma^*$.

In the next section, OSSs and ARE are addressed while the two forms of outputs, desirable and undesirable ones, are detected.

3. The Proposed Approach

In this section, the two-stage DEA-based approach is introduced to determine OSSs and ARE of entities in the presence of undesirable outputs. Assume there are n DMUs, $DMU_j\,(j=1,...,n)$ that use m inputs $x_{ij}\,(i=1,...,m)$, produce s desirable outputs $y_{rj}\,(r=1,...,s)$ and emit P undesirable outputs $b_{pj}\,(p=1,...,P)$. Prices of desirable and undesirable outputs are also shown by $\mathbf{q}=(q_1,...,q_s)$ and $\mathbf{w}=(w_1,...,w_p)$.

Definition 4. The average-revenue efficiency of DMU_o with undesirable outputs is determined as follows:

$$R_o^{(RB)}=\frac{\mathbf{q}\bar{\mathbf{y}}-\mathbf{w}\bar{\mathbf{b}}}{\mathbf{q}\mathbf{y}_o-\mathbf{w}\mathbf{b}_o}.\xi_o \tag{6}$$

in which ξ_o is the radial scaling factor achieved from the differentiation between the input vector of o th DMU and the reference unit, i.e. $\xi_o=Max\{\frac{x_{io}}{\bar{x}_i},i=1,...,m\}$.

Also, $(\bar{\mathbf{x}},\bar{\mathbf{y}},\bar{\mathbf{b}})$ is an arbitrary reference point.

Definition 5. A solution for the following problem is determined as an OSS:

$$\begin{aligned} &Max\ \ R_o^{(RB)}\\ &s.t.\quad (\hat{x},\hat{y},\hat{b})\in\bar{T}. \end{aligned} \tag{7}$$

In other words, a production possibility $(\hat{x},\hat{y},\hat{b})\in\bar{T}$ that maximizes $R_o^{(RB)}$ is defined as an OSS for given DMU_o. $\bar{T}$ denotes a set of all feasible vectors related to inputs, desirable outputs and undesirable outputs, in other words, $\bar{T}=\{(x,y,b)\,|\,x\ can\ \ produce\ (y,b)\}$.

Thus, we propose the following two-stage approach in order to estimate OSSs of DMUs in the presence of undesirable outputs. Taking the technology (1) into account, the revenue of DMUs with undesirable outputs is maximized in the following way that is deemed as the leader stage:

$$
\begin{aligned}
& Max \sum_{r=1}^{s} q_r \bar{y}_r - \sum_{p=1}^{P} w_p \bar{b}_p \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \geq x_{io},\ i = 1,\ldots,m, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} \geq \bar{y}_r,\ r = 1,\ldots,s, \\
& \qquad \sum_{j=1}^{n} \lambda_j b_{pj} \leq \bar{b}_p,\ p = 1,\ldots,P, \\
& \qquad \sum_{j=1}^{n} \lambda_j = 1, \\
& \qquad \lambda_j \geq 0, \forall j, \bar{y}_r, \bar{b}_p \geq 0, \forall r, \forall p.
\end{aligned} \tag{8}
$$

By examining the results of model (8), the revenue efficiency can be calculated as follows:

$$
RE_o^* = \frac{\sum_{r=1}^{s} q_r \bar{y}_r^* - \sum_{p=1}^{P} w_p \bar{b}_p^*}{\sum_{r=1}^{s} q_r y_{ro} - \sum_{p=1}^{P} w_p b_{po}} \tag{9}
$$

Under the assumption $\sum_{r=1}^{s} q_r y_{ro} > \sum_{p=1}^{P} w_p b_{po}$, the values achieved from the expression (9) are greater than or equal to one. To express the revenue, efficiency scores between zero and one, $\frac{1}{RE_o^*}$ is calculated. The unit under consideration is called revenue efficient provided that $\frac{1}{RE_o^*}$ is obtained equal to one. Otherwise, it is revenue inefficient.

