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A multi-parametric approach for solid transportation problem with uncertainty fuzzy flexible conditions

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The most convenient models of Solid Transportation (ST) problems have been justly considered a kind of uncertainty in their parameters such as fuzzy, grey, stochastic, etc. and usually, they suggest solving the main problems by solving some crisp equivalent model/models based on their proposed approach such as using ranking functions, embedding problems etc. Furthermore, there exist some shortcomings in formulating the main model for the realistic situations, since it omitted the flexibility conditions in their studies. Hence, to overcome these shortages, we formulate these conditions for the mentioned these problems with fuzzy flexible constraints, where there are no exact predictions for the values of the resources. In particluar, numerical investigation shows that each increasing for the values of the supply and demand is not effective for improving the objective function. The value of the objective function is sensitive when supply and demand change, so we conduct a new study to diversify the value of the objective function, due to changes in resource and demand levels simultaneously. 3 to 6 italic words that describe the focus and contribution of the paper.

Keywords*: Multi-parametric Flexible Fuzzy Transportation Problem, Solid Transportation, Membership Function.*

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1. Introduction

The transportation problem is one of the most important special structures in linear programming. The world of commerce today is a world of uncertainty, especially uncertainty in transportation systems. Optimization in the transportation process can increase customer satisfaction and profit and make the economy better. Uncertainty exists in variables related to the demand, supply, costs and other variables that investigating these problems in uncertain and fuzzy conditions is of great importance for researchers. Always finding the best answer according to the criteria for lightening and weighing costs and profits is of interest. In recent decades, achieving the optimal answer has been considered. A Solid transportation problem is linear programming in operations research that can optimize the cost of products that are transferred from several destinations. Have been different methods studiedies in classical transportation, but given that the actual data is not correct, we need a fuzzy model. Leo and Chen [7] proposed a multi-objective programming algorithm by using goal programming. Gao, Wang, and Zhou [7] introduced the transportation problem-solving algorithm with imprecise supply and random demand. Zhang and Peng [22] checked the solid transportation problem in which resources, demands and conveyances capacities are inaccurate direct costs and fixed charges and objective function is to minimize transportation costs. Their proposed algorithm is based on uncertainty theory and tabu search algorithm to solve these models, which also made a comparison between this method and some existing methods. Kochen et al. [9] proposed a three-step algorithm for the solid transportation problem with triangular fuzzy supply and demand, conveyances, and fuzzy cost. In the first step, the feasibility space is found to be a triangular fuzzy constraint, and in the second stage, using the defined membership function, it finds the breaking points of the cost coefficients that change the optimal solution. Finally, they obtained the maximum feasibility level and the breaking points using the introduced algorithm. Dos et al. [4] investigated solid transportation problems with type 2 fuzzy parameters that aimed to minimize the cost and time. They developed transportation problems in two models. The first model with cost and time of fuzzy type 2 and the second model cost and time

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and all parameters of fuzzy type 2. To solve this method, the weighting method and global criteria, and CV-based reduction method were used. Tootooni et al. [21] proposed a fuzzy type I and II programming approach for a new model presented in the literature, i.e., the single allocation ordered median problem. The aim of the study by Alizadeh et al. [1] is to design a cold multi-cycle supply chain based on multiple connections taking uncertainty into account. Fallah-Tafti et al. [5] developed a capacitated hub location-routing model to design a rapid transit network under uncertainty. Soleimani Kourandeh et al. [20] investigated the goal Weber location problem in which the location of some of demand points on a plane is given, and the ideal is locating the facility in the distance R_i, from the i-th demand point. Rivaz et al. [17] considered a multi-objective transportation problem which has proposed a new model based on fuzzy goal programming to solve Multi-Objective Transportation Problem. By varying the weights in the new model, different solutions could be obtained. Then, a comparison has been made with some existing methods. Chhibber et al. [3], considered a fuzzy solid transportation problem under an intuitionistic fuzzy environment. They obtained a Pareto-optimal solution for a multi-objective fixed-charge solid transportation problems with linear, hyperbolic, and exponential membership as well as non-membership functions. Sahoo [18] solved the transportation problem where supply, demand and transportation costs are Fermatean Fuzzy Numbers (FFNs). He proposed an algorithm for solving the transportation problem with Fermatean fuzzy parameters, proposed an algorithm, and then used arithmetic operations of Fermatean fuzzy number to obtain the optimal solution. Samanta et al. [19] considered a solid transportation problem in which the transportation is accomplished in two stages – firstly, from the origin(s) to the nearby station(s) of the destination(s) and then, from the nearby station(s) to the main destination(s). A fuzzy discount policy has been introduced based on the amount of transportation along with a fuzzy fixed charge and fuzzy unit transportation cost. To solve the model, Genetic Algorithm (GA) proposed has been used. Nasseri et al. [14] has solved a linear program with flexible fuzzy numbers by the goal programming method. They used different cuts to convert the original problem to a crisp multi-parametric multi-objective linear programming problem. They found the optimal pareto solution for the reduced multi parametric multi-objective linear programming by goal programming model. After that they used this approach for the various kinds of the flexible linear programming models based on their approach, we investigate a Solid Transportation (ST) problem in a fuzzy environment with triangular fuzzy cost coefficients and flexible constraints. For this purpose, we use a multi-parametric approach and goal programming according to the membership function which is defined to solve the ST problem with flexible constraints. We then compare the optimal solutions of the proposed model and the goal programming. For both proposed cases, we also offer two algorithms, due to fuzzy flexibility and parametric approach and we reach the maximum level of $\alpha-$satisfaction. This research has been prepared in five sections. In section 2, we give some fundamental concepts such as the lemma and the orans as our key tools to prepare the study to include the models and method. In section 3, we express proposed approaches and an algorithm. In section 4, we use an illustrative example. Finally, section 5 is devoted to the conclusion of the study.

2. Fuzzy Solid transportation problem with flexible conditions

One of the types of transportation models is the solid transportation problem, which is a generalization for classic transportation. Essentially, in the solid transportation model, it is assumed that each supplier has a supply capacity of $s_i, i=1,2,...,m$, and customers have a demand of $d_j, j=1,2,...,n$, and e_k vehicle have a capacity of $e_k, 1\le k \le K$, while c_{ijk} is the cost of sending a product unit from supplier i to customer j, and x_{ijk} is the decision variable.

Solid Transportation problems with fuzzy constraints and fuzzy costs are among realistic transportation models, which are aimed at minimizing the transportation cost. The product is moved from m suppliers, $1\le i \le m$, to n customers $1\le j \le n$ by k vehicle $1\le k \le K$. e_k, capacity vehicle transportation goods from origin of supply to product. When supply has changed due to the covid-19 conditions and demand is sent online, supply, demand and transportation conditions will no longer have a definitive structure.

