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A Vehicle Location-Routing Model for Waste Management Problem under Fuzzy Flexible Conditions

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One of the most important and widely used problems in the logistics part of any supply chain is the Location-Routing Problem (LRP) of vehicles. The purpose is to select distribution centers to supply goods for customers and create suitable travel routes for vehicles to serve customers. Studies conducted in the field of supply chain logistics systems have shown that if vehicle travel routing is neglected when locating supply centers, the costs of the logistics system may increase dramatically. Therefore, in the LRP problem, the location of supply centers and the routing of vehicles are considered simultaneously. In this paper, we will present a multi-objective model for vehicle location-routing problems with a flexible fuzzy approach. Its' goals are to make strategic decisions to deploy candidate supply centers at the beginning of the planning horizon, as well as form the vehicle travel at the tactical level to serve the customers in short-term periods of time. Therefore, in order to adapt the mathematical model to the real conditions, the constraints related to the capacity of the vehicles have been considered in a flexible fuzzy state, and also the problem has been modeled in a multi-period state along with the presence of the distance limit and the accessibility factor for each vehicle. The evaluation criterion is to minimize costs related to the establishment of candidate supply centers, the fixed cost of using vehicles and transportation costs, as well as maximizing customer satisfaction by reducing shortage costs and reducing harmful environmental effects. To solve the model, it is first converted into a single-objective model using the weight method and then solved using the proposed algorithm. Finally, using a numerical example in the field of waste management, the effectiveness of the proposed solution method is shown. It should be mentioned that the model was solved using GAMS software and the results are shown.

Keywords*: Supply Chain; Location-routing problem; Fuzzy flexible programming; Multi-objective modeling; Waste management.*

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1. Introduction

Competition between companies and factories for the supply of goods and services is one of the undeniable facts, especially in the last three decades. Today, companies seek to create integration and coordination between their production activities from raw material procurement to delivery of goods to the final consumer. Supply chain management is an integrative approach to planning and controlling materials and information. Supply chain refers to activities related to raw material suppliers, product manufacturers and consumers, and its components include suppliers, manufacturers, wholesalers, distributors and customers (consumers). On the other hand, activities related to distribution warehouses and means of transportation are related to the field of logistics of each supply chain. One of the most important and widely used issues in the logistics sector is

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the selection of distribution centers for the supply of goods to customers, as well as the creation of suitable routes for vehicles to serve customers. The location-routing problem (LRP) with the aim of locating facilities and routing vehicles is one of the most widely used problems in the logistics sector, which is related to three levels of the supply chain, namely wholesalers, distributors and customers. Since the costs related to the selection of appropriate supply centers and their establishment are more significant than the costs related to vehicle travel routing, for this reason, the use of the location- vehicle routing, Is taken into consideration. Some researchers were against the idea of considering these two issues simultaneously and in the form of a model. From their point of view, the location of facilities is a strategic decision, which means that related decisions are made for a long time, while the routing of vehicle travel is a tactical issue. In fact, it is a decision that can be taken alternately in different periods of time. Despite this, the simultaneous examination of these two issues in a long-term time horizon can significantly reduce costs and improve productivity.

In general, the vehicle Location-Routing Problem (LRP) has the following conditions: There are certain candidate places for establishing supply centers (depots). A number of customers are located in certain places and are ready to receive services from supply centers. There is a fleet of vehicles that serve the customers and start their journey from the supply centers and return to the same supply centers after serving the customers. According to the capacity of each depot, several vehicles can be considered for it, and each vehicle is only allowed to make one trip and can only be assigned to one depot, and the higher the capacity of the vehicle, the more customers it can serve. provide services Establishing candidate supply centers in designated locations and forming vehicle trips to serve customers creates costs for the transportation system, which include: the fixed cost of establishing supply centers (depots), the fixed cost of using vehicles, and transportation costs. and quote And the main goal of LRP problems is to minimize these costs.

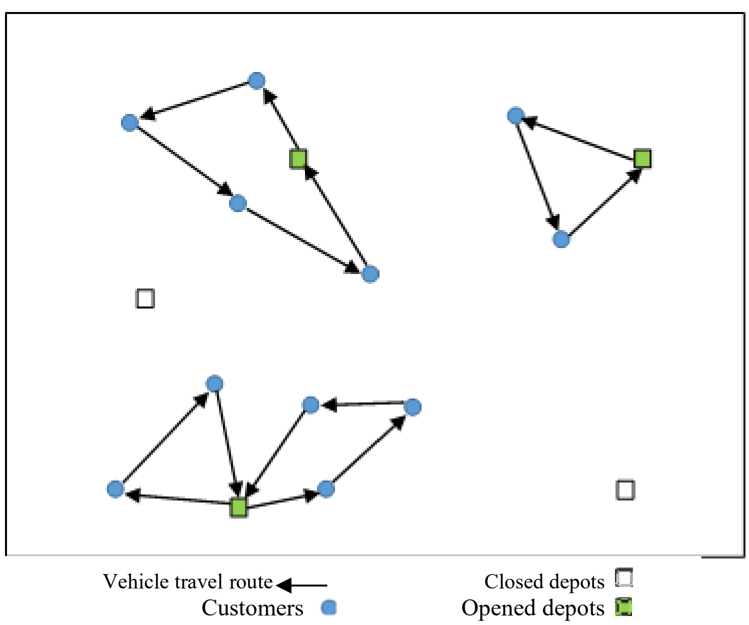

Figure 1. General form of LRP problem

Figure 1 shows a general view of the vehicle location-routing problem, where there are five candidate locations for deployment, three candidate locations are opened and two other candidate locations are closed according to customer demand. Also, eleven customers have been stationed in certain places to receive the service and are ready to receive the service from the candidate places by vehicles. As you can see, several vehicles can be assigned to each depot according to its capacity, and each vehicle starts its travel from the depot and returns to the same depot after serving customers, and each vehicle is only allowed one trip. The objective of the problem in Figure 1 is to minimize the costs of setting up three depots, the fixed costs of using four vehicles, and also the transportation costs.

In this paper, in addition to the assumptions of the location-routing problem, other conditions are also taken into account, which we will describe below:

• The problem has several planning periods (the problem is in dynamic mode).

• The number of candidate locations and their locations and the number of customers are known at the beginning of the planning period.

• Candidate places have limited capacity.

• Vehicles have limited capacity and the related constraints are fuzzy flexible mode.

• The number of vehicles in each period is different.

• Each vehicle is assigned an accessibility factor in each period.

• There is a distance limit for vehicles.

The research problem with the stated conditions is modeled as a multi-objective problem whose objectives are as follows:

The first objective function related to the costs of establishing the candidate locations, the fixed costs of using vehicles and the transportation costs that should be minimized. And also, the second objective function is related to maximizing customer satisfaction, which is done by minimizing shortage costs caused by not serving the demand of some customers. Also, we seek to minimize the adverse environmental effects by maximizing the utility related to vehicle capacity in the case of using fuzzy flexible constraints. In fact, in this paper, in addition to considering the economic and social goals, we also consider the environmental goals, which are among the goals of the sustainable supply chain.

2. Related work

The vehicle location-routing problem has many different types and modes, usually derived from case studies on real-world problems. Changes in some parameters of the problem in terms of being deterministic or non-deterministic, taking into account limitations such as the existence of vehicles with different capacities, the type of objective function, etc., have caused changes in the type of problem. Researchers have provided various formulations for different models of the LRP problem. Table 1 shows a list of researchers and the type of problem they have modeled. In the following, we briefly describe the presented models.

