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Inverse Data Envelopment Analysis on the Base of Non-Convex Cost Efficiency

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Cost efficiency in which cost coefficients are given for some inputs (cost coefficients can be different for disparate decision-making units (DMUs)) is one of the most important concepts in data envelopment analysis (DEA) to analyze the performance. Moreover, in some occasions, the cost performance and changes of input measures should be addressed while the convexity property is violated. Therefore, in this paper, first a DEA model is provided to assess cost efficiency based on the free disposal hull (FDH) model. Then, by considering cost and technical efficiencies achieved, a multi-objective problem called the inverse FDH cost model is presented to determine input values based on output changes while the cost and technical efficiency levels are preserved. The multi-objective problem is computed applying two approaches. Also, a dataset from the literature is presented to show the performance of the proposed method. For this purpose, we used the data of six banks in different countries. We added 2% to the outputs and analyzed the inputs with two models. In the first model, we used cost coefficients for weights, and in the second model, we used the same weights. Contrary to forecasts, some entries have decreased and others have increased. But from the results, we have noticed that the first model is more realistic because most of the solutions have increased in this model.

***Keywords**: Cost efficiency, DEA, Inverse DEA, FDH.*

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1. Introduction

Data Envelopment Analysis (DEA) as a non-parametric technique includes various models for evaluating the relative efficiency of decision-making units (DMUs) concerning multiple inputs and multiple outputs. The first DEA paper was presented by Charnes et al. [9], and then, many researchers addressed the performance of systems based on various extended DEA models such as [1-3, 6-7, 22]. One of the most significant information obtained from DEA models is the cost efficiency of DMUs. In fact, one of the most major aspects of analysing the production of organizations is measuring costs and incomes [14].

The cost efficiency model attempts to find the lowest cost for inputs [4, 15, 16, 27, 29, 32]. Cost efficiency calculations contain cases where the prices of some inputs in each decision-making unit are precisely known and even cases where the price information in each decision-making unit is vague and imprecise [5, 8, 11, 18, 19, 23, 25, 26, 31]. These facts show that DEA models can provide a robust approximation of cost efficiency even when prices are unknown. Cost efficiency was first developed by Farrell [14] and then by Fare et al. [12, 13]. Where input price information is

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available in each DMU, cost efficiency evaluation can be addressed based on Farrell's method, and in other conditions where the exact input prices in each unit are not known and only the upper and lower bounds of these prices are available, it can be used some existing related approaches to calculate efficiency scores. Studies on cost efficiency estimation with the unknown and imprecise prices were primarily provided by Thompson et al. [31] and Schaffnit et al. [26]. Furthermore, Khanjani Shiraz et al. [28] presented a rough cost efficiency under convex DEA and free disposal hull (FDH) technologies. Leleu [20] introduced a linear structure for FDH technologies and FDH cost functions. The FDH model is one of the most widely used models in DEA that the convexity principle is ignored. Pourmahmoud et al. [34] evaluated cost efficiency using the fuzzy DEA method. Also Pourmahmoud et al. [35] calculated the cost efficiency using prices dependent on time via approximate method.

In addition to efficiency analysis, the estimation of changes in some outputs (inputs) for changes in some inputs (outputs) when the efficiency value is maintained is an important aspect for decision makers. Accordingly, in the DEA literature, one can find studies such as [21, 33] that pay attention to this issue. For further explanation, Wei et al. [32] originally developed an inverse DEA approach to consider inputs (outputs). Lertworasirikul et al. [21] presented the inverse BCC model to deal with the resource allocation problem while some outputs increase and others decrease. Asadi et al. [36] presented inverse free disposal hull models from optimistic and pessimistic aspects. Ghiyasi [15] provided inverse DEA models founded on cost and revenue efficiencies. Moreover, Soleimani-Chamkhorami et al. [30] planned alternative inverse DEA models to investigate the changes of data while cost and revenue efficiencies are maintained. Some studies [37, 38] addressed the changes of performance measures in two-stage processes where price information is presented. However, there is no DEA study to estimate the changes of inputs for the modifications of outputs while input prices are available and the convexity property is not held.