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In this case, a suitable structure representing the real conditions of the problem should be designed for the mathematical model. The following model assumes the total constraints to be modeled as fuzzy flexible, in which $\lesssim$ stands for "lower than or equal to" and $\gtrsim$ stands for "upper than or equal to" is used for inaccurate data. The mathematical model is represented below:

Model I:

$$Min\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\tilde{c}_{ijk}x_{ijk} \tag{1}$$

s.t.

$$\sum_{j=1}^{n}\sum_{k=1}^{K}x_{ijk} \lesssim s_i\,, \quad \forall i \tag{2}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk} \gtrsim d_j\,, \quad \forall j \tag{3}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk} \lesssim e_k\,, \quad \forall k \tag{4}$$

$$x_{ijk} \geq 0, \quad i=1,2,...,m\,, \quad j=1,2,...,n\,, \quad k=1,2,...,K. \tag{5}$$

For Eq. (3), Nasseri et al. (2018) proposed the following fuzzy constraint membership functions:

$$\mu_j(x)=\begin{cases} 1, & x \leq d_j \\ 1-\dfrac{d_j-x}{q_j}, & d_j \leq x \leq d_j+q_j \\ 0, & x \geq d_j+q_j \end{cases} \tag{6}$$

For Eq. (2) and (4), Nasseri et al. (2018) proposed the following fuzzy constraint membership functions:

$$\mu_i(x)=\begin{cases} 1, & x \leq s_i \\ 1-\dfrac{x-s_i}{p_i}, & s_i \leq x \leq p_i+s_i \\ 0, & x \geq p_i+s_i \end{cases} \tag{7}$$

Considering the lack of clear definition for conditions (2), (3) and (4), the equivalent model is constructed to solve this model from a parametric approach.

Assume that the tolerance of the $i-th$ constraint of supply is p_i. Based on tolerance p_i and its flexibility in its range, we have $\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \lesssim s_i$, $i=1,2,...,m$, and $\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le s_i + \theta p_i$, in which $\theta \in [0,1]$. The tolerance of the $j-th$ constraint of demand is q_j. Based on tolerance q_j and its flexibility in its range, we have $\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \gtrsim d_j$ and $\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge d_j - \theta q_j$, $j=1,2,...,n$, and the tolerance of the $k-th$ constraint of vehicle is r_k. Based on tolerance r_k and its flexibility in its range, we have $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \lesssim e_k$, and $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_k + \theta r_k$, $k=1,2,...,K$.

The following lemmas will be useful for our discussion.

Lemma 2.1: The constraint $\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \gtrsim d_j$ is equivalent with the constraint $\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge d_j - \theta q_j$, for $\theta \in [0,1]$.

Proof: Each feasible solution x_{ijk} which is satisfied in $\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \gtrsim d_j$ is indeed a fuzzy set with the following membership function:

$$\mu_j\{t_j\} = \begin{cases} 1, & t_j \le d_j \\ 1 - \dfrac{d_j - t_j}{q_j}, & d_j \le t_j \le d_j + q_j \\ 0, & t_j \ge d_j + q_j \end{cases} \tag{8}$$

where $t_j = \sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}$.

We named this feasible solution as $\beta-$feasible solution of the mentioned constraint.

Now consider the three following cases:

A) If $\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} - d_j \le 0$, then the $j-th$ Constraint is satisfied and so equal to 1;

B) If $0 \le \sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} - d_j \le q_j$ then the membership function for $j-th$ Constraint is monotonically increasing. This case means that the satisfaction degree (level) of the $j-th$ Constraint is reducing.

C) If $\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} - d_j \ge q_j$, the tolerance accepted range is larger than the value which is determined by the decision-maker; Thus the $j-th$ Constraint has been completely violated, and its membership function is equal to 0.

Hence, because of the membership function is continuous, the right-hand-sides of the flexible constraint form d_j to $d_j - q_j$ base on the continuous value for θ, from $\theta = 0$ to $\theta = 1$ can be achieved. Therefore, the fuzzy flexible relation can be shown by the following equivalent parametric form, $d_j(\theta) = d_j - \theta q_j$ where $\theta \in [0,1]$.

Lemma 2.2: Problem I is equivalent to the following multi-parametric linear programming problem:

Model II:

$$Min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{c}_{ijk} x_{ijk} \quad (9)$$

s.t.

$$\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \le s_i + (1-\alpha_i) p_i, \quad i = 1,2,...,m, \quad (10)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \ge d_j - (1-\beta_j) q_j, \quad j = 1,2,...,n, \quad (11)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \le e_k + (1-\gamma_k) r_k, \quad k = 1,2,...,K, \quad (12)$$

$$x_{ijk} \ge 0, \quad i = 1,2,...,m, \quad j = 1,2,...,n, \quad k = 1,2,...,K, \quad 0 \le \alpha, \beta, \delta \le 1. \quad (13)$$

Proof: It is enough to prove that the relation (2), (3) and (4) are respectively equivalent to (10), (11) and (12). Because of all three above cases have the same process, we just focus on the second one and omit others to the readers. The result is clear based on Lemma 2.1.

Now, we clearly may write the main problem in the following equivalent form:

Model III:

$$Min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{c}_{ijk} x_{ijk} \quad (14)$$

s.t.

$$X \in X_{\alpha} \quad (15)$$

$$x_{ijk} \ge 0, \quad i = 1,2,...,m, \quad j = 1,2,...,n, \quad k = 1,2,...,K, \quad 0 \le \alpha, \beta, \gamma \le 1. \quad (16)$$

where the set of all feasible solutions of the problem is defined as follows.

Definition 2.1: Suppose $\alpha = (\alpha_1,...,\alpha_m, \beta_1,...,\beta_n, \gamma_1,...,\gamma_k) \in [0,1]^{m+n+k}$ and

$$X_{\alpha} = \left\{ x_{ijk} \in \mathbb{R} \,\middle|\, \begin{array}{l} \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \le s_i + (1-\alpha_i) p_i, \sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \ge d_j - (1-\beta_j) q_j, \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \le e_k + (1-\gamma_k) r_k \\ x_{ijk} \ge 0, \quad i = 1,2,...,m, \quad j = 1,2,...,n, \quad k = 1,2,...,K, \quad 0 \le \alpha, \beta, \delta \le 1. \end{array} \right\}$$

So that $X = (x_{ijk}) \in X_{\alpha}, x_{ijk} \in \mathbb{R}$, is an $\alpha - \beta - \gamma$ feasible solution for Modell III.