Melechovsky et al. [9] considered the LRP problem with nonlinear setup costs. In the model presented by them, depots and vehicles have limited capacity and a non-linear cost is considered for each depot based on the amount of demands that each depot covers for its connected customers. The problem is formulated by mixed integer linear programming and two-phase method along with variable neighborhood search (VNS) meta-heuristic algorithms are used to solve it.

Salhi and Negi [19] have considered the Planar LRP problem in multi-depot mode. The condition of the problem is that P number of depots should be established in a continuous space with unlimited capacity. There are fleets of vehicles of similar capacity that have similar maximum travel times. In this article, the goal of the problem is to deploy P depot in a continuous space and organize vehicle trips so that the costs of the distribution system are minimized. In their research, Salhi and Negi presented an innovative method in two stages. These two phases are repeated regularly until no significant improvement is found in the answers.

Tavakkli-Moghaddam et al. [21] studied the LRP problem by considering two objective functions. In this issue, there are a set of candidate depots with specific capacity for deployment. Each customer with specific demand is ready to receive service by vehicles at designated locations, and there is a fleet of similar vehicles with limited capacity that also have travel time restrictions. In this article, the first goal is to minimize the costs of setting up depots, using vehicles, and the costs of vehicle travel, and the second goal is to maximize the total amount of customer demand that has been answered. The mentioned problem is solved in the form of multi-objective mixed linear programming formulation using an innovative method. And finally, the proposed method has been solved on a number of problem examples.

Zarandi et al. [24] investigated the LRP problem with a fuzzy consideration of vehicle travel time. In this problem, the travel time between two customers is considered in a fuzzy way, and the problem is modeled in the form of fuzzy programming of chance constraints, and the refrigeration simulation algorithm is used to solve it.

Marinakis [8] presented a new version of the particle swarm optimization (PSO) meta-heuristic algorithm for solving vehicle location and routing problems and LRP problems with stochastic demands. In this research, the vehicles are dissimilar and have limited capacity. Each customer can only receive service by one vehicle. Every vehicle must end its journey from any depot it started to the same depot. The goal of the problem is to find a suitable place for the establishment of depots and the formation of vehicle trips so that the costs are minimized.

Fazayeli et al. [3] evaluated the location-routing problem in the transportation network with demand and fuzzy time window. The location of the customers and the location of the depots, as well as the location of the manufacturer, are known and determined. Each vehicle is assigned to only one depot and has limited capacity. In this article, in addition to considering customer demands in a fuzzy manner, the time window limit was also applied to maximize customer satisfaction.

Pekel and Kara [17] studied the fuzzy LRP problem using variable neighborhood search algorithm. In this article, customer demands are considered as fuzzy and the travel time between the depot and the customer is definite. The transportation fleet is heterogeneous and the goal of the problem is to minimize transportation costs, waiting costs, additional costs, vehicle costs, and delay costs.

Almouhanna et al. [2] investigated the location-routing problems of electric vehicles. In this problem, the vehicles have a limited travel distance. The goal of the problem is to find a suitable place for the establishment of depots, to allocate customers to the depots, and also to organize the journey of the vehicles so that the costs are minimized. In this article, the researchers first proposed a multi-step innovative method to solve the problem and in the next step, they combined this method with the variable neighborhood search method to solve the model.

Nucamendi et al. [16] evaluated the open routing-location problem in a multi-depot mode with a heterogeneous fixed fleet and seek to reduce the fixed costs of using vehicles and the variable costs related to the traveled distance. The problem is formulated as mixed integer programming and a meta-heuristic method is used to solve the model.

Alamatsaz et al. [1] presented a multi-objective model for the green capacitated routing-location problem with a time window and the existence of customer demand in the state of uncertainty.

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Table1. General classification of researches in the LRP

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No	Authors			Type of data entry					Programming period		number of depots		Type of depots		Type of vehicles		Number of vehicles		Time window			Objective function		Solution method		
				Crisp	Possible	Fuzzy	Grey	Stochastic	Static	Dynamic	Single Depot	Multi-depot	Capable	Unlimited capacity	Similar	dissimilar	Single trip	Multi-trip	Crisp	Fuzzy	possible	Single-objective	Multi-objective	Exact	Heuristic	Meta-heuristic
1	[19]	2009	Salhi and Negi	❊	–	–	–	–	❊	–	–	❊	–	❊	❊	–	❊	–	–	–	–	❊	–	–	❊	–
2	[21]	2010	Tavakkoli Moghaddam et al.	❊	–	–	–	–	❊	–	–	❊	❊	–	❊	–	❊	–	–	–	–	–	❊	–	❊	–
3	[24]	2011	Zarandi et al.	–	❊	–	–	–	❊	–	–	❊	❊	–	❊	–	❊	–	❊	–	–	❊	–	–	–	❊
4	[4]	2012	Ghaffari-Nasab et al.	–	❊	–	–	–	❊	–	–	❊	❊	–	❊	–	❊	–	–	–	–	–	❊	–	–	❊
5	[25]	2013	Zarandi et al.	–	–	❊	–	–	❊	–	–	❊	❊	–	❊	–	❊	–	❊	–	–	❊	–	–	–	❊
6	[5]	2013	Golzari et al.	–	–	❊	–	–	❊	–	–	❊	❊	–	❊	–	❊	–	–	–	–	❊	–	❊	–	❊
7	[7]	2014	.Lopes et al	❊	–	–	–	–	❊	–	–	❊	❊	–	❊	–	❊	–	–	–	–	❊	–	–	–	❊
8	[10]	2014	Nadizade et al.	–	–	❊	–	–	–	❊	–	❊	❊	–	–	❊	❊	–	–	–	–	❊	–	–	❊	❊
9	[8]	2015	Marinakis	–	–	–	–	❊	❊	–	–	❊	❊	–	❊	–	❊	–	–	–	–	❊	–	–	–	❊
10	[18]	2016	Ponboon et al.	❊	–	–	–	–	❊	–	–	❊	❊	–	❊	–	❊	–	❊	–	–	❊	–	❊	–	–
11	[22]	2016	Tavakkoli-Moghaddam et al.	–	–	❊	–	–	–	❊	–	❊	❊	–	–	❊	❊	–	–	–	–	–	❊	❊	–	–

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				Crisp	Possible	Fuzzy	Grey	Stochastic	Static	Dynamic	Single Depot	Multi-depot	Capable	Unlimited capacity	Similar	dissimilar	Single trip	Multi-trip	Crisp	Fuzzy	possible	Single-objective	Multi-objective	Exact	Heuristic	Meta-heuristic
12	[15]	2017	Nedjati et al.	❉	–	–	–	–	❉	–	–	❉	❉	–	❉	–	–	❉	–	–	–	–	❉	❉	❉	–
13	[3]	2018	Fazayeli et al.	–	–	❉	–	–	❉	–	–	❉	–	❉	–	❉	❉	–	❉	–	–	❉	–	–	–	❉
14	[17]	2019	Pekel et al.	–	–	❉	–	–	❉	–	–	❉	❉	–	–	❉	–	❉	–	–	–	❉	–	–	–	❉
15	[2]	2019	Almouhanna et al.	❉	–	–	–	–	❉	–	–	❉	❉	–	❉	–	❉	–	–	–	–	❉	–	–	–	❉
16	[16]	2020	Nucamendi-Guille et al.	❉	–	–	–	–	❉	–	–	❉	❉	–	–	❉	–	❉	–	–	–	❉	–	–	–	❉
17	[1]	2022	Alamatsaz et al.	–	–	–	–	❉	❉	–	–	❉	❉	–	❉	–	❉	–	❉	–	–	–	❉	–	–	❉
18	[23]	2022	Tirkolaee et al.	–	–	❉	–	–	❉	–	–	❉	❉	–	–	❉	–	❉	❉	–	–	–	❉	❉	–	–
19	[9]	2020	Wei et al.	❉	–	–	–	–	❉	–	❉	–	–	❉	❉	–	❉	–	–	❉		❉	–	–	❉	–