For this reason, in this research, after presenting the FDH cost model, an inverse FDH cost model is proposed to assess inputs for changes of outputs when the input prices are specified, and cost and technical efficiencies are kept. The proposed inverse FDH cost approach is a multi-objective problem and two plans is applied to address it. Moreover, a set of data from the literature is given to demonstrate the introduced procedure.

The rest of this paper organized as follows. A review of the FDH model, cost efficiency, and inverse DEA is declared in Section 2. The main procedure to estimate inputs with known prices for the changes of outputs in accordance with the non-convex technology and the preservation of technical and cost efficiencies is described in Section 3. A set of data is given in Section 4 to clarify the rendered approach. Finally, conclusions and suggestions are presented in Section 5.

2. Preliminaries

In this section, some primary items connected to the next sections are examined. Specifically, the FDH model, cost efficiency, and inverse DEA are described.

The terms applied in this research are outlined as follows:

$DMU_j\,(j=1,...,n)$: j th decision making unit,

DMU_o: The unit under consideration,

x_{ij}: i th input of DMU_j,

y_{rj}: r th output of DMU_j,

x_{io}: i th input of DMU_o,

y_{ro}: r th output of DMU_o,

λ_j: The intensity variables,

$i = 1,...,m$: The subscript that shows inputs
$r = 1,...,s$: The subscript that shows outputs
γ : Nonnegative variable,
c_{io} : Prices related to i th input of DMU_o ,
M : A positive large number,
Δx_{io} : The changes of inputs related to DMU_o ,
Δy_{ro} : The changes of outputs related to DMU_o ,
θ_o^* : The optimal value achieved that is considered as the efficiency level of DMU_o .

2.1. FDH Model

DEA includes five basic principles, envelopment, convexity, free disposability, constant returns to scale (CRS), minimum extrapolation. Without considering the convexity principle, the FDH model was rendered by Deprins et al. [10]. The FDH model under CRS is as follows:

$$
\begin{aligned}
& \min \ \theta_o \\
& s.t. \sum_{j=1}^{n} \gamma \lambda_j x_{ij} \le \theta_o x_{io}, \quad i = 1,2,...,m, \\
& \quad \sum_{j=1}^{n} \gamma \lambda_j y_{rj} \ge y_{ro}, \quad r = 1,2,...,s, \\
& \quad \sum_{j=1}^{n} \lambda_j = 1, \\
& \quad \lambda_j \in \{0,1\}, \quad j = 1,2,...,n, \\
& \quad \gamma \ge 0.
\end{aligned}
\tag{1}
$$

The value of the objective function in model (1) is less than or equal to one. If this value is equal to one, the unit o, DMU_o , is called efficient and otherwise, it is inefficient. Of course, this problem is a non-linear programming one involving binary variables. To solve it, the following approach has been introduced by Podinovski [24].

$$\begin{aligned} &\theta_o^* = \min\ \theta_o \\ &s.t. \sum_{j=1}^{n} \Lambda_j x_{ij} \le \theta_o x_{io}, \quad i = 1, 2, \ldots, m, \\ &\sum_{j=1}^{n} \Lambda_j y_{rj} \ge y_{ro}, \quad r = 1, 2, \ldots, s, \\ &\sum_{j=1}^{n} \lambda_j = 1, \\ &0 \le \Lambda_j \le M \lambda_j, \\ &\lambda_j \in \{0,1\}, \quad j = 1, 2, \ldots, n, \end{aligned} \tag{2}$$

where $\Lambda_j = \gamma \lambda_j$.

2.2. Cost efficiency

In the presence of input prices, the cost efficieny can be applied to estimate the performance. Farrell [14] proposed the subsequent approach to measure the cost efficiency. Model (5) measures the minimum cost.

$$\begin{aligned} &\min \sum_{i=1}^{m} c_{io} x_i \\ &s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le x_i, \quad i = 1, 2, \ldots, m, \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r = 1, 2, \ldots, s, \\ &\lambda_j \ge 0, \quad j = 1, 2, \ldots, n, \end{aligned} \tag{3}$$

in which $x_i (i = 1, \ldots, m)$ are the decision variables. Accordingly, the cost efficiency can be defined as follows:

$$\frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}} \tag{4}$$

where x_i^* is the optimal solution obtained from model (3).