3. Transformed model

Consider the problem with supply constraints in the flexible range of $\left[s_i, s_i + p_i\right]$, demand constraints in the flexible range of $\left[d_j - q_j, d_j\right]$ and the constraint of vehicle's capacity in the range of $\left[e_k, e_k + r_k\right]$. Using the membership function for cost coefficients, we convert the problem into a Multi-Parametric Solid Transportation (MPST) problem.

Assume that the cost coefficients of the objective function are in the form of triangular fuzzy numbers. Due to the fuzzy coefficients of the objective function, it is impossible to solve it directly, and hence we suggest to convert it to the crisp objective function such as Yager's method as well as used in [mahdavi- amiri, nasseri 2007].

<u>*Model IV:*</u>

$$Min \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\Re(\tilde{c}_{ijk})x_{ijk} \quad (17)$$

s.t.

$$\sum_{j=1}^{n}\sum_{k=1}^{K}x_{ijk} \le s_i + (1-\alpha_i)p_i\,, \quad i = 1,2,...,m, \quad (18)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk} \ge d_j - (1-\beta_j)q_j\,, \quad j = 1,2,...,n, \quad (19)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk} \le e_k + (1-\gamma_k)r_k\,, \quad k = 1,2,...,K\,, \quad (20)$$

$$x_{ijk} \ge 0, \quad i = 1,2,...,m\,, \quad j = 1,2,...,n\,, \quad k = 1,2,...,K\,, \quad 0 \le \alpha, \beta, \delta \le 1. \quad (21)$$

where $\Re(\tilde{c}_{ijk})$ means that the corresponding crisp value of the cost coefficient based on a linear ranking function. Now by solving this problem, the optimal values of the decision variables and also the optimal value of the objective function will be obtained.

3.1 two-step multi parametric method

In this section, we use a multi parametric approach to propose a new method for solving flexible fuzzy solid transportation problem. After solving the problem IV, the optimal solution of the problem is obtained as $(x^*, \alpha^*, \beta^*, \gamma^*)$, and also z^* as the value of the objective function. Thus, we solve the following problem in order to achieve the maximum satisfaction degree.

Formulate the multi-parametric linear programming problem as the following Model V.

Model V:

$$Max\sum_{i=0}^{m}\alpha_i+\sum_{j=1}^{n}\beta_j+\sum_{k=1}^{K}\gamma_k \quad (22)$$

s.t.

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}c_{ijk}x_{ijk}\le z^*+(1-\alpha_0)p_0 \quad (23)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K}x_{ijk}\le s_i+(1-\alpha_i)p_i\,,\quad i=1,2,...,m, \quad (24)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K}x_{ijk}\ge d_j-(1-\beta_j)q_j\,,\quad j=1,2,...,n, \quad (25)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk}\le e_k+(1-\gamma_k)r_k\,,\quad k=1,2,...,K, \quad (26)$$

$$x_{ijk}\ge 0,\quad i=1,2,...,m,\quad j=1,2,...,n,\quad k=1,2,...,K,\quad 0\le\alpha,\beta,\gamma\le 1. \quad (27)$$

$$\alpha_i^*\le\alpha_i\le 1,\quad \beta_j^*\le\beta_j\le 1,\quad \gamma_k^*\le\gamma_k\le 1 \quad (28)$$

By solving the second step, the optimal solution is achieved as x^{**} with the optimal value of the objective function as z^{**} and also the degree of efficiency $(\alpha^{**},\beta^{**},\gamma^{**})$, which the second phase creates the maximum satisfaction degree.

The algorithm for solving the main transportation problem is as follows:

Algorithm (STPFFC Solver):

Assumption: The Solid Transportation Problem with Flexible Constraints (STPFFC) is given which includes the parameters: $s_i, d_j, e_k, p_i, q_j, r_k$.

Step 1: Use a linear ranking function to convert the fuzzy cost coefficients to the associated crisp values as: $Min\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\tilde{c}_{ijk}x_{ijk}$ which is equivalent to $Min\mathcal{R}\left(z\left(\tilde{c},x\right)\right)=Min\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{k}\mathcal{R}\left(\tilde{c}_{ijk}\right)x_{ijk}$

Step 2: Write the problem to the multi-parametric form and then solve Model IV to obtain the optimal values of satisfaction parameters as $\alpha_i^*,\beta_j^*,\gamma_k^*$ and also optimal value of the objective function z^* and finally the optimal feasible solution as x^*.

Step 3: Solve Model V based on the associated Model IV by considering $\alpha_i^*,\beta_j^*,\gamma_k^*$, which are selected by an expert decision maker from the first phase, in terms of their maximum level of satisfaction, in order to obtain the optimal solution x^{**} with the optimal objective function value.

In the next section, we give an example to illustrate the proposed approach. The models solved by Lingo software.

4. Numerical example

Example 4.1: Consider the parameters of the Solid Transportation (ST) problem as follows.

Table 4.1: Demand and transportation data

	$k=1$			$k=2$			$k=3$			
$i \setminus j$	1	2	3	1	2	3	1	2	3	
1	(8,9,10)	(3,6,9)	(2,3,4)	(10,12,14)	(8,9,10)	(6,7,8)	(7,9,11)	(5,7,9)	(5,7,9)	8
2	(4,5,6)	(7,9,11)	(5,6,7)	(5,6,7)	(8,11,14)	(6,8,10)	(3,5,7)	(1,3,5)	(5,6,7)	9
3	(1,2,3)	(1,2,3)	(1,1,1)	(1,2,3)	(6,7,8)	(8,9,10)	(1,1,1)	(6,7,8)	(1,3,5)	5
	10			5			6			

$$\sum_{i=1}^{3}\sum_{k=1}^{3} x_{i1k} \geq 7, \quad \sum_{i=1}^{3}\sum_{k=1}^{3} x_{i2k} \geq 8, \quad \sum_{i=1}^{3}\sum_{k=1}^{3} x_{i3k} \geq 6$$

According to the given data, we have a solid transportation problem including fuzzy costs and fuzzy flexible constraints. We are going to solve this problem by Algorithm STPFFC.
consider the ST problem based on the given information in Table 4.1. By use of Yager's ranking function (see in [2]), we have the following equivalent Model with fuzzy flexible constraints.