For situations that $\sum_{r=1}^{s} q_r y_{ro} - \sum_{p=1}^{P} w_p b_{po}$ is negative and it may be difficult to work with, the definition of the revenue efficiency and related expressions should be reconsidered. For more detail, the ratio form can be used to estimate the most revenue. Also, the linear fractional problem can be linearized applying Charnes and Cooper’s transformation [12].

Afterward, the following model is examined as the follower stage:

$$
\begin{aligned}
& R_o^{(\mathrm{Re})} = Max\{Max\{\frac{x_{io}}{\widehat{x}_i}, i=1,\ldots,m\}\} \\
& s.t. \ \sum_{j=1}^{n} \lambda_j x_{ij} = \widehat{x}_i \ , \ i=1,\ldots,m, \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} = \widehat{y}_r \, , r=1,\ldots,s, \\
& \quad \sum_{j=1}^{n} \lambda_j b_{pj} = \widehat{b}_p \, , \ p=1,\ldots,P, \\
& \quad RE_o^* = \frac{\sum_{r=1}^{s} q_r \widehat{y}_r - \sum_{p=1}^{P} w_p \widehat{b}_p}{\sum_{r=1}^{s} q_r y_{ro} - \sum_{p=1}^{P} w_p b_{po}}, \\
& \quad \sum_{j=1}^{n} \lambda_j = 1, \\
& \quad \lambda_j \geq 0, \forall j, \widehat{x}_i \, , \widehat{y}_r \, , \widehat{b}_p \geq 0, \forall i, \forall r, \forall p.
\end{aligned}
\tag{10}
$$

This model is not linear, but it can be transformed into the linear model (11) using some changes in the following way:

$$
0 \neq Max\{\frac{x_{io}}{\overline{x}_i}, i=1,\ldots,m\} = \frac{1}{Min\{\frac{\overline{x}_i}{x_{io}}, i=1,\ldots,m\}} \Rightarrow
$$

$$
\psi = Min\{\frac{\overline{x}_i}{x_{io}}, i=1,\ldots,m\} \Rightarrow \psi \leq \frac{\overline{x}_i}{x_{io}}, i=1,\ldots,m \Rightarrow \psi x_{io} \leq \overline{x}_i, i=1,\ldots,m
$$

It should be noted that in order to maintain the optimality of the leader stage, the fourth constraint has been added to model (10).

Thus, we have:

$$
\begin{aligned}
&\psi^* = Max\ \psi \\
&s.t.\ \ \psi x_{io} \le \hat{x}_i\ ,\ i=1,\ldots,m, \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij} = \hat{x}_i\ ,\ i=1,\ldots,m, \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj} = \hat{y}_r\ ,r=1,\ldots,s, \\
&\quad \sum_{j=1}^{n} \lambda_j b_{pj} = \hat{b}_p\ ,\ p=1,\ldots,P, \\
&\quad \sum_{j=1}^{n} \lambda_j = 1, \\
&\quad RE_o^* = \frac{\sum_{r=1}^{s} q_r \hat{y}_r - \sum_{p=1}^{P} w_p \hat{b}_p}{\sum_{r=1}^{s} q_r y_{ro} - \sum_{p=1}^{P} w_p b_{po}}, \\
&\quad \lambda_j \ge 0, \forall j, \hat{x}_i\ , \hat{y}_r\ , \hat{b}_p \ge 0, \forall i, \forall r, \forall p.
\end{aligned}
\qquad (11)
$$

The production possibility $(\hat{x}_i^*, \hat{y}_r^*, \hat{b}_p^*)$ as the optimal solution of model (11) is appraised as the OSS and $R_o^{(RB)} = RE_o^* \times \psi^*$.

Theorem 1. The ARE measure of a firm is not less than its revenue efficiency.