2.3. Inverse DEA

In many situations, DMUs need to change, so changes are made to the inputs and then the amount of change in outputs is measured (or conversely, the amount of the output is changed, then the amount of input is estimated). In this case, new units are made based on their needs. For this purpose, first the efficiency in the initial model is calculated, then the inverse model is designed so that the efficiency value remains the same as the original one. Thus, the relative efficiency can be evaluated using the CCR (Charnes, Cooper and Rhodes) model as follows:

$$\begin{aligned} &\min\ \theta_o \\ &s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o x_{io}, \quad i=1,2,\dots,m, \\ &\quad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r=1,2,\dots,s, \\ &\quad \lambda_j \ge 0, \quad j=1,2,\dots,n. \end{aligned} \tag{5}$$

Suppose that θ_o^* is the optimal value achieved from model (5). By following [21, 33], for changing outputs as much as Δy_{ro}, the changes of the inputs are calculated using the next inverse problem (6):

$$\begin{aligned} &\min\ (\Delta x_{1o}, \Delta x_{2o}, \dots, \Delta x_{mo}) \\ &s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o^* (x_{io} + \Delta x_{io}), \quad i=1,2,\dots,m, \\ &\quad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \Delta y_{ro}, \quad r=1,2,\dots,s, \\ &\quad \lambda_j \ge 0, \quad j=1,2,\dots,n. \end{aligned} \tag{6}$$

Also, some conditions can be added to problem to control Δx_{io}. For example, if Δy_{ro} be nonnegative, we can add the nonnegativity condition for Δx_{io}. To solve the multi-objective model (6), the weighted sum approach can be used.

3. Main Model

In this section, we have proposed a model based on FDH technology. In cost efficiency models, instead of going radially towards the efficient frontier, we use the direction related to the costs of each DMU. This model is based on constant returns to scale and binary variables. Therefore, a non-linear programming includes binary variables is made that we have linearized, but it still includes binary variables. After that, the inverse model and the solution of the related inverse problem have been investigated. The inverse model is a multi-objective problem that has been solved using two different methods. In inverse models, the efficiency is preserved. The objective function of its basic model uses some data to consider costs for each input. Therefore, it reduces the inputs in the direction that the lowest cost occurs. In the inverse model, changes in the output are given and based on those changes in the input are measured. In inverse models, a multi-objective problem arises which is solved by two approaches. In one of them, the weighted sum approach is used, and in the other, the same costs are used to unify the objective.

3.1. FDH Cost Efficiency

To calculate the cost efficiency based on the FDH, model (7) can be utilized.

$$\begin{aligned} \min\ & \frac{\sum_{i=1}^{m} c_{io} x_i}{\sum_{i=1}^{m} c_{io} x_{io}} \\ s.t. & \sum_{j=1}^{n} \gamma \lambda_j x_{ij} \le x_i, \quad i=1,2,\dots,m, \\ & \sum_{j=1}^{n} \gamma \lambda_j y_{rj} \ge y_{ro}, \quad r=1,2,\dots,s, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \in \{0,1\}, \quad j=1,2,\dots,n, \\ & \gamma \ge 0, x_i \ge 0, \end{aligned} \tag{7}$$

in which $\sum_{i=1}^{m} c_{io} x_{io}$ is a fixed number. The value $\frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}}$ is called the cost efficiency of FDH that is between zero to one for each unit under estimation. It is supposed that $\upsilon_o^* = \frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}}$. But the model (7) is non-linear and includes binary variables. By following [24], the change of variable $\Lambda_j = \gamma \lambda_j$ is applied to transform the non-linear model (7) into the mixed integer linear problem (8). Therefore, we have:

$$\begin{aligned}
&\min \frac{\sum_{i=1}^{m} c_{io} x_i}{\sum_{i=1}^{m} c_{io} x_{io}} \\
&s.t. \sum_{j=1}^{n} \Lambda_j x_{ij} \le x_i, \quad i = 1, 2, \dots, m, \\
&\quad \sum_{j=1}^{n} \Lambda_j y_{rj} \ge y_{ro}, \quad r = 1, 2, \dots, s, \\
&\quad \sum_{j=1}^{n} \lambda_j = 1, \\
&\quad \lambda_j \in \{0,1\}, \quad j = 1, 2, \dots, n, \\
&\quad 0 \le \Lambda_j \le M \lambda_j, \\
&\quad x_i \ge 0,
\end{aligned} \tag{8}$$

where M is a positive large enough number.

