*Iranian Journal of Operations Research*
Vol. 13, No.1, 2022, pp. 184-197

# A Three-Stage Process for Fuzzy Stochastic Network Data Envelopment Analysis Models

Seyed Hadi Nasseri[*,1], Parastoo Niksefat[2]

*One of the most useful tools in Operations Research (OR) which is essentially applied to evaluate the performance of treated Decision-Making Units (DMUs) is Data Envelopment Analysis (DEA). Because of in the current decades, DEA models have been used and extended in many disciplines and hence attracted much interests. The traditional DEA treats DMUs as black boxes and calculates their efficiencies by considering their initial inputs and their final outputs. Since, in the real situations, input data are included some uncertainties, hence in this study we consider a DEA with fuzzy stochastic data and suggest a three-stage DEA model by taking into account undesirable output. To achieve this aim, an extended probability approach is applied to the reform of three-stage DEA models. Finally, we give an illustrative example by considering a case study on the banking industry.*

***Keywords***: *Data Envelopment Analysis, Network DEA, Fuzzy random variable, Multi-stage method Undesirable output.*
Manuscript was received on 28/01/2022, revised on 20/05/2022 and accepted for publication on 20/05/2022.

## 1. Introduction

Data Envelopment Analysis (DEA), initially introduced by Charnes et al. [2], requires crisp input and output data, whereas real-life decisions are usually made in a state of uncertainty. In such situations, we often face uncertain programming in the DEA model, where in the data could possess randomness and fuzziness. On the other hand, in a production system, the input usually goes through several processes before it becomes the output. Traditional DEA models treat the system as a whole unit, disregarding the interactions of the processes in the system when calculating the efficiency. This two progress in network and uncertainty DEA models need to be handeled together. This paper solves a case of the network DEA model in which the input and output data are assumed to be characterized by fuzzy random variables. The first study concerning to the network DEA was prepared by Charnes et al. [3]. Several models for measuring the efficiency of network systems have been proposed. Halkos et al. [9] provided a unified classification of the two-stage DEA model. This study was similarly presented by Zhou et al. [45]. Kwon and Lee [19] propose a new approach to model a two-stage production process supported by using data from large U.S. banks.

Liu et al. [24] proposed a two-stage DEA model with undesirable input–intermediate-outputs. Carrillo and Jorge [1] give a new model for ranking alternatives that use common weight DEA under a multi-objective optimization approach. Soleimani Kourandeh et al. [34] investigated the goal Weber location problem in which the location of some of demand points on a plane is given, and the ideal is locating the facility in the distance $R_i$, from the i-th demand point. Nasseri et al. [26] suggested a new ranking method based on the extension of PPS by virtual units named relatively similar units. Wu et al. [43] introduced a cross-efficiency approach based on Pareto optimality which can be generated by

[*,1] Department of Applied Mathematics, University of Mazandaran, Babolsar, IranE-mail: nasseri@umz.ac.ir
[2] Department of Applied Mathematics, University of Mazandaran, Babolsar, Iran , E-mail: parastooniksefat@gmail.com

[ Downloaded from iors.ir on 2026-01-29 ]

only a common set of weights. Hanafizade et al. [10] used neural network DEA for measuring the efficiency of mutual funds. Tootooni et al. [40] proposed a fuzzy type I and II programming approach for a new model presented in the literature, i.e., the single allocation ordered median problem. Sahoo et al. [32] discussed the return to scale and most productivity scale sizes in DEA with negative data. Hatami-Marbini et al. [11] classified the fuzzy DEA methods in the literature into five general groups, the tolerance approach [33],[39], the α-level based approach, the fuzzy ranking approach [11], the possibility approach [20], and the fuzzy arithmetic approach [42]. Among these approaches, the α-level based approach is probably the most relevant fuzzy DEA model in the literature. Nevertheless, the possibility approach seems to be more efficient in hybrid uncertainty, especially with a twofold fuzzy-random environment. Saati et al. [31] proposed a fuzzy CCR model as a possibilistic programming problem by applying an alternative α-cut approach. Puri and Yadav [29] applied the suggested methodology by Saati et al. [31] to solve the fuzzy DEA model with undesirable outputs. Khanjani et al. [14] proposed fuzzy-free disposal hull models under possibility and credibility measures. Khodabakhshi et al. [16] proposed a fuzzy DEA model with an optimistic and pessimistic performance and congestion analysis in fuzzy DEA. Kwakernaak [17,18] introduced the concept of the fuzzy random variable, and then this idea was enhanced by some researchers in the literature ([8],[21],[22],[30]). Qin and Liu [30] developed a Fuzzy Random DEA (FRDEA) model where randomness and fuzziness exist simultaneously. The authors characterized the fuzzy random data with known possibility and probability distributions. Tavana et al. [38] also introduced three different fuzzy stochastic DEA models consisting of probability-possibility, probability-necessity, and probability-credibility constraints in which input and output data entailed fuzziness and randomness at the same time.Also, Tavana et al. [37] and [39] provided a chance-constrained DEA model with random fuzzy inputs and outputs with Poisson, uniform and normal distributions. Khanjani et al. [15] proposed fuzzy rough DEA models based on the expected value and possibility approaches. Nasseri et al. [27] proposed a fuzzy stochastic DEA model. They formulated a linear and feasible model with an extension of normal distribution to deal with fuzzy random data. Miguel Sarmiento and Jorge E. Galan [25] show a stochastic frontier model with random inefficiency parameters to a sample of Colombian banks. Their model provides accurate cost and profit efficiency estimates. Ebrahimnejad et al. [7] solved dual DEA problems with fuzzy stochastic data. This approach overcomes the shortcomings of linearity and normal efficiency score relative to corresponding approaches. However, few studies have investigated the problem of allocating limited medical resources allocation among hospitals during public health emergencies ([22], [28], [35],[42]). This study tries to incorporate fuzzy random inputs and outputs in a network model with undesirable output. We apply extended probability measures to deal with the fuzzy random environments. The achievement of the present study is three items: (1) To formulate a new version of the network DEA model equipped with undesirable output, (2) To formulate a linear model for solving fuzzy stochastic two-stage DEA model, and (3) To demonstrate the applicability of the proposed model using a case study for the banking industry.

The remainder of the paper is organized as follows: Next section presents some approaches to a two-stage model and proposes our proposed network model equipped with fuzzy stochastic input and output data. In Section 3, the results of the case was conducted for the banking industry to evaluate the efficiency of 10 branches. Section 4 presents our conclusions and future research directions.

## ۲. Traditional DEA model

Data Envelopment Analysis (DEA) was originally proposed by Charnes, Cooper, and Rhodes [2]. DEA has been widely exploited to evaluate the efficiency of excess activities. DEA evaluates the relative efficiency of a set of DMUs using the ratio of the weighted sum of outputs to the weighted sum of inputs. Specifically, DEA determines a set of weights such that the efficiency of the undervalued

[ Downloaded from iors.ir on 2026-01-29 ]

DMU is maximized instead of other DMUs. The efficiency score varies in the interval [0,1], and a DMU with an efficiency score equal to 1 is called efficient.

