Iranian Journal of Operations Research
Vol. 13, No. 1, 2022, pp. 1-12

Calculating cost efficiency using prices dependent on time via approximate method

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Abstract

In the traditional cost-efficiency model, the information about each decision unit includes inputs, outputs, and the input prices are fixed and specific. In practice, the price of the inputs often fluctuates at different times, and these prices for the decision-making unit are time-dependent. By the traditional method, the efficiency of decision units is impossible in the presence of time-dependent input prices. On the other hand, the exact method of cost-efficiency calculation is also difficult and time-consuming. In this study, a new method for calculating cost efficiency of decision making units with time-dependent prices during a period of time using numerical integral is presented. As the information of the decision-making units varies over time, a method for calculating their cost efficiency accurately is presented. however, the exact method is difficult or impossible to be solved in some cases. Therefore, in this study, an approximate method for calculating the cost efficiency in the given state is presented. This is a suitable replacement for the precise method. The efficiency of decision making units at different time is measured and the units are ranked using the proposed method. Finally, a numerical example is provided to indicate the method and compare it with the precise method. This study shows that the efficiency obtained by the approximate method is very close to the efficiency obtained by the exact method, and at the same time, the calculation speed increases.

Keywords: *Data envelopment analysis, Time-dependent prices, Time-dependent cost efficiency, Ranking.*

The manuscript was received on 08/12/2022, revised on 10/19/2022- and accepted for publication on 11/16/2022.

١. Introduction

Data Envelopment Analysis (DEA) is a nonparametric method for evaluating the performance of Decision-Making Units (DMUs) and calculating their efficiency, first proposed by Charnes et al [3]. Today, DEA is quickly growing and being used for evaluation of organizations, and various industries such as banking, mailing, sports clubs, hospitals, educational centers, power plants, refineries and so on. One of the most recent studies in the field of DEA can be mentioned by Komijani et al [14], Modhej et al. [16] and Fallah et al. [6].

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With the help of DEA models, the weaknesses of the organizations in different indices are found and the progressive policies of the organizations are determined and some suggestions are made to overcome these weaknesses. One of the most important goals of executives is to produce their products with minimum costs. When the input prices are available, DEA can compute the cost efficiency of DMUs. In cost efficiency models, the ability to produce current output with minimum cost is computed. The concept of cost efficiency is first introduced by Farel [9] and then developed by Fare et al. [8] using a linear programing model. They considered cost efficiency of a DMU as the proportion of the minimum possible cost for producing current output to the actual cost paid for that DMU. Then, Tone [21] improved Fare`s model by considering production possibility set based on costs instead of the traditional production possibility set. Other studies in this area include Kholmuminov et al., [13] and Haralayya et al. [10]. Then, numerous studies on cost efficiency have been conducted by various researchers, some of which are presented in the following sections. As mentioned above, the first profitability models calculate the cost efficiency of the DMUs when all data of the DMU, including the input data and output values, the costs of the inputs are determined and fixed. In practice, however, this detailed information is sometimes unavailable for various reasons. To solve these problems, various types of data are input, including fuzzy data, interval data, qualitative data, ordinal data, and stochastic data. Below are some studies on different types of data in DEA.

Many methods have been offered to measure the efficiency of DUMs in the presence of fuzzy data and Hatami et al. [11] classified them and examined most of them. Moreover, a lot of researchers carried out research on cost efficiency in the presence of fuzzy data. In a pure fuzzy environment, Pourmahmood et al. [18] computed cost efficiency of DMUs. One of the most recent studies in the field of cost efficiency in the fuzzy environment can be mentioned by Pourmahmoud et al [19].

Many scientists examined computing cost efficiency, profit and revenue in the presence of interval data. Camanho et al. [1] and Hosseinzade Lotfi et al. [12] are considered as first researchers who calculated cost efficiency and/or profit of DMUs in the presence of interval data. Camanho et al. introduced a method for calculating cost efficiency of DMUs with interval input prices and the certain values of input and output. Hosseinzade et al calculated cost efficiency of DMUs that the values of input, output and the price of their outputs are interval. Dyvak et al. (2022) [5] are the most recent studies in the field of interval data.

