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A Three-Stage Process for Fuzzy Stochastic Network Data Envelopment Analysis Models

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One of the most useful tools in Operations Research (OR) which is essentially applied to evaluate the performance of treated Decision-Making Units (DMUs) is Data Envelopment Analysis (DEA). Because of in the current decades, DEA models have been used and extended in many disciplines and hence attracted much interests. The traditional DEA treats DMUs as black boxes and calculates their efficiencies by considering their initial inputs and their final outputs. Since, in the real situations, input data are included some uncertainties, hence in this study we consider a DEA with fuzzy stochastic data and suggest a three-stage DEA model by taking into account undesirable output. To achieve this aim, an extended probability approach is applied to the reform of three-stage DEA models. Finally, we give an illustrative example by considering a case study on the banking industry.

***Keywords**: Data Envelopment Analysis, Network DEA, Fuzzy random variable, Multi-stage method Undesirable output.*
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۱. Introduction

Data Envelopment Analysis (DEA), initially introduced by Charnes et al. [2], requires crisp input and output data, whereas real-life decisions are usually made in a state of uncertainty. In such situations, we often face uncertain programming in the DEA model, where in the data could possess randomness and fuzziness. On the other hand, in a production system, the input usually goes through several processes before it becomes the output. Traditional DEA models treat the system as a whole unit, disregarding the interactions of the processes in the system when calculating the efficiency. This two progress in network and uncertainty DEA models need to be handeled together. This paper solves a case of the network DEA model in which the input and output data are assumed to be characterized by fuzzy random variables. The first study concerning to the network DEA was prepared by Charnes et al. [3]. Several models for measuring the efficiency of network systems have been proposed. Halkos et al. [9] provided a unified classification of the two-stage DEA model. This study was similarly presented by Zhou et al. [45]. Kwon and Lee [19] propose a new approach to model a two-stage production process supported by using data from large U.S. banks.

Liu et al. [24] proposed a two-stage DEA model with undesirable input–intermediate-outputs. Carrillo and Jorge [1] give a new model for ranking alternatives that use common weight DEA under a multi-objective optimization approach. Soleimani Kourandeh et al. [34] investigated the goal Weber location problem in which the location of some of demand points on a plane is given, and the ideal is locating the facility in the distance R_i, from the i-th demand point. Nasseri et al. [26] suggested a new ranking method based on the extension of PPS by virtual units named relatively similar units. Wu et al. [43] introduced a cross-efficiency approach based on Pareto optimality which can be generated by

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only a common set of weights. Hanafizade et al. [10] used neural network DEA for measuring the efficiency of mutual funds. Tootooni et al. [40] proposed a fuzzy type I and II programming approach for a new model presented in the literature, i.e., the single allocation ordered median problem. Sahoo et al. [32] discussed the return to scale and most productivity scale sizes in DEA with negative data. Hatami-Marbini et al. [11] classified the fuzzy DEA methods in the literature into five general groups, the tolerance approach [33],[39], the α-level based approach, the fuzzy ranking approach [11], the possibility approach [20], and the fuzzy arithmetic approach [42]. Among these approaches, the α-level based approach is probably the most relevant fuzzy DEA model in the literature. Nevertheless, the possibility approach seems to be more efficient in hybrid uncertainty, especially with a twofold fuzzy-random environment. Saati et al. [31] proposed a fuzzy CCR model as a possibilistic programming problem by applying an alternative α-cut approach. Puri and Yadav [29] applied the suggested methodology by Saati et al. [31] to solve the fuzzy DEA model with undesirable outputs. Khanjani et al. [14] proposed fuzzy-free disposal hull models under possibility and credibility measures. Khodabakhshi et al. [16] proposed a fuzzy DEA model with an optimistic and pessimistic performance and congestion analysis in fuzzy DEA. Kwakernaak [17,18] introduced the concept of the fuzzy random variable, and then this idea was enhanced by some researchers in the literature ([8],[21],[22],[30]). Qin and Liu [30] developed a Fuzzy Random DEA (FRDEA) model where randomness and fuzziness exist simultaneously. The authors characterized the fuzzy random data with known possibility and probability distributions. Tavana et al. [38] also introduced three different fuzzy stochastic DEA models consisting of probability-possibility, probability-necessity, and probability-credibility constraints in which input and output data entailed fuzziness and randomness at the same time.Also, Tavana et al. [37] and [39] provided a chance-constrained DEA model with random fuzzy inputs and outputs with Poisson, uniform and normal distributions. Khanjani et al. [15] proposed fuzzy rough DEA models based on the expected value and possibility approaches. Nasseri et al. [27] proposed a fuzzy stochastic DEA model. They formulated a linear and feasible model with an extension of normal distribution to deal with fuzzy random data. Miguel Sarmiento and Jorge E. Galan [25] show a stochastic frontier model with random inefficiency parameters to a sample of Colombian banks. Their model provides accurate cost and profit efficiency estimates. Ebrahimnejad et al. [7] solved dual DEA problems with fuzzy stochastic data. This approach overcomes the shortcomings of linearity and normal efficiency score relative to corresponding approaches. However, few studies have investigated the problem of allocating limited medical resources allocation among hospitals during public health emergencies ([22], [28], [35],[42]). This study tries to incorporate fuzzy random inputs and outputs in a network model with undesirable output. We apply extended probability measures to deal with the fuzzy random environments. The achievement of the present study is three items: (1) To formulate a new version of the network DEA model equipped with undesirable output, (2) To formulate a linear model for solving fuzzy stochastic two-stage DEA model, and (3) To demonstrate the applicability of the proposed model using a case study for the banking industry.