3.2. Inverse FDH Cost Efficiency

Suppose that the values of technical efficiency θ_o^* and cost efficiency υ_o^* have been obtained using models (2) and (8), respectively. At this time, the purpose is to estimate inputs for the perturbations of outputs while the FDH efficiency and the FDH cost efficiency levels are preserved. The amount of change related to outputs is shown by Δy_{ro}. Also, Δx_{io} indicates the amount of changes of inputs. As can be seen in model (9), it has been tried to include both efficiency values. Thus, we have:

$$
\begin{aligned}
&\min\ (\Delta x_{1o}, \Delta x_{2o}, \ldots, \Delta x_{mo}) \\
&s.t. \sum_{j=1}^{n} \gamma \lambda_j x_{ij} \le x_i, \quad i = 1, 2, \ldots, m, \\
&\sum_{j=1}^{n} \gamma \lambda_j x_{ij} \le \theta_o^* (x_{io} + \Delta x_{io}), \quad i = 1, 2, \ldots, m, \\
&\sum_{j=1}^{n} \gamma \lambda_j y_{rj} \ge y_{ro} + \Delta y_{ro}, \quad r = 1, 2, \ldots, s, \\
&\sum_{i=1}^{m} c_{io} x_i = \upsilon_o^* \sum_{i=1}^{m} c_{io} (x_{io} + \Delta x_{io}), \\
&\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j \in \{0, 1\}, \quad j = 1, 2, \ldots, n, \\
&\gamma \ge 0, x_i \ge 0, x_{io} + \Delta x_{io} \ge 0.
\end{aligned} \tag{9}
$$

As can be seen, model (9) is a multi-objective programming problem, accordingly, two methods for solving it are stated in the following. Also, notice that that the problem (9) is non-linear and includes binary variable that can be linearized with the before-mentioned technique. Therefore, we have:

$$
\begin{aligned}
&\min\ (\Delta x_{1o}, \Delta x_{2o}, \ldots, \Delta x_{mo}) \\
&s.t. \sum_{j=1}^{n} \Lambda_j x_{ij} \le x_i, \quad i = 1, 2, \ldots, m, \\
&\sum_{j=1}^{n} \Lambda_j x_{ij} \le \theta_o^* (x_{io} + \Delta x_{io}), \quad i = 1, 2, \ldots, m, \\
&\sum_{j=1}^{n} \Lambda_j y_{rj} \ge y_{ro} + \Delta y_{ro}, \quad r = 1, 2, \ldots, s, \\
&\sum_{i=1}^{m} c_{io} x_i = \upsilon_o^* \sum_{i=1}^{m} c_{io} (x_{io} + \Delta x_{io}), \\
&\sum_{j=1}^{n} \lambda_j = 1, \\
&0 \le \Lambda_j \le M \lambda_j, \\
&\lambda_j \in \{0, 1\}, \quad j = 1, 2, \ldots, n, \\
&\gamma \ge 0, x_i \ge 0, x_{io} + \Delta x_{io} \ge 0.
\end{aligned} \tag{10}
$$

In which M is a large enough number. Therefore, the problem is no longer nonlinear, but still has binary variables and it is a mixed integer linear problem.