Some of the studies done on computation of cost efficiency of DMUs in the presence of ordinal, qualitative and stochastic data will be discussed. The first study was done by Cook et al. [4] on ordinal data. In the literature, less attention has been paid to qualitative data and sometimes it is discussed together with interval, ordinal or fuzzy data. Developed models for evaluating the efficiency of DMUs are shown with random variables when the data is random and they are introduced based on the idea of chance-constrained plan by Charnes et al., [2] and land et al. [15].

Additionally, A kind of data known as time-dependent data are first introduced by Taeb et al. [20] They computed DMUs in which the values of input and outputs are functions of time. Their proposed model was nonlinear which was impossible or too hard to be converted into a linear model. Therefore, they used approximate methods to solve this model. They obtained the efficiency of DMUs under evaluation as a function of time through interpolation.

In many every-day life situations, the prices of different things such as gold, oil, stocks etc. fluctuates as time changes ,that is they are time-dependent and this fluctuation in prices leads to changes in cost efficiency of DMUs. Sometimes these changes are so fast that we need to quickly calculate the cost efficiency at any time so that DMUs can be evaluated during an interval. As a result, we need to introduce a new definition for cost efficiency in this interval. This study aims to calculate cost efficiency of DMUs in which the market prices for input are time-dependent and the values of input and output are fixed and definite. It is assumed that input prices are a function of time and values of input and output for DMUs are positive numbers. Approximate methods are used to compute the efficiency of each DMU and our proposed method is compared with other methods.

Table 1. Literature Gap

Researches	Method used
Previous Researches [20, 11, 12, 2, 15, 4, and 17]	Calculating the efficiency of DMUs or their cost efficiency in the presence of inputs and outputs time-dependent/ fuzzy/ interval/ stochastic/ ordinal/ qualitative. Accurate calculation of cost efficiency in the presence of time-dependent input prices
Current study	Calculating the efficiency of DMUs in the presence of time-dependent prices
Comparison	Approximate calculation of cost efficiency in the presence of time-dependent input prices

In the case that the inputs and outputs are accurate and the price of the inputs is a function of time, the cost efficiency of DMUs has been accurately calculated by Pourmahmoud et al. [17]. while in practice, in the evaluation of DMUs, cost efficiency is obtained as a function of time, and solving these types of problems using different software is sometimes either difficult or impossible. To deal with this defect, an approximate method is presented in this study. which is a suitable alternative to the exact method. via a numerical example, it is shown that the results of both exact and approximate methods are very close to each other. This is the superiority of the method presented in this study compared to previous models. Literature gap is given in the table 1.

This study is organized as follows. Section 2 presents traditional cost efficiency from Fare viewpoint. Section3 proposes a new model for calculating cost efficiency of DMUs in the presence of time-dependent market prices. Section 4 discusses a numerical example. Finally, the last section deals with conclusion.

٢. Traditional cost efficiency

Assume that there are n DMUs being evaluated and each one consumes m input and produces s output. In other words, by consuming input $x_o=(x_{1o},x_{2o},...,x_{mo})$, DMU_o , o= 1,2,…,n produces output $y_o=(y_{1o},y_{2o},...,y_{so})$ (all input and outputs are considered non-negative). Also suppose that price vectors of inputs for DMU is as $w_o=(w_{1o},w_{2o},...,w_{mo})$. Therefore, the current cost by DMU_o equals:

$$C_o=\sum_{i=1}^{m} w_{io}x_{io} \tag{1}$$

DMU_o is efficient of cost when its current cost is the lease cost that y_o can produce. To calculate the lowest possible cost for producing y_o , cost minimization problem in variable return to scale (VRS) ,introduced by Far et al, [7] can be solved:

$$\begin{aligned} C_o^* = \min & \sum_{i=1}^{m} w_{io}x_i \\ s.t. & \sum_{j=1}^{n}\lambda_j x_{ij} \le x_i, i=1,2,...,m, \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro}, r=1,2,...,s, \\ & \sum_{j=1}^{n}\lambda_j = 1, \\ & \lambda_j \ge 0, j=1,2,...,n. \end{aligned} \tag{2}$$

If $x^*=(x_1^*,x_2^*,...,x_m^*)$ is the answer of optimizing problem (2), the minimum cost for producing y_o equals:

$$C_o^* = \sum_{i=1}^{m} w_{io} x_i^*$$

Fare et al introduced cost efficiency of DMU_o as follows:

$$CE_o = \frac{C_o^*}{C_o}$$

Where the closer CE_o is to one, the more efficient the DMU being evaluated is.