The remainder of the paper is organized as follows: Next section presents some approaches to a two-stage model and proposes our proposed network model equipped with fuzzy stochastic input and output data. In Section 3, the results of the case was conducted for the banking industry to evaluate the efficiency of 10 branches. Section 4 presents our conclusions and future research directions.

٢. Traditional DEA model

Data Envelopment Analysis (DEA) was originally proposed by Charnes, Cooper, and Rhodes [2]. DEA has been widely exploited to evaluate the efficiency of excess activities. DEA evaluates the relative efficiency of a set of DMUs using the ratio of the weighted sum of outputs to the weighted sum of inputs. Specifically, DEA determines a set of weights such that the efficiency of the undervalued

DMU is maximized instead of other DMUs. The efficiency score varies in the interval [0,1], and a DMU with an efficiency score equal to 1 is called efficient.

Recall that DEA uses the ratio of the weighted sum of outputs to the weighted sum of inputs to measure efficiency. Since this ratio cannot exceed the value of 1, if each DMU has s outputs and m inputs, and x_{ij} and y_{rj} represent the value of the first input to DMUj and the value of the rth output of that DMU, respectively, the fractional form of the model DEA evaluates the efficiency

$DMU_{\cdot}$ is as follows:

$$\begin{aligned} \max \quad & h_0 = \frac{\sum_{r=1}^{s} u_r y_{rj0}}{\sum_{i=1}^{m} v_i x_{ij0}}, \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1; \quad j = 1,\dots,n. \\ & u_r, v_i \ge 0; \quad \text{for all} \quad r \quad \text{and} \quad j. \end{aligned}$$

In this non-linear and non-convex problem, h_0 is the efficiency score of DMU_0 and the weights v_i and u_r are the decision variables of the given problem. There is a problem with this model, that is, it has countless solutions because if the optimal value of the variables is v^* and u^*, there are other optimal solutions such as αv^* and αu^*. To avoid this problem, a classic linear DEA model is obtained after two variable transformations as follows:

$$\begin{aligned} \max \quad & h_0 = \sum_{r=1}^{s} \mu_r y_{r0}, \\ s.t. \quad & \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} \le 0; \quad j = 1,\dots,n, \\ & \sum_{i=1}^{m} w_i x_{ij} = 1, \\ & \mu_r, w_i \ge 0; \quad \text{for all} \quad r \quad \text{and} \quad j. \end{aligned} \tag{1}$$

where $\mu_r = u_r \Big/ \sum_{r=1}^{s} u_r y_{r0}$ and $w_i = v_i \Big/ \sum_{i=1}^{m} v_i x_{i0}$.

۳. The proposed model

3.1 Two- stage model

Consider the two-stage process illustrated in Figure 1. We have n DMUs that each $DMUj$ $(j = ۱,۲, \dots, n)$ has m inputs $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ and D outputs $Z_j = (z_{1j}, z_{2j}, \dots, z_{Dj})$ to the first stage. These D outputs known as the intermediate measures then are consumed in the second stage. The outputs from the second stage are $Y_j = (y_{1j}, y_{2j}, \dots, y_{rj})$. Chen and Zhu [5] developed an efficiency model that identified the efficient frontier of a two-stage production process linked by intermediate measures. They used a set of firms in the banking industry to illustrate how the new model could be utilized. Model (2) is the two-stage model proposed by Chen and Zhu.

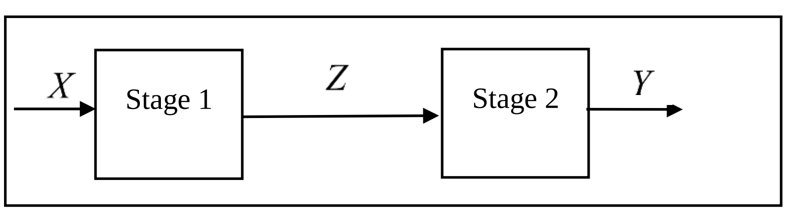

Figure 1. A two-stage DEA system

$$
\begin{aligned}
&\max \quad w_1\alpha - w_2\beta \\
&s.t. \\
&(\text{Stage}\,1) \\
&\sum_{j=1}^{n}\lambda_j x_{ij} \le \alpha x_{io},\ i=1,2,\dots,m \qquad (2.1) \\
&\sum_{j=1}^{n}\lambda_j z_{dj} \ge \tilde{z}_{do},\ d=1,2,\dots,D \qquad (2.2) \\
&\lambda_j \ge 0,\ j=1,2,\dots,n \\
&(\text{Stage}\,2) \\
&\sum_{j=1}^{n}\mu_j z_{dj} \le \tilde{z}_{do},\ d=1,2,\dots,D \qquad (2.3) \\
&\sum_{j=1}^{n}\mu_j y_{rj} \ge \beta y_{ro},\ r=1,2,\dots,s \qquad (2.4) \\
&\mu_j \ge 0,\ j=1,2,\dots,n
\end{aligned}
\tag{2}
$$

where α and β are the efficiency scores corresponding to Stage 1 and Stage 2, respectively.