$$11x_{222}+8x_{223}+5x_{213}+3x_{223}+6x_{233}+2x_{311}+2x_{321}+x_{331}+2x_{312}+7x_{322}+9x_{323}+x_{313}+7x_{323}+3x_{333}$$

$$s.t.$$

$$x_{111}+x_{121}+x_{131}+x_{112}+x_{122}+x_{132}+x_{113}+x_{123}+x_{133} \lesssim 8$$
$$x_{211}+x_{221}+x_{231}+x_{212}+x_{222}+x_{232}+x_{213}+x_{223}+x_{233} \lesssim 9$$
$$x_{311}+x_{321}+x_{331}+x_{312}+x_{322}+x_{332}+x_{313}+x_{323}+x_{333} \lesssim 5$$
$$x_{111}+x_{211}+x_{311}+x_{121}+x_{221}+x_{321}+x_{131}+x_{231}+x_{331} \gtrsim 7$$
$$x_{112}+x_{212}+x_{312}+x_{122}+x_{222}+x_{322}+x_{132}+x_{232}+x_{332} \gtrsim 8$$
$$x_{113}+x_{213}+x_{313}+x_{123}+x_{223}+x_{323}+x_{133}+x_{233}+x_{333} \gtrsim 6$$
$$x_{111}+x_{211}+x_{311}+x_{121}+x_{221}+x_{321}+x_{131}+x_{231}+x_{331} \lesssim 10$$
$$x_{112}+x_{212}+x_{312}+x_{122}+x_{222}+x_{322}+x_{132}+x_{232}+x_{332} \lesssim 5$$
$$x_{113}+x_{213}+x_{313}+x_{123}+x_{223}+x_{323}+x_{133}+x_{233}+x_{333} \lesssim 6$$
$$x_{ijk} \geq 0, \quad i=1,2,3, \quad j=1,2,3, \quad k=1,2,3, \quad 0 \leq \alpha_i \leq 1, \quad 0 \leq \beta_j \leq 1, \quad 0 \leq \gamma_k \leq 1$$

Now, Lemma 2.1 and 2.2 lead us to have the following multi-parametric linear programming problem, where we knew how solve it by the convenient approach and in particular, using Lingo software.
In the process of solving the above problem, as the following minutes.

$$Min\ z = 9x_{111} + 8x_{121} + 3.5x_{131} + 12x_{112} + 9x_{122} + 7x_{123} + 9x_{113} + 7x_{123} + 7x_{133} + 5x_{211} + 9x_{221} + 6x_{231} + 6x_{212} +$$

$$11x_{222} + 8x_{223} + 5x_{213} + 3x_{223} + 6x_{233} + 2x_{311} + 2x_{321} + x_{331} + 2x_{312} + 7x_{322} + 9x_{323} + x_{313} + 7x_{323} + 3x_{333}$$

$$s.t.$$

$$x_{111} + x_{121} + x_{131} + x_{112} + x_{122} + x_{132} + x_{113} + x_{123} + x_{133} \le 8 + 3(1-\alpha_1)$$
$$x_{211} + x_{221} + x_{231} + x_{212} + x_{222} + x_{232} + x_{213} + x_{223} + x_{233} \le 9 + 4(1-\alpha_2)$$
$$x_{311} + x_{321} + x_{331} + x_{312} + x_{322} + x_{332} + x_{313} + x_{323} + x_{333} \le 5 + 2(1-\alpha_3)$$
$$x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} \ge 7 - 3(1-\beta_1)$$
$$x_{112} + x_{212} + x_{312} + x_{122} + x_{222} + x_{322} + x_{132} + x_{232} + x_{332} \ge 8 - 3(1-\beta_2)$$
$$x_{113} + x_{213} + x_{313} + x_{123} + x_{223} + x_{323} + x_{133} + x_{233} + x_{333} \ge 6 - 2(1-\beta_3)$$
$$x_{111} + x_{211} + x_{311} + x_{121} + x_{221} + x_{321} + x_{131} + x_{231} + x_{331} \le 10 + 4(1-\gamma_1)$$
$$x_{112} + x_{212} + x_{312} + x_{122} + x_{222} + x_{322} + x_{132} + x_{232} + x_{332} \le 5 + 2(1-\gamma_2)$$
$$x_{113} + x_{213} + x_{313} + x_{123} + x_{223} + x_{323} + x_{133} + x_{233} + x_{333} \le 6 + 2(1-\gamma_3)$$
$$x_{ijk} \ge 0, \quad i = 1,2,3, \quad j = 1,2,3, \quad k = 1,2,3, \quad \alpha_i^* \le \alpha_i \le 1, \quad \beta_j^* \le \beta_j \le 1, \quad \gamma_k^* \le \gamma_k \le 1$$

Solving the above problem by Lingo software for different values α, β, γ, (but a same value) we have the following tables and graphs, and different solution which are obtained according to the decision-making priorities. In the first evaluation, we change the parameters α_i, β_j and γ_k 's from zero to one, and then by solving Problem VI, we obtain the results as given in the following table including the values of the objective function.

Table 4.2: The optimal values of the objective function based on the different values of α, β, γ.

α, β, γ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Z	25.5	29.55	33.6	37.65	41.7	45.75	49.8	54.3	59.2	64.1	73

Note that for $\alpha_1^* = 0.5, \alpha_2^* = 0.5, \alpha_3^* = 0.5, \beta_1^* = 0.5, \beta_2^* = 0.5, \beta_3^* = 0.5, \gamma_1^* = 0.5, \gamma_2^* = 0.5, \gamma_3^* = 0.5$ we have the objective function value: $z_1^* = 45.75$.

The diagram of changes in the objective function is given in Figure 4.1, in which we see that by increasing the parameter value, the objective function value does not get better and it increases.