The multi-table location-routing problem with fuzzy parameters in steady state was investigated by Irfan Babaei et al. [23] for the management of medical waste during the outbreak of Covid-19. In this paper, vehicles are dissimilar and can have multiple trips. The demand of customers is in the form of a triangular phase number and must be provided by only one vehicle. There is a time window for each customer. In this paper, the researchers presented a mixed integer programming model whose purpose is to minimize the travel time of vehicles and time window violation, as well as to minimize site risk, i.e. to minimize the number of people around waste disposal sites. The problem is modeled using chance constraints.

3. Conditions of the capacitated location-routing problem with fuzzy flexible constraints

The general condition of the capacitated location-routing problem with flexible fuzzy constraints is that a time horizon with several planning periods is considered. Also, a number of candidate locations with specific capacity are available for deployment, and a number of customers are also deployed in specific locations with definite requests to receive service from depots. There is also a fleet of unique vehicles to serve customers who are only allowed to make one trip.

In each time period, the demand of customers is provided by only one vehicle, and the total demand of customers who receive service by each vehicle should not exceed the capacity of the vehicle. On the other hand, it is possible for depots to include several vehicle trips and the total demand of customers assigned to each depot should not exceed the capacity of the depot.

It should be noted that each vehicle is assigned an accessibility coefficient in the range [0,1] in each period. The existence of this coefficient means that due to some unforeseen problems such as the unavailability of vehicles, vehicle breakdowns, etc., any vehicle may not be ready for service all the time. For this reason, the presence of this coefficient for each vehicle indicates that the higher the coefficient value (closer to one), the longer the vehicle will be available for servicing. In the same way, the lower the value of this coefficient (closer to zero), the less time the vehicle will be available. In this case, there is a possibility that some customers will not receive service from the depot and the system will suffer from a lack of cost. In this article, considering the capacity of vehicles in the state of uncertainty, it has been tried to present and solve a problem close to the real world conditions. For this purpose, the flexible fuzzy approach has been used, which will be fully described below. In fact, the objectives of the problem are to meet customer demand with minimum costs, which include fixed costs of using vehicles, fixed costs of establishing depots, and transportation costs, as well as maximizing customer satisfaction by minimizing shortage costs and at the same time minimizing harmful environmental impacts.

In order to model the CLRP problem with fuzzy flexible constraints, suppose the constraints related to the capacity of vehicles are fuzzy flexible, which is expressed as follows:

$$\sum_{i \in V}\sum_{j \in J} d_j^t x_{ijk}^t \lesssim Q_k \qquad \forall k \in K, t \in T \tag{1}$$

where the membership function of the fuzzy answer set is defined as follows:

$$\mu_k(u) = \begin{cases} 1, & u \le Q_k \\ 1 - \dfrac{u - Q_k}{r_k}, & Q_k \le u \le Q_k + r_k \\ 0, & u \ge Q_k + r_k \end{cases} \tag{2}$$

Where $u = \sum_{i \in V}\sum_{j \in J} d_j^t x_{ijk}^t$.

Considering that for constraint (1) the space of feasible solutions of the problem is not well defined, a parametric approach is used to solve this model [13].

Lemma 1. The flexible constraint $\sum_{i \in V}\sum_{j \in J} d_j^t x_{ijk}^t \lesssim Q_k$ is equivalent to the arametric constraint $\sum_{i \in V}\sum_{j \in J} d_j^t x_{ijk}^t \lesssim Q_k + \theta r_k$ where $\theta \in [0,1]$.[13]

Acording to the definition of the membership function (2), which is a monotonic and continuous function, a problem with constraints as $\sum_{i \in V}\sum_{j \in J} d_j^t x_{ijk}^t \lesssim Q_k$ with a problem with constraints As follows it will be equivalent to the membership function defined in (2)

$$\sum_{i \in V}\sum_{j \in J} d_j^t x_{ijk}^t \leq \tilde{Q}_k \qquad \forall k \in K, t \in T \tag{3}$$

By using the membership function defined in (2), in order to facilitate the solution of the model with flexible fuzzy constraint, we convert it to the exact parametric mode. Since the data is presented in the state of uncertainty, we interpret the information as flexible fuzzy with parameters in the interval $[Q_k, Q_k + r_k]$. Since it is impossible to solve the model with flexible fuzzy constraint, we will use its exact parametric form which is given as

$$\sum_{i \in V}\sum_{j \in J} d_j^t x_{ijk}^t \leq Q_k + (1-\alpha_k) r_k \qquad \forall k \in K, t \in T \tag{4}$$

where $\alpha_k \in [0,1]$ and the values of α_k are specified by the decision maker.

In the relation (4), whenever the value of α_k is closer to one, the value of the utility will increase, and in a similar way, if their value is closer to zero, the utility will decrease. Applying the flexible fuzzy approach in the limitation of vehicle capacity is because the use of vehicles has costs for the system, and the use of methods to reduce these costs is of interest to decision makers. On the other hand, we will look for an approach to reduce the amount of harmful environmental effects caused by the use of vehicles.

Each vehicle has a certain capacity, which due to the uncertainty in the data in the real world, sometimes the capacity of vehicles can be increased to a certain amount. In fact, a maximum threshold can be considered for the capacity of the vehicle. This may reduce the number of vehicles used according to customer demand. On the other hand, with the increase in the capacity of vehicles, the amount of utility will decrease because imposing an additional load on the vehicle in the long term will reduce the life or breakdown of the vehicle, which will reduce its performance and further reduce access to the vehicle. will be. In addition, reducing the life of the vehicle will result in harmful environmental effects, including the increase in vehicle depreciation, which will increase environmental pollution. Also, the breakdown of the vehicle may reduce its accessibility in different periods, and the demand of all customers may not be met and the shortage cost will be created. Therefore, it is important to consider the situation in which, while reducing the number of vehicles used, it improves the utility.

4. Mathematical modeling of the problem

In order to introduce the mathematical model, first define the signs, parameters and variables of the problem, and then the objective functions and their constraints are stated [11].

Definition of symbols:

J :The collection of all customers in which $J=\{1,2,...,N\}$ and N the number of customers are shown with an index $j \in J$.

I : The collection of all candidate depots in which $I=\{1,2,...,M\}$ and M the number of candidate depots are shown with an index $i \in I$.

V : The set of all customers and candidate depots as $V = I \cup J$ shown in $V=\{1,2,...,M,M+1,...,M+N\}$.

K : The set of all vehicles where $K=\{1,2,..., K'\}$ and K' is the number of vehicles and is indicated by the index $k \in K$.

T : The set of all planning horizons where $T=\{1,2,..., T'\}$ and T' is the number of courses and with index $t \in T$ is denoted.

O : Is the set of all arcs (i,j) where $i,j \in V$.