Proof. The revenue efficiency of DMU_o is equal to the maximization of

$$\frac{\mathbf{q}\bar{\mathbf{y}} - \mathbf{w}\bar{\mathbf{b}}}{\mathbf{q}\mathbf{y}_o - \mathbf{w}\mathbf{b}_o} . \xi_o$$

subject to $\xi_o = 1$ and $\mathbf{x} \ge \mathbf{x}_o$. Thus, the following suffices to show that $RE_o^* \le ARE_o^*$:

$\xi_o = Max\{\frac{x_{io}}{\bar{x}_i}, i=1,\ldots,m\}$. □

Theorem 2. An OSS must be revenue efficient.

Proof. Suppose that the production possibility k, i.e. $(\mathbf{x}_k, \mathbf{y}_k, \mathbf{b}_k)$ is an OSS for *j*th DMU. If it is not revenue efficient, there is $(\mathbf{x}_v, \mathbf{y}_v, \mathbf{b}_v)$ that $x_v \ge x_k, qy_v > qy_k$ and $wb_v \le wb_k$.

Therefore, $\xi_v = Max\{\frac{x_{iv}}{\bar{x}_i}, i=1,...,m\} \geq \xi_k = Max\{\frac{x_{ik}}{\bar{x}_i}, i=1,...,m\}$ and we have

$\frac{qy_v - wb_v}{qy_j - wb_j}.\xi_v > \frac{qy_k - wb_k}{qy_j - wb_j}.\xi_k$ that contradicts the assumption that $(\mathbf{x}_k, \mathbf{y}_k, \mathbf{b}_k)$ maximizes $R_o^{(RB)}$. □

4. Illustrative Examples

In this part, two datasets are applied to demonstrate the introduced technique.

4.1. Numerical Example

In this subsection, we consider 7 DMUs with one input, one desirable output and one undesirable output. The dataset is presented in Table 1. Prices of the desirable output and the undesirable output are 1.5 and 0.5, respectively. OSSs and ARE are assessed using the proposed approach. The findings are shown in Table 2.

Table 1. Dataset

DMU	Input	Desirable output	Undesirable output
1	145	3780	10
2	140	3245	20
3	255	4069	25
4	70	2043	16
5	401	8463	18
6	246	4345	50
7	542	2341	28

Column 2 indicates the revenue efficiency scores and ARE measures are presented in column 3.

Table 2. Results

DMU	Revenue efficiency	ARE
1	0.4466	2.7655
2	0.3829	2.8643
3	0.4802	1.5725
4	0.2409	5.7286
5	1	1
6	0.5118	1.6301
7	1	1

As can be found from Table 2, OSSs for two DMUs, 5 and 7, that are also revenue efficient, are as follows:

$(\hat{x}_{15}^*, \hat{y}_{15}^*, \hat{b}_{15}^*) = (401, 8463, 18)$ and $(\hat{x}_{17}^*, \hat{y}_{17}^*, \hat{b}_{17}^*) = (542, 2341, 28)$.

In the next subsection, OSSs of Japanese electric power companies are found using the introduced model in this research.

4.2. Determining ARE and OSSs of Japanese Electric Power Companies

In this subsection, the performance of nine electric power companies of Japan with two inputs, assets and employees, two desirable outputs, sales and customers, and one undesirable output, CO_2 emission, is examined. The dataset that firstly has been used in [13] is shown in Table 3.

Table 3. The dataset of electric power companies [13]

DMU	Company	x_1	x_2	y_1	y_2	b_1
1	Hokkaido	15.6	5.7	318.4	39.4	167.8
2	Tohoku	36.8	12.4	811.0	76.8	397.9
3	Tokyo	129.9	37.9	2889.6	284.9	1265.0
4	Chubu	51.1	16.2	1297.3	104.6	646.7
5	Hokuriku	14.2	4.6	281.5	20.8	185.2
6	Kansai	62.4	22.1	1458.7	134.0	549.9
7	Chugoku	26.1	9.9	612.2	51.9	430.7
8	Shikoku	13.5	6.0	287.0	28.3	114.6
9	Kyushu	38.3	12.5	858.8	84.0	341.0