3.3. Solving Multi-Objective Inverse FDH Cost Model

To solve the multi-objective problem (10), two approaches can be considered. The first approach is to use the weighted sum method and place weights according to their importance. In this research, equal weights are considered for all DMUs. The second approach is to use the same cost weights to solve the problem. Note that the second approach is not a special case of the first approach, because the cost weights are different for each unit, but in the weighted sum method, the same weights are considered for all units. Therefore, the next two problems can be computed:

$$
\begin{aligned}
\min\ & \omega_1 \Delta x_{1o} + \omega_2 \Delta x_{2o} + \ldots + \omega_m \Delta x_{mo} \\
s.t.\ & \sum_{j=1}^{n} \Lambda_j x_{ij} \le x_i, \quad i = 1,2,\ldots,m, \\
& \sum_{j=1}^{n} \Lambda_j x_{ij} \le \theta_o^* (x_{io} + \Delta x_{io}), \quad i = 1,2,\ldots,m, \\
& \sum_{j=1}^{n} \Lambda_j y_{rj} \ge y_{ro} + \Delta y_{ro}, \quad r = 1,2,\ldots,s, \\
& \sum_{i=1}^{m} c_{io} x_i = \upsilon_o^* \sum_{i=1}^{m} c_{io} (x_{io} + \Delta x_{io}), \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& 0 \le \Lambda_j \le M \lambda_j, \\
& \lambda_j \in \{0,1\}, \quad j = 1,2,\ldots,n, \\
& \gamma \ge 0, x_i \ge 0, x_{io} + \Delta x_{io} \ge 0.
\end{aligned} \tag{11}
$$

Where ω_i are constant positive weights for all units.

$$
\begin{aligned}
\min\ & c_{1o} \Delta x_{1o} + c_{2o} \Delta x_{2o} + \ldots + c_{mo} \Delta x_{mo} \\
s.t.\ & \sum_{j=1}^{n} \Lambda_j x_{ij} \le x_i, \quad i = 1,2,\ldots,m, \\
& \sum_{j=1}^{n} \Lambda_j x_{ij} \le \theta_o^* (x_{io} + \Delta x_{io}), \quad i = 1,2,\ldots,m, \\
& \sum_{j=1}^{n} \Lambda_j y_{rj} \ge y_{ro} + \Delta y_{ro}, \quad r = 1,2,\ldots,s, \\
& \sum_{i=1}^{m} c_{io} x_i = \upsilon_o^* \sum_{i=1}^{m} c_{io} (x_{io} + \Delta x_{io}), \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& 0 \le \Lambda_j \le M \lambda_j, \\
& \lambda_j \in \{0,1\}, \quad j = 1,2,\ldots,n, \\
& \gamma \ge 0, x_i \ge 0, x_{io} + \Delta x_{io} \ge 0.
\end{aligned} \tag{12}
$$

4. Numerical Result

In this section, we present a numerical example and analyze the results.

4.1 Example

In this section, the dataset of six banks from different countries is used to examine the introduced approach in this research. These details have been derived from [17, 30] and summarized from 1994 to 2006. The inputs and outputs are as follows. Inputs consist of fixed costs (x_1), labor (x_2), and borrowed funds (x_3). Input prices (c_i) are extracted from each bank as the depreciation relative to fixed assets, personnel expenses relative to full time equivalent and interest expenses relative to total borrowed funds. Outputs consist of the volume of customer deposits (y_1), the volume of customer credits (y_2) and the bank's net fee and commission incomes (y_3). The data are given in Table 1.

Table 1. The data set

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6
	Germany	Spain	US	France	Italy	UK
x_1	1965591	1983462	2785838	2225689	1402750	3974915
x_2	23885	25844	74740	30651	19389	57392
x_3	84700000	14100000	31100000	39700000	17600000	32600000
y_1	94700000	48000000	151000000	94900000	33800000	153000000
y_2	138000000	56500000	142000000	89500000	50600000	162000000
y_3	1621068	918988.5	2902312	1349212	967738.4	2661434
c_1	28.52	12.56	161.59	18.14	15.88	26.44
c_2	87297.62	47728.55	55525.5	73338.93	61359.3	4721128
c_3	24.61	81.7	62.24	50.21	241.7	12434

Now, the technical efficiency and cost efficiency based on the presented FDH model are calculated. The results are shown in Table 2.