According to the definitions of indices, it can be seen that the data of the problem can be considered on a weighted and undirected graph such as $G=(V,O,C)$, where V is the set of all nodes including the subset I of M Candidate depot and subset J of N customers. Also, O is the set of all arcs between existing nodes and C shows the cost of navigation between nodes and is symmetric.

The parameters of the problem are defined as follows:

d_j^t : The amount of demand of the j th customer in the t th period.

p_i : The capacity of the i th candidate depot.

Q_k : The capacity of vehicle k .

E_i : The fixed cost of establishing a depot in the i -th candidate location.

H_k : Fixed cost of using vehicle k .

B_j^t : Cost of deficiency caused by not serving the j th customer in the t th period.

C_{ij} : Transportation cost between two nodes i and j . (will be converted into distance unit)

A_k^t : Accessibility coefficient of the k th vehicle in the t th period where $A_k^t \in [0,1]$.

DT^t : The maximum allowed travel distance of vehicles in the period t.

S_J : The duration of service to the j th customer (it will be converted into a distance unit)

r_k : Amount of violation of the capacity of vehicle k .

α_k : The maximum degree of membership of fuzzy flexible constraints such that $\alpha_k \in [0,1]$.

Definition of decision variables: The decision variables of the model are defined as follows:

$$x_{ijk}^{t} = \begin{cases} 1, & \text{if the kth vehicle move from node i to node j during tth period} \\ 0, & \text{o.w} \end{cases}$$

$$y_{ij}^{t} = \begin{cases} 1, & \text{if costomer j recieves service from depot i in tth period} \\ 0, & \text{o.w.} \end{cases}$$

$$z_{i} = \begin{cases} 1, & \text{if a depot is established in the ith condidate location} \\ 0, & \text{o.w.} \end{cases}$$

rs_{jk}^{t} : Auxiliary variable to remove vehicle travel subtours.

And finally, the proposed model according to the conditions and data presented will be as follows:

$$Min\, Z_1 = \sum_{i\in I} E_i Z_i + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T} H_k x_{ijk}^{t} + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K}\sum_{t\in T} C_{ij} x_{ijk}^{t} \quad (5)$$

$$Min\, Z_2 = \sum_{j\in J}\sum_{t\in T}(1 - \sum_{i\in V}\sum_{k\in K} x_{ijk}^{t}) B_j^{t} \quad (6)$$

$$\sum_{i\in V}\sum_{j\in J} d_j^{t} x_{ijk}^{t} \le Q_k + \left(1-\alpha_k\right) r_k \qquad \forall k \in K, t \in T \quad (7)$$

$$\sum_{j\in J} d_j^{t} y_{ij}^{t} \le P_i z_i \qquad \forall i \in I, t \in T \quad (8)$$

$$\sum_{i\in V}\sum_{k\in K} x_{ijk}^{t} \le 1 \qquad \forall j \in J, t \in T \quad (9)$$

$$rs_{lk}^{t} - rs_{jk}^{t} - N x_{ljk}^{t} \le N - 1 \qquad \forall j, l \in J, k \in K, t \in T \quad (10)$$

$$\sum_{j\in V} x_{ijk}^{t} - \sum_{j\in V} x_{jik}^{t} = 0 \qquad \forall i \in V, k \in K, t \in T \quad (11)$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk}^{t} \le 1 \qquad \forall k \in K, t \in T \quad (12)$$

$$\sum_{u\in V} x_{iuk}^{t} + \sum_{u\in V} x_{ujk}^{t} \le 1 + y_{ij}^{t} \qquad \forall i \in I, j \in J, k \in K, t \in T \quad (13)$$

$$\sum_{i\in V}\sum_{j\in V}\left(C_{ij} + S_J\right) x_{ijk}^{t} \le A_k^{t} DT^{t} \qquad \forall k \in K, t \in T \quad (14)$$

$$x_{ijk}^{t} \in \{0,1\} \qquad \forall i \in I, j \in J, k \in K, t \in T \quad (15)$$

$$y_{ij}^{t} \in \{0,1\} \qquad \forall i \in I, j \in J, t \in T \quad (16)$$

$$z_i \in \{0,1\} \qquad \forall i \in I \quad (17)$$

$$rs_{jk}^{t} \ge 0, Integer \qquad \forall j \in J, k \in K, t \in T \quad (18)$$

In the presented model, relation (5) shows the first objective function of the problem, which includes the total fixed costs of establishing candidate locations, the total fixed costs of using vehicles, and the cost of transportation, which must be minimized. The relation (6) is related to the second objective function of the problem and states that if the demand of some customers is not answered, the shortage cost will be created. It should be noted that the shortage cost is due to the lack of service to some customers, which is caused by reasons such as lack of vehicle capacity, lack of depot capacity, unavailability of all vehicles, distance Constraints, etc. In fact, the higher the amount of these costs, the lower the customer satisfaction. For this reason, it is necessary to seek to maximize customer satisfaction by minimizing these costs. Constraint (7), states that the total demand of customers assigned to each vehicle should not exceed the capacity of the vehicle. Also, restriction (8) is related to not exceeding the total demand of customers assigned to each depot, from the capacity of that depot. Constraint (9), guarantees that if a customer receives a service, that service will be provided by only one vehicle. Constraint (10), which is known as subtours Constraint, eliminates possible tours or rings. The condition of vehicle travel continuity is shown in constraint (11). Constraint (12) indicates that the available vehicle can not be used in a period. The necessity of allocating each customer to a depot that has a route to that depot is shown in constraint (13). Constraint (14) is related to the limitation of vehicle travel distance according to their accessibility factor. Constraints (15) to (17) represent binary variables (zero and one). Constraint (18) is related to integer and non-negative variables.

5. The solution Method

Considering that the research problem is a multi-objective linear programming problem that has flexible fuzzy constraints, to solve the model it is necessary to pay attention to the two definitions of multi-objectiveness and fuzzy flexibility of constraints. There are many methods to solve multi-objective linear programming problems, one of the most practical methods is the weighting method.

In this approach, the decision maker assigns a weight to each objective function based on his criteria and the importance of each objective for him, and considers several objectives in the form of one objective and as a weighted sum of objectives. In this case, the multi-objective linear programming problem becomes a single-objective linear programming problem. The important point in using the weighted approach is that the sum of the weights assigned to each objective function must be equal to one. Also, the objective functions may not be the same, in which case it is necessary to transform the objectives into the same form before they are added together.

On the other hand, the presence of flexible fuzzy constraints convert the problem into a flexible fuzzy linear programming problem, and there are several solution methods to solve such problems, all of which want to maximize the level of utility. Considering the combination of these two problems and the solution of the presented model, a solution algorithm is proposed. Before presenting the proposal solution algorithm, some definitions and concepts that are needed to solve the problem are stated.

When there is an assumption of flexibility of constraints in the model, due to the model not being well-defined, the solution space of the problem cannot be clearly defined, and the problem in its initial form does not reflect the real nature of the decision-making system. Therefore, in order to solve the problem, we will first convert it from a flexible fuzzy state to a deterministic parametric state, and then, in order to reach the optimal solution, we will seek to maximize the utility.