We suppose $(q_1, q_2, w_1) = (3, 2, 0.5)$ are prices related to desirable outputs and the undesirable output. To estimate ARE and OSSs of these companies, statements (8), (9) and (11) are computed. Results are denoted in Table 4. Column 2 shows the revenue efficiency obtained from expression (8) and (9). As can be seen, only Tokyo is revenue efficient with the value one. It can be found from column 3 that this company is, furthermore, average-revenue efficient. The OSS point for this company located in Tokyo, which is also revenue efficient, is as the next expression:

$$(\hat{x}_{15}^{*}, \hat{x}_{25}^{*}, \hat{y}_{15}^{*}, \hat{y}_{25}^{*}, \hat{b}_{15}^{*}) = (129.9, 37.9, 2889.6, 284.9, 1265.0).$$

Table 4. Results

DMU	Revenue efficiency	ARE	Model (4)	Model (5)
1	0.1118	6.6491	1	1
2	0.2755	3.0565	0.9023	1
3	1	1	1	1
4	0.4307	2.3395	1	1
5	0.0888	8.2391	1	1
6	0.5073	1.7149	0.9642	1
7	0.1941	3.8283	0.9769	1
8	0.1002	6.3167	1	1
9	0.2998	3.032	0.9226	1

Now, to compare the consequences of the proposed approach with the existing approaches, models (4) and (5) are solved that exclude undesirable outputs and consider the strong disposability. The findings appear in columns 4-5. As shown, five companies, Hokkaido, Tokyo, Chubu, Hokuriku and Shikoku are determined as revenue efficient using model (4) whilst only Tokyo is deemed as revenue efficient applying the proposed model. For more clarification, the revenue efficiency values resulted from two approaches are depicted in Figure 1. As can be found, model (4) that does not include undesirable outputs overestimates the revenue efficiencies. Actually the revenue efficiency obtained from model (4) is not less than the proposed method. Moreover, ARE values achieved from model (5) are shown in column 5 that for all companies have been achieved equal to one that are different from the results gained from model (11) except for Tokyo. The comparison of two approaches indicates including undesirable outputs and considering the technology under managerial disposability have remarkable influences on the results. It is clear that the outcomes of the proposed method that contains undesirable outputs are more realistic and rational.

It should be noted that all models have been run using GAMS (General Algebraic Modeling System) software on an Intel (R) Core 2, 3 GB RAM, 2.20 GHz PC.