Recall that DEA uses the ratio of the weighted sum of outputs to the weighted sum of inputs to measure efficiency. Since this ratio cannot exceed the value of 1, if each DMU has $s$ outputs and $m$ inputs, and $x_{ij}$ and $y_{rj}$ represent the value of the first input to DMUj and the value of the $r$th output of that DMU, respectively, the fractional form of the model DEA evaluates the efficiency

$DMU.$ is as follows:

$$\max \quad h_0 = \frac{\sum_{r=1}^{s} u_r y_{rj0}}{\sum_{i=1}^{m} v_i x_{ij0}},$$

$$s.t. \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1; \quad j = 1, \dots, n.$$

$$u_r, v_i \ge 0; \quad \text{for all} \quad r \quad \text{and} \quad j.$$

In this non-linear and non-convex problem, $h_0$ is the efficiency score of $DMU_0$ and the weights $v_i$ and $u_r$ are the decision variables of the given problem. There is a problem with this model, that is, it has countless solutions because if the optimal value of the variables is $v^*$ and $u^*$, there are other optimal solutions such as $\alpha v^*$ and $\alpha u^*$. To avoid this problem, a classic linear DEA model is obtained after two variable transformations as follows:

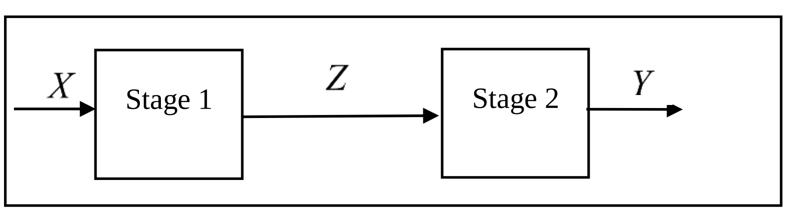
$$\begin{aligned} \max \quad & h_0 = \sum_{r=1}^{s} \mu_r y_{r0}, \\ s.t. \quad & \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} \le 0; \quad j = 1, \dots, n, \\ & \sum_{i=1}^{m} w_i x_{ij} = 1, \\ & \mu_r, w_i \ge 0; \quad \text{for all} \quad r \quad \text{and} \quad j. \end{aligned} \tag{1}$$

where $\mu_r = u_r \Big/ \sum_{r=1}^{s} u_r y_{r0}$ and $w_i = v_i \Big/ \sum_{i=1}^{m} v_i x_{i0}$.

# ٣. The proposed model

### 3.1 Two- stage model

Consider the two-stage process illustrated in Figure 1. We have $n$ DMUs that each $DMUj$ ($j =$ ١،٢$, \dots, n$) has $m$ inputs $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ and D outputs $Z_j = (z_{1j}, z_{2j}, \dots, z_{Dj})$ to the first stage. These D outputs known as the intermediate measures then are consumed in the second stage. The outputs from the second stage are $Y_j = (y_{1j}, y_{2j}, \dots, y_{rj})$. Chen and Zhu [5] developed an efficiency model that identified the efficient frontier of a two-stage production process linked by intermediate measures. They used a set of firms in the banking industry to illustrate how the new model could be utilized. Model (2) is the two-stage model proposed by Chen and Zhu.




[ Downloaded from iors.ir on 2026-01-29 ]

**Figure 1.** A two-stage DEA system

$$
\begin{aligned}
&\max \quad w_1\alpha - w_2\beta \\
&s.t. \\
&(\text{Stage}\,1) \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \le \alpha x_{io},\ i = 1,2,\ldots,m \qquad (2.1) \\
&\sum_{j=1}^{n} \lambda_j z_{dj} \ge \tilde{z}_{do},\ d = 1,2,\ldots,D \qquad (2.2) \\
&\lambda_j \ge 0,\ j = 1,2,\ldots,n \\
&(\text{Stage}\,2) \\
&\sum_{j=1}^{n} \mu_j z_{dj} \le \tilde{z}_{do},\ d = 1,2,\ldots,D \qquad (2.3) \\
&\sum_{j=1}^{n} \mu_j y_{rj} \ge \beta y_{ro},\ r = 1,2,\ldots,s \qquad (2.4) \\
&\mu_j \ge 0,\ j = 1,2,\ldots,n
\end{aligned}
\tag{2}
$$

where $\alpha$ and $\beta$ are the efficiency scores corresponding to Stage 1 and Stage 2, respectively.

In addition $z_{dj}$ are the intermediary inputs which are outputs of Stage 1 and inputs of Stage 2 and the values of $\tilde{z}_{do}$ are unknown. Moreover, $w_1$ and $w_2$ are the weights reflecting the total preference over the two stages. The values of $w_1$ and $w_2$ will be equal when two stages 1 and 2 have the same importance, and they add up to 1. In this approach, DMUs that achieve an efficiency score of 1 in both stages are considered efficient.

Kao [12] proposed a relational approach to model network systems. The underlying assumption is that the virtual multiplier associated with the same factor should be the same no matter whether it is the output of one process or the input of another. This approach requires that the aggregated output be less than or equal to the aggregated input for all processes in addition to the usual requirement for the system. A special case of the series system is the one in which all processes, except the first, are not allowed to utilize exogenous inputs , and all processes, except the last, are not allowed to produce exogenous outputs. Kao and Hwang [13] have shown that, in this case, system efficiency is the product of process efficiencies. Chen et al. [4] have shown that the model which is proposed by Chen and Zhu [5] is equivalent to Kao-Hwang's model under constant returns to scale. Below, we adopt the last assumption to construct the proposed network model.

### 3.2. Three-stage system

Let us consider the open system depicted in Figure 2 and use Kao and Hwang's [13] approach to present the mathematical model (3) for this system as follows:

[ Downloaded from iors.ir on 2026-01-29 ]

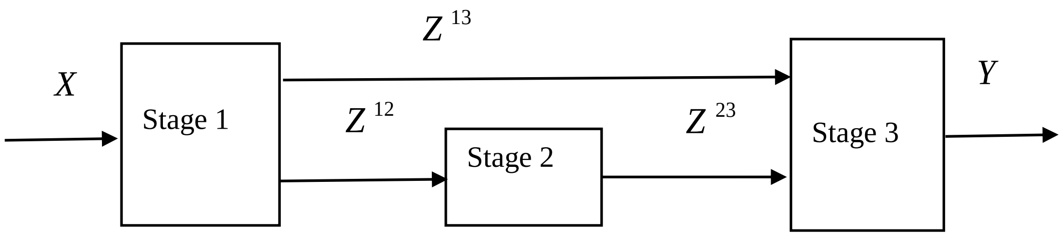



**Figure 2.** The network system of three stages.

$$
\begin{aligned}
&\min \quad \theta \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{io},\ i = 1,2,\ldots,I \\
&\sum_{j=1}^{n} \lambda_j^3 y_{rj} \ge y_{ro},\ r = 1,2,\ldots,R \qquad\qquad (3.1) \\
&\sum_{j=1}^{n} \lambda_j^1 z_{ej}^{13} - \sum_{j=1}^{n} \lambda_j^3 z_{ej}^{13} \ge 0,\ e = 1,2,\ldots,E \\
&\sum_{j=1}^{n} \lambda_j^1 z_{sj}^{12} - \sum_{j=1}^{n} \lambda_j^2 z_{sj}^{12} \ge 0,\ s = 1,2,\ldots,S \qquad\qquad (3) \\
&\sum_{j=1}^{n} \lambda_j^1 z_{kj}^{23} - \sum_{j=1}^{n} \lambda_j^3 z_{kj}^{23} \ge 0,\ k = 1,2,\ldots,K \qquad\qquad (3.2) \\
&\lambda_j^1,\ \lambda_j^2, \lambda_j^3 \ge 0
\end{aligned}
$$

