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integrated Slacks-based Measure of Efficiency and Super-efficiency improving in additive Data Envelopment Analysis

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Tone [29] proposed a method of super-efficiency slack-based measures (SBM) for ranking efficient decision-making units (DMUs), so that this model would rank efficient DMUs. The established model was able to measure radially. It calculates and measuring the efficiency of inefficient DMUs and the amount of super-efficiency of efficient DMUs. Du et al. [11] developed the Charens et al. [6] model in to the additive DEA model, as well as the additive super performance model. Turn et al. [32] used a linear SBM and S-SBM integrated model that had the properties of both models and reduced the time factor compared to previous models. In order to be able to calculate the amount of additive super efficiency; First we identify the efficient DMUs and then apply the additive super-efficiency model to the efficient DMUs. In this paper, the proposed model obtains the additive efficiency value of inefficient DMUs and the additive super efficiency value of efficient DMUs with less computation time. The amount of DMUs calculated from the integrated model in this article can be compared to the Guo et al. [15] article in comparison with the time table of the text of the article.

Keywords: *Additive super-efficiency, Data envelopment analysis (DEA), Efficiency, Slacks-based measure (SBM), Super-efficiency.*

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1. Introduction

Data Envelopment Analysis (DEA) measures the relative efficiency of heterogeneous decision-making units (DMU) with multiple inputs and outputs. The decision-making units that get a efficiency score of one are considered the DEA (with the best performance). DEA is a research method in operations and economics for efficiency evaluation and benchmark with multiple efficiency measures. As for the perspective of practical application for the first 20 years of DEA development, Emrouznejad and Yang [12] provide a comprehensive survey and analysis of the first 40 years of DEA related studies. The top five application fields include banking, healthcare, agriculture and farming, transportation, and education Liu et al. [20]. In

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addition, some novel DEA applications include the corporate management of securities Pourmahmoud, and Kaheh [23], the automotives Tan et al. [27], tourism in the Coral Triangle region Huang and Coelho [16], the thermal power generation Song et al. [26], etc. In this method, research was quickly developed and published among various disciplines of engineering sciences. For a survey of methodological development of the various models for measuring efficiency, readers can refer to Cook and seiford [10] and Emrouzenjad et. Al. [13]. First, the efficiency score was introduced radially. A radius efficiency measure with a value of one might be called weak efficiency. DMU weak efficiency with zero slacks variables is usually strong efficiency. Banker et al. [2] was evaluated on the input-oriented model to evaluate the efficiency of a DMU by solving a linear program with a separate variable with constant scale returns. It can be upgraded to multiple inputs and multiple outputs by developing returns to scale. Charnes et al. [4] and [5] developed additive measuer the efficient models of a DMU based on all simultaneous input and output slacks to reach the efficient boundary point made by all efficient DMUs. The additive models have the advantage that they can be estimated as a combination of all inefficient DMUs, which was not used in previous models. The additive models do not explicitly show the efficiency measure of the objective function. Tone [28] showed the additive models using the slacks-based measuer (SBM) in which slack variables excesses in inputs and short falls in output whit structure defined the efficiency score it in the objective function for DMU. The SBM model is a non-radial efficiency measure for evaluating DMUs with a efficiency score of one, is strong efficient. Anderson and Petersen [1] studied a radial super-efficient model in the additive of inefficient DMUs. Issues of super-efficiency models can be found in literatures (chen [7], chen and Liang [8], lovell and Rouse [21]). The specificity of this additive super-efficiency model in constant returns to scale (CRS) efficiency has infeasible been studied. Tone [29] examines the non-radial super efficiency model for evaluating efficient DMUs with input and output in the objective function. To obtain super-efficiency from DMUs, one must first identify the efficient DMUs and then apply the additive super-efficiency model to the efficient DMUs. Fang et al. [14] was a two-stage non-radial model that created the super-efficient amount of efficient DMUs and the efficient amount of inefficient DMUs. The model first solved super SBM and then applied SBM. The results of the stronger pareto efficient super SBM model and the results can be compared with the Tone [28] and Tone [29], Du et al. [11] developed the Tone [29] super-efficiency model. The additive non-radial super-function model is always feasible under variable returns to scale (VRS) compared to the DEA radial additive super-efficiency models. These detail discussions can be found in (cook et al. [9], seiford and Zhu [25]). So that the ranking of efficient DMUs can be concluded. However, the slacks-based in the objective function are different in all models. After calculating the efficiency of inefficient DMUs, the model determines the efficient DMUs by calculating the time and calculates the super-efficiency measure of efficient DMUs on a large scale. Therefore, Guo et al. [15] designed an integration (slacks-based) model that proposed a additive non-radial model capable of calculating the efficiency scores of inefficient DMUs and the super-efficiency scores of efficient DMUs in one step. Time can be saved in calculating the application of a single-stage model. The one-stage solution approach can save computational time for large-scale practical applications, for example, computing the SBM-based Malmquist productivity index used to evaluate the efficiency change over time Tone [29]. In addition, strongly efficient can be Concluded from the model. In the one-step model for calculating efficiency and super efficiency, all DMUs are used except the evaluated DMU. The model in this paper saves time on large data scales. Another advantage of the Guo et al. [15] model is that the direct objective function specifies the amount of efficiency and super-efficiency of the DMUs without prior calculation. Tran et al. [32] proposed and targeted a binary linear programming model that combines the SBM and super SBM models. Tran et al. [32] introduced binary integer variables that can be switched between SBM and Super SBM models. Combining SBM and super SBM models is essential in computing super efficient DMUs. See Torabi and Salahi [31] for practical examples. With the combined Tran et al. [32] model, she easily calculated the amount of super efficiency, such as the Andersen and Petersen [1] radial model. The combination model not only simplifies the implementation of the model, but also acts as a benchmark model for other methods with an efficiency index. When dealing with large volumes of data for processing, the combination model can simply examine the data. As a result, Pourmahmoud and Kaheh [23] Lee [19], Lee et al. [17] and Lee and Zhu [18] proposed a programming model that integrates SBM and Super-SBM with a new approach. If the value of the result obtained by the integrated model is less than one, the DMU is evaluated as inefficient, which results in the same combined model as the SBM model. If the efficiency score is greater than or equal to the integrated model, the DMU under evaluation is efficient. Under such a condition, our model is the same as the Super-SBM model. Computation time is important, so in calculating Super-efficiency, it is essential that the SBM and Super-SBM hybrid model be integrated. That calculations are easier and more efficient. We use a new method to compute the additive integrated model and show that our method is more efficient than any other paper. In the article Tran et al. [32] the results show that only two-thirds of the time was spent. This means that the integrated model is more efficient than previous models. In this method, the additive integrated model is reduced by changing the variable, first the variables of the model, then it is easier to work with the model. Calculations are done with less time and time is saved. These are the results that have been done on the new model. Section 2 briefly reviews the SBM and Super SBM models proposed by Tone [28] In Section 3, the additive super-efficiency model proposed by Du et al. [11] is reviewed. Section 4 presents our integrated additive super-efficiency model. Illustrative examples are demonstrated in Section 5. Finally, some remarks are made in Section 6.