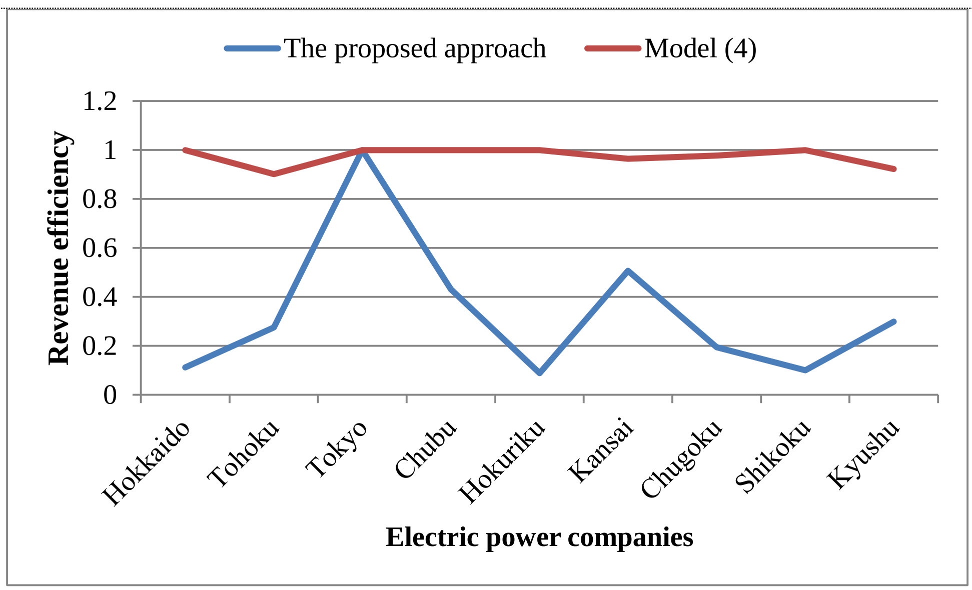

Figure 1. The revenue efficiency values

5. Conclusions

In many processes, undesirable outputs are presented, while they have been not included in many existing DEA studies related to OSSs and average-economic efficiencies. Therefore, in this paper, the concepts of OSSs and ARE under managerial disposability have been dealt with when there are undesirable outputs. The suggested DEA-based approach can be used to estimate optimal scale sizes and average-revenue efficiency of DMUs in the presence undesirable outputs and where the prices of outputs are known. Illustrative examples have also been provided to clarify the proposed technique. The results show the suggested method is beneficial to estimate optimal scale sizes and average-revenue efficiency of entities in environmental assessments. In this study, the relationship between OSSs and MPSS of systems with undesirable outputs is not investigated.

Therefore, this topic can be considered for future examinations. The extension of the models to find OSSs, average-cost efficiency and average-profit efficiency when undesirable outputs are presented is also an interesting topic for further research. The approach can, moreover, be generalized for situations that there are uncertain and negative measures.

References

[1] Charnes, A., Cooper, W. W. and Rhodes, E. (1978), Measuring the efficiency of decision making units, *European Journal of Operational Research*, 2(6), 429-444.

[2] Banker, R.D. (1984), Estimating most productive scale size using data envelopment analysis, *European journal of operational research*, 17(1), 35-44.

[3] Podinovski, V. (2004), Efficiency and global scale characteristics on the “no free lunch” assumption only, *Journal of Productivity Analysis*, 22(3), 227-257.

[4] Førsund, F. R. and Hjalmarsson, L. (2004), Are all scales optimal in DEA? Theory and empirical evidence, *Journal of Productivity Analysis*, 21, 25–48.

[5] Førsund, F.R. and Hjalmarsson, L. (2004), Calculating scale elasticity in DEA models, *Journal of the Operational Research Society*, 55(10) , 1023-1038.

[6] Cesaroni, G. and Giovannola, D. (2015), Average-cost efficiency and optimal scale sizes in non-parametric analysis, *European Journal of Operational Research*, 242(1), 121-133.

[7] Haghighatpisheh, H., Kordrostami, S. and Amirteimoori, A. and Lotfi, F.H. (2019), Optimal scale sizes in input–output allocative data envelopment analysis models, *Annals of Operations Research*,1-22.

[8] Liu, W. B., Meng, W., Li, X. X., and Zhang, D. Q. (2010), DEA models with undesirable inputs and outputs. *Annals of Operations Research*, 173(1), 177-194.

[9] Halkos, G., & Petrou, K. N. (2019), Treating undesirable outputs in DEA: A critical review. *Economic Analysis and Policy*, 62, 97-104.

[10] Sueyoshi, T. and Goto, M. (2012), Weak and strong disposability vs. natural and managerial disposability in DEA environmental assessment: comparison between Japanese electric power industry and manufacturing industries, *Energy Economics*, 34(3), 686-699.

[11] Sueyoshi, T., Goto, M. and Snell, M.A. (2013), DEA environmental assessment: measurement of damages to scale with unified efficiency under managerial disposability or environmental efficiency, *Applied Mathematical Modelling*, 37(12-13), 7300-7314.

[12] Charnes, A. and Cooper, W.W. (1962). Programming with linear fractional functionals. *Naval Research logistics quarterly*, 9(3-4), 181-186.

[13] Sueyoshi, T., and Goto, M. (2011), Measurement of returns to scale and damages to scale for DEA-based operational and environmental assessment: how to manage desirable (good) and undesirable (bad) outputs?. *European journal of operational research*, 211(1), 76-89.