The constraint set (3.1) correspons to the system inputs, X, and the final output, Y, which are the constraints for the conventional envelopment-form DEA model. The constraint set (3.2) correspond to intermediate products.

### 2.3. Fuzzy Stochastic model

This section aims to equip the proposed model (3) for evaluating the efficiencies of DMUs with fuzzy stochastic (intermediate) inputs and fuzzy stochastic (intermediate) outputs.To this end, consider *n* DMUs, each unit consumes fuzzy stochastic inputs, denoted by $\tilde{\tilde{X}}_j = \left(\tilde{X}_j, X_j^{\alpha}, X_j^{\beta}\right)_{LR}$ and intermediate measure vectors $\tilde{\tilde{Z}}_j = \left(\tilde{Z}_j, Z_j^{\alpha}, Z_j^{\beta}\right)_{LR}$ to the first stage, and produces fuzzy stochastic outputs, denoted by $\tilde{\tilde{Y}}_j = \left(\tilde{Y}_j^{g}, Y_j^{g,\alpha}, Y_j^{g,\beta}\right)_{LR}$ as desirable outputs and $\tilde{\tilde{Y}}_j^{b} = (\tilde{y}_j^{b}, y_j^{b,\alpha}, y_j^{b,\beta})_{LR}$ as undesirable outputs. Let, each component of $\tilde{X}_j$, $\tilde{Z}_j$, $\tilde{Y}_j^{g}$,

[ Downloaded from iors.ir on 2026-01-29 ]

and $\tilde{Y}_j^b$ be normally distributed by $\tilde{X}_j \sim N(X_j, \sigma_j)$, $\tilde{Z}_{dj} \sim N(Z_j, \sigma_j)$, $\tilde{Y}_j^g \sim N(Y_j^g, \sigma_j^g)$, and $\tilde{Y}_j^b \sim N(Y_j^b, \sigma_j^b)$, respectively.

The Chance-Constrained Programming (CCP) developed by Cooper et al. [6] is a stochastic optimization approach suitable for solving optimization problems with uncertain parameters. Building on CCP and possibility theory as the principal techniques, the following $\overline{\Pr}-$CCR model is proposed:

$$
\begin{aligned}
& E_o(\delta,\gamma) = \min \quad \theta \\
& s.t.\overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{io}\Big) \ge \gamma,\ i = 1,2,\dots,I \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^3 y_{rj}^g \ge y_{ro}^g\Big) \ge \gamma,\ r = 1,2,\dots,R \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^3 y_{r'j}^b \le y_{r'o}^b\Big) \ge \gamma,\ r' = 1,2,\dots,R' \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 z_{ej}^{13} - \sum_{j=1}^{n} \lambda_j^3 z_{ej}^{13} \ge 0\Big) \ge \gamma,\ e = 1,2,\dots,E \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 z_{sj}^{12} - \sum_{j=1}^{n} \lambda_j^2 z_{sj}^{12} \ge 0\Big) \ge \gamma,\ s = 1,2,\dots,S \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 z_{kj}^{23} - \sum_{j=1}^{n} \lambda_j^3 z_{kj}^{23} \ge 0\Big) \ge \gamma,\ k = 1,2,\dots,K \\
& \lambda_j^1,\ \lambda_j^2, \lambda_j^3 \ge 0
\end{aligned} \tag{4}
$$

where $\gamma \in [0,1]$ is the predetermined thresholds defined by DM and $\overline{\Pr}[\cdot]$ in Model (4) denote the fuzzy stochastic measure.
To get a linear form of solving Model (4), we consider the following substitutions:

$$
\begin{aligned}
& \hat{x}_{ij} = \lambda_j^1 x_{ij},\ \hat{y}_{rj}^g = \lambda_j^3 y_{rj}^g,\ \hat{y}_{r'j}^b = \lambda_j^3 y_{r'j}^b \\
& \hat{z}_{ej}^{113} = \lambda_j^1 z_{ej}^{13},\ \hat{z}_{ej}^{313} = \lambda_j^3 z_{ej}^{13} \\
& \hat{z}_{sj}^{112} = \lambda_j^1 z_{sj}^{12},\ \hat{z}_{sj}^{212} = \lambda_j^2 z_{sj}^{12} \\
& \hat{z}_{kj}^{223} = \lambda_j^2 z_{kj}^{23},\ \hat{z}_{kj}^{323} = \lambda_j^3 z_{kj}^{23}
\end{aligned} \tag{5}
$$

By substituting these variables, model (4) changing to the following model:

[ Downloaded from iors.ir on 2026-01-29 ]