Table 2. Technical and Cost Efficiencies

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6
	Germany	Spain	US	France	Italy	UK
Tech. Eff.	1	0.8071	1	1	1	1
Cost Eff.	1	0.7890	0.9774	0.8573	0.7626	1

Because the number of data is minor compared to the number of inputs and outputs, most of the DMUs are technically efficient, and only DMU 2 that is Spain with the score 0.8071 is not efficient. However, since the cost efficiency is dependent on costs, most of the banks are not efficient. For more illustration, Germany and UK are determined as cost efficient. Also, Italy with the score 0.7626 is the most cost inefficient bank.

In this part, an amount of two percent of outputs is added and input values are investigated. Two different perspectives are used to solve the multi-objective inverse cost FDH problem. The first viewpoint is to apply cost coefficients and the second view is to use the weighted sum method (we have considered the weights the same). The results of both aspects are given in Tables 3 and 4. Table 3 is related to the coefficients of the cost function and Table 4 is for constant coefficients. For more explanation in detail by considering cost coefficients, for the increase of outputs by two percent, three inputs, fixed costs, labor, and borrowed funds increase in Germany and UK as shown in Table 3. In Spain and US, labor decreases while the borrowed funds decrease in France and Italy.
Furthermore, for equal coefficients and the expansion of outputs by two percent, inputs of the US and the UK that are fixed costs, labor, and borrowed funds increase. Fixed costs and borrowed funds decrease in two countries, France and Italy. In Spain, fixed costs decrease and labor and borrowed funds increase. Moreover, fixed costs, labor increase and borrowed funds reduce in Germany.

Table 3. The difference between the new input and the previous one with cost coefficients

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6
	Germany	Spain	US	France	Italy	UK
Δx_1	39549.81	1192539	1923358	5719634	5381090	13258969
Δx_2	477.6757	-66.4804	-9868.08	10990.5	14675.35	1132.34
Δx_3	1694061	445548.7	5910654	-16414172.84	-3626821	653714.2

Table 4. The difference between the new input and the previous one with equal coefficients

	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6
	Germany	Spain	US	France	Italy	UK
Δx_1	1488168	-231564	55716.73	-434709	-390196	79498.24
Δx_2	25982.38	612.079	4268.075	27746.86	30170.76	1147.877
Δx_3	-56374222.68	268070.7	621999.7	-19706204.65	-6296245	651999.6

It is clear that some numbers are negative because there is a constraint that new inputs must be non-negative, not Δx_i. In the similar way, the variations of inputs can be addressed for different changes of outputs while the FDH and FDH cost efficiencies are maintained.
The comparison of the results of the FDH cost method with the cost efficiency scores presented in [30] shows that there is the difference between the efficiency level for Italy. Actually, for Italy, the cost efficiency is equal to 0.5949 in [30] while the value 0.7626 has been obtained in this research. Also, only Spain with the score 0.9998 is determined as inefficient, using the CCR model. Thus, the non-convexity assumption is effective on results. Moreover, comparing changes achieved from convex and non-convex methods is not rational in Spain and Italy due to the disparities of technical and cost efficiencies.

4.2 Sensitivity analyses

Because we increase the amount of outputs in this model, we expect the amount of inputs to increase as well, but this does not happen in the numerical results and some inputs have decreased. In the first model, that is, the model that we have used cost coefficients, is more appropriate because the number of inputs that have been reduced in it is less than the second model. Of course, in the

second model, weights can be chosen based on the decision maker opinion. We have used equal weights for the objective functions. As can be found, the proposed approach in this study is applicable to analyze the cost efficiency of DMUs and the changes of inputs while the convexity assumption is not held.
For future work, we suggest using variable returns to scale. It seems that it must have very different solutions than the proposed model, because the data are so different. It is also necessary to mention that in variable returns to scale in FDH models, the space is very small and most of the DMUs are efficient.

5. Conclusion

In many real-world studies, investigating the changes of performance measures is a significant aspect for managers while input prices are certain and the convexity property is violated. Therefore, in this paper, a method for calculating the cost efficiency based on the FDH model was first presented, and then an inverse FDH cost model was rendered for addressing the changes of performance measures. The presented inverse FDH cost model is a multi-objective programming problem that has been solved using two different approaches. Also, an example from the real world has been utilized to show the performance of the introduced method. In the proposed procedure, all measures were considered to be precise.
The extension of the suggested technique for situations that uncertain inputs and outputs are presented is an interesting topic for more investigation. Also, the development of the inverse FDH cost model to estimate performance measures of multi-stage processes is a prevailing topic for future research.