2. slack-based measure efficienc or super-efficiency SBM and Super-SBM Models

We assume that the Tone [28] model is evaluated as follows for n $DMUs$ with inputs $X = \left(X_{ij}\right) \in R^{m\times n}$ and output $Y = \left(y_{ij}\right) \in R^{s\times n}$: Assuming $x_i^1 = x_{ik} - z_i^-$and $y_r^1 = y_{rk} + z_r^+$ the Tone [29] SBM model can be used to evaluate the DMU_k Wrote as follows:

$$
\begin{aligned}
& min\rho^1 = \frac{\frac{1}{m}\sum_{i=1}^{m}\frac{x_i^1}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s}\frac{y_r^1}{y_{rk}}} \\
& s.t.\ x_i^1 \geq \sum_{j=1}^{n}\lambda_j^1 x_{ij}, i = 1,\dots,m \\
& \sum_{j=1}^{n}\lambda_j^1 y_{rj} \geq y_r^1, r = 1,\dots,s \\
& \lambda_j^1 \geq 0, j = 1,..,n \\
& x_{ik} \geq x_i^1 \geq 0, i = 1,\dots,m \\
& y_r^1 \geq y_{rk}, r = 1,\dots,s.
\end{aligned}
\tag{1}
$$

Tone [29] also studied the following model. DMU_k was targeted to evaluate super-efficiency by SBM. By placing and assuming $x_i^2 = \overline{x}_{ik}$and $y_r^2 = \overline{y}_{rk}$, we get the following model:

$$
\begin{aligned}
& min\delta_k^2 = \frac{\frac{1}{m}\sum_{i=1}^{m}\frac{x_i^2}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s}\frac{y_r^2}{y_{rk}}} \\
& s.t. \sum_{j=1, j\neq k}^{n}\lambda_j x_{ij} \leq x_i^2, i = 1,\dots,m \\
& \sum_{j=1, j\neq k}^{n}\lambda_j y_{rj} \geq y_r^2, r = 1,\dots,s \\
& \lambda_j^2 \geq 0, j = 1,..,n, j \neq k \\
& x_{ik} \leq x_i^2, i = 1,\dots,m \\
& 0 \leq y_r^2 \leq y_{rk}, r = 1,\dots,s.
\end{aligned}
\tag{2}
$$

We know that the value of the objective function of model (2) is greater than or equal to one under optimal conditions. That is $\delta^{2*} \geq 1$. The amount of super -efficiency for DMU_k is one even if the DMU_k is inefficient. To determine if the $DMU_k$is efficient or inefficient, we use both (1) and (2) models together. We conclude from the above two models; If the DMU_k is efficient by model (1), now we use model (2) to calculate the super efficiency. Therefore, efficient DMU_k and inefficient DMU_kcan not be distinguished from the (2) model. Evaluate the amount of efficient and super-efficient for all DMU_k with model (1) And

(2); We usually use model (1) for all $DMUs$ and then use model (2) for efficient DMUs output from model (1) for the super-efficiency scores. Note that Fang et al. [14] changed the calculation command. That is, first the model (2) was applied to all DMUs. If the super-efficiency score is greater than one, then the DMU is efficient. All DMUs that give a super-efficiency value equal to 1 are single output, which may be efficient or inefficient. Now we use model (1) for all DMUs. If the efficiency score is less than one, the DMU_k is inefficient.

3. additive effiency and super-effiency

Du et al. [11] considered a additive DEA model for DMU_k based on Charans et al. [6] By developing this model of supere fficiency in DEA and based on the additive model and assuming $x_{ik}^1 = x_{ik} - s_{ik}^-$ and $y_{rk}^1 = y_{rk} + s_{rk}^+$, we apply the variable change and the following model, we obtain the additive SBM efficiency.

$$
\begin{aligned}
& max\acute{\alpha}_k^1 = \sum_{i=1}^{m} s_{ik}^- + \sum_{r=1}^{s} s_{rk}^+ \\
& s.t \sum_{j=1}^{n} x_{ij}\, \acute{\lambda}_j^1 = x_{ik}^1, i = 1,\dots,m \\
& \sum_{j=1}^{n} y_{rj}\, \acute{\lambda}_j^1 = y_{rk}^1, r = 1,\dots,s \\
& \acute{\lambda}_j^1 \geq 0, j = 1,\dots,n \\
& x_{ik} \geq x_{ik}^1 \geq 0, i = \dots,m \\
& y_{rk}^1 \geq y_r, r = 1,\dots,s.
\end{aligned}
\tag{3}
$$