٣. The proposed time-dependent cost efficiency model

In reality, market prices such as prices of gold, oil, etc. often fluctuates at different time , so cost efficiency of DMUs using these inputs changes at different time. Sometimes the evaluation of cost efficiency of DMUs during a given interval is needed. so these kinds of evaluation are time dependent. Suppose n DMUs are being evaluated in which inputs, outputs, definite data and input prices are time dependent. Assume that for DMU_o, the inputs, the outputs and input prices are $x_o = (x_{1o}, x_{2o}, ..., x_{mo})$, $y_o = (y_{1o}, y_{2o}, ..., y_{so})$ and $w_o(t) = (w_{1o}(t), w_{2o}(t), ..., w_{mo}(t))$, respectively. $w_{io}(t)$ is a function of time which shows input prices at time t. the current cost of DMU_o (at time t) equals:

$$C_o(t) = \sum_{i=1}^{m} w_{io}(t) x_{io} \qquad (3)$$

DMU_o at time t with current cost $C_o(t)$ is the cost efficient if at this time there is no DMU with current cost less than $C_o(t)$ that can produce y_o . model 2 is used for evaluating cost efficiency when input prices are fixed at different time. Therefore, this model cannot be used for when the prices are changing. In this state, for evaluating cost efficiency of DMU_o in VRS at time interval $t \in [t_1, t_2]$ the following model is suggested:

$$C_o^*(t) = \min \sum_{i=1}^{m} w_{io}(t) x_i \tag{4}$$
$$s.t. \ \sum_{j=1}^{n} \lambda_j x_{ij} \le x_i, i = 1, 2, ..., m,$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, r = 1, 2, ..., s,$$
$$\sum_{j=1}^{n} \lambda_j = 1,$$
$$\lambda_j \ge 0, \ j = 1, 2, ..., n.$$

Since the input prices are constantly changing, cost efficiency of DMUs is changing too. In other words, cost efficiency is dependent on time. As a result, for when the prices of inputs are dependent on time, time-dependent cost efficiency can be defined as:

Definition 3.1. time-dependent cost efficiency of DMU_o is a function of time as is defined as:

$$CE_o(t) = \frac{C_o^*(t)}{C_o(t)} \tag{5}$$

In relation (5), $C_o(t) \ne 0$. because current cost of DMU_o never becomes zero. Since, based on Relation (3), for DMU_o there is always at least one DMU with a fixed price dependent on time entering the system.

Theorem 3.1.

A) Model (٤) is feasible.

B) time-dependent cost efficiency of DMU_o is always less than or equal to one. That is, $CE_o(t) \le 1$

Proof.

A) for feasibility of Model (4) try to provide a feasible answer for it. Answer $x_i = x_{io}, \lambda_i = 1, \forall j, \ j \ne i; \lambda_j = 0$ is within the limitations (constraints) of the problem so it is a feasible answer for Model (4).

B) Based on the feasible answer provided in A and Relation 3, $C_o(t)$ is one of values of objective function of model (4). Hence, regarding the kind of objective function of Model 4: $C_o^*(t) \le C_o(t)$. so the proof is done.

When for DMU_o at time t_i , $CE_o(t_i)=1$, then this DMU at time t_i is cost efficient. Obviously, DMU_o can be efficient at a time and inefficient at another time. Relation (5) is a fractional function in terms of t which can be used at any given t . As a result Relation (5) is converted into $CE_o = C_o^* / C_o$ at special time which was presented by Fare et al for fixed prices. However, for calculating cost efficiency in terms of time, it is not possible to use relation (5) and it is only presented theoretically. Thus, a new definition for time-dependent cost efficiency is offered:

Definition 3.2. the relative accumulative cost efficiency of DMU_o during time interval $T=\left[t_1,t_2\right]$ is shown as CE_o^T and it is defined as :

$$CE_o^T = \frac{\int_{t_1}^{t_2} CE_O(t)dt}{\max\limits_{1\le j\le n} \int_{t_1}^{t_2} CE_j(t)dt} \tag{6}$$

Theorem 3.2. for each time interval $T=\left[t_1,t_2\right]$, $CE_o^T \le 1$.