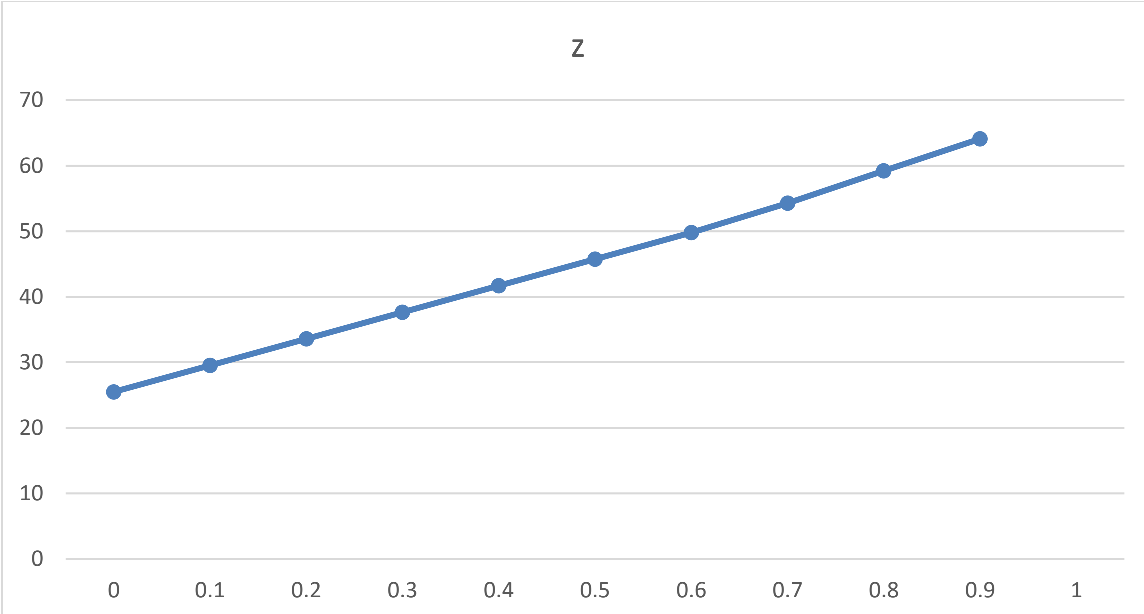

Figure 4.1 Objective function based on different parameters of α, β and γ (but same values of all parameters)

In the second evaluation, we change the supply parameters according to the constant considering the demand and vehicle parameters from zero to one to check the behavior of the objective function of the MPST problem based on tolerance changes.

Table 4.3 Objective function values based on the various values of α_i , while $\beta_j = 1$and $\gamma_k = 1$.

α_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Z	63	63.8	64.6	65.4	66.2	67	67.8	68.6	69.4	71	73

Considering the values which are obtained in the above table, we see that the minimum value of the objective function is much higher than the previous one. The change diagram of the Objective function is as follows.

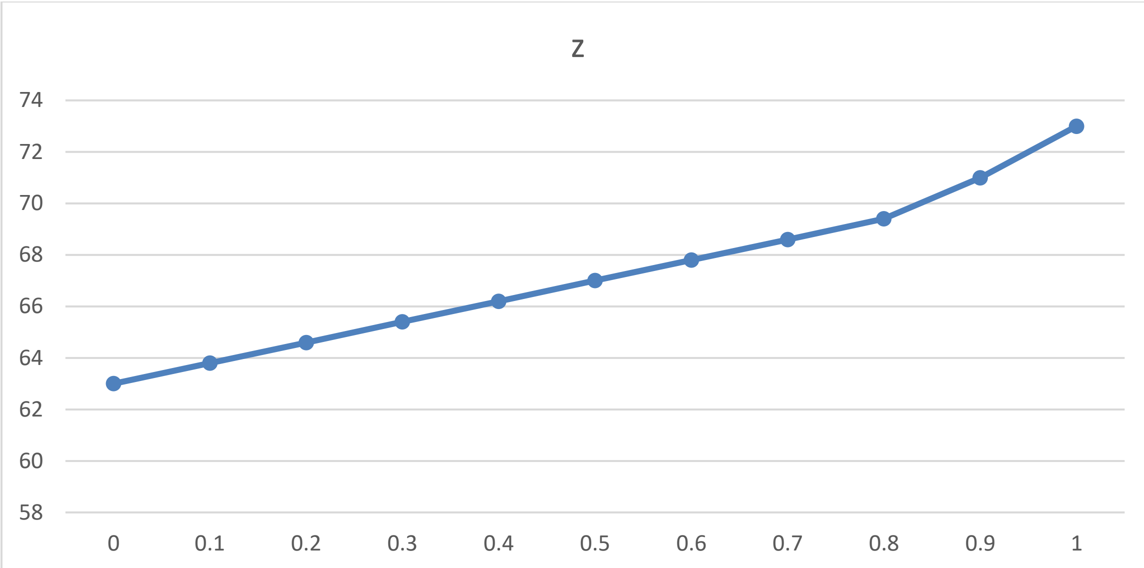

Figure 4.2 Objective function based on different parameters of α_i , while $\beta_j = 1$ and $\gamma_k = 1$

In the third evaluation, we change the demand parameters according to the constant considering the supply and vehicle parameters from zero to one to investigate the behavior of the objective function of the problem based on tolerance changes.

Table 4.4 Objective function values for various values of β_j , while $\alpha_i = 1$ and $\gamma_k = 1$

β_j	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Z	33.5	36.75	40	43.25	46.8	50.5	54.2	57.9	61.6	67	73

According to the values of the table above, we see that increasing supply and demand does not improve the objective function and increases its value. The change diagram of the objective function is as follows.

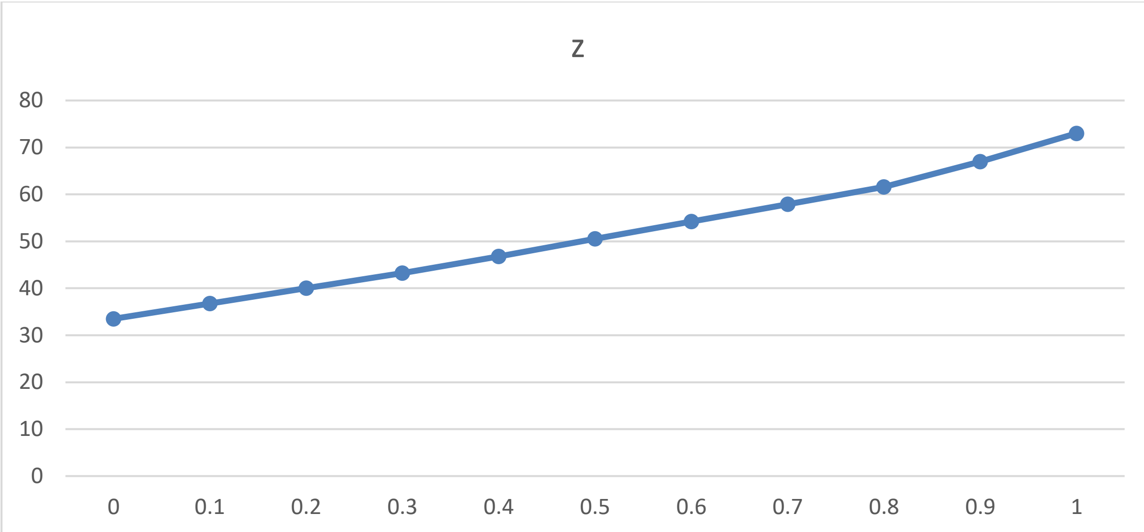

Figure 4.3 Objective function values for various values of β_j, while $\alpha_i = 1$ and $\gamma_k = 1$

In the fourth evaluation, we change the vehicle parameters according to the constant of supply and demand parameters from zero to one to check the behavior of the objective function of the MPSTP problem based on tolerance changes.