References

[1] Afsharian, M., Ahn, H., & Alirezaee, M. (2015). Developing selective proportionality on the FDH models: New insight on the proportionality axiom. *International journal of information and decision sciences*, *7*(2), 99-114.

[2] Amirteimoori, A., and Yang, F. (2014). A DEA model for two-stage parallel-series production processes. *RAIRO-Operations Research*, *48*(1), 123-134.

[3] Amirteimoori, A., Kordrostami, S., and Azizi, H. (2016). Additive models for network data envelopment analysis in the presence of shared resources. *Transportation Research Part D: Transport and Environment*, *48*, 411-424.

[4] Ashrafi, A., and Kaleibar, M. M., (2017). Cost, Revenue and Profit Efficiency Models in Generalized Fuzzy Data Envelopment Analysis. *Fuzzy Information and Engineering*, 9(2), 237- 246.

[5] Bagherzadeh Valami, H., (2009). Cost efficiency with triangular fuzzy number input prices: An application of DEA. *Chaos, Solitons and Fractals*, 42(3), 1631-1637.

[6] Banker, R.D., Charnes, A., Cooper, W.W. and Schinnar, A.P., (1981). A bi-extremal principle for frontier estimation and efficiency evaluations. *Management Science*, 27(12), 1370-1382.

[7] Banker, R.D., Cooper, W.W., Seiford, L.M., Thrall, R.M. and Zhu, J., (2004). Returns to scale in different DEA models. *European Journal of Operational Research*, 154(2), 345-362.

[8] Camanho, A. S., and Dyson, R. G., (2005). Cost efficiency measurement with price uncertainty: a DEA application to bank branch assessments. *European journal of operational research*, 161(2), 432- 446.

[9] Charnes, A., Cooper, W.W. and Rhodes, E., (1987). Measuring the efficiency of decision making units. *European journal of operational research*, 2(6), 429-444.

[10] Deprins, D., Simar, L., & Tulkens, H. (2006). Measuring labor-efficiency in post offices. Public goods, environmental externalities and fiscal competition. Springer, 285–309.

[11] Fang, L., and Hecheng, L., (2013). Duality and efficiency computations in the cost efficiency model with price uncertainty. *Computers & Operations Research*, 40(2), 594-602.

[12] Färe, R., Grosskopf, S., & Lovell, C. K., (2013). The measurement of efficiency of production (Vol. 6). Springer Science & Business Media.

[13] Färe, R., Grosskopf, S., Lovell, C.A.K., (1985). The Measurement of Efficiency of Production. Klumer-Nijhoff Publishing: Boston.

[14] Farrell, M. J., (1957). The measurement of productive efficiency. Journal of the Royal Statistical Society. Series A (General), 120(3), 253-281.

[15] Ghiyasi, M., (2017). Inverse DEA based on cost and revenue efficiency. *Computers & Industrial Engineering*, 114, 258-263.

[16] Jahanshahloo, G. R., Soleimani-Damaneh, M., & Mostafaee, A., (2007). On the computational complexity of cost efficiency analysis models. *Applied mathematics and computation*, 188(1), 638-640.

[17] Jimborean, R., & Brack, E. (2010). The cost-efficiency of French banks. MPRA Paper, 23471, University Library of Munich, Germany.

[18] Kuosmanen, T., & Post, T., (2001). Measuring economic efficiency with incomplete price information: With an application to European commercial banks. *European journal of operational research*, 134(1), 43-58.

[19] Kuosmanen, T., & Post, T., (2003). Measuring economic efficiency with incomplete price information. *European Journal of Operational Research*, 144(2), 454-457.

[20] Leleu, H. (2006). A linear programming framework for free disposal hull technologies and cost functions: Primal and dual models. *European journal of operational research*, *168*(2), 340-344.

[21] Lertworasirikul, S., Charnsethikul, P., & Fang, S. C. (2011). Inverse data envelopment analysis model to preserve relative efficiency values: The case of variable returns to scale. *Computers & Industrial Engineering*, *61*(4), 1017-1023.