In addition z_{dj} are the intermediary inputs which are outputs of Stage 1 and inputs of Stage 2 and the values of $\tilde{z}_{do}$ are unknown. Moreover, w_1 and w_2 are the weights reflecting the total preference over the two stages. The values of w_1 and w_2 will be equal when two stages 1 and 2 have the same importance, and they add up to 1. In this approach, DMUs that achieve an efficiency score of 1 in both stages are considered efficient.

Kao [12] proposed a relational approach to model network systems. The underlying assumption is that the virtual multiplier associated with the same factor should be the same no matter whether it is the output of one process or the input of another. This approach requires that the aggregated output be less than or equal to the aggregated input for all processes in addition to the usual requirement for the system. A special case of the series system is the one in which all processes, except the first, are not allowed to utilize exogenous inputs , and all processes, except the last, are not allowed to produce exogenous outputs. Kao and Hwang [13] have shown that, in this case, system efficiency is the product of process efficiencies. Chen et al. [4] have shown that the model which is proposed by Chen and Zhu [5] is equivalent to Kao-Hwang’s model under constant returns to scale. Below, we adopt the last assumption to construct the proposed network model.

3.2. Three-stage system

Let us consider the open system depicted in Figure 2 and use Kao and Hwang's [13] approach to present the mathematical model (3) for this system as follows:

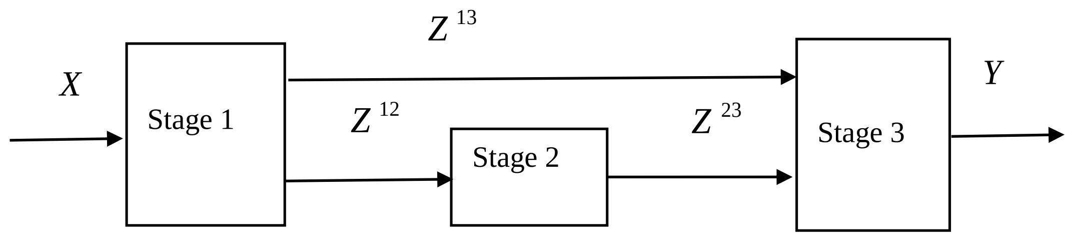

Figure 2. The network system of three stages.

$$
\begin{aligned}
&\min \quad \theta \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{io},\ i = 1,2,\ldots,I \\
&\sum_{j=1}^{n} \lambda_j^3 y_{rj} \ge y_{ro},\ r = 1,2,\ldots,R && (3.1) \\
&\sum_{j=1}^{n} \lambda_j^1 z_{ej}^{13} - \sum_{j=1}^{n} \lambda_j^3 z_{ej}^{13} \ge 0,\ e = 1,2,\ldots,E \\
&\sum_{j=1}^{n} \lambda_j^1 z_{sj}^{12} - \sum_{j=1}^{n} \lambda_j^2 z_{sj}^{12} \ge 0,\ s = 1,2,\ldots,S && && (3) \\
&\sum_{j=1}^{n} \lambda_j^1 z_{kj}^{23} - \sum_{j=1}^{n} \lambda_j^3 z_{kj}^{23} \ge 0, k = 1,2,\ldots,K && (3.2) \\
&\lambda_j^1,\ \lambda_j^2, \lambda_j^3 \ge 0
\end{aligned}
$$

The constraint set (3.1) correspons to the system inputs, X, and the final output, Y, which are the constraints for the conventional envelopment-form DEA model. The constraint set (3.2) correspond to intermediate products.

2.3. Fuzzy Stochastic model

This section aims to equip the proposed model (3) for evaluating the efficiencies of DMUs with fuzzy stochastic (intermediate) inputs and fuzzy stochastic (intermediate) outputs.To this end, consider n DMUs, each unit consumes fuzzy stochastic inputs, denoted by $\tilde{\tilde{X}}_j = \left(\tilde{X}_j, X_j^{\alpha}, X_j^{\beta}\right)_{LR}$ and intermediate measure vectors $\tilde{\tilde{Z}}_j = \left(\tilde{Z}_j, Z_j^{\alpha}, Z_j^{\beta}\right)_{LR}$ to the first stage, and produces fuzzy stochastic outputs, denoted by $\tilde{\tilde{Y}}_j = \left(\tilde{Y}_j^{g}, Y_j^{g,\alpha}, Y_j^{g,\beta}\right)_{LR}$ as desirable outputs and $\tilde{\tilde{Y}}_j^{b} = (\tilde{y}_j^{b}, y_j^{b,\alpha}, y_j^{b,\beta})_{LR}$ as undesirable outputs. Let, each component of $\tilde{X}_j$, $\tilde{Z}_j$, $\tilde{Y}_j^{g}$, and $\tilde{Y}_j^b$ be normally distributed by $\tilde{X}_j \sim N(X_j, \sigma_j)$, $\tilde{Z}_{dj} \sim N(Z_j, \sigma_j)$, $\tilde{Y}_j^g \sim N(Y_j^g, \sigma_j^g)$, and $\tilde{Y}_j^b \sim N(Y_j^b, \sigma_j^b)$, respectively.