In the model we have s_{ik}^- and s_{rk}^+ slacks inputs and outputs. The DMU_k efficient if and only if the slacks variables are zero. It can be easily shown that the DMU_k is efficient under model (3) if and only if the DMU is efficient under the Tone's slacks-based measure model (1) Suppose the DMU_k is efficient. The DMU_k additive super-efficiency results under Model (3) We can not simply improve model (3) by removing the DMU_k in the reference set. If we do this, it is possible that the infeasible for the model can be inferred. Du et al. [11] targeted the additive super-efficiency model. Suppose $x_{ik}^2 = x_{ik} + t_{ik}^-$ and $y_{rk}^2 = y_{rk} - t_{rk}^+$. is the placement values in the model of Du et al. [11] :

$$
min\acute{\alpha}_k^2 = \sum_{i=1}^{m} t_{ik}^- + \sum_{r=1}^{s} t_{rk}^+ \tag{4}
$$

$$s.t \sum_{j=1, j\neq k}^{n} x_{ij} \acute{\lambda}_j^2 \leq x_{ik}^2, i = 1, \dots, m$$
$$\sum_{j=1, j\neq k}^{n} y_{rj} \acute{\lambda}_j^2 \geq y_{rk}^2, r = 1, \dots, s$$
$$\acute{\lambda}_j^2 \geq 0. j = 1, \dots, n, j \neq k$$
$$x_{ik}^2 \geq x_{ik}, i = 1, \dots, m$$
$$y_{rk} \geq y_{rk}^2, r = 1, \dots, s.$$

Where t_{ik}^{+}, t_{rk}^{-} are slacks representing the input savings and output surplus from the frontier respectively. According to the definition of model (1) SBM super-efficiency, the objective function of the model and the constraints of model (3) for optimization were written as follows.

$$\sum_{j=1}^{n} x_{ij} \acute{\lambda}_j^1 = x_{ik}^1, i = 1, \dots, m$$

$$\sum_{j=1}^{n} y_{rj} \acute{\lambda}_j^1 = y_{rk}^1, r = 1, \dots, s$$

$$\rho^1 = \frac{\frac{1}{m}\sum_{i=1}^{m} \frac{x_i^1}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s} \frac{y_r^1}{y_{rk}}}$$

From the combination of the above two parts, the optimal values can be defined as follows:

$$\alpha_k^{*1} = \frac{\frac{1}{m}\sum_{i=1}^{m} \frac{x_{ik}^{1*}}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s} \frac{y_{rk}^{1*}}{y_{rk}}} \qquad (5)$$

where x_{ik}^{1*} and y_{rk}^{1*} are the optimal solutions of (3) Similarly, according to the definition of the (2) model, the SBM super-efficiency and the objective function and the constraints of the (4) model for optimization can be defined as follows:

$$\delta_k^2 = \frac{\frac{1}{m}\sum_{i=1}^{m}\frac{x_i^2}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s}\frac{y_r^2}{y_{rk}}}$$
$$\sum_{j=1, j\neq k}^{n} x_{ij}\,\acute{\lambda}_j^2 \leq x_{ik}^2, i = 1, \ldots, m$$
$$\sum_{j=1, j\neq k}^{n} y_{rj}\,\acute{\lambda}_j^2 \geq y_{rk}^2, r = 1, \ldots, s \tag{6}$$
$$\alpha_k^{*2} = \frac{\frac{1}{m}\sum_{i=1}^{m}\frac{x_{ik}^{2*}}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s}\frac{y_{rk}^{2*}}{y_{rk}}}$$

where x_{ik}^{2*} and y_{rk}^{2*} are the optimal solutions of (4). To evaluate theDMU_k under evaluation, we use two models of additive super-efficiency and efficiency score of (3) and (4) models simultaneously. First, we use the additive efficiency model for the DMU set, and we apply the resulting efficiency DMUs to the (4) additive super-efficiency model. we can also use a different objective function for model (3) and (4) so that the resulting model is unit invariant, for example, The additive efficiency model can be written as follows:

$$max\widehat{\beta_k}^1 = \frac{1}{m+s}\left(\sum_{i=1}^{m}\frac{s_{ik}^-}{x_{ik}} + \sum_{r=1}^{s}\frac{s_{rk}^+}{y_{rk}}\right)$$
$$s.t\sum_{j=1}^{n} x_{ij}\,\widehat{\lambda_j}^1 = \widehat{x_{ik}}^1, i = 1, \ldots, m$$
$$\sum_{j=1}^{n} y_{rj}\,\widehat{\lambda_j}^1 = \widehat{y_{rk}}^1, r = 1, \ldots, s \tag{7}$$
$$\widehat{\lambda_j^1} \geq 0, j = 1, \ldots, n$$
$$x_{ik} \geq \widehat{x_{ik}}^1 \geq 0, i = \ldots, m$$
$$\widehat{y_{rk}}^1 \geq y_r, r = 1, \ldots, s.$$

The collective performance super model can be written as follows:

$$
\begin{aligned}
& min\acute{\alpha}_k^2 = \frac{1}{m+s}\left(\sum_{i=1}^{m}\frac{t_{ik}^-}{x_{ik}} + \sum_{r=1}^{s}\frac{t_{rk}^+}{y_{rk}}\right) \\
& s.t \sum_{j=1, j\neq k}^{n} x_{ij}\,\acute{\lambda}_j^2 \le \widehat{x_{ik}}^2, i = 1,\ldots,m \\
& \sum_{j=1, j\neq k}^{n} y_{rj}\,\acute{\lambda}_j^2 \ge \widehat{y_{rk}}^2, r = 1,\ldots,s \\
& \acute{\lambda}_j^2 \ge 0. j = 1,\ldots,n, j \neq k \\
& \widehat{x_{ik}}^2 \ge x_{ik}, i = 1,\ldots,m \\
& y_{rk} \ge \widehat{y_{rk}}^2, r = 1,\ldots,s.
\end{aligned}
\tag{8}
$$

4. Integrated model

Guo et al. [15] proposed the integrated model, Model (3) and model (4) can be integrated into the following model:

Let $\bar{x}_{ik} = x_{ik} - s_{ik}^- + t_{ik}^-, i = 1,\ldots,m$ and $\bar{y_{rk}} = y_{rk} + s_{rk}^+ - t_{rk}^+, r = 1,\ldots s.$, we can combine the additive model and additve super-efficiencies (3) , (4) be rewritten as:

$$\max \bar{\alpha}_k = \left(\left(\sum_{i=1}^{m} s_{ik}^- + \sum_{r=1}^{s} s_{rk}^+ \right) - \epsilon \left(\sum_{i=1}^{m} t_{ik}^- + \sum_{r=1}^{s} t_{rk}^+ \right) \right)$$
$$s.t \sum_{j=1, j\neq k}^{n} x_{ij} \bar{\lambda}_j = \bar{x}_{ik}, i = 1, \ldots, m$$
$$\sum_{j=1, j\neq k}^{n} y_{rj} \bar{\lambda}_j = \bar{y}_{rk}, r = 1, \ldots, s$$
$$\bar{\lambda}_j \geq 0, j = 1, \ldots, n, j \neq k$$
$$if s_{ik}^- \geq 0, t_{ik}^- \geq 0, \{ \bar{x}_{ik} \leq x_{ik} + t_{ik}^-, \quad \bar{x}_{ik} + s_{ik}^- \geq x_{ik}, i = \ldots, m$$
$$if s_{rk}^+ \geq 0, t_{rk}^+ \geq 0, \{ \bar{y}_{rk} + t_{rk}^+ \geq y_{rk}, \quad \bar{y}_{rk} \leq y_{rk} + s_{rk}^+, r = 1, \ldots, s$$
$$if s_{ik}^- \leq 0, t_{ik}^- \leq 0, \{ \bar{x}_{ik} \geq x_{ik} + t_{ik}^-, \quad \bar{x}_{ik} + s_{ik}^- \leq x_{ik}, i = \ldots, m$$
$$if s_{rk}^+ \geq 0, t_{rk}^+ \leq 0, \{ \bar{y}_{rk} + t_{rk}^+ \leq y_{rk}, \quad \bar{y}_{rk} \geq y_{rk} + s_{rk}^+, r = 1, \ldots, s. \tag{9}$$

As noted by Fang et al. [14] the sequence of applying SBM and Super-SBM can be reversed. Hence we optimize the objections of the models (3) and (4) in an opposite order. We identify the super-efficiency slacks first and then the inefficiency slacks. That is, $(\sum_{i=1}^{m} t_{ik}^- + \sum_{r=1}^{s} t_{rk}^+)$ is first minimized and then $(\sum_{i=1}^{m} s_{ik}^- + \sum_{r=1}^{s} s_{rk}^+)$ is maximized:

$$\min \widehat{\bar{\alpha}_k} = \left(\left(\sum_{i=1}^{m} t_{ik}^- + \sum_{r=1}^{s} t_{rk}^+ \right) - \epsilon \left(\sum_{i=1}^{m} s_{ik}^- + \sum_{r=1}^{s} s_{rk}^+ \right) \right)$$
$$s.t \sum_{j=1, j\neq k}^{n} x_{ij} \bar{\lambda}_j = \bar{x}_{ik}, i = 1, \ldots, m$$
$$\sum_{j=1, j\neq k}^{n} y_{rj} \bar{\lambda}_j = \bar{y}_{rk}, r = 1, \ldots, s$$
$$\bar{\lambda}_j \geq 0, j = 1, \ldots, n, j \neq k$$
$$if s_{ik}^- \geq 0, t_{ik}^- \geq 0, \{ \bar{x}_{ik} \leq x_{ik} + t_{ik}^-, \quad \bar{x}_{ik} + s_{ik}^- \geq x_{ik}, i = \ldots, m$$
$$if s_{rk}^+ \geq 0, t_{rk}^+ \geq 0, \{ \bar{y}_{rk} + t_{rk}^+ \geq y_{rk}, \quad \bar{y}_{rk} \leq y_{rk} + s_{rk}^+, r = 1, \ldots, s$$
$$if s_{ik}^- \leq 0, t_{ik}^- \leq 0, \{ \bar{x}_{ik} \geq x_{ik} + t_{ik}^-, \quad \bar{x}_{ik} + s_{ik}^- \leq x_{ik}, i = \ldots, m$$
$$if s_{rk}^+ \geq 0, t_{rk}^+ \leq 0, \{ \bar{y}_{rk} + t_{rk}^+ \leq y_{rk}, \quad \bar{y}_{rk} \geq y_{rk} + s_{rk}^+, r = 1, \ldots, s. \tag{10}$$

If the slacks variables of the (10) model of super efficiency are taken as zero values, several modes can be considered. First, if the DMU_k evaluated on the boundary is formed by all $DMUs$ except DMU_k. In this case, the DMU_k is efficient and the efficiency score is equal to one. Other modes DMU_k within the boundary formed by all $DMUs$ except $DMUs$ which is also within the boundary formed by all. In this case, the DMU_k is inefficient. This is the case with the super-efficincy slacks by model (10) when $t_{ik}^{-*} = t_{rk}^{+*} = 0$. In either case, the inefficient slacks variables (s_{ik}^{-*}, s_{rk}^{+*}) can be calculated for the DMU_k efficiency score. The integrated model (10) can easily calculate efficient values as well as the amount of super-efficiency of efficient $DMUs$ when the slacks variables of the super-efficiency are zero by switching between models (3) and (4) for inefficient $DMUs$. Let ($\bar{x_{ik}}^*$,$\bar{y_{rk}}^*$,$\bar{\lambda_j}^*$,$\bar{s_{ik}}^*$,$\bar{s_{rk}}^*$,$\bar{t_{ik}}^*$,$\bar{t_{rk}}^*$) be the optimal solution of (9). If $\bar{\alpha}^*$<1, the projection of SBM is ($\bar{x_{ik}}^*$,$\bar{y_{rk}}^*$,$\bar{\lambda_j}^*$,$\bar{s_{ik}}^*$,$\bar{s_{rk}}^*$). Otherwise, the projection of Super-SBM is ($\bar{x_{ik}}^*$,$\bar{y_{rk}}^*$,$\bar{\lambda_j}^*$,$\bar{t_{ik}}^*$,$\bar{t_{rk}}^*$).