$$
\begin{aligned}
& E_o(\delta,\gamma) = \min \quad \theta \\
& s.t. \\
& \sum_{j=1}^{n} \hat{x}_{ij} \le \theta x_{io}\,,\ i = 1,2,\ldots,I \\
& \sum_{j=1}^{n} \hat{y}^{g}_{rj} \ge y^{g}_{ro}\,,\ r = 1,2,\ldots,R \\
& \sum_{j=1}^{n} y^{b}_{r'j} \le y^{b}_{r'o}\,,\ r' = 1,2,\ldots,R' \\
& \sum_{j=1}^{n} \hat{z}^{113}_{ej} - \sum_{j=1}^{n} \hat{z}^{313}_{ej} \ge 0\,,\ e = 1,2,\ldots,E \\
& \sum_{j=1}^{n} \hat{z}^{112}_{sj} - \sum_{j=1}^{n} \hat{z}^{212}_{sj} \ge 0\,,\ s = 1,2,\ldots,S \\
& \sum_{j=1}^{n} \hat{z}^{223}_{kj} - \sum_{j=1}^{n} \hat{z}^{323}_{kj} \ge 0\,,\ k = 1,2,\ldots,K \\
& \overline{\Pr}(\lambda^1_j x_{ij} \le \hat{x}_{ij} \le \lambda^1_j x_{ij}) \ge \gamma\ ;\ \overline{\Pr}(\lambda^3_j z^{13}_{ej} \le \hat{z}^{313}_{ej} \le \lambda^3_j z^{13}_{ej}) \ge \gamma \\
& \overline{\Pr}(\lambda^3_j y^{g}_{rj} \le \hat{y}^{g}_{rj} \le \lambda^3_j y^{g}_{rj}) \ge \gamma\ ;\ \overline{\Pr}(\lambda^1_j z^{12}_{sj} \le \hat{z}^{112}_{sj} \le \lambda^1_j z^{12}_{sj}) \ge \gamma \\
& \overline{\Pr}(\lambda^3_j y^{b}_{rj} \le \hat{y}^{b}_{rj} \le \lambda^3_j y^{b}_{rj}) \ge \gamma\ ;\overline{\Pr}(\lambda^2_j z^{12}_{sj} \le \hat{z}^{212}_{sj} \le \lambda^2_j z^{12}_{sj}) \ge \gamma \\
& \overline{\Pr}(\lambda^1_j z^{13}_{ej} \le \hat{z}^{113}_{ej} \le \lambda^1_j z^{13}_{ej}) \ge \gamma ; \overline{\Pr}(\lambda^2_j z^{23}_{kj} \le \hat{z}^{223}_{kj} \le \lambda^2_j z^{23}_{kj}) \ge \gamma \\
& \overline{\Pr}(\lambda^3_j z^{23}_{kj} \le \hat{z}^{323}_{kj} \le \lambda^3_j z^{23}_{kj}) \ge \gamma \\
& \lambda^1_j, \lambda^2_j, \lambda^3_j \ge 0
\end{aligned}
\tag{6}
$$

To solve model (6), we utilize Theorem 1 and give Definition 1.

**Theorem 1** (Nasseri et al. [27])**.** If $\tilde{\tilde{X}} \sim \overline{\mathrm{N}}(\bar{\mu},\sigma)$ with $\bar{\mu} = (\mu,\alpha,\beta)$, then

a. $\overline{\Pr}(\tilde{\tilde{X}} \le \mathrm{r}) > \gamma \ iff\ \dfrac{\mathrm{r}-\bar{\mu}}{\sigma} \ge (\Phi^{-1}(\gamma), \dfrac{\alpha+\beta}{\sigma}, \dfrac{\alpha+\beta}{\sigma})$

b. $\overline{\Pr}(\tilde{\tilde{X}} \ge \mathrm{r}) > \gamma \ iff\ \dfrac{\mathrm{r}-\bar{\mu}}{\sigma} \le (\Phi^{-1}(1-\gamma), \dfrac{\alpha+\beta}{\sigma}, \dfrac{\alpha+\beta}{\sigma})$

And so $\overline{\Pr}(r \le \tilde{\tilde{X}} \le \mathrm{r}) > \gamma \ iff\ (\Phi^{-1}(\gamma), \dfrac{\alpha+\beta}{\sigma}, \dfrac{\alpha+\beta}{\sigma}) \le \dfrac{\mathrm{r}-\bar{\mu}}{\sigma} \le (\Phi^{-1}(1-\gamma), \dfrac{\alpha+\beta}{\sigma}, \dfrac{\alpha+\beta}{\sigma})$

Notably, the fuzzy ranking method adopted in this study is Tanaka's approach at the such a threshold $\delta$ [36]. Hence, we have:

[ Downloaded from iors.ir on 2026-01-29 ]

$$
\begin{aligned}
& E_o(\delta,\gamma) = \min \quad \theta \\
& s.t. \sum_{j=1}^{n} \hat{x}_{ij} \le \theta (x_{io} + R^{-1}(\delta) x_{io}^{\beta} + \sigma_{ij} \Phi_{1-\gamma}^{-1}), \; i = 1,2,\dots,I \\
& \sum_{j=1}^{n} \hat{y}_{rj}^{g} \ge (y_{ro}^{g} - L^{-1}(\delta) y_{ro}^{g,\alpha} - \sigma_{rj} \Phi_{1-\gamma}^{-1}), \; r = 1,2,\dots,R \\
& \sum_{j=1}^{n} \hat{y}_{r\hat{j}}^{b} \le (y_{r'o}^{g} + R^{-1}(\delta) y_{ro}^{g,\alpha} + \sigma_{r\hat{j}} \Phi_{1-\gamma}^{-1}), \; r' = 1,2,\dots,R' \\
& \sum_{j=1}^{n} \hat{z}_{ej}^{113} - \sum_{j=1}^{n} \hat{z}_{ej}^{313} \ge 0, \; e = 1,2,\dots,E \\
& \sum_{j=1}^{n} \hat{z}_{sj}^{112} - \sum_{j=1}^{n} \hat{z}_{sj}^{212} \ge 0, \; s = 1,2,\dots,S \\
& \sum_{j=1}^{n} \hat{z}_{kj}^{223} - \sum_{j=1}^{n} \hat{z}_{kj}^{323} \ge 0, \; k = 1,2,\dots,K \\
& \lambda_j^1 (x_{ij} - L^{-1}(\delta) x_{ij}^{\alpha} - \sigma_{ij} \Phi_{1-\gamma}^{-1}) \le \hat{x}_{ij} \le \lambda_j^1 (x_{ij} + R^{-1}(\delta) x_{ij}^{\beta} + \sigma_{ij} \Phi_{1-\gamma}^{-1}) \qquad (7) \\
& \lambda_j^3 (y_{rj} - L^{-1}(\delta) y_{rj}^{g,\alpha} - \sigma_{rj} \Phi_{1-\gamma}^{-1}) \le \hat{y}_{rj}^{g} \le \lambda_j^3 (y_{rj}^{g} + R^{-1}(\delta) y_{rj}^{g,\beta} + \sigma_{rj} \Phi_{1-\gamma}^{-1}) \\
& \lambda_j^3 (y_{r\hat{j}}^{b} - L^{-1}(\delta) y_{r\hat{j}}^{b,\alpha} - \sigma_{rj} \Phi_{1-\gamma}^{-1}) \le \hat{y}_{r\hat{j}}^{b} \le (\lambda_j^3 y_{r\hat{j}}^{b} + R^{-1}(\delta) y_{r\hat{j}}^{b,\beta} + \sigma_{r\hat{j}} \Phi_{1-\gamma}^{-1}) \\
& \lambda_j^1 (z_{ej}^{13} - L^{-1}(\delta) z_{ej}^{13,\alpha} - \sigma_{ej}^{13} \Phi_{1-\gamma}^{-1}) \le \hat{z}_{ej}^{113} \le \lambda_j^1 (z_{ej}^{13} + R^{-1}(\delta) z_{ej}^{13,\beta} + \sigma_{ej}^{13} \Phi_{1-\gamma}^{-1}) \\
& \lambda_j^3 (z_{ej}^{13} - L^{-1}(\delta) z_{ej}^{13,\alpha} - \sigma_{ej}^{13} \Phi_{1-\gamma}^{-1}) \le \hat{z}_{ej}^{313} \le \lambda_j^3 (z_{ej}^{13} + R^{-1}(\delta) z_{ej}^{13,\beta} + \sigma_{ej}^{13} \Phi_{1-\gamma}^{-1}) \\
& \lambda_j^1 (z_{sj}^{12} - L^{-1}(\delta) z_{sj}^{12,\alpha} - \sigma_{sj}^{12} \Phi_{1-\gamma}^{-1}) \le \hat{z}_{sj}^{112} \le \lambda_j^1 (z_{sj}^{12} + R^{-1}(\delta) z_{sj}^{12,\beta} + \sigma_{sj}^{12} \Phi_{1-\gamma}^{-1}) \\
& \lambda_j^2 (z_{sj}^{12} - L^{-1}(\delta) z_{sj}^{12,\alpha} - \sigma_{sj}^{12} \Phi_{1-\gamma}^{-1}) \le \hat{z}_{sj}^{212} \le \lambda_j^2 (z_{sj}^{12} + R^{-1}(\delta) z_{sj}^{12,\beta} + \sigma_{sj}^{12} \Phi_{1-\gamma}^{-1}) \\
& \lambda_j^2 (z_{kj}^{23} - L^{-1}(\delta) z_{kj}^{23,\alpha} - \sigma_{kj}^{23} \Phi_{1-\gamma}^{-1}) \le \hat{z}_{kj}^{223} \le \lambda_j^2 (z_{kj}^{23} + R^{-1}(\delta) z_{kj}^{23,\beta} + \sigma_{kj}^{23} \Phi_{1-\gamma}^{-1}) \\
& \lambda_j^3 (z_{kj}^{23} - L^{-1}(\delta) z_{kj}^{23,\alpha} - \sigma_{kj}^{23} \Phi_{1-\gamma}^{-1}) \le \hat{z}_{kj}^{323} \le \lambda_j^3 (z_{kj}^{23} + R^{-1}(\delta) z_{kj}^{23,\beta} + \sigma_{kj}^{23} \Phi_{1-\gamma}^{-1}) \\
& \lambda_j^1, \lambda_j^2, \lambda_j^3 \ge 0
\end{aligned}
$$