Proof. regarding relation (6), it is true without needing a proof.

Model (4) is a parametric programing issue due to the presence of $w_{io}(t)$ which needs to be solved through algorithms related to parametric programing. On the other hand, solving parametric problems is time-consuming due to the existing amount of data in evaluating cost efficiency of DMUs, increase in number of DMUs, inputs and outputs. Therefore, calculating $C_o^*(t)$ and, in turn, calculating $CE_o(t)$ and CE_o^T gets harder and more time-consuming. CE_o^T can be calculated via approximate methods. In this study, numerical integration methods are used for computing CE_o^T. There are many approximate methods for computing integration. Since Composite Simpson Rule has high accuracy, this method is used for computing $\int_{t_1}^{t_2} CE_j(t)dt$ in this study. The formula for integration using Composite Simpson Rule for f(x) at [a,b] is:

$$\int_a^b f(x)dx = \int_{x_0}^{x_{2n}} f(x)dx \approx S(h) = \frac{h}{3}(f(x_0) + 4\sum_{i=0}^{n-1} f(x_{2i+1}) + 2\sum_{i=1}^{n-1} f(x_{2i}) + f(x_{2n})) \quad (7)$$

Where [a,b] is divided into 2n subsets of equal distances $[x_i, x_{i+1}]$ and $h = \frac{b-a}{2n}$. The proposed algorithm for computing relative accumulative cost efficiency of DMUs is:

Algorithm 3.1. Computing CET

Step 1. Enter the following items:

- Number of DMUs for n
- Number of inputs for m
- Number of outputs for s
- The value of inputs and outputs
- The prices of inputs
- Values of t_1 and t_2 for the time interval and the even natural number for L

Step 2. Divide $T = [t_1, t_2]$ into L subsets of $[a_0, a_1], [a_1, a_2], ..., [a_{L-1}, a_L]$ with equal distances

Step 3. Set j=1

Step 4. Apply algorithm (2) for j and save the result in CE_j .

Step 5. If $j < n$, add a unit to j and return to Step 4.

Step 6. Put the maximum of CE_j , j=1,2,…,n in MCE.

Step7. Set o=1

Step8. Calculate CE_o^T using $CE_o^T = \frac{CE_j}{MCE}$.

Step 9. If $o < n$, add a unit to o and then go back to Step 8.

Step 10. Finish.

Algorithm 3.2. Calculating $\int_{t_1}^{t_2} CE_j(t)dt$

Step 1. Set $l = 0$

Step 2. Calculate $C_j(a_l)$ from $C_j(a_l) = \sum_{i=1}^{m} w_{ij}(a_l) x_{ij}$.

Step 3. Solve Model (4) for $t = a_l$ and save its optimal value in $C_j^*(a_l)$.

Step 4. Calculate $CE_j(a_l)$ from $CE_j(a_l) = \dfrac{C_j^*(a_l)}{C_j(a_l)}$.

Step 5. If $l < L$, add a unit to l and go back to Step 4.

Step 6. Compute $\int_{t_1}^{t_2} CE_j(t)dt$ using the following Relation:

$$\int_{t_1}^{t_2} CE_j(t)dt = \frac{t_2 - t_1}{3L}(CE_j(a_0) + 4CE_j(a_1) + 2CE_j(a_2) + \ldots + 2CE_j(a_{L-2}) + 4CE_j(a_{L-1}) + CE_j(a_L))$$

Step 7. Finish.

Definition 3.3. When for DMU_k and DMU_q during $T = [t_1, t_2]$ we have $CE_q^T < CE_k^T$, the efficiency of DMU_k is more than DMU_q, it is said that DMU_k is more efficient than DMU_q.

٤. Numerical example

In this section, a numerical example is presented in order to explain the proposed method. Assume that the information on five DMUs with two inputs and one output is given in Table 2.