The Chance-Constrained Programming (CCP) developed by Cooper et al. [6] is a stochastic optimization approach suitable for solving optimization problems with uncertain parameters. Building on CCP and possibility theory as the principal techniques, the following $\overline{\Pr}-$CCR model is proposed:

$$
\begin{aligned}
& E_o(\delta,\gamma) = \min \quad \theta \\
& s.t.\ \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{io}\Big) \ge \gamma,\ i = 1,2,\dots,I \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^3 y_{rj}^{g} \ge y_{ro}^{g}\Big) \ge \gamma,\ r = 1,2,\dots,R \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^3 y_{r'j}^{b} \le y_{r'o}^{b}\Big) \ge \gamma,\ r' = 1,2,\dots,R' \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 z_{ej}^{13} - \sum_{j=1}^{n} \lambda_j^3 z_{ej}^{13} \ge 0\Big) \ge \gamma,\ e = 1,2,\dots,E \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 z_{sj}^{12} - \sum_{j=1}^{n} \lambda_j^2 z_{sj}^{12} \ge 0\Big) \ge \gamma,\ s = 1,2,\dots,S \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 z_{kj}^{23} - \sum_{j=1}^{n} \lambda_j^3 z_{kj}^{23} \ge 0\Big) \ge \gamma,\ k = 1,2,\dots,K \\
& \lambda_j^1,\ \lambda_j^2, \lambda_j^3 \ge 0
\end{aligned} \tag{4}
$$

where $\gamma \in [0,1]$ is the predetermined thresholds defined by DM and $\overline{\Pr}[\cdot]$ in Model (4) denote the fuzzy stochastic measure.

To get a linear form of solving Model (4), we consider the following substitutions:

$$
\begin{aligned}
& \hat{x}_{ij} = \lambda_j^1 x_{ij},\ \hat{y}_{rj}^{g} = \lambda_j^3 y_{rj}^{g},\ \hat{y}_{r'j}^{b} = \lambda_j^3 y_{r'j}^{b} \\
& \hat{z}_{ej}^{113} = \lambda_j^1 z_{ej}^{13},\ \hat{z}_{ej}^{313} = \lambda_j^3 z_{ej}^{13} \\
& \hat{z}_{sj}^{112} = \lambda_j^1 z_{sj}^{12},\ \hat{z}_{sj}^{212} = \lambda_j^2 z_{sj}^{12} \\
& \hat{z}_{kj}^{223} = \lambda_j^2 z_{kj}^{23},\ \hat{z}_{kj}^{323} = \lambda_j^3 z_{kj}^{23}
\end{aligned} \tag{5}
$$

By substituting these variables, model (4) changing to the following model:

$$
\begin{aligned}
& E_o(\delta,\gamma) = \min \quad \theta \\
& s.t. \\
& \sum_{j=1}^{n} \hat{x}_{ij} \le \theta x_{io}\,,\ i = 1,2,\dots,I \\
& \sum_{j=1}^{n} \hat{y}_{rj}^{g} \ge y_{ro}^{g}\,,\ r = 1,2,\dots,R \\
& \sum_{j=1}^{n} y_{r'j}^{b} \le y_{r'o}^{b}\,,\ r' = 1,2,\dots,R' \\
& \sum_{j=1}^{n} \hat{z}_{ej}^{113} - \sum_{j=1}^{n} \hat{z}_{ej}^{313} \ge 0\,,\ e = 1,2,\dots,E \\
& \sum_{j=1}^{n} \hat{z}_{sj}^{112} - \sum_{j=1}^{n} \hat{z}_{sj}^{212} \ge 0\,,\ s = 1,2,\dots,S \qquad (6) \\
& \sum_{j=1}^{n} \hat{z}_{kj}^{223} - \sum_{j=1}^{n} \hat{z}_{kj}^{323} \ge 0\,,\ k = 1,2,\dots,K \\
& \overline{\Pr}(\lambda_j^1 x_{ij} \le \hat{x}_{ij} \le \lambda_j^1 x_{ij}) \ge \gamma\ ;\ \overline{\Pr}(\lambda_j^3 z_{ej}^{13} \le \hat{z}_{ej}^{313} \le \lambda_j^3 z_{ej}^{13}) \ge \gamma \\
& \overline{\Pr}(\lambda_j^3 y_{rj}^{g} \le \hat{y}_{rj}^{g} \le \lambda_j^3 y_{rj}^{g}) \ge \gamma\ ;\ \overline{\Pr}(\lambda_j^1 z_{sj}^{12} \le \hat{z}_{sj}^{112} \le \lambda_j^1 z_{sj}^{12}) \ge \gamma \\
& \overline{\Pr}(\lambda_j^3 y_{r'j}^{b} \le \hat{y}_{r'j}^{b} \le \lambda_j^3 y_{r'j}^{b}) \ge \gamma\ ;\ \overline{\Pr}(\lambda_j^2 z_{sj}^{12} \le \hat{z}_{sj}^{212} \le \lambda_j^2 z_{sj}^{12}) \ge \gamma \\
& \overline{\Pr}(\lambda_j^1 z_{ej}^{13} \le \hat{z}_{ej}^{113} \le \lambda_j^1 z_{ej}^{13}) \ge \gamma\ ;\ \overline{\Pr}(\lambda_j^2 z_{kj}^{23} \le \hat{z}_{kj}^{223} \le \lambda_j^2 z_{kj}^{23}) \ge \gamma \\
& \overline{\Pr}(\lambda_j^3 z_{kj}^{23} \le \hat{z}_{kj}^{323} \le \lambda_j^3 z_{kj}^{23}) \ge \gamma \\
& \lambda_j^1, \lambda_j^2, \lambda_j^3 \ge 0
\end{aligned}
$$

To solve model (6), we utilize Theorem 1 and give Definition 1.