The production possibility set P is defined as $p = \{(x, y)|x \geq X\lambda, y \leq y\lambda, \lambda \geq 0\}$, where λ is a negative vector in R^n. Let p/DMU_K denote the production possibility set (X_0, Y_0) spanned by (X, Y) excluding (X_0, Y_0), i.e. $\overline{p}/(x_0, y_0) = \{(\overline{x}, \overline{y})|\overline{x} \geq \sum_{j=1,j\neq k}^{n} x_{ij}\, \lambda_j, i = 1,.., m, \overline{y} \leq \sum_{j=1,j\neq k}^{n} y_{rj}\, \lambda_j, 1,\dots m, \lambda_j, j = 1,\dots n, \lambda \geq 0\}$. Further, we define a subset $\overline{p}/(x_0, y_0)$ of $p/(x_0, y_0)$ as $\overline{p}/(x_0, y_0) = p/(x_0, y_0) \cap \{\overline{x} > x_0 and \overline{y} < y_0\}$.

Theorem 4.1. If DMU_k is inefficient in P, then $\acute{\alpha}_k^{1^*} = \widehat{\bar{\alpha}_k}^* < 1$. If DMU_k is efficient in P, then $\acute{\alpha}_k^{2^*} = \widehat{\bar{\alpha}_k}^* > 1$.

Proof. If DMU_k is inefficient in P, we have $\acute{\alpha}_k^1 = \sum_{i=1}^{m} s_{ik}^- + \sum_{r=1}^{s} s_{rk}^+ < 1$ and $\acute{\alpha}_k^2 = \sum_{i=1}^{m} t_{ik}^- + \sum_{r=1}^{s} t_{rk}^+ = 1$. Therefore, we have $\alpha_k^{1^*} = \acute{\alpha_k^{1^*}} + \acute{\alpha_k^{2^*}} - 1 = \widehat{\bar{\alpha}_k}^* < 1$. If DMU_k is efficient in P, we have $\acute{\alpha}_k^1 = \sum_{i=1}^{m} s_{ik}^- + \sum_{r=1}^{s} s_{rk}^+ = 1$ and $\acute{\alpha}_k^2 = \sum_{i=1}^{m} t_{ik}^- + \sum_{r=1}^{s} t_{rk}^+ > 1$ Therefore, we have $\acute{\alpha}_k^{2^*} = \acute{\alpha_k^{1^*}} + \acute{\alpha_k^{2^*}} - 1 = \widehat{\bar{\alpha}_k}^* > 1$.

Theorem 4.2. Let $s_{ik}^{-*\prime}$ and $s_{rk}^{+*\prime}$ be the optimal solutions of the model (3). If DMU_k is inefficient, then $s_{ik}^{-*} = s_{rk}^{-*\prime}$ and $s_{ik}^{+*} = s_{rk}^{+*\prime}$.

Proof. If DMU_k is inefficient, DMU_k lies in $(\sum_{j=1,j\neq k}^{n} x_{ij}\, \overline{\lambda_j} = \overline{x_{ik}}, \sum_{j=1,j\neq k}^{n} y_{rj}\, \lambda_j = \overline{y_{rk}})$. Therefore, $t_{ik}^{-*} = t_{rk}^{+*} = 0$. Model (10) can be further reduced in to the following:

$$min\widehat{\bar{\alpha}_k} = -\epsilon\left(\sum_{i=1}^{m} s_{ik}^- + \sum_{r=1}^{s} s_{rk}^+\right)$$

$$s.t \sum_{j=1,j\neq k}^{n} x_{ij}\, \bar{\lambda}_j = \bar{x}_{ik}, i = 1,\dots, m$$

$$\sum_{j=1,j\neq k}^{n} y_{rj}\, \lambda_j = \bar{y}_{rk}, r = 1,\dots, s$$

$$\bar{\lambda}_j \geq 0, j = 1,\dots, n, j \neq k$$

$$s_{ik}^- \geq 0, i =\dots, m$$

$$s_{rk}^{+} \geq 0, r = 1, \dots, s.$$

which is equivalent to model (3). Hences $s_{ik}^{-*} = s_{rk}^{-*\prime}ands_{ik}^{+*} = s_{rk}^{+*\prime}$.

For an efficient $DMUk$, model (10) identifies the same superefficiency slacks that model (4) does, which is stated in the following Theorem 3.

Theorem 4.3. Let $t_{ik}^{-*\prime}$and $t_{rk}^{+*\prime}$ be the optimal solutions of the model (4) If $DMUk$ is efficient, $t_{ik}^{-*} = t_{ik}^{-*\prime}$ and $t_{rk}^{+*} = t_{rk}^{+*\prime}$.

Proof. If $DMUk$ is efficient, let $t_{ik}^{-} = t_{ik}^{-*\prime}$ and $t_{rk}^{+} = t_{rk}^{+*\prime}$. It would be also feasible for model (10) If $t_{ik}^{-} = t_{ik}^{-\,*\prime}$ and $t_{rk}^{+} = t_{rk}^{+*\prime}$ are not optimal, they can be further improved. Assume the optimal solutions are $t_{ik}^{\prime -*}$ and $t_{rk}^{\prime +*}$, where $t_{rk}^{\prime +*} \leq t_{rk}^{+*\prime}$ and $t_{ik}^{\prime -*} \leq t_{ik}^{-*\prime}$. The solutions $t_{ik}^{\prime -*}$and $t_{rk}^{\prime +*}$will be also feasible for model (4) which contradicts that $t_{ik}^{-\,*\prime}$ and$t_{rk}^{+*\prime}$ are the optimal solutions for (4).