Definition1. Let the relation $\lesssim$ be a fuzzy representation of the inequality relation $\leq$ in The following problem. In this case, the vector $x_j \in R$ is an **α-**efficient solution with the objective function of minimization, if there is no $x'_j \in X_\alpha$ such that $z(x_j) > z'(x_j)$.

$$
\begin{aligned}
& Min\ z = \sum_{j=1}^{n} c_j x_j \\
& s.t. \\
& \sum_{j=1}^{n} a_{ij} x_j \leq b_i + (1-\alpha_i) p_i \qquad i = 1,2,\cdots,m \qquad (19) \\
& x_j \geq 0 \qquad j = 1,2,\cdots,n \\
& 0 \leq \alpha_i \leq 1 \qquad i = 1,2,\cdots,m
\end{aligned}
$$

It should be mentioned that every **α**-efficient solution for problem (19) is also an α-efficient solution for it [12,13].

Theorem 1. Suppose $\alpha' = (\alpha'_1, \alpha'_2, \cdots, \alpha'_m)$ and $\alpha'' = (\alpha''_1, \alpha''_2, \cdots, \alpha''_m)$ where $\alpha'_i \leq \alpha''_i$ for each $i = 1,2,\cdots,m$. In this case, every α'' feasible solution will be an α' feasible solution.

Theorem 2. Suppose $\alpha = (\alpha_1, \alpha_2, ..., \alpha_m) \in [0,1]^m$ and $x_j^* \in X_\alpha, j = 1,2,\cdots,n$ and also $x_j^* \geq 0,\ j = 1,\cdots,n$ be an α-feasible solution of problem (19). Then the vector $x_j^* \in \mathbb{R}^n$ is an α-efficient solution of problem (19) with the objective function of minimization, if and only if x_j^* is an optimal solution of problem (19).

Now we solve the problem (19) for different α_i (according to the opinion of the decision maker) and obtain an α-efficient solution for the mentioned problem. This effective answer has two features:

1- The answer has a degree of desirability proportional to each limitation.

2- The answer is optimal.

This answer gives the possibility to the decision maker to reach an answer with a higher degree of desirability according to the environmental conditions and by determining the desired priority.

After solving problem (19), we assume that (x_j^*, α_i^*) is the optimal solution and z^* is the optimal value of the objective function. In order to find the maximum utility, we consider the objective function of the first problem as a constraint with a penalty p_0 in the second step and solve the problem presented below using the optimal value of the objective function obtained from the previous step [14]:

$$
\begin{aligned}
& Max \sum_{i=0}^{m} \alpha_i \\
& s.t. \\
& z \le z^* + (1-\alpha_0) p_0 \\
& \sum_{j=1}^{n} a_{ij} x_j \le b_i + (1-\alpha_i) p_i \qquad i = 1,2,\ldots,m \\
& x_j \ge 0 \qquad j = 1,\ldots,n \\
& \alpha_i \in [0,1],\ \alpha_i^* \le \alpha_i \le 1 \qquad i = 1,2,\ldots,m \\
& \alpha_0 \in [0,1]
\end{aligned} \tag{20}
$$

Obviously, the presented problem is a multi-parameter linear programming problem and by solving it, the optimal solution $(x_j^{**}, \alpha_i^{**})$ and the optimal value of the objective function z^{**} it will be obtained.

It is important to note that the optimal value of the objective function will not improve as the solution area shrinks. For this reason, the decision-maker seeks to increase the optimal value of the objective function to a maximum of $(1-\alpha_0)p_0$ by observing the minimum acceptability of the restrictions. It is obvious that the presented model is a linear programming problem, and by solving it, we will get an optimal solution with a higher degree of desirability.

6. Main Steps algorithm

Consider the multi-objective linear programming problem with fuzzy flexible constraints (MOLPFFC) as follows [11]:

$$
\begin{aligned}
& Min\ Z_1 = \sum_{i \in I} E_i Z_i + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{t \in T} H_k x_{ijk}^t + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} \sum_{t \in T} C_{ij} x_{ijk}^t \\
& Min\ Z_2 = \sum_{j \in J} \sum_{t \in T} (1 - \sum_{i \in V} \sum_{k \in K} x_{ijk}^t) B_j^t
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
& s.t. \\
& \sum_{i \in V} \sum_{j \in J} d_j^t x_{ijk}^t \lesssim Q_k \qquad \forall k \in K, t \in T
\end{aligned}
$$

Constraints (8) to (18)**.**

Step 1: Obtain the multi-parametric multi-objective linear programming (MPMOLP) problem equivalent to the multi-objective linear programming problem with fuzzy flexible constraints:

$$Min\, Z_1 = \sum_{i \in I} E_i Z_i + \sum_{i \in I}\sum_{j \in J}\sum_{k \in K}\sum_{t \in T} H_k x^t_{ijk} + \sum_{i \in V}\sum_{j \in V}\sum_{k \in K}\sum_{t \in T} C_{ij} x^t_{ijk}$$

$$Min\, Z_2 = \sum_{j \in J}\sum_{t \in T}(1 - \sum_{i \in V}\sum_{k \in K} x^t_{ijk}) B^t_j \tag{22}$$

$s.t.$

$$\sum_{i \in V}\sum_{j \in J} d^t_j x^t_{ijk} \le Q_k + (1 - \alpha_k) r_k \qquad \forall k \in K, t \in T$$

Constraints (8) to (18).

Step 2: With the weight approach and the selection of appropriate weights (with expert opinion), we will convert the multi-parameter multi-objective linear programming problem into a Multi-Parameter Single-Objective Linear Programming (MPSOLP) problem.

$$Min\, Z_3 = \lambda_1 (\sum_{i \in I} E_i Z_i + \sum_{i \in I}\sum_{j \in J}\sum_{k \in K}\sum_{t \in T} H_k x^t_{ijk} + \sum_{i \in V}\sum_{j \in V}\sum_{k \in K}\sum_{t \in T} C_{ij} x^t_{ijk})$$

$$+\lambda_2 (\sum_{j \in J}\sum_{t \in T}(1 - \sum_{i \in V}\sum_{k \in K} x^t_{ijk}) B^t_j) \tag{23}$$

$s.t.$

$$\sum_{i \in V}\sum_{j \in J} d^t_j x^t_{ijk} \le Q_k + (1 - \alpha_k) r_k \qquad \forall k \in K, t \in T$$

Constraints (8) to (18).

where λ_1 and λ_2 are respectively the weight of the costs related to the establishment of depots and transportation costs and the use of vehicles and the weight of the shortage cost so that: $\lambda_1 + \lambda_2 = 1$.

Step 3: Solve the MPSOLP problem presented in the second step based on different α_k to obtain the optimal values of the variables and the optimal value of the objective function z^*_3:

$\alpha_k^*, \qquad \forall k \in K$

$x^{t\,*}_{ijk}, \qquad \forall i \in I, j \in J, k \in K, t \in T$

$y^{t\,*}_{ij}, \qquad \forall i \in I, j \in J, t \in T$

$z_i^*, \qquad \forall i \in I$

Step 4: Using the optimal values of α^*_k and z^*_3 obtained from the third step, solve the AMPLP problem with the aim of maximizing the utility.

$$Max\ (\alpha_0 + \sum_{k \in K} \alpha_k)$$

$$s.t.$$

$$Z_3 \le Z_3^* + (1-\alpha_0) r_0$$

$$\sum_{i \in V} \sum_{j \in J} d_j^t x_{ijk}^t \le Q_k + (1-\alpha_k) r_k \qquad \forall k \in K, t \in T \qquad (24)$$

$$\alpha_k^* \le \alpha_k \le 1 \qquad \forall k \in K$$

$$\alpha_k \epsilon [0,1] \qquad \forall k \in K$$

$$\alpha_0 \epsilon [0,1]$$

Constraints (8) to (18)

where the optimal values are:

$$\alpha_k^{**}, \qquad \forall k \in K$$

$$x_{ijk}^{t\ **}, \qquad \forall i \in I, j \in J, k \in K, t \in T$$

$$y_{ij}^{t\ **}, \qquad \forall i \in I, j \in J, t \in T$$

$$z_i^{**}, \qquad \forall i \in I$$

Note: As stated earlier, due to the lack of well-defined model in the case of flexible constraints, the solution space of the problem cannot be clearly defined. Therefore, in order to solve the model, we must first convert the problem from a flexible fuzzy state to a precise parametric state and then use maximization of utility levels to reach the optimal solution. We transform the multi-objective linear programming problem with flexible fuzzy constraints into a parametric multi-objective linear programming problem.