Table 4.5 Objective function values for the various values of γ_k, while $\alpha_i = 1$ and $\beta_j = 1$

γ_k	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Z	68	68.3	68.6	68.9	69.2	69.5	69.8	70.2	70.8	71.5	73

The change diagram of the objective function is as follows.

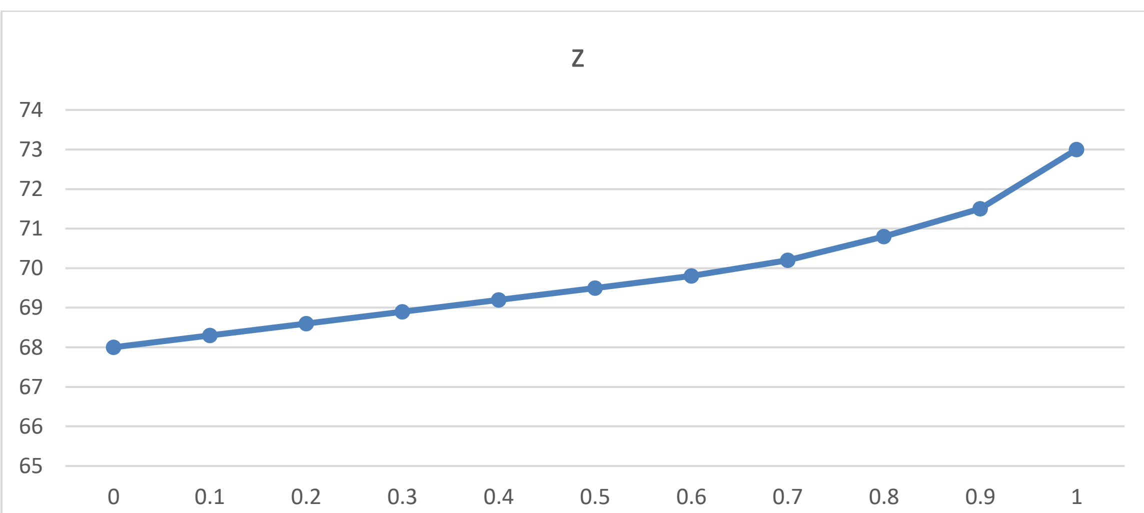

Figure 4.4 Objective function values for the various values of γ_k, while $\alpha_i = 1$ and $\beta_j = 1$

Numerical investigation shows that increasing for the values of the supply and demand is not effective for improving the Objective function. The value of the objective function is sensitive when supply and demand change, so we conduct a new study to diversify the value of the objective function, due to changes in resource and demand levels simultaneously.

Table 4.6 Objective function values for the various values of α_i and β_j while $\gamma_k = 1$.

α_i \ β_j	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	28.5	31.75	35	38.25	41.6	45	48.4	51.9	55.6	59.3	63
0.1	29	32.25	35.5	38.75	42.1	45.5	48.9	52.5	56.2	59.9	63.8
0.2	29.5	32.75	36	39.25	42.6	46	49.4	53.1	56.8	60.5	64.6
0.3	30.5	33.25	36.5	39.75	43.1	46.5	50	53.7	57.4	61.1	65.4
0.4	31	33.75	37	40.25	43.6	47	50.6	54.3	58	61.7	66.2
0.5	31.5	34.25	37.5	40.75	44.1	47.5	51.2	54.9	58.6	62.3	67
0.6	32	34.75	38	41.25	44.6	48.1	51.8	55.5	59.2	63	67.8
0.7	32.5	35.25	38.5	41.75	45.1	48.7	52.4	56.1	59.8	63.8	68.6
0.8	33	35.75	39	42.25	45.6	49.3	53	56.7	60.4	64.6	69.4
0.9	33.5	36.25	39.5	42.75	46.2	49.9	53.6	57.3	61	65.4	71
1	34	36.75	40	43.25	46.8	50.5	54.2	57.9	61.4	67	73

The above table shows that the simultaneous changes in the supply and demand values according to the constant of the vehicle and its effect on the behavior of the objective function. And also, its diagram is shown in the below.

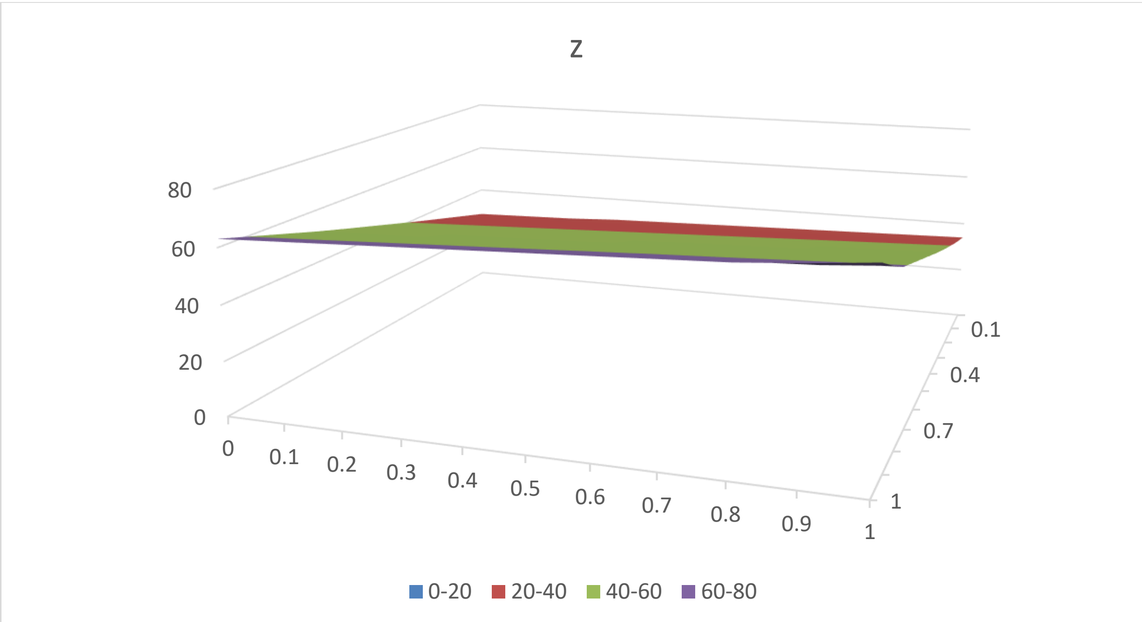

Figure 4.3 Objective function values for the various values of α_i and β_j while $\gamma_k = 1$.