Theorem 1 (Nasseri et al. [27])**.** If $\tilde{\tilde{X}} \sim \overline{\mathrm{N}}(\bar{\mu},\sigma)$ with $\bar{\mu} = (\mu,\alpha,\beta)$, then

a. $\overline{\Pr}(\tilde{\tilde{X}} \le \mathrm{r}) > \gamma \ iff\ \dfrac{\mathrm{r}-\bar{\mu}}{\sigma} \ge (\Phi^{-1}(\gamma), \dfrac{\alpha+\beta}{\sigma}, \dfrac{\alpha+\beta}{\sigma})$

b. $\overline{\Pr}(\tilde{\tilde{X}} \ge \mathrm{r}) > \gamma \ iff\ \dfrac{\mathrm{r}-\bar{\mu}}{\sigma} \le (\Phi^{-1}(1-\gamma), \dfrac{\alpha+\beta}{\sigma}, \dfrac{\alpha+\beta}{\sigma})$

And so $\overline{\Pr}(r \le \tilde{\tilde{X}} \le \mathrm{r}) > \gamma \ iff\ (\Phi^{-1}(\gamma), \dfrac{\alpha+\beta}{\sigma}, \dfrac{\alpha+\beta}{\sigma}) \le \dfrac{\mathrm{r}-\bar{\mu}}{\sigma} \le (\Phi^{-1}(1-\gamma), \dfrac{\alpha+\beta}{\sigma}, \dfrac{\alpha+\beta}{\sigma})$

Notably, the fuzzy ranking method adopted in this study is Tanaka's approach at the such a threshold δ [36]. Hence, we have:

$$
\begin{aligned}
&E_o(\delta,\gamma) = \min \quad \theta \\
&s.t. \sum_{j=1}^{n} \hat{x}_{ij} \le \theta(x_{io} + R^{-1}(\delta)x_{io}^{\beta} + \sigma_{ij}\Phi_{1-\gamma}^{-1}),\ i = 1,2,...,I \\
&\sum_{j=1}^{n} \hat{y}_{rj}^{g} \ge (y_{ro}^{g} - L^{-1}(\delta)y_{ro}^{g,\alpha} - \sigma_{rj}\Phi_{1-\gamma}^{-1}),\ r = 1,2,...,R \\
&\sum_{j=1}^{n} \hat{y}_{r'j}^{b} \le (y_{r'o}^{g} + R^{-1}(\delta)y_{ro}^{g,\alpha} + \sigma_{r'j}\Phi_{1-\gamma}^{-1}),\ r' = 1,2,...,R' \\
&\sum_{j=1}^{n} \hat{z}_{ej}^{113} - \sum_{j=1}^{n} \hat{z}_{ej}^{313} \ge 0,\ e = 1,2,...,E \\
&\sum_{j=1}^{n} \hat{z}_{sj}^{112} - \sum_{j=1}^{n} \hat{z}_{sj}^{212} \ge 0,\ s = 1,2,...,S \\
&\sum_{j=1}^{n} \hat{z}_{kj}^{223} - \sum_{j=1}^{n} \hat{z}_{kj}^{323} \ge 0,\ k = 1,2,...,K \\
&\lambda_j^1(x_{ij} - L^{-1}(\delta)x_{ij}^{\alpha} - \sigma_{ij}\Phi_{1-\gamma}^{-1}) \le \hat{x}_{ij} \le \lambda_j^1(x_{ij} + R^{-1}(\delta)x_{ij}^{\beta} + \sigma_{ij}\Phi_{1-\gamma}^{-1}) \qquad (7) \\
&\lambda_j^3(y_{rj} - L^{-1}(\delta)y_{rj}^{g,\alpha} - \sigma_{rj}\Phi_{1-\gamma}^{-1}) \le \hat{y}_{rj}^{g} \le \lambda_j^3(y_{rj}^{g} + R^{-1}(\delta)y_{rj}^{g,\beta} + \sigma_{rj}\Phi_{1-\gamma}^{-1}) \\
&\lambda_j^3(y_{r'j}^{b} - L^{-1}(\delta)y_{r'j}^{b,\alpha} - \sigma_{rj}\Phi_{1-\gamma}^{-1}) \le \hat{y}_{r'j}^{b} \le (\lambda_j^3 y_{r'j}^{b} + R^{-1}(\delta)y_{r'j}^{b,\beta} + \sigma_{r'j}\Phi_{1-\gamma}^{-1}) \\
&\lambda_j^1(z_{ej}^{13} - L^{-1}(\delta)z_{ej}^{13,\alpha} - \sigma_{ej}^{13}\Phi_{1-\gamma}^{-1}) \le \hat{z}_{ej}^{113} \le \lambda_j^1(z_{ej}^{13} + R^{-1}(\delta)z_{ej}^{13,\beta} + \sigma_{ej}^{13}\Phi_{1-\gamma}^{-1}) \\
&\lambda_j^3(z_{ej}^{13} - L^{-1}(\delta)z_{ej}^{13,\alpha} - \sigma_{ej}^{13}\Phi_{1-\gamma}^{-1}) \le \hat{z}_{ej}^{313} \le \lambda_j^3(z_{ej}^{13} + R^{-1}(\delta)z_{ej}^{13,\beta} + \sigma_{ej}^{13}\Phi_{1-\gamma}^{-1}) \\
&\lambda_j^1(z_{sj}^{12} - L^{-1}(\delta)z_{sj}^{12,\alpha} - \sigma_{sj}^{12}\Phi_{1-\gamma}^{-1}) \le \hat{z}_{sj}^{112} \le \lambda_j^1(z_{sj}^{12} + R^{-1}(\delta)z_{sj}^{12,\beta} + \sigma_{sj}^{12}\Phi_{1-\gamma}^{-1}) \\
&\lambda_j^2(z_{sj}^{12} - L^{-1}(\delta)z_{sj}^{12,\alpha} - \sigma_{sj}^{12}\Phi_{1-\gamma}^{-1}) \le \hat{z}_{sj}^{212} \le \lambda_j^2(z_{sj}^{12} + R^{-1}(\delta)z_{sj}^{12,\beta} + \sigma_{sj}^{12}\Phi_{1-\gamma}^{-1}) \\
&\lambda_j^2(z_{kj}^{23} - L^{-1}(\delta)z_{kj}^{23,\alpha} - \sigma_{kj}^{23}\Phi_{1-\gamma}^{-1}) \le \hat{z}_{kj}^{223} \le \lambda_j^2(z_{kj}^{23} + R^{-1}(\delta)z_{kj}^{23,\beta} + \sigma_{kj}^{23}\Phi_{1-\gamma}^{-1}) \\
&\lambda_j^3(z_{kj}^{23} - L^{-1}(\delta)z_{kj}^{23,\alpha} - \sigma_{kj}^{23}\Phi_{1-\gamma}^{-1}) \le \hat{z}_{kj}^{323} \le \lambda_j^3(z_{kj}^{23} + R^{-1}(\delta)z_{kj}^{23,\beta} + \sigma_{kj}^{23}\Phi_{1-\gamma}^{-1}) \\
&\lambda_j^1,\ \lambda_j^2,\ \lambda_j^3 \ge 0
\end{aligned}
$$