In order to make it easier to do the work, we turned the multi-objective problem into a single-objective problem. There are many methods for solving multi-objective problems, one of which is the balanced sum method. In this method, a coefficient between zero and one will be assigned to each function according to the indicators and priorities of the decision maker, and the sum of the coefficients assigned to the target functions should be equal to one.

Considering that by solving the problem presented in the third step, the optimal solution α_k^* and the optimal value of the objective function z_3^* were obtained. Therefore, by using it and by changing the objective function of the problem in order to maximize the level of usefulness, he solved the following problem to obtain the following optimal solution with the objective function value z_3^{**}.

Theorem 3. If the multi-objective linear programming problem with fuzzy flexible constraints (MOLPFFC) has an optimal solution, then the AMPLP problem is feasible.

Proof. Given that $\alpha_k \in [0,1]$, every optimal solution of the multi-objective linear programming problem with flexible fuzzy constraints (MOLPFFC) is a feasible solution of the AMPLP problem. Especially for $\alpha_k = 1$, the optimal solution of MOLPFFC is an obvious feasible solution to the problem.

Theorem 4. The AMPLP problem is never infinite if it is feasible.

Proof. According to Theorem 3, because the problem of the AMPLP problem is feasible, and the objective function of the problem is always bounded considering that $\alpha_k \in [0,1]$, so the infinite state does not occur.

7. Numerical example

In this section, in order to explain the performance of the model and its solution method, we present a numerical example on a small scale.

In this example, there are 2 candidate depots (waste disposal site) with different certain capacities and certain costs, which are mentioned in Table 3. Eight customers are stationed at defineded locations and are ready to receive service. Also, considering the dynamics of the problem, the number of two planning periods has been considered. The demand of customers and the cost of shortage caused by not providing service to each customer in each planning period are also stated in Table 4. On the other hand, there are 3 different vehicles with different accessibility coefficients that serve customers. Each vehicle has a cost of using it and also a maximum distance that can be navigated, which is stated in Table 2. The cost of transportation, the information of which is given in Table 5. Also, the duration of service by each vehicle to each customer is presented in Table 4. Since the capacity of the vehicles is different due to their dissimilarity, and on the other hand, due to the fuzzy flexibility of the related constraints, a maximum threshold of capacity violation is also stated for each vehicle. It should be mentioned that the amount of violation of each vehicle from the capacity defined for it by the decision maker is determined according to the indicators he wants to increase the capacity of each vehicle. Since we want to convert the problem from multi-objective mode to single-objective mode using the weighting method, we consider the weight of the first objective function to be 0.55 and the weight of the second objective function to be 0.45. It should be mentioned that the weighting of the objective functions is according to the opinion of the decision maker and based on his desired indicators.

Table 2. Condition of available vehicles.

Vehicle	Capacity	Fixed cost of use (currency)	accessibility coefficient in the first period	Accessibility coefficient in the second period	Maximum distance in the first period	Maximum distance in the second period	Maximum acceptable threshold
1	450	5	1	0.9	100	120	60

2	300	8	0.8	0.2	100	120	50
3	700	6	0.7	1	100	120	140

Table 3. Conditions of existing candidate depots**.**

Depot	capacity in each period	fixed cost of establishment (currency)
1	700	7
2	1000	10

Table 4. Customer conditions**.**

Customer	Demand in the first period	Demand in the second period	The duration of receiving the service	Shortage cost in the first period	Shortage cost in the second period
1	250	200	4	25	15
2	100	200	5	20	30
3	300	100	5	60	40
4	100	150	8	40	30
5	200	200	4	40	25
6	150	150	1	30	50
7	150	400	4	30	40
8	300	100	4	60	60

Table 5. Transportation cost**.**

node	1	2	3	4	5	6	7	8	9	10
1	0	8	10	12	10	12	9	5	11	10
2	8	0	10	14	2	13	9	4	7	14
3	10	10	0	4	7	6	8	10	10	10
4	12	14	4	0	10	5	7	10	12	15
5	10	14	7	10	0	13	18	5	4	5
6	12	13	6	5	13	0	10	6	8	9

7	9	9	8	7	18	10	0	9	5	4
8	5	4	10	10	5	6	9	0	10	11
9	11	7	10	12	4	8	5	10	0	5
10	10	14	10	15	5	9	4	11	5	0

According to the data of the problem and using the solution algorithm presented in the previous section, the value of the objective function was obtained for different degrees of desirability and reported in Table 6.

Table 6. Values of the objective function for different values of α_k .

	a	***B***	***c***	***d***	***e***	***G***
$\bar{\alpha}$	(0.8,0.8,0.8)	(0.2,0.5,0.8)	(0.5,0.5,0.5)	(0.5,0.5,0.2)	(0.8,0.5,0.2)	(0.2,0.2,0)
z_3	115.35	115.35	114.45	112.80	112.80	98.30
0.7	0.8	0.2	0.5	0.5	0.8	0.2
0.3	0.8	0.5	0.5	0.5	0.5	0.2
0.7	0.8	0.8	0.5	0.2	0.2	0

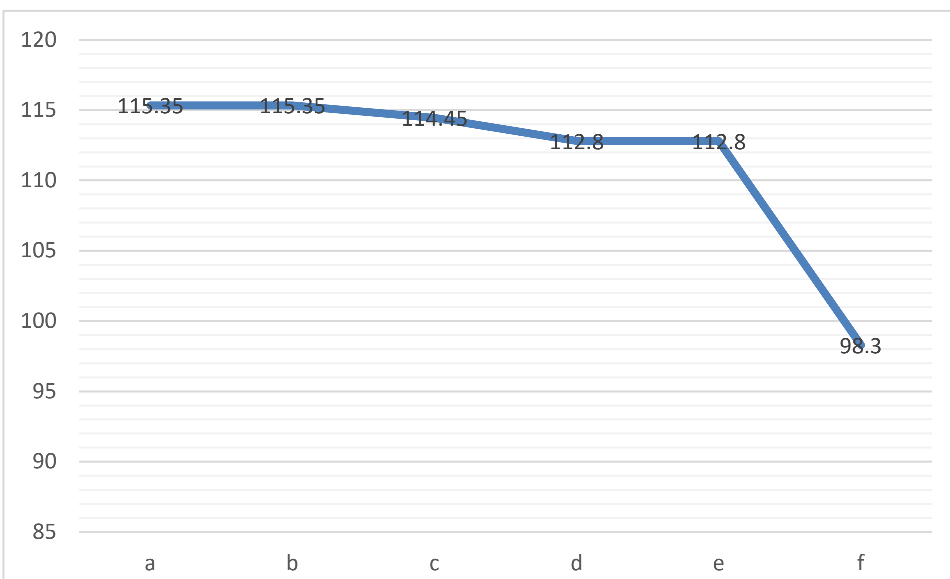

Figure 2. Values of the objective function for different values of α_k .