The above model is linear. This model is an extension of the Nasseri et al. 's model to the proposed network CCR model when undesirable outputs are considered [27].

**Definition 1**. For the given level $\delta$ and $\gamma$, we define $\mathrm{E}_o^T(\delta,\gamma) = \mathrm{E}_o\ (\delta, \frac{\gamma}{2})$ as efficiency score of $DMU_o$ in the fuzzy random DEA Model.

**Theorem 2.** If $E_k(\delta,\gamma)$ is the optimum objective function value of Model (7), then $E_k(\delta_1,\gamma) \ge E_k(\delta_2,\gamma)$ and $E_k(\delta,\gamma_1) \ge E_k(\delta,\gamma_2)$, where $\delta_1 \le \delta_2$ and $\gamma_1 \le \gamma_2$.

***Proof.*** Denote the feasible space of Model (7) by $S_{\delta,\gamma}$. We need to prove that $S_{\delta_2,\gamma_2} \subseteq S_{\delta_1,\gamma_1}$.To this, consider the following constraint of Model (7)

$$v_i(x_{ij} - L^{-1}(\delta) x_{ij}^{\alpha} - \sigma_{ij} \Phi_{1-\gamma}^{-1}) \le \hat{x}_{ij} \le v_i(x_{ij} + R^{-1}(\delta) x_{ij}^{\beta} + \sigma_{ij} \Phi_{1-\gamma}^{-1})$$

[ Downloaded from iors.ir on 2026-01-29 ]

Let $\Phi^{-1}(\gamma)=\Phi_{\gamma}^{-1}$. As $\Phi^{-1}(1-\gamma)$ , $L^{-1}(\delta)$ and $R^{-1}(\delta)$ are decreasing functions and, the functions $-\Phi^{-1}(1-\gamma), -L^{-1}(\delta)$ and $-R^{-1}(\delta)$ will be increasing. It is concluded that

$$\left[x_{ij}-L^{-1}(\delta_2)x_{ij}^{\alpha}-\sigma_{ij}\Phi^{-1}(1-\gamma_2), x_{ij}+R^{-1}(\delta_2)x_{ij}^{\beta}+\sigma_{ij}\Phi^{-1}(1-\gamma_2)\right]\subseteq \left[x_{ij}-L^{-1}(\delta_1)x_{ij}^{\alpha}-\sigma_{ij}\Phi^{-1}(1-\gamma_1), x_{ij}+R^{-1}(\delta_1)x_{ij}^{\beta}+\sigma_{ij}\Phi^{-1}(1-\gamma_1)\right]$$

similarly, we can conclude that

$$\left[y_{rj}-L^{-1}(\delta_2)y_{rj}^{\alpha}-\sigma_{rj}\Phi^{-1}(1-\gamma_2), y_{rj}+R^{-1}(\delta_2)y_{rj}^{\beta}+\sigma_{rj}\Phi^{-1}(1-\gamma_2)\right]\subseteq \left[y_{rj}-L^{-1}(\delta_1)y_{rj}^{\alpha}-\sigma_{rj}\Phi^{-1}(1-\gamma_1), y_{rj}+R^{-1}(\delta_1)y_{rj}^{\beta}+\sigma_{rj}\Phi^{-1}(1-\gamma_1)\right]$$

This completes the proof.☐

Now, we can present the following definition to define the efficiency of each DMU.

**Definition 2**. For the given level $\delta$ and $\gamma$, we define $\mathrm{E}_k^T(\delta,\gamma)=\mathrm{E}_k\ (\delta,\frac{\gamma}{2})$ as probabilistic-possibilistic efficiency score of $DMU_k$ in the fuzzy random DEA Model.

The corresponding model $\mathrm{E}_k^T(\delta,\gamma)$ is as follows:

$$\begin{aligned}
&E_k^T(\delta,\gamma)=\max\ \varphi\\
&s.t.\\
&\left.\begin{aligned}
&\varphi\le\sum_{r=1}^{s_1}\hat{y}_{rk}^{g}-\sum_{p=1}^{s_2}\hat{y}_{pk}^{b}\\
&\sum_{i=1}^{m}\hat{x}_{ik}=1\\
&\sum_{r=1}^{s_1}\hat{y}_{rj}^{g}-\sum_{p=1}^{s_2}\hat{y}_{pj}^{b}-\sum_{d=1}^{D}\hat{z}_{dj}\le 0,\ \ j=1,2,\ldots,n\\
&\sum_{d=1}^{D}\hat{z}_{dj}-\sum_{i=1}^{m}\hat{x}_{rj}\le 0,\ \ j=1,2,\ldots,n
\end{aligned}\right\}(i)\\
&u_r^g(y_{rj}^g-L^{-1}(\delta)y_{rj}^{\alpha,g}-\sigma_{rj}\Phi^{-1}_{1-\frac{\gamma}{2}})\le\hat{y}_{rj}^{g}\le u_r^g(y_{rj}^g+R^{-1}(\delta)y_{rj}^{\beta,g}+\sigma_{rj}\Phi^{-1}_{1-\frac{\gamma}{2}}),\ \ \forall r,j\\
&u_p^b(y_{pj}^b-L^{-1}(\delta)y_{pj}^{\alpha,b}-\sigma_{pj}\Phi^{-1}_{1-\frac{\gamma}{2}})\le\hat{y}_{rj}^{b}\le u_p^b(y_{pj}^b+R^{-1}(\delta)y_{pj}^{\beta,b}+\sigma_{pj}\Phi^{-1}_{1-\frac{\gamma}{2}}),\ \ \forall p,j\\
&w_d(z_{dj}-L^{-1}(\delta)z_{dj}^{\beta}-\sigma_{dj}\Phi^{-1}_{1-\frac{\gamma}{2}})\le\hat{z}_{rj}\le w_d(z_{dj}+R^{-1}(\delta)z_{dj}^{\beta}+\sigma_{dj}\Phi^{-1}_{1-\frac{\gamma}{2}}),\ \ \forall d,j\\
&v_i(x_{ij}-L^{-1}(\delta)x_{ij}^{\alpha}-\sigma_{ij}\Phi^{-1}_{1-\frac{\gamma}{2}})\le\hat{x}_{ij}\le v_i(x_{ij}+R^{-1}(\delta)x_{ij}^{\beta}+\sigma_{ij}\Phi^{-1}_{1-\frac{\gamma}{2}}),\ \forall i,j\\
&u_r^g\ge 0,u_p^b\ge 0,v_i\ge 0,w_d\ge 0,\varphi\ge 0,\\
&r=1,2,\ldots,s_1;\ p=1,2,\ldots,s_2\ i=1,2,\ldots,m;d=1,2,\ldots,D
\end{aligned}\tag{8}$$

**Theorem 3.** Consider $E_k^{\mathrm{T}}(\delta,\gamma)$ as the optimum objective function value of Model (8) for $\mathrm{DMU_k}$, then

[ Downloaded from iors.ir on 2026-01-29 ]

a. $E_k^T(\delta_1,\gamma) \ge E_k^T(\delta_2,\gamma)$ and $E_k^T(\delta,\gamma_1) \ge E_k^T(\delta,\gamma_2)$ where $\delta_1 \le \delta_2$ and $\gamma_1 \le \gamma_2$.
b. $0 < E_j^T(\delta,\gamma) \le 1,\ (j = 1,2,\ldots,n)$.
c. Model (13) is feasible for any $\delta$ and $\gamma$.
***Proof:*** **a.** It is straightforward using Theorem 2 and Definition 2.
In assertion b is followed immediately by the restriction $\varphi \ge 0$ and four constraints in part (i) of the model (8) as follows:

$$\varphi \le \sum_{r=1}^{s_1} \hat{y}_{rk}^{g} - \sum_{p=1}^{s_2} \hat{y}_{pk}^{b} \le \sum_{d=1}^{D} \hat{z}_{dk} \le \sum_{i=1}^{m} \hat{x}_{rk} = 1.$$

To prove assertion c, Let $\delta = 1$ and $\gamma=1$, then $L^{-1}(1) = R^{-1}(1) = 0$ and $\Phi^{-1}(0.5) = 0$. Hence, we have $\hat{x}_{ij} = v_i x_{ij}$, $\hat{y}_{rj}^{g} = u_r^g y_{rj}^g$, $\hat{y}_{pj}^{b} = u_p^b y_{pj}^b$ and $\hat{z}_{dj} = w_d z_{dj}$ in Model (8). Therefore, The correspondig model with $E_k^T(1,1)$ will be as follows:

$$\begin{aligned}
&E_k^T(1,1) = Max \sum_{r=1}^{s_1} u_r^g y_{rk}^g - \sum_{p=1}^{s_2} u_p^b y_{pk}^b \\
&s.t. \sum_{i=1}^{m} v_i x_{ik} = 1 \\
&\quad \sum_{r=1}^{s_1} u_r^g y_{rj}^g - \sum_{p=1}^{s_2} u_p^b y_{pj}^b - \sum_{d=1}^{D} w_d z_{dj} \le 0, \quad j = 1,2,\ldots,n \\
&\quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{rj} \le 0, \quad j = 1,2,\ldots,n \\
&\quad u_r^g \ge 0, u_p^b \ge 0, v_i \ge 0, w_d \ge 0, \quad r = 1,2,\ldots,s_1; p = 1,2,\ldots,s_2; i = 1,2,\ldots,m; d = 1,2,\ldots,D
\end{aligned} \tag{9}$$

To prove assertion c, denote the feasible space of Model (8) by $S_{\delta,\gamma}^T$. According to the proof of Theorem 2, $S_{1,1}^T \subseteq S_{\delta,\gamma}^T$. Therefore, it is sufficient to show that the feasible space $S_{1,1}^T$ is nonempty. Suppose that $u_p^b = 0\,(p = 1,2,\ldots,s_2)$, then $E_k^T(1,1)$ is converted to the model (9) as a two-stage model. Chen et al. [4] explicitly showed the feasibility of the model (9). This completes the proof of part (c).□
Now, we are going to apply the given model to the banking industry as a real case study.

## ٤. Case Study

We focused on the banking Industry has a comprehensive network of over 300 branches and 30000 employees in Iran. Countrywide coverage in Iran, service quality ,and experienced multi-lingual staff are important factors of their success. In this section, we apply the proposed approach in this study to some commercial bank branches in Mazandaran province. Here the data sources consist of the reports of some selected branches. The inputs for the first stage are personnel score, cost, location, and branch facilities with intermediate output service and Total of Deposits (TDs) (of current, short duration , and long duration accounts). The second stage's input is TDs and the loan is as intermediate output. Finally, in the third stage service and TDs as intermediate input and recovered loans as desirable outputs, and non-performing loans (delay in delivering loans and other facilities) as undesirable output. However, there always exist some degrees of uncertainty in the data which can be represented

[ Downloaded from iors.ir on 2026-01-29 ]

by fuzzy stochastic numbers. In banks, uncertainty occurs due to the difference between the actual data and the available data. Then the difference between actual data and possible data results in the occurrence of uncertainty in the data which further may affect. Therefore, in the present study, we fuzzify the data as TFNs. The collected crisp data in Table 1 are considered as the mean of TFNs. On the other hand, the inputs and outputs are supposed as random variables. By using the goodness of fit tests, normal distributions have been fit on the random variables. The corresponding expected value is the observed inputs (outputs) data and the standard deviation is one. Hence, each DMU is considered a fuzzy variable with a randomized mean. This fuzzy random input–intermediate-output data of each bank is available in Table 1. Finally, Table 2 presents the average efficiency scores and the final rankings of the 10 bank branches. However, the average efficiency can be an appropriate overall index to indicate the efficiency variations.