The above model is linear. This model is an extension of the Nasseri et al. 's model to the proposed network CCR model when undesirable outputs are considered [27].

Definition 1. For the given level δ and γ, we define $E_o^T(\delta,\gamma) = E_o(\delta, \frac{\gamma}{2})$ as efficiency score of DMU_o in the fuzzy random DEA Model.

Theorem 2. If $E_k(\delta,\gamma)$ is the optimum objective function value of Model (7), then $E_k(\delta_1,\gamma) \ge E_k(\delta_2,\gamma)$ and $E_k(\delta,\gamma_1) \ge E_k(\delta,\gamma_2)$, where $\delta_1 \le \delta_2$ and $\gamma_1 \le \gamma_2$.

Proof. Denote the feasible space of Model (7) by $S_{\delta,\gamma}$. We need to prove that $S_{\delta_2,\gamma_2} \subseteq S_{\delta_1,\gamma_1}$. To this, consider the following constraint of Model (7)

$$v_i(x_{ij} - L^{-1}(\delta)x_{ij}^{\alpha} - \sigma_{ij}\Phi_{1-\gamma}^{-1}) \le \hat{x}_{ij} \le v_i(x_{ij} + R^{-1}(\delta)x_{ij}^{\beta} + \sigma_{ij}\Phi_{1-\gamma}^{-1})$$

Let $\Phi^{-1}(\gamma)=\Phi_{\gamma}^{-1}$. As $\Phi^{-1}(1-\gamma)$, $L^{-1}(\delta)$ and $R^{-1}(\delta)$ are decreasing functions and, the functions $-\Phi^{-1}(1-\gamma), -L^{-1}(\delta)$ and $-R^{-1}(\delta)$ will be increasing. It is concluded that

$$\left[x_{ij}-L^{-1}(\delta_2)x_{ij}^{\alpha}-\sigma_{ij}\Phi^{-1}(1-\gamma_2), x_{ij}+R^{-1}(\delta_2)x_{ij}^{\beta}+\sigma_{ij}\Phi^{-1}(1-\gamma_2)\right]\subseteq \left[x_{ij}-L^{-1}(\delta_1)x_{ij}^{\alpha}-\sigma_{ij}\Phi^{-1}(1-\gamma_1), x_{ij}+R^{-1}(\delta_1)x_{ij}^{\beta}+\sigma_{ij}\Phi^{-1}(1-\gamma_1)\right]$$

similarly, we can conclude that

$$\left[y_{rj}-L^{-1}(\delta_2)y_{rj}^{\alpha}-\sigma_{rj}\Phi^{-1}(1-\gamma_2), y_{rj}+R^{-1}(\delta_2)y_{rj}^{\beta}+\sigma_{rj}\Phi^{-1}(1-\gamma_2)\right]\subseteq \left[y_{rj}-L^{-1}(\delta_1)y_{rj}^{\alpha}-\sigma_{rj}\Phi^{-1}(1-\gamma_1), y_{rj}+R^{-1}(\delta_1)y_{rj}^{\beta}+\sigma_{rj}\Phi^{-1}(1-\gamma_1)\right]$$

This completes the proof.☐

Now, we can present the following definition to define the efficiency of each DMU.

Definition 2. For the given level δ and γ , we define $\mathrm{E}_k^T(\delta,\gamma)=\mathrm{E}_k\ (\delta,\frac{\gamma}{2})$ as probabilistic-possibilistic efficiency score of DMU_k in the fuzzy random DEA Model.