[22] Liu, L.C., Lee, C. and Tzeng, G.H., (2004). DEA approach for the current and the cross period efficiency for evaluating the vocational education. *International Journal of Information Technology Decision Making*, 3(02), 353-374.

[23] Mostafaee, A., & Saljooghi, F. H., (2010). Cost efficiency measures in data envelopment analysis with data uncertainty. *European Journal of Operational Research*, 202(2), 595-603.

[24] Podinovski, V. V. (2004). On the linearisation of reference technologies for testing returns to scale in FDH models. *European Journal of Operational Research*, *152*(3), 800-802.

[25] Puri, J., & Yadav, S. P., (2016). A fully fuzzy DEA approach for cost and revenue efficiency measurements in the presence of undesirable outputs and its application to the banking sector in India. *International Journal of Fuzzy Systems*, 18(2), 212-226.

[26] Schaffnit, C., Rosen, D., & Paradi, J. C., (1997). Best practice analysis of bank branches: an application of DEA in a large Canadian bank. *European Journal of Operational Research*, 98(2), 269-289.

[27] Seyedboveir, S., Kordrostami, S., Daneshian, B., & Amirteimoori, A., (2017). Cost Efficiency Measurement in Data Envelopment Analysis with Dynamic Network Structures: A Relational Model. *Asia-Pacific Journal of Operational Research*, 34(05), 1750023.

[28] Shiraz, R. K., Fukuyama, H., Tavana, M., & Di Caprio, D. (2016). An integrated data envelopment analysis and free disposal hull framework for cost-efficiency measurement

using rough sets. *Applied Soft Computing*, *46*, 204-219.

[29] Shiraz, R. K., Hatami-Marbini, A., Emrouznejad, A., & Fukuyama, H., (2018). Chance-constrained cost efficiency in data envelopment analysis model with random inputs and outputs. *Operational Research*, 1-36.

[30] Soleimani-Chamkhorami, K., Hosseinzadeh Lotfi, F., Jahanshahloo, G. R., & Rostamy-Malkhalifeh, M. (2020). Preserving cost and revenue efficiency through inverse data envelopment analysis models. *INFOR: Information Systems and Operational Research*, *58*(4), 561-578.

[31] Thompson, R. G., Dharmapala, P. S., Humphrey, D. B., Taylor, W. M., & Thrall, R. M., (1996). Computing DEA/AR efficiency and profit ratio measures with an illustrative bank application. *Annals of Operations Research*, 68(3), 301-327.

[32] Tone, K., (2002). A strange case of the cost and allocative efficiencies in DEA. *Journal of the Operational Research Society*, 53(11), 1225-1231.

[33] Wei, Q., Zhang, J., & Zhang, X. (2000). An inverse DEA model for inputs/outputs estimate. *European Journal of Operational Research*, *121*(1), 151-163.

[34] Pourmahmoud, J., & Bafekr Sharak, N., (2020). Evaluating Cost Efficiency Using Fuzzy Data Envelopment Analysis method. *Iranian Journal of Operations Research*, 11(1), 25-42.

[35] Pourmahmoud, J., and N. Kaheh., (2022). Calculating cost efficiency using prices dependent on time via approximate method. *Iranian Journal of Operations Research*, 13(1), 1-12.

[36] Asadi, F., Kordrostami, S., Amirteimoori, A. & Bazrafshan, M., (2022). Inverse data envelopment analysis without convexity: double frontiers. *Decisions in Economics and Finance*. https://doi.org/10.1007/s10203-022-00377-8

[37] Shiri Daryani, Z., Tohidi, G., Daneshian, B., Razavyan, S., & Hosseinzadeh Lotfi, F. (2021). Inverse DEA in two-stage systems based on allocative efficiency. Journal of Intelligent & Fuzzy Systems, 40(1), 591-603.

[38] Shiri Daryani, Z., & Razavyan, S. (2021). Input Estimation in Two-Stage Systems with Undesirable Outputs Based on Cost Efficiency. International Journal of Data Envelopment Analysis, 9(4).