**Table 1**. The fuzzy random input and output data[۳]

| DMU | Personnel Score | Cost | Branch Facilities | Interest Income | Location | Loans | User fee income | Deposit | Non-performing loans |
|---|---|---|---|---|---|---|---|---|---|
| **1** | 17,014,781 | 354,133 | 28,347 | 796,832 | 715 | 3,648,031 | 95,045 | 5,981,048 | 301,779 |
| **2** | 14,297,944 | 287,066 | 17,889 | 879,802 | 879 | 4,317,806 | 46,845 | 6,323,772 | 175,162 |
| **3** | 16,252,095 | 384,871 | 28,001 | 1,116,566 | 2,087 | 4,522,011 | 111,225 | 7,950,451 | 415,303 |
| **4** | 16,342,530 | 424,974 | 19,630 | 1,210,623 | 1,292 | 6,278,297 | 91,316 | 8,851,770 | 312,750 |
| **5** | 16,687,868 | 377,789 | 23,508 | 836,644 | 1,164 | 3,491,101 | 159,909 | 5,992,871 | 658,208 |
| **6** | 14,765,164 | 249,487 | 20,307 | 627,658 | 913 | 2,524,526 | 45,740 | 5,116,146 | 126,437 |
| **7** | 16,933,047 | 366,048 | 25,208 | 1,003,786 | 2,671 | 3,991,867 | 188,924 | 7,588,686 | 284,899 |
| **8** | 10,583,687 | 245,834 | 7,146 | 589,456 | 480 | 2,953,722 | 119,975 | 5,414,472 | 460,950 |
| **9** | 8,183,284 | 210,688 | 13,514 | 530,537 | 417 | 3,511,138 | 54,141 | 5,559,826 | 179,385 |
| **10** | 5,439,440 | 131,682 | 7,881 | 345,072 | 347 | 2,044,424 | 18,125 | 2,952,701 | 130,017 |

**Table 2.** The fuzzy random efficiency scores and final ranking

| DMU | ($\gamma$=0.9,$\delta$=0.7) | ($\gamma$=0.9,$\delta$=0.4) | ($\gamma$=0.7,$\delta$=0.7) | ($\gamma$=0.5,$\delta$=0.5) | Overall efficiency | Ranking |
|---|---|---|---|---|---|---|
| **1** | **0.5603** | **0.5673** | **0.5650** | **0.5744** | **0.5668** | **8** |
| **2** | **0.6112** | **0.6151** | **0.6162** | **0.6240** | **0.6166** | **7** |
| **3** | **0.7448** | **0.7495** | **0.7500** | **0.7585** | **0.7507** | **4** |
| **4** | **0.5560** | **0.5627** | **0.5593** | **0.5672** | **0.5613** | **10** |
| **5** | **0.5606** | **0.5690** | **0.5643** | **0.5738** | **0.5669** | **9** |
| **6** | **0.5495** | **0.5610** | **0.5496** | **0.5572** | **0.5543** | **11** |
| **7** | **0.6791** | **0.6883** | **0.6842** | **0.6955** | **0.6868** | **6** |
| **8** | **1.0000** | **1.0000** | **1.0000** | **1.0000** | **1.0000** | **1** |
| **9** | **0.5005** | **0.5077** | **0.5045** | **0.5134** | **0.5065** | **14** |
| **10** | **0.5372** | **0.5449** | **0.5413** | **0.5505** | **0.5435** | **12** |

## ۵. Conclusions and future works

This paper formulated the DEA model handling the three-stage process and undesirable outputs in a fuzzy random environment. the extended model depicts the influence of the presence of fuzziness and randomness in the data over the efficiency values. To do this, we have first incorporated an undesirable output in the three-stage DEA model. The resulting model was converted into a new model with some variable substitutions. Then, to solve the uncertainty part of the model, we applied

[۳]The prices are in million Rials.

[ Downloaded from iors.ir on 2026-01-29 ]

the $\overline{\Pr}(.)$ measure that led to a linear model. Furthermore, the proposed approach can be used in many practical situations such as Insurance Industry, Supply Chain, etc.

**Acknowledgement.** The authors would like to appreciate the anonymous referees who deliver their valuable comments to help us for improving the earlier versions of the current manuscript.

## References

[1] Carrillo, M., Jorge, J. N., (2016). A multi objective DEA approach to ranking alternative, *Expert Systems with Applications*, 50(15) 130-139.

[2] Charnes, A., Cooper, W. W., Rhodes, E. (1978). Measuring the efficiency of decisionmaking units. *European Journal of Operational Research*, 2(6), 429–444.

[3] Charnes, A., Cooper, W. W., Golany, B., Halek, R., Klopp, G., Schmitz, E., et al. (1986). Two-phase data envelopment analysis approaches to policy evaluation and management of army recruiting activities: Tradeoffs between joint services and army advertising. *Research Report CCS #532*, Center for Cybernetic Studies, University of Texas-Austin, Austin, TX.

[4] Chen, Y., Liang, L., Zhu, J., (2009). Equivalence in two-stage DEA approaches. *European Journal of Operational Research*, 193, 600–604.

[5] Chen, Y., Zhu, J. (2004). Measuring information technology`s indirect impact on firm performance. *Information Technology and Management*, 5, 9-22.

[6] Cooper, W.W., Deng, H., Huang, Z.M., Li, S.X., (2004). Chance constrained programming approaches to congestion in stochastic data envelopment analysis, *European Journal of Operational Research*, 155, 487-501.

[7] Ebrahimnejad, A., Tavana. M., Nasseri., S.H., Gholami, O., (2018). A New Method for Solving Dual DEA Problems with Fuzzy Stochastic Data, *International Journal of Information Technology and Decision Making*, 18(1), 147-170.

[8] Feng, X., Liu,Y.K. , (2006). Measurability criteria for fuzzy random vectors, *Fuzzy Optimization and Decision Making*, 5, 245–253.

[9] Halkos, G.E, Tzeremes, N.G, Kourtzidis, S.A., ( 2014). A unified classification of two-stage DEA models.*Surv Op Res Manag Sci*,19(1), 1–16.

[10] Hanifzade, P., Khedmatgozar, H. R., Emrouznejad, A., Derakhshan, M., (2014). Neuoral network DEA for measuring the efficiency of mutual funds. *International Journal of Applied Decision Sciences*, 7(3), 225-269.

[11] Hatami-Marbini, A., Emrouznejad, A., Tavana, M. (2011a). A taxonomy and review of the fuzzy data envelopment analysis literature: Two decades in the making. *European Journal of Operational Research*, 214, 457–472.

[12] Kao, C., (2009). Efficiency decomposition in network data envelopment analysis: a relational model, *European Journal of Operational Research*, 192, 949–962.

[13] Kao, C., Hwang, S.N., (2008). Efficiency decomposition in two-stage data envelopment analysis: an application to non-life insurance companies in Taiwan, *European Journal of Operational Research*, 185, 418–429.