The corresponding model $\mathrm{E}_k^T(\delta,\gamma)$ is as follows:

$$\begin{aligned}
&E_k^T(\delta,\gamma)=\max\ \varphi\\
&s.t.\\
&\left.\begin{aligned}
&\varphi\le\sum_{r=1}^{s_1}\hat{y}_{rk}^{g}-\sum_{p=1}^{s_2}\hat{y}_{pk}^{b}\\
&\sum_{i=1}^{m}\hat{x}_{ik}=1\\
&\sum_{r=1}^{s_1}\hat{y}_{rj}^{g}-\sum_{p=1}^{s_2}\hat{y}_{pj}^{b}-\sum_{d=1}^{D}\hat{z}_{dj}\le 0,\ \ j=1,2,\dots,n\\
&\sum_{d=1}^{D}\hat{z}_{dj}-\sum_{i=1}^{m}\hat{x}_{rj}\le 0,\ \ j=1,2,\dots,n
\end{aligned}\right\}(i)\\
&u_r^g(y_{rj}^g-L^{-1}(\delta)y_{rj}^{\alpha,g}-\sigma_{rj}\Phi^{-1}_{1-\frac{\gamma}{2}})\le\hat{y}_{rj}^g\le u_r^g(y_{rj}^g+R^{-1}(\delta)y_{rj}^{\beta,g}+\sigma_{rj}\Phi^{-1}_{1-\frac{\gamma}{2}}),\ \ \forall r,j\\
&u_p^b(y_{pj}^b-L^{-1}(\delta)y_{pj}^{\alpha,b}-\sigma_{pj}\Phi^{-1}_{1-\frac{\gamma}{2}})\le\hat{y}_{rj}^b\le u_p^b(y_{pj}^b+R^{-1}(\delta)y_{pj}^{\beta,b}+\sigma_{pj}\Phi^{-1}_{1-\frac{\gamma}{2}}),\ \ \forall p,j\\
&w_d(z_{dj}-L^{-1}(\delta)z_{dj}^{\beta}-\sigma_{dj}\Phi^{-1}_{1-\frac{\gamma}{2}})\le\hat{z}_{rj}\le w_d(z_{dj}+R^{-1}(\delta)z_{dj}^{\beta}+\sigma_{dj}\Phi^{-1}_{1-\frac{\gamma}{2}}),\ \ \forall d,j\\
&v_i(x_{ij}-L^{-1}(\delta)x_{ij}^{\alpha}-\sigma_{ij}\Phi^{-1}_{1-\frac{\gamma}{2}})\le\hat{x}_{ij}\le v_i(x_{ij}+R^{-1}(\delta)x_{ij}^{\beta}+\sigma_{ij}\Phi^{-1}_{1-\frac{\gamma}{2}}),\ \forall i,j\\
&u_r^g\ge 0, u_p^b\ge 0, v_i\ge 0, w_d\ge 0, \varphi\ge 0,\\
&r=1,2,\dots,s_1;\ p=1,2,\dots,s_2\ i=1,2,\dots,m;\ d=1,2,\dots,D
\end{aligned}\tag{8}$$

Theorem 3. Consider $E_k^{\mathrm{T}}(\delta,\gamma)$ as the optimum objective function value of Model (8) for DMU$_k$, then

a. $E_k^T(\delta_1,\gamma) \geq E_k^T(\delta_2,\gamma)$ and $E_k^T(\delta,\gamma_1) \geq E_k^T(\delta,\gamma_2)$ where $\delta_1 \leq \delta_2$ and $\gamma_1 \leq \gamma_2$.
b. $0 < E_j^T(\delta,\gamma) \leq 1,\ (j=1,2,\ldots,n)$.
c. Model (13) is feasible for any δ and γ .
Proof: **a.** It is straightforward using Theorem 2 and Definition 2.
In assertion b is followed immediately by the restriction $\varphi \geq 0$ and four constraints in part (i) of the model (8) as follows:

$$\varphi \leq \sum_{r=1}^{s_1} \hat{y}_{rk}^{g} - \sum_{p=1}^{s_2} \hat{y}_{pk}^{b} \leq \sum_{d=1}^{D} \hat{z}_{dk} \leq \sum_{i=1}^{m} \hat{x}_{rk} = 1.$$

To prove assertion c, Let $\delta=1$ and $\gamma=1$, then $L^{-1}(1)=R^{-1}(1)=0$ and $\Phi^{-1}(0.5)=0$. Hence, we have $\hat{x}_{ij}=v_i x_{ij}$, $\hat{y}_{rj}^{g}=u_r^{g} y_{rj}^{g}$, $\hat{y}_{pj}^{b}=u_p^{b} y_{pj}^{b}$ and $\hat{z}_{dj}=w_d z_{dj}$ in Model (8). Therefore, The correspondig model with $E_k^T(1,1)$ will be as follows:

$$\begin{aligned}
E_k^T(1,1) = Max\ & \sum_{r=1}^{s_1} u_r^g y_{rk}^g - \sum_{p=1}^{s_2} u_p^b y_{pk}^b \\
s.t.\ & \sum_{i=1}^{m} v_i x_{ik} = 1 \\
& \sum_{r=1}^{s_1} u_r^g y_{rj}^g - \sum_{p=1}^{s_2} u_p^b y_{pj}^b - \sum_{d=1}^{D} w_d z_{dj} \leq 0,\ \ j=1,2,\ldots,n \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{rj} \leq 0,\ \ j=1,2,\ldots,n \\
& u_r^g \geq 0, u_p^b \geq 0, v_i \geq 0, w_d \geq 0,\ \ r=1,2,\ldots,s_1; p=1,2,\ldots,s_2; i=1,2,\ldots,m; d=1,2,\ldots,D
\end{aligned} \tag{9}$$

To prove assertion c, denote the feasible space of Model (8) by $S_{\delta,\gamma}^T$. According to the proof of Theorem 2, $S_{1,1}^T \subseteq S_{\delta,\gamma}^T$. Therefore, it is sufficient to show that the feasible space $S_{1,1}^T$ is nonempty. Suppose that $u_p^b = 0\,(p=1,2,\ldots,s_2)$, then $E_k^T(1,1)$ is converted to the model (9) as a two-stage model. Chen et al. [4] explicitly showed the feasibility of the model (9). This completes the proof of part (c).□
Now, we are going to apply the given model to the banking industry as a real case study.