Using the optimal utility α^* and the optimal value of the objective function z_3^* obtained from the previous step and considering the maximum acceptable threshold for the limitation of the objective function to the value $r_0 = 14$, the direction To maximize the utility of vehicles, we solve the problem stated in the fourth step of the proposed solution algorithm and get the following optimal solution:

$$\alpha^{**} = (0.8, 0.8, 0.5)$$

$$z_3^{**} = 114.45$$

After solving the problem and reaching the optimal value of the objective function, as can be seen in Figure 3, due to the existence of two planning periods for vehicle travel, in the first period both depots were established to serve customers and customers number 6 , 7 and 8 are assigned to the first depot and customers number 5, 9 and 10 are assigned to the second depot. The work of serving customers from the first depot was done by vehicle number 1 and also vehicle 2 serves the customers of the second depot. It should be noted that customers 3 and 4 do not receive service during this period, and in fact, the cost of shortage is created.

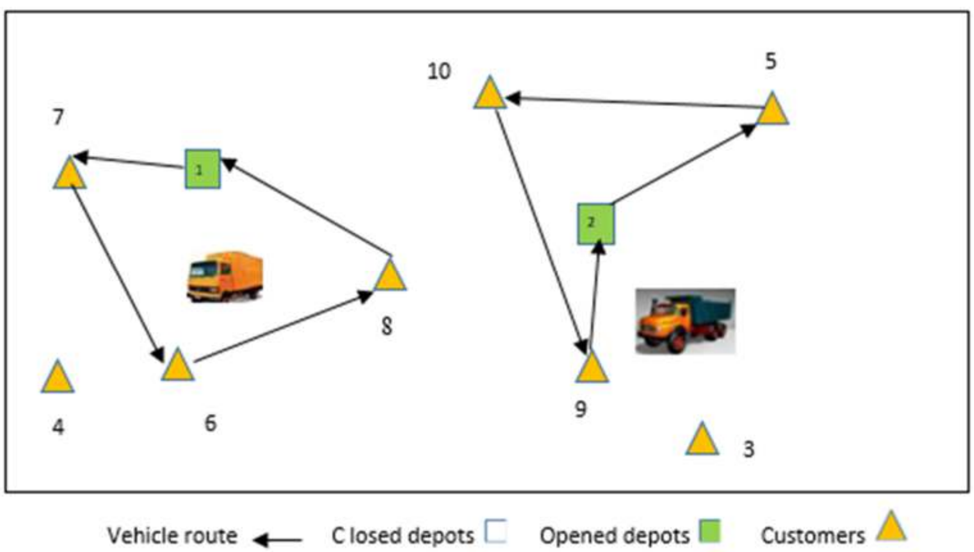

Figure 3. Vehicle travel planning in the first period.

As shown in Figure 4, in the second period, vehicle number 1 and 2 are assigned to depot number 2, vehicle number 1 serves the 9th customer and vehicle number 2 serves customers 5 and 8. Also, customers 4, 6, 7 and 10 also receive service from depot number 2 by vehicle number 1. During this period, the third customer did not receive any service and the cost of shortage is imposed on the system.

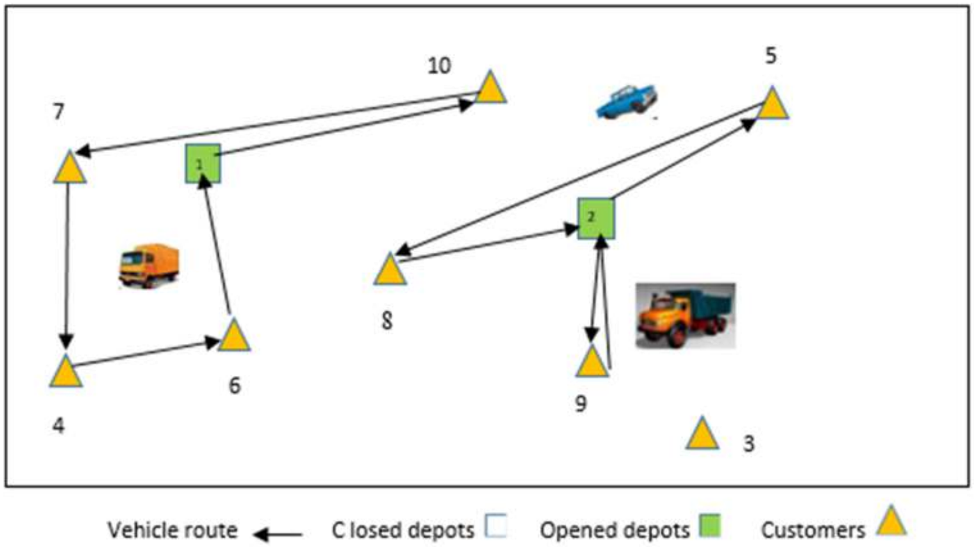

Figure 4. Vehicle travel planning in the second period.

According to the optimal value obtained, it can be seen that its value after solving the problem in the second stage is the same as before and actually remains constant. But the satisfaction of customers has increased. In fact, it means that the utility for the first and second vehicle has increased and remained constant for the third vehicle. Therefore, we were able to increase the utility of vehicle capacity so that the value of the objective function of the problem does not deteriorate. As stated in the previous sections, the utility of vehicle capacity will increase when the amount of additional load imposed on vehicles is reduced. In fact, the lower the violation of the defined capacity for vehicles, the higher the utility. In this case, it is possible that due to the use of more vehicles, the amount of costs will increase and finally the value of the objective function will increase. But as can be seen, according to the presented model and proposed solution method, we were able to keep the value of the objective function constant while increasing the utility. Therefore, according to the obtained results, it seems justified to use the proposed model and its solution approach

8. Conclusion

Considering the importance of location-routing problem in the field of logistics systems of each supply chain, in this research, a multi-objective mathematical model for the location-routing problem of vehicles in dynamic mode, assuming the stability of the supply chain and the existence of restrictions related to the capacity of vehicles as The flexible phase, which was not considered in the literature of the location-routing problem, was investigated. The assumptions of the presented model are: the existence of capacity limits for depots and vehicles, the multi-period nature of the problem, the limitation of travel distance and the existence of accessibility coefficients for vehicles in each time period, as well as the constraints related to the capacity of vehicles, are flexible fuzzy. The problem consists of two objective functions. Considering the importance of economic goals in the sustainable supply chain, in order to minimize the costs that are related to the first objective function, the costs related to the establishment of depots, the fixed cost of using vehicles and transportation costs have been examined and in order to increase the satisfaction of customers who It is related to the function of the second goal and is considered among the social goals in the sustainable supply chain, the cost of shortages caused by not providing services to some customers should be minimized. On the other hand, due to the fuzzy flexibility of vehicle capacity constraints in the problem in a two-stage approach to solve the model, we seek to maximize the value of utility, which is also considered as an environmental goal in every supply chain. Because as much as the value of the utility increases, the amount of violation of the defined capacity for each vehicle decreases and finally the adverse environmental effects caused by the additional load imposed on the vehicles, which may result in vehicle breakdown, increase in fuel consumption, increase Depreciation etc. will be reduced. In order to solve the problem, according to the multi-objective nature of the proposed model, first the model is converted into a single-objective model using the weight method, and then, using the presented certain solution algorithm, the model is solved in the presence of flexible fuzzy constraints. Finally, a numerical example is given to show the performance of the proposed solution method, which according to the results of the solution shown in the available tables and graphs, the value of the objective function of the problem in the case where flexible fuzzy constraints are used, with increasing the degree of desirability, It will not get worse. This issue justifies the use of this approach. In fact, the results show that the presented model is valuable among the existing models in the field of vehicle location-routing, considering the conditions and assumptions closer to the real world, and it is also more efficient than some existing models.