We can assume the efficiency value for inefficient DMU_k results in model (5) to ensure the ranking of model (10) and also the amount of super efficiency in model DMU_k results in model (7) Model (10) can be defined as follows:

$$\widehat{\delta_k^*} = \begin{cases} \dfrac{\frac{1}{m}\sum_{i=1}^{m} \frac{x_{ik}^{1*}}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s} \frac{y_{rk}^{1*}}{y_{rk}}} & if \left(\sum_{i=1}^{m} t_{ik}^{-} + \sum_{r=1}^{s} t_{rk}^{+}\right) = 0 \\ \dfrac{\frac{1}{m}\sum_{i=1}^{m} \frac{x_{ik}^{2*}}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s} \frac{y_{rk}^{2*}}{y_{rk}}} & otherwise \end{cases} \quad (11)$$

Theorem 4.4. If DMU_k is inefficient, then $\widehat{\delta_k^*} = \alpha_k^{*1}$.

Proof. Let $s_{ik}^{-*}, t_{ik}^{-*}, s_{rk}^{+*}$ and t_{rk}^{+*} be the optimal solutions of the model (10) Let $s_{ik}^{-*\prime}$ and $s_{rk}^{+*\prime}$ denote the optimal solutions of the model (3.1). If DMU_k is inefficient,$t_{ik}^{-} = t_{rk}^{+} = 0$ and $s_{ik}^{-*} = s_{ik}^{-*\prime} and s_{rk}^{+*\prime} = s_{rk}^{+*\prime}$.hence $\widehat{\delta_k^*} = \alpha_k^{*1}$.

Theorem 4.5. If DMU_k is efficient, then $\widehat{\delta_k^*} = \alpha_k^{*2}$.

Proof. Let $s_{ik}^{-*}, t_{ik}^{-*}, s_{rk}^{+*}$ and t_{rk}^{+*} be the optimal solutions of the model (10) Let $t_{rk}^{+*\prime}$ and $t_{ik}^{-*\prime}$ denote the optimal solutions of the model (4). Following Theorem 2, If DMU_k is efficient,$t_{ik}^{-*} = t_{ik}^{-*\prime} and t_{rk}^{+*} = t_{rk}^{+*\prime}$.

$if(\sum_{i=1}^{m} t_{ik}^{-*} + \sum_{r=1}^{s} t_{rk}^{+*}) \gneqq 0$, $\widehat{\delta_k^*} = \frac{\frac{1}{m}\sum_{i=1}^{m}\frac{x_{ik}^{1*}}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s}\frac{y_{rk}^{1*}}{y_{rk}}} = \alpha_k^{*1}$. If $(\sum_{i=1}^{m} t_{ik}^{-} + \sum_{r=1}^{s} t_{rk}^{+}) = 0$, $s_{ik}^{-*} = s_{rk}^{+*} = 0$. we have $\alpha_k^{*2} = \frac{\frac{1}{m}\sum_{i=1}^{m}\frac{x_{ik}^{2*}}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s}\frac{y_{rk}^{2*}}{y_{rk}}} = 1 = \frac{\frac{1}{m}\sum_{i=1}^{m}\frac{x_{ik}^{1*}}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s}\frac{y_{rk}^{1*}}{y_{rk}}} = \widehat{\delta_k^*}$.

5. numerical exampel or Illustation

A In the section, we investigate the computational efficiency of measuring the efficiency scores of DMUs by our one-stage model. We evaluate the performance of the proposed model on several data sets in the literature and a case study. The obtained results are compared with those from other models, such as Tone [28] and Tone [29] , Guo et al. [15] , Lee [19] . numerical examples will be examined to demonstrate our contributions. Namely, our method identifies the strongly efficient projection and obtains the efficiency scores and super-efficiency scores in a single step. Consider the simple data set in Table 10 and Table 12 shows the results of the additive super-efficiency model proposed by Du et al. [11] . First, model (3) is applied. DMU A and DMU D are identified as the efficient DMUs. Second, the super-efficiency model is applied to DMU A and DMU D. The super-efficiencies of DMU A and DMU D Du et al. [11] both 2. The projection of DMU D is (1, 1.5, 0.5). Table 13 presents the results of the integrated model. As shown in Table 13, for in efficient DMUs, model (10) identifies the same efficiency scores that model (3) does, and for efficient DMUs, model (10) identifies the same super-efficiency scores that model (4) does. In other words, the ranking of those inefficient DMUs obtained by model (3) is the same as the proposed model (10), and the ranking of those efficient DMUs obtained by model (4) is the same as our model (10). However, the projection of DMU D identified by model (10) is strongly efficient and different from the projection identified by model (4) as shown in Table 12.

Two examples in Tone [29] are used for demonstration. The data sets are shown in Tables 4 and Table 7 and Table 1 respectively. Table 5 presents the results of Table 4 yielded by the additive model proposed by Du et al. [11] and Table 6 presents the results yielded by our approach. It demonstrates that our approach yields the same results as model (5) and model (4) except for the projections. Four DMUs (DMU A, B, F and G) are inefficient and three DMUs (DMU C, D and E) are efficient. The efficiency scores yielded by our approach are presented in the eighth column of Table 6. From Tables 5 and 6, it can be seen that the integrated model (10) provides the same efficiency scores as those obtained by model (5) when the DMUs are inefficient. Tables 5 and 6 also indicate that the integrated model (10) yields the same super-efficiency scores as those obtained by model (4) when the DMUs are efficient. From Tables 5 and 6, we find that the projection of the DMU E identified by model (4) is weakly efficient while the projection of the DMU E identified by model (10) is strongly efficient. The DMUs in Table 7 are all efficient. From Tables 8 and 9, it can be seen that the super-efficiency scores yielded by the integrated model (10) are the same as those obtained by model (4). Note that projections identified by model (10) are the same as the projections identified by model (4). Model (10) assumes constant returns to scale (CRS). To demonstrate the effects of the integrated model under variable returns to scale (VRS), we provide another data set in Table 10.

Table 1. A data set of 5 DMUs (2 inputs, 2 outputs) in Tone [28].