٤. Case Study

We focused on the banking Industry has a comprehensive network of over 300 branches and 30000 employees in Iran. Countrywide coverage in Iran, service quality ,and experienced multi-lingual staff are important factors of their success. In this section, we apply the proposed approach in this study to some commercial bank branches in Mazandaran province. Here the data sources consist of the reports of some selected branches. The inputs for the first stage are personnel score, cost, location, and branch facilities with intermediate output service and Total of Deposits (TDs) (of current, short duration , and long duration accounts). The second stage's input is TDs and the loan is as intermediate output. Finally, in the third stage service and TDs as intermediate input and recovered loans as desirable outputs, and non-performing loans (delay in delivering loans and other facilities) as undesirable output. However, there always exist some degrees of uncertainty in the data which can be represented by fuzzy stochastic numbers. In banks, uncertainty occurs due to the difference between the actual data and the available data. Then the difference between actual data and possible data results in the occurrence of uncertainty in the data which further may affect. Therefore, in the present study, we fuzzify the data as TFNs. The collected crisp data in Table 1 are considered as the mean of TFNs. On the other hand, the inputs and outputs are supposed as random variables. By using the goodness of fit tests, normal distributions have been fit on the random variables. The corresponding expected value is the observed inputs (outputs) data and the standard deviation is one. Hence, each DMU is considered a fuzzy variable with a randomized mean. This fuzzy random input–intermediate-output data of each bank is available in Table 1. Finally, Table 2 presents the average efficiency scores and the final rankings of the 10 bank branches. However, the average efficiency can be an appropriate overall index to indicate the efficiency variations.

Table 1. The fuzzy random input and output data[۳]

DMU	Personnel Score	Cost	Branch Facilities	Interest Income	Location	Loans	User fee income	Deposit	Non-performing loans
1	17,014,781	354,133	28,347	796,832	715	3,648,031	95,045	5,981,048	301,779
2	14,297,944	287,066	17,889	879,802	879	4,317,806	46,845	6,323,772	175,162
3	16,252,095	384,871	28,001	1,116,566	2,087	4,522,011	111,225	7,950,451	415,303
4	16,342,530	424,974	19,630	1,210,623	1,292	6,278,297	91,316	8,851,770	312,750
5	16,687,868	377,789	23,508	836,644	1,164	3,491,101	159,909	5,992,871	658,208
6	14,765,164	249,487	20,307	627,658	913	2,524,526	45,740	5,116,146	126,437
7	16,933,047	366,048	25,208	1,003,786	2,671	3,991,867	188,924	7,588,686	284,899
8	10,583,687	245,834	7,146	589,456	480	2,953,722	119,975	5,414,472	460,950
9	8,183,284	210,688	13,514	530,537	417	3,511,138	54,141	5,559,826	179,385
10	5,439,440	131,682	7,881	345,072	347	2,044,424	18,125	2,952,701	130,017

Table 2. The fuzzy random efficiency scores and final ranking

DMU	(γ=0.9,δ=0.7)	(γ=0.9,δ=0.4)	(γ=0.7,δ=0.7)	(γ=0.5,δ=0.5)	Overall efficiency	Ranking
1	**0.5603**	**0.5673**	**0.5650**	**0.5744**	**0.5668**	**8**
2	**0.6112**	**0.6151**	**0.6162**	**0.6240**	**0.6166**	**7**
3	**0.7448**	**0.7495**	**0.7500**	**0.7585**	**0.7507**	**4**
4	**0.5560**	**0.5627**	**0.5593**	**0.5672**	**0.5613**	**10**
5	**0.5606**	**0.5690**	**0.5643**	**0.5738**	**0.5669**	**9**
6	**0.5495**	**0.5610**	**0.5496**	**0.5572**	**0.5543**	**11**
7	**0.6791**	**0.6883**	**0.6842**	**0.6955**	**0.6868**	**6**
8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1**
9	**0.5005**	**0.5077**	**0.5045**	**0.5134**	**0.5065**	**14**
10	**0.5372**	**0.5449**	**0.5413**	**0.5505**	**0.5435**	**12**

۵. Conclusions and future works

This paper formulated the DEA model handling the three-stage process and undesirable outputs in a fuzzy random environment. the extended model depicts the influence of the presence of fuzziness and randomness in the data over the efficiency values. To do this, we have first incorporated an undesirable output in the three-stage DEA model. The resulting model was converted into a new model with some variable substitutions. Then, to solve the uncertainty part of the model, we applied

[۳]The prices are in million Rials.

the $\overline{\Pr}(.)$ measure that led to a linear model. Furthermore, the proposed approach can be used in many practical situations such as Insurance Industry, Supply Chain, etc.

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