Now we investigate the simultaneous changes in demand and vehicles according to the constant supply and its effect on the behavior of the objective function.

Table 4.7 Objective function values for the various values of β_j and γ_k while $\alpha_i = 1$.

γ_k / β_j	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	30.5	30.8	31.1	31.4	31.7	32	32.3	32.6	32.9	33.2	33.5
0.1	33.75	34.5	34.35	34.65	34.95	35.25	35.55	35.85	36.15	36.45	36.75
0.2	37	37.3	37.6	37.9	38.2	38.5	38.8	39.1	39.4	39.7	40
0.3	40.25	40.55	40.85	41.15	41.45	41.75	42.05	42.35	42.46	42.95	43025
0.4	43.6	43.9	44.2	44.5	44.8	45.1	45.4	45.7	46	46.3	46.8

0.5	*47*	*47.3*	*47.6*	*47.9*	*48.2*	*48.5*	*48.8*	*49.1*	*49.4*	*49.9*	*50.5*
0.6	*50.4*	*50.7*	*51*	*51.3*	*51.6*	*51.9*	*52.2*	*52.5*	*53*	*53.6*	*54.2*
0.7	*53.8*	*54.1*	*54.4*	*54.7*	*55*	*55.3*	*55.6*	*56.1*	*56.7*	*57.3*	*57.9*
0.8	*57.2*	*57.5*	*57.8*	*58.1*	*58.4*	*58.7*	*59.2*	*59.8*	*60.4*	*61*	*61.6*
0.9	*62.2*	*62.5*	*62.8*	*63.1*	*63.4*	*63.7*	*64.1*	*64.7*	*65.3*	*65.9*	*67*
1	*68*	*69.3*	*68.6*	*68.9*	*69.2*	*69.5*	*69.8*	*70.2*	*70.8*	*71.5*	*73*

The objective function changes chart is given in Figure 4.7.

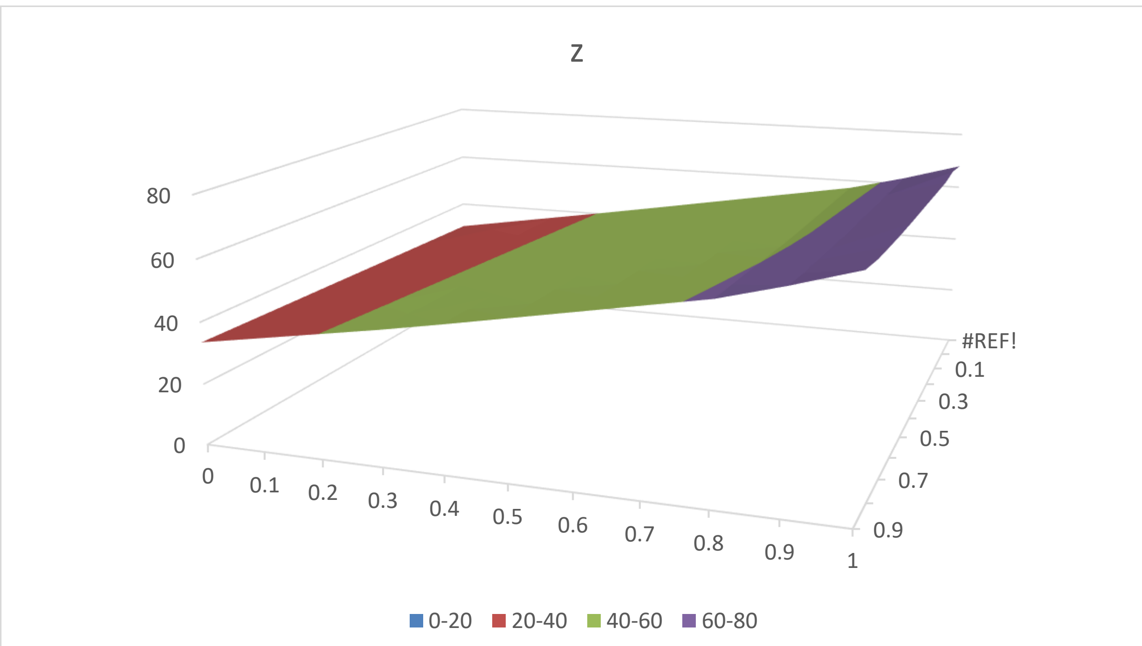

Figure 4.6 Objective function values for the various values of β_j and γ_k while $\alpha_i = 1$.

Table 4.8 shows the simultaneous changes in the supply and vehicle values according to the constant demand and its effect on the changes of the objective function.

Table 4.8 Objective function values for the various values of α_i and γ_k while $\beta_j = 1$.

α_i \ γ_k	0	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1*
0	58	58.3	58.6	58.9	59.4	60	60.6	61.2	61.8	62.4	63
0.1	58.6	58.9	59.2	59.5	60	60.6	61.2	61.8	62.4	63	63.8
0.2	59.2	59.5	59.8	60.1	60.6	61.2	61.8	62.4	63	63.6	64.6
0.3	59.8	60.1	60.4	60.7	61.2	61.8	62.4	63	63.6	64.2	65.4
0.4	60.4	60.7	61	61.3	61.8	62.4	63	63.6	64.2	64.8	66.2
0.5	61	61.3	61.6	61.9	62.4	63	63.6	64.2	64.8	65.5	67
0.6	61.6	61.9	62.2	62.5	63	63.6	64.2	64.8	65.4	66.3	67.8
0.7	62.2	62.5	62.8	63.1	63.6	64.2	64.8	65.4	66	67.1	68.6
0.8	62.8	63.1	63.4	63.7	64.2	64.8	65.4	66	66.6	67.9	69.4
0.9	65	65.3	65.6	65.9	66.2	66.6	67.2	67.8	68.4	69.5	71
1	68	68.3	68.6	68.9	69.2	69.5	69.8	70.2	70.8	71.5	73

The Objective function changes chart based on the above results is as follows.