DMU	x_1	x_2	y_1	y_2
A	4	3	2	3
B	6	3	2	3
C	8	1	6	2
D	8	1	6	1
E	2	4	1	4

Table 2. Results of the SBM model and the SupSBM model for the dataset of Table 1.

	model (SBM)					model (SUPSBM)					Tran et al. [32]
DMU	s_{1k}^{-*}	s_{2k}^{-*}	s_{1k}^{+*}	s_{2k}^{+*}	ρ^{1*}	$\overline{x_1}^*$	$\overline{x_2}^*$	$\overline{y_1}^*$	$\overline{y_2}^*$	δ_k^{2*}	Efficiency
A	0	0.303	0.6061	0	0.798	4	3	2	3	1	1
B	0	0.4091	1.455	0	0.5682	6	3	2	3	1	1
C	0	0	0	0	1	10.67	1.333	8	1.333	1.333	1.333
D	0	0	0	0.6667	0.6667	8	1	6	1	1	1
E	0	0	0	0	1	2.909	5.818	1.455	2.182	1.455	1.455

Table 3. The results of the integrated model for Table 1:

DMU	model integrated (10)									projection			
	s_{1k}^{-*}	s_{2k}^{-*}	s_{1k}^{+*}	s_{2k}^{+*}	t_{1k}^{-*}	t_{2k}^{-*}	t_{1k}^{+*}	t_{2k}^{+*}	$\widehat{\delta_k^*}$	x_1	x_2	y_1	y_2
A	0	0.33	0.57	1	0	0	0.5	0	0.79	4	2.77	2.25	4
B	0	0	1.48	0.5	0	0	0.5	0	0.56	6	3	3.98	3.50
C	0	0	0	0	0	0	0	1	1.33	8	1	1	1
D	0	0.45	0	0	0	0	0	1	0.66	8	0.55	6	0
E	0	0	1	0	0	0	0	1	1.45	2	4	0	3

Table 4. A dataset of 7 DMUs (2 inputs, 1 output) in Tone [29] .

DMU	x_1	x_2	y_1
A	4	3	1
B	7	3	1
C	8	1	1
D	4	2	1
E	2	4	1
F	10	1	1
G	12	1	1

Table 5. The results of the additive super-efficiency model for Table 4.

	model (additive efficiency)				model (additive super-efficiency)				projection		
DMU	s_{1k}^{-*}	s_{2k}^{-*}	s_{1k}^{+*}	α_k^{*1}	t_{1k}^{-*}	t_{2k}^{-*}	t_{1k}^{+*}	α_k^{*2}	x_1	x_2	y_1
A	1	0	0	0.875	4	3	1	1	3	3	1
B	4	0	0	0.712	7	3	1	1	3	3	1
C	0	0	0	1	10	1	1	1.125	8	1	0.875

	model (additive efficiency)				model (additive super-efficiency)				projection		
D	0	0	0	1	6	2	1	1.25	4	2	0.8
E	0	0	0	1	4	4	1	1.50	2	1.5	0.5
F	2	0	0	0.9	10	1	1	1	8	1	1
G	4	0	0	0.833	12	1	1	1	8	1	1

Table 6. The results of the additive super-efficiency model for Table 4.

DMU	integrated model (10)							projection		
	s_{1k}^{-*}	s_{2k}^{-*}	s_{1k}^{+*}	t_{1k}^{-*}	t_{2k}^{-*}	t_{1k}^{+*}	$\widehat{\delta_k^*}$	$\bar{x}_1$	$\bar{x}_2$	$\bar{y}_1$
A	1	0	0	0	0	0	0.875	2	3	1
B	4	0	0	0	0	0	0.712	2	3	1
C	0	0	0	0.125	0	0	1.14	8.125	1.125	0.87
D	0	0	0	0.2	0	0	1.25	4.2	2.2	0.8
E	0	0	0	0.5	0	0	2	2.5	2	0.5
F	2	0	0	0	0	0	0.9	6	1	1
G	4	0	0	0	0	0	0.83	4	1	1

Table 7. A dataset of 6 DMUs (4 inputs, 2 outputs) in Tone [29] .

DMU	x_1	x_2	x_3	x_4	y_1	y_2
D_1	80	600	54	8	90	5
D_2	65	200	97	1	58	1
D_3	83	400	72	4	60	7
D_4	40	1000	75	7	80	10
D_5	52	600	20	3	72	8
D_6	94	700	36	5	96	6

Table 8. Results of the SupSBM model for the dataset of Table 7.

	model (additive super-efficiency)							projection					
DMU	t_{1k}^{-*}	t_{2k}^{-*}	t_{3k}^{-*}	t_{4k}^{-*}	t_{1k}^{+*}	t_{2k}^{+*}	α_k^{*2}	x_1	x_2	x_3	x_4	y_1	y_2
D_1	0.00	0.00	0.00	0.00	2.47	0.00	1.01	80.00	600.00	54.00	3.56	87.52	6.20
D_2	0.00	0.00	0.00	1.00	28.00	0.00	1.64	41.50	200.00	36.00	2.00	30.00	3.50
D_3	0.00	0.00	0.00	0.00	0.00	2.25	1.19	51.49	400.00	45.21	2.00	60.00	4.74
D_4	17.77	0.00	0.00	0.00	0.00	1.11	1.17	57.77	666.66	22.22	3.33	80.00	8.88
D_5	0.00	0.00	0.00	0.00	18.83	4.67	1.73	52.00	388.23	20.00	2.77	53.16	3.33
D_6	0.00	0.00	0.00	0.36	5.29	0.00	1.04	104.45	779.41	48.70	6.95	109.76	6.62

Table 9. The results of the integrated model for Table 7.