9. Suggestions and future works

In the future, research can be done in line with this research, which consists of:

1. Constraints related to the capacity of supply centers should be fuzzy flexible.
2. Customer demands should be considered in the form of delivery and then picked up.
3. Considering the duration of customer service in the form of fuzzy numbers.
4. Having a time window to serve each customer.
5. Considering the accessibility factor of each vehicle in the form of fuzzy numbers.
6. Using gray or fuzzy-interval data in cases where the parameters were uncertain.
7. Making it possible to communicate and exchange goods between supply centers to meet the maximum demand of customers.
8. Considering the possibility of not returning the vehicle to the depots.
9. Adding goals such as maximum coverage of customer demand and reducing the extra distance traveled due to failure on the route in case of uncertainty of customer demand or vehicle capacity.
10. Location of supply centers in a continuous space. In fact, it means that there are infinite candidate supply centers.
11. Considering the mode of multiple products to send to customers.
12. Using heuristic and meta-heuristic algorithms to solve the model in larger dimensions.

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References

[1] Alamatsaz, K., Ahmadi, A., Mirzapour Al-e-hashem, S. (2022). A multi-objective model for the green capacitated location-routing problem considering drivers' satisfaction and time window with uncertain demand. *Environmental Science and Pollution Research*. 29 (4): p. 5052-5071.

[2] Almouhanna, A., Quintero-Araujo, C.L., Panadero, J., Juan, A.A., Khosravi, B., Ouelhadj, D. (2019). The location routing problem using electric vehicles with constrained distance. *Computers and Operations Research*. 115: p. 104851.

[3] Fazayeli, S., Eydi A., Kamalabadi, I.N. (2018). Location-routing problem in multimodal transportation network with time windows and fuzzy demands: Presenting a Two-Part Genetic Algorithm. *Computers and Industrial Engineering*. 119: p. 233-246.

[4] Ghaffari-Nasab, N., Jabalameli, M.S., Aryanezhad, M.B., Makui, A. (2012). Modeling and solving the bi-objective cZapacitated location-routing problem with probabilistic travel times. *International Journal of Advanced Manufacturing Technology*. P. 1–13.

[5] Golozari, F., Jafari, A., Amiri, M. (2013). Application of a hybrid simulated annealing-mutation operator to solve fuzzy capacitated location-routing problem. *International Journal of Advanced Manufacturing Technology*. 67: p. 1791–1807.

[6] Keshteli, G.R., and Nasseri, S.H. (2019). Solving flexible fuzzy multi-objective linear programming problems: Feasibility and efficiency concept of solutions. *Punjab University Journal of Mathematics*. 51(6): p. 19-31.

[7] Lopes, R.B., Plastria, F., Ferreira, C., Santos, B.S. (2014). Location-arc routing problem: heuristic approaches and test instances. *Computers and Operations Research*, 43: p. 309–317.

[8] Marinakis, Y. (2015). An improved particle swarm optimization algorithm for the capacitated location routing problem and for the location routing problem with stochastic demands. *Applied Soft Computing*. P. 680-701.

[9] Melechovsky, J., Prins, C., Wolfler Calvo, R. (2005). A metaheuristic to solve a location-routing problem with non-linear costs. *Journal of Heuristics*. 11: p. 375-391.

[10] Nadizadeh, A., Hosseini Nasab, H. (2014). Solving the dynamic capacitated location-routing problem with fuzzy demands by hybrid heuristic algorithm. *European Journal of Operational Research*. P. 458-470.

[11] Nasseri, S.H., Ghaffari-far, F. (2023). A multi-objective mathematical model for vehicle Location-routing problem with flexible fuzzy constraints. *Journal of Operational Research in Its Applications*. P. 149-169.

[12] Nasseri, S.H., Ramzannia-Keshteli, G.A. (2018). A goal programming approach for fuzzy flexible linear programming problems. *Iranian Journal of Operations Research*. 9: p. 1-28.

[13] Nasseri, S.H., et al. (2019). *A*pplication for the Flexible Linear Programming. *Solution Techniques and Applications, Studies in Fuzziness and Soft Computing*, *springer*. P. 223-232.

[14] Nasseri, S., Zavieh, H. (2018). A Multi-objective Method for Solving Fuzzy Linear Programming Based on Semi-infinite Model. *Fuzzy Information and Engineering*. 10: p. 91-98.

[15] Nedjati, A., Izbirak, G., Arkat, J. (2017). Bi-objective covering tour location routing problem with replenishment at intermediate depots: Formulation and Meta-heuristics. *Computers & Industrial Engineering*, 110: p. 458-470.

[16] Nucamendi-Guillén, S., Gómez Padilla, A., Olivares-Benitez, E. and Moreno-Vega, J. (2021). The multi-depot open location routing problem with a heterogeneous fixed fleet. *Expert Systems with Applications.* 165: p. 113846.

[17] Pekel, P., Kara, S. (2019). Solving fuzzy capacitated location routing problem using hybrid variable neighborhood search and evolutionary local search. *Applied Soft Computing Journal*. 83: p. 105665.

[18] Ponboon, S., Qureshi, A.G., Taniguchi, E. (2016). Branch-and-price algorithm for the location-routing problem with time windows. *Transportation Research Part E*. 86: p. 1–19.

[19] Salhi, S., Nagy, G. (2009). Local improvement in planar facility location using vehicle routing. *Annals of Operations Research*. 167: p. 287-296.

[20] Stenger, A., Schneider, M., Schwind, M., Vigo, D. (2012). Location routing for small package shippers with subcontracting options. *International Journal of Production Economics*. 140(2): p. 702–712.

[21] Tavakkoli-Moghaddam, R., Makui, A., Mazloomi, Z. (2010). A new integrated mathematical model for a bi-objective multi-depot location-routing problem solved by a multi-objective scatter search algorithm. *Journal of Manufacturing Systems*. 29: p. 111-119.

[22] Tavakoli-Moghaddam, R., Raziei, Z. (2016). A new bi-objective location-routing-inventory problem with fuzzy demands. *International Federation of Automatic Control*, 49: p. 1116–1121.

[23] Tirkolaee, E., Abbasian. P., Weber, G. (2021). Sustainable fuzzy multi-trip location-routing problem for medical waste management during the COVID-19 outbreak. *Science of the Total Environment*. 756: p. 143607.

[24] Zarandi, M.H.F., Hemmati, A., Davari, S. (2011). The multi-depot capacitated location-routing problem with fuzzy travel times. *Expert Systems wit Applications*. 38(8): p. 10075-10084.

[25] Zarandi, M., Hemmati, A., Davari, S., Turksen, I. (2013). Capacitated location routing problem with time windows under uncertainty. *Knowledge-Based Systems*. 37: p. 480–489.