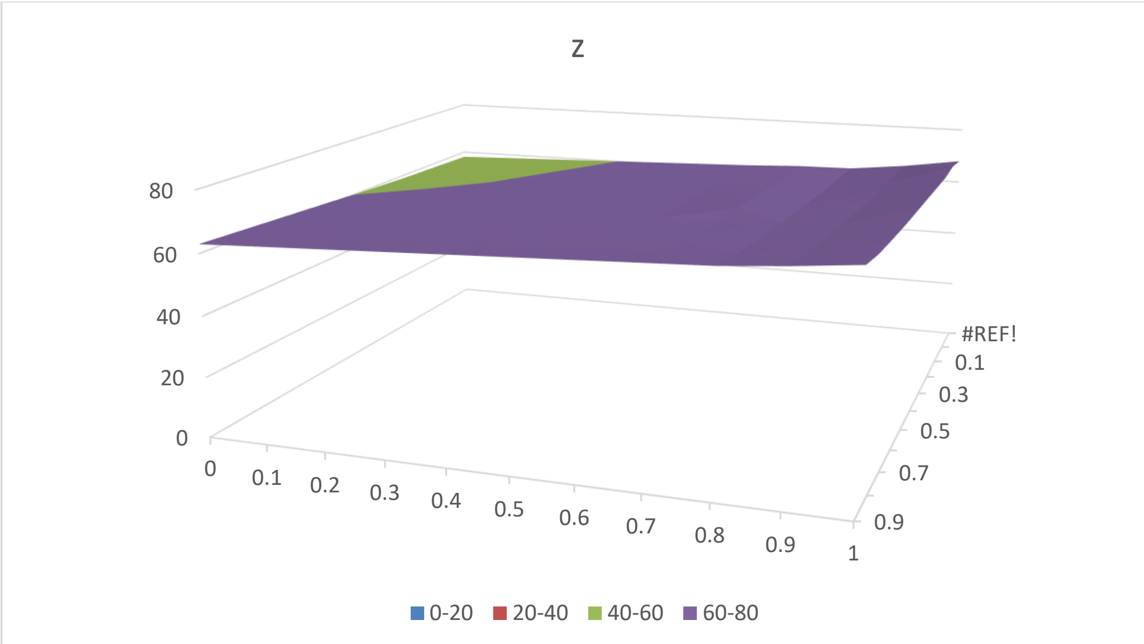

Figure 4.7 Objective function values for the various of α_i and γ_k while $\beta_j = 1$.

Currently, due to sensitivity analysis for the decision-maker, we want to improve the level of satisfaction levels by changing all parameters. According to the decision maker's opinion, we consider the levels of the following parameters and then solve the next problem to find the optimal solution with the best decision parameters.

$$\alpha_1^* = 0.5, \alpha_2^* = 0.5, \alpha_3^* = 0.5, \beta_1^* = 0.5, \beta_2^* = 0.5, \beta_3^* = 0.5, \gamma_1^* = 0.5, \gamma_2^* = 0.5, \gamma_3^* = 0.5 \text{ and } z^* = 45.75$$

$$
\begin{aligned}
&Max\, \alpha_0+\alpha_1+\alpha_2+\alpha_3+\beta_1+\beta_2+\beta_3+\gamma_1+\gamma_2+\gamma_3\\
&s.t.\\
&9x_{111}+8x_{121}+3.5x_{131}+12x_{112}+9x_{122}+7x_{123}+9x_{113}+7x_{123}+7x_{133}+5x_{211}+9x_{221}+6x_{231}+6x_{212}+\\
&11x_{222}+8x_{223}+5x_{213}+3x_{223}+6x_{233}+2x_{311}+2x_{321}+x_{331}+2x_{312}+7x_{322}+9x_{323}+x_{313}+7x_{323}+\\
&3x_{333}\le z^*+4(1-\alpha_0)\\
&x_{111}+x_{121}+x_{131}+x_{112}+x_{122}+x_{132}+x_{113}+x_{123}+x_{133}\le 8+3(1-\alpha_1)\\
&x_{211}+x_{221}+x_{231}+x_{212}+x_{222}+x_{232}+x_{213}+x_{223}+x_{233}\le 9+4(1-\alpha_2)\\
&x_{311}+x_{321}+x_{331}+x_{312}+x_{322}+x_{332}+x_{313}+x_{323}+x_{333}\le 5+2(1-\alpha_3)\\
&x_{111}+x_{211}+x_{311}+x_{121}+x_{221}+x_{321}+x_{131}+x_{231}+x_{331}\ge 7-3(1-\beta_1)\\
&x_{112}+x_{212}+x_{312}+x_{122}+x_{222}+x_{322}+x_{132}+x_{232}+x_{332}\ge 8-3(1-\beta_2)\\
&x_{113}+x_{213}+x_{313}+x_{123}+x_{223}+x_{323}+x_{133}+x_{233}+x_{333}\ge 6-2(1-\beta_3)\\
&x_{111}+x_{211}+x_{311}+x_{121}+x_{221}+x_{321}+x_{131}+x_{231}+x_{331}\le 10+4(1-\gamma_1)\\
&x_{112}+x_{212}+x_{312}+x_{122}+x_{222}+x_{322}+x_{132}+x_{232}+x_{332}\le 5+2(1-\gamma_2)\\
&x_{113}+x_{213}+x_{313}+x_{123}+x_{223}+x_{323}+x_{133}+x_{233}+x_{333}\le 6+2(1-\gamma_3)\\
&x_{ijk}\ge 0,\quad i=1,2,3,\quad j=1,2,3,\quad k=1,2,3,\quad \alpha_i^*\le\alpha_i\le 1,\quad \beta_j^*\le\beta_j\le 1,\quad \gamma_k^*\le\gamma_k\le 1
\end{aligned}
$$

The optimal value of the objective function is $z^{**}=47.5$ with the best decision parameters as follows:

$$\alpha_0=0.5625,\alpha_1=1,\alpha_2=1,\alpha_3=0.5,\beta_1=0.5,\beta_2=1,\beta_3=0.5,\gamma_1=1,\gamma_2=1,\gamma_3=1$$

In the example above, the better satisfaction is achieved and better use of the resources is done.

5. Conclusions and future work

In this study for adapting the real situations in the formulated problem as a correspondence mathematical model where the model involves the flexibility conditions, in particular, in the limited available resources values, a new fuzzy mathematical program suggested. The illustrative example explained how the decision makers may improve their satisfaction level by optimizing in using the associated resource values. Numerical investigation showed that each increasing for the values of the supply and demand is not effective for improving the objective function. The value of the objective function is sensitive when supply and demand change, so we conduct a new study to diversify the value of the objective function, due to changes in resource and demand levels simultaneously. In the continuing this study, we suggest that an interested reader extends this approach to the problems with multi commodity and multi-objective cases.

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