Table 2. Five DMUs' data

DMU	Input		Output	Time Dependent Input Prices	
	Input 1	Input 2		Input Price 1	Input Price 2
A	5	1	2	$9t^2 + 2$	$2t^2 + 3$
B	3	2	3	$1 + 6t$	$1 + 2t$
C	4	5	1	$t^2 + 1$	$2t^2 + 1$
D	2	4	2	$1 + 5t$	$1 + t$
E	2	6	1	$7t + 2$	$3t + 3$

The proposed method by Algorithms 3.1 and 3.2 is applied to the information in Table 2 as follows. To this end, in the calculation of CE_j^T for *j=A, B, C, D, E,* an interval of [0,1] is selected and it is assumed L=10. Therefore, [0,1] is divided into 10 subsets of equal distances. By calculating $CE_j(t)$ for each *t=0*, *t=0.1*, *t=0.2*,…,*t=1*, the value of CE_j for each of the DMUs is obtained. Note that in this example $CE_j = CE_j^T$, since MCE=1. The results of implementing algorithms 3.1 and 3.2 in calculating the time-dependent cost efficiency of DMUs are listed in Table 3.

Table 3. Five decision making units' data

	DMU_A	DMU_B	DMU_C	DMU_D	DMU_E
$CE_j(0)$	0.923	1.000	0.556	0.833	0.545
$CE_j(0.1)$	0.914	1.000	0.555	0.905	0.683
$CE_j(0.2)$	0.890	1.000	0.552	0.955	0.613
$CE_j(0.3)$	0.858	1.000	0.549	0.990	0.636
$CE_j(0.4)$	0.827	1.000	0.544	1.000	0.655
$CE_j(0.5)$	0.798	1.000	0.540	1.000	0.671
$CE_j(0.6)$	0.774	1.000	0.536	1.000	0.684
$CE_j(0.7)$	0.755	1.000	0.532	1.000	0.696
$CE_j(0.8)$	0.739	1.000	0.528	1.000	0.706
$CE_j(0.9)$	0.726	1.000	0.525	1.000	0.715
$CE_j(1)$	0.700	1.000	0.522	1.000	0.722
CE_j^T	0.810	1.000	0.540	0.977	0.660

The second to twelfth rows of Table 3 show the cost efficiency of each of the DMUs for nodal points *t=0*, *t=0.1*, *t=0.2*,…,*t=1*. Finally, the amount of time-dependent cost efficiency of DMUs in the [0,1] is calculated by an approximate method and is listed in the 13th row of Table 3. DMUs can be ranked via the values in the 13th row of Table 3. Ranking of the DMUs in this example are: B, D, A, E, C. When the time-dependent cost efficiency of the DMUs of this example are calculated in a precise way, the ranking of the DMUs are: B, D, A, E, C. Therefore, as can be seen, the ranking of DMUs is the same in both exact and approximate methods. But in real problems, the results of both methods will be very close. Here, the difference of CE_j^T values between two methods is less than 0.001 to three decimal places. in this study, with such a difference, the proposed method is faster and easier than the exact method. The other important feature of this method is that it is easy to solve with different programming languages, while this feature does not present in the exact method. Therefore, it is suggested to use the proposed method instead of the exact method to solve similar problems.

٥. Conclusion

In evaluating DMUs, one of the issues presented by DEA is their cost efficiency. Cost efficiency evaluates the ability of a DMU for producing the current outputs with the lowest possible cost. In traditional models, cost efficiency of DMUs are calculated at a fixed time so the prices are considered fixed too. However, in practice, input prices at different time are function of time and fluctuate. In other words, input prices are dependent on time. Our proposed method is based on time-dependent input prices. Since this is model is a parametric problem and it is hard to solve when the number of DMUs are high so it is recommended to solve it with approximate methods. In this study, cost efficiency of DMUs are calculated using approximate methods in the presence of time-dependent input prices. Some definitions are provided for time-dependent cost efficiency. Finally, an example of cost efficiency of DMUs during a definite time period is measured and then the DMUs are ranked. This example shows that the presented approximate method gives similar results compared to the exact method. In addition, the approximation method is simpler and has a higher calculation speed. The proposed plan can be utilized in organizations and factories as well as for similar issues. The recommended model in this study was VRS. This problem can also be solved in constant return to scale (CRS). Additionally, numerical integration was used to calculate relative accumulative cost efficiency which can be replaced with interpolation methods to calculate time-dependent cost efficiency and then the integration of the existing function is calculateed accurately. This model is based the model of Fare et al. and it is recommended to design a model based on Tone` model. The mentioned points can open up new research opportunities for researches.

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