[14] Khanjani Shiraz, R., Tavana, M., Paryab, K., (2014a). Fuzzy free disposal hull models under possibility and credibility measures, *International Journal of Data Analysis Techniques and Strategies*, 6(3), 286-306.

[15] Khanjani Shiraz, R., Charles, V., Jalalzadeh, L., (2014b). Fuzzy Rough DEA Model: a possibility and expected value approaches. *Expert System with Applications*, 41(2), 434–444.

[16] Khodabakhshi, M., Tavana, M., Baghbani, F., (2016). Optimistic and pessimistic performance and congestion analysis in fuzzy Data Envelopment Analysis. *International Journal of Logestic Systems and Management*, 24(1), 1-17.

[ Downloaded from iors.ir on 2026-01-29 ]

[17] Kwakernaak, H., (1978). Fuzzy random variables. Part I: Definitions and theorems. *Information Sciences*, 15(1), 1–29.

[18] Kwakernaak, H., (1979). Fuzzy random variables. Part II: Algorithms and examples for the discrete case. *Information Sciences*, 17(3), 253–278.

[19] Kwon, H.B., Lee, J., (2015). Two-stage production modeling of large U.S. banks: A DEA-neural network approach. *Expert Systems with Applications*, 42(19), 6758-6766.

[20] Lertworasirikul, S., Shu-Cherng, F., Joines, J.A., Nuttle, H.L.W., (2003). Fuzzy data envelopment analysis (DEA): A possibility approach, *Fuzzy Sets and Systems*, 139(2), 379-394.

[21] Liu, B., (2004). Uncertainty Theory, *Springer-Verlag*, Berlin.

[22] Liu, H., & Liu, Y. (2021). Construction of a Medical Resource Sharing Mechanism Based on Blockchain Technology: Evidence from the Medical Resource Imbalance of China. *Healthcare*, 9(1). 1-16.

[23] Liu, Y., Liu,B., (2003). Fuzzy random variable: a scalar expected value operator, *Fuzzy Optimization and Decision Making*, 2(1), 43–160.

[24] Liu,W., Zhou, Z., Ma, C, Liu, D., Shen, W., (2015). A two-stage DEA models with undesirable input –intermediate-outputs. *Omega*, 56, 74-87.

[25] Miguel, S., Jorge, E.G. (2017). The influence of risk-taking on bank efficiency: Evidence from Colombia, *Emerging Markets Review*, 32, 52-73.

[26] Nasseri, S. H., Gholami, O., Ebrahimnejad, A., (2014). On ranking decision making science using relative similar units in data envelopment analysis. *International Journal of Applied Decision Sciences*, 7(4) 424-436.

[27] Nasseri, S.H., Ebrahimnejad, A., Gholami, O., (2016). Fuzzy stochastic input-oriented primal data envelopment analysis models with application to insurance industry. *International Journal of Applied Decision Sciences*, 9(3), 259-282.

[28] Nepomuceno, T. C., Silva, W., Nepomuceno, K. T., & Barros, I. K. (2020). A DEA-based complexity of needs approach for hospital beds evacuation during the COVID-19 outbreak. *Journal of Healthcare Engineering*, 2020. 1-9.

[29] Puri, J., Yadav, S. P., (2014). A fuzzy DEA model with undesirable fuzzy outputs and its application. *Expert Systems with Applications*, 41(14), 6419-6432.

[30] Qin, R., Liu, Y.K., (2010). Modeling data envelopment analysis by chance method in hybrid uncertain environments. *Mathematics and Computers in Simulation*, 80(5), 922–95.

[31] Saati, S., Memariani, A., Jahanshahloo, G.R., (2002). Efficiency analysis and ranking of DMUs with fuzzy data. *Fuzzy Optimization and Decision Making*, 1, 255–267.

[32] Sahoo, B. K., Khoveyni, M., Eslami, R., Chauhury, P., (2016). return to scale and most productivity scale size in DEA with negative data. *European Journal of Operational Research*, 255(2), 545-558.

[33] Sengupta, J. K., (1992). A fuzzy systems approach in data envelopment analysis. *Computers and Mathematics with Applications*, 24(8), 259–266.

[34] Soleimani Kourandeh, A., Fathali, J. & Taherifard, S. (2021). Solving single facility goal Weber location problem using stochastic optimization methods. *Iranian Journal of Operations Research*, *12*(1), 1-19.

[35] Sun, J., & Luo, H. (2017). Evaluation on equality and efficiency of health resources allocation and health services utilization in China. *International Journal for Equity in Health*, 16(1), 1-8.

[36] Tanaka, H., Ichihasi, H. , Asai, K., (1984). A formulation of fuzzy linear programming problem based on comparison of fuzzy numbers, *Control and Cybernetics*, 13, 185–194.

[37] Tavana, M., Khanjani Shiraz, R., Hatami-Marbini, A., (2014b). A New Chance-Constrained DEA Model with Birandom Input and Output Data. *Journal of the Operational Research Society*, 27, 1824-1839.

[ Downloaded from iors.ir on 2026-01-29 ]

[38] Tavana, M., Khanjani Shiraz, R., Hatami-Marbini, A., Per J. Agrell, Paryab, P., (2012). Fuzzy stochastic data envelopment analysis with application to base realignment and closure (BRAC). *Expert Systems with Applications*, 39(15), 12247–12259.

[39] Tavana, M., Khanjani Shiraz, R., Hatami-Marbini, A., Per J. Agrell, Paryab, P., (2013). Chance-constrained DEA models with random fuzzy inputs and outputs. *Knowledge-Based Systems*, 52, 32–52.

[40] Tootooni1, B., Sadegheih, A., Khademi Zare, H. & Vahdatzad, M. A. (2020). A novel type I and II fuzzy approach for solving single allocation ordered median hub location problem. *Iranian Journal of Operations Research, 11*(2), 65-79.

[41] Triantis, K.P., Girod, O., (1998). A mathematical programming approach for measuring technical efficiency in a fuzzy environment. *Journal of Productivity Analysis*, 10(1), 85–102.

[42] Wang, Y. M., Luo, Y., and Liang, L., (2009). Fuzzy data envelopment analysis based upon fuzzy arithmetic with an application to performance assessment of manufacturing enterprises. *Expert Systems with Applications*, 36(3), 5205–5211.

[43] Wu, J., Chu, J., S, J., Zhu, Q., (2016). DEA-cross efficiency evalution based on pareto improvement, *European Journal of Operational Research*, 248(2) 571-579.

[44] Yang, C. C. (2017). Measuring health indicators and allocating health resources: A DEAbased approach. *Health Care Management Science*, 20(3), 365–378.

[45] Zhou. Z., Lin, L., Xiao, H., Ma, C., Wu, S., (2017). Stochastic network DEA models for two-stage systems under the centralized control organization mechanism. *Computers & Industrial Engineering*, 110, 404-412.

[ Downloaded from iors.ir on 2026-01-29 ]