	integrated model												
DMU	s^{-*}_{1k}	s^{-*}_{2k}	s^{-*}_{3k}	s^{-*}_{4k}	s^{+*}_{1k}	s^{+*}_{2k}	t^{-*}_{1k}	t^{-*}_{2k}	t^{-*}_{3k}	t^{-*}_{4k}	t^{+*}_{1k}	t^{+*}_{2k}	$\widehat{\delta^*_k}$
D_1	0.00	0.00	0.00	4.43	0.00	1.20	0.00	0.00	0.00	0.00	2.47	0.00	1.01
D_2	23.51	0.00	61.00	0.00	0.00	2.50	0.00	0.00	0.00	1.00	28.00	0.00	1.64
D_3	31.51	0.00	026.78	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.25	1.19
D_4	0.00	333.33	52.77	3.66	0.00	0.00	17.77	0.00	0.00	0.00	0.00	1.11	1.17
D_5	0.00	211.76	0.00	0.22	0.00	0.00	0.00	0.00	0.00	0.00	18.83	4.67	1.73
D_6	22.902	0.00	0.00	0.00	0.00	2.21	0.00	0.00	0.00	0.36	5.29	0.00	1.04

The continuation of Table (9) is given below.

	Projection					
DMU	x_1	x_2	x_3	x_4	y_1	y_2
D_1	80.00	600.00	54.00	3.56	85.05	7.40
D_2	12.49	126.49	12.49	3.00	2.00	5.00
D_3	19.88	368.49	13.70	0.00	60.00	2.49
D_4	17.77	684.43	39.99	1.33	80.00	7.77
D_5	52.00	388.23	20.00	2.77	34.33	1.50
D_6	81.55	756.50	45.80	10.01	106.68	8.83

Table 10. A simple data set.

DMU	x_1	x_2	y_1
A	2	1	1
B	2	3	1
C	3	4	1
D	1	2	1

Table 11. Results of the SBM model and the SupSBM model for the dataset of Table 10.

DMU			integrated model by Guo et al. [15]				integrated model by Lee [19]				
	SBM	SupSBM		projection			projection				
	ρ^1	δ^2_k	Efficiency	x_1	x_2	y_1	x_1	x_2	y_1		efficiency
A	1	1.5	2	0.5	1	0.5	0	0.303	0.6061		1.5
B	0.583	1	0.58	1	2	1	0	0.4091	1.455		0.58
C	0.416	1	0.54	1	2	1	0	0	0		0.41

DMU			integrated model by Guo et al. [15]				integrated model by Lee [19]				
D	1	1.5	2	1	1.5	0.5	0	0	0		1.5

Table 12. The results of the additive super-efficiency model for Table 10 .

DMU	model (additive efficiency)				model (additive super-efficiency)				projection		
	s_{1k}^{-*}	s_{2k}^{-*}	s_{1k}^{+*}	α_k^{*1}	t_{1k}^{-*}	t_{2k}^{-*}	t_{1k}^{+*}	α_k^{*2}	x_1	x_2	y_1
A	0	0	0	1	0	0	0.5	2	0.5	1	0.5
B	1	1	0	0.583	0	0	0	1	1	2	1
C	2	2	0	0.416	0	0	0	1	1	2	1
D	0	0	0	1	0	0	0.5	2	1	1.5	0.5

Table 13. The results of the additive super-efficiency model for Table 10.

	model (10)							projection		
DMU	s_{1k}^{-*}	s_{2k}^{-*}	s_{1k}^{+*}	t_{1k}^{-*}	t_{2k}^{-*}	t_{1k}^{+*}	$\widehat{\delta_k^*}$	x_1	x_2	y_1
A	0	0	0	0	0	0.00	2	0.50	1.00	0.50
B	1	1	0	0	0	0.50	0.50	0.00	1.00	0.50
C	2	2	0	0	0	0.50	0.50	0.00	0.00	0.50
D	0	0	0	0	0	0.00	2	1.00	1.50	0.50

6. concluding remarks

The traditional solution approaches in DEA require identification of the efficient DMUs before applying the super-efficiency DEA models for the DMUs to achieve their super-efficiency scores, and vice versa. Therefore, the approaches entail a relatively high computational cost to obtain the scores of all DMUs, especially in large scale practical applications. Guo et al. [15] proposed the one-stage solution approach in which two efficiency and supper-efficiency measure models are integrated into a single model. This paper extends the work of Du et al. [11] and develops an integrated model based on the additive DEA. Our new model differs from the additive super-efficiency model proposed by Du et al. [11] in two aspects. However, this is an integrated additive (slacks-based) DEA model that requires a post-computation process to obtain the efficiency scores of DMUs. Our new model differs from the additive super-efficiency model proposed by Du et al. [11] in two aspects. The first is that our model calculates the super-efficiencies in one stage instead of two stages. The second is that the projections identified by our model are strongly efficient. In addition to the formal proofs of the related theorems, we also provide numerical examples to demonstrate that our integrated model gives the same super-efficiency scores in one stage instead of two stages, which are required by the method proposed by Du et al. [11]. Our objective function can directly obtain the efficiency and super-efficiency scores of DMUs without the post-computation process. A case study, along with several examples in the literature, are constructed to evaluate the proposed model. Note that the discussion in this paper is based on the constant returns to scale assumption. The integrated model can be extended to the variable returns to scale assumption and similar results can be obtained. The experimental results demonstrate the accuracy and the computation effectiveness of our model as compared with other models. The method proposed by Du et al. [11] involves three steps. First, we apply the additive model to all DMUs. Then we single out the efficient DMUs, and finally apply the super-efficiency model to the efficient DMUs. This is time-consuming. It will be cumbersome and unmanageable in large scale applications involving huge volume of data. In the case study, our focus is exclusively on firms' financial functioning. However, we can include in puts/outputs relevant to environmental and social aspects (e.g., CO2 emission, waste management, etc.) for a more realistic application. In addition, since using uniform weights for inputs and outputs may be unrealis tic, we should engage with stakeholders (e.g., city council) to obtain the appropriate weights by multi-criteria decision analysis. We may also enrich the methodology to represent firm's responses to policy measures. Our method overcomes the problem of switching between different models and provides an efficient approach toward the problem of evaluating efficiency and super-efficiency scores when the problem size is large.

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