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Fully fuzzy solid transportation problems with k-scale trapezoidal fuzzy numbers

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A transportation problem involving three constraints: source, destination, and conveyance, where all parameters of the problem are fuzzy is called Fully Fuzzy Solid Transportation Problem (FFSTP). In this paper, a new method is proposed to find an optimal solution of an unbalanced FFSTP which the fuzzy numbers are considered to be k-scale trapezoidal fuzzy numbers. The k-scale trapezoidal fuzzy numbers are a generalization of symmetric trapezoidal fuzzy numbers which are considered recently in the literature. In this method, using a new ranking method, we transform the unbalanced FFSTP into a crisp linear programming formulation and find a fuzzy optimal solution for it. The considered model is not necessary balanced and introduced method will solve that without convert it to a balanced model. The advantages of the proposed method are also discussed.

***Keywords**: Fuzzy solid transportation problem, k-scale trapezoidal fuzzy numbers, ranking methods, trapezoidal fuzzy numbers.*

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1 Introduction

An important case of linear programming problem is the Solid Transportation Problem (STP). The STP is used when there is more than one conveyance such as trucks, cargo flights, goods trains, ships, etc, to transfer goods from supplies to demands and we want to minimize the cost of transporting [Ahuja et al. (1993)]. To the best of our knowledge, Shell (1955) is the pioneer of studying TP. After that, many scholars investigated this solution concept from different standpoints; see e.g. [Haley (1962), Baidya et al. (2014), Pandian and Anuradha (2010), Pramanik et al (2013)] and the references therein. In the mentioned works, the authors have assumed that the parameters of the problem are specified in a precise way while an inevitable complexity which exists in the real problems is the impreciseness of values of coefficients of the variables in the objective functions, availability, and demand for the products. Therefore, overcoming the impreciseness, we use the fuzzy set theory introduced by Zadeh (1965). The fuzzy transportation problem and fuzzy solid transportation problem is investigated by many authors; see e.g. [Bit et al (1993), Jiménez and Verdegay (1999), Liu and Kao (2004), Gani and Razak (2006), Dinagar and Palanivel (2009), Pandian and Natarajan (2010), Kumar and Kaur (2012), Rani et al. (2015), Kocken and Sivri (2016), Nasseri et al. (2016)] and the references therein. One of very important problems which are appeared in fuzzy optimization is optimization using symmetric trapezoidal fuzzy numbers; see e.g. [Ganesan and Veeramani (2006), Christi and Kumari (2015), and the references therein. Recently, Nasseri and Khabiri (2017) introduced a generalization of symmetric

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trapezoidal fuzzy numbers named k-scale trapezoidal fuzzy numbers and fined a method to solve a fully fuzzy linear programming using k-scale trapezoidal fuzzy numbers. They showed that their method is also good when one want to solve a linear programming using symmetric trapezoidal fuzzy numbers. In this paper, we deal with an unbalanced FFSTP using k-scale trapezoidal fuzzy numbers and develop Nasseri and Khabiri (2017) method for this problem. We convert FFSTP model to a crisp linear optimization problem using a ranking function which is introduced by Nasseri and Khabiri (2017). This rankig is in fact corresponding to Hadi ranking method for trapezoidal fuzzy numbers [Nasseri (2010)]. Hadi method is developed by Nasseri (2010) for ranking trapezoidal fuzzy numbers to overcome shortcomings which are found in ranking fuzzy numbers with some convenient methods such as Asady, Chen method, Cheng distance and Chu-Tsao methods. Furthermore, the mentioned method will be useful for solving fuzzy linear programming problems by using ranking functions.

A new scoring function has been proposed for ranking INNVs, and then a Topsis method based on this function has been suggested for MAGDM problems in a neutrosophic environment. In this method, features are examined by different DMs and may have different priorities. Using neutrosophic sets , this method is capable of dealing with issues with uneven, uncertain, and imperfect information that have inseparable elements in the decision-making process[Nafei et al (2021)].A fuzzy interval linear programming model is introduced using triangular fuzzy numbers. This model is recognized as an interval Nonlinear Programming (INLP) problem. A ranking function is used to transform the INLP problem into a deterministic model, which can then be solved using standard methods. In this model, all aspects of real-life conditions are covered, including accuracy degrees, uncertainty, and falsehood [Nafei et al (2020)]. They presented a solution for solving integer programming problems in the neutrosophic triangular numbers, and transformed the integer model into a crisp model using a scoring function, enhancing its applications[Nafei and Nasseri (2019)].

The MAGDM approach presented neutrosophic autocratic to address challenges in selecting construction materials. Neutrosophic triplets are used to represent evaluation values, allowing designers to handle inconsistent and imprecise information. The autocratic method is chosen of decision makers, making it suitable for commercial and administrative concerns. this method recalculates decision-maker weights until a predetermined consensus threshold is reached[Nafei et al (2023)].

Using this method, we do not need to convert our unbalanced model to a balanced one, because it may be a so expensive process.

This paper is organized into 6 sections. In the next section, some preliminaries of fuzzy numbers, k-scale trapezoidal fuzzy numbers and Hadi method reviewed. In Section 3, we describe our model and a formulation of FFSTP is introduced. In Section 4 the new method is proposed and we illustrate this method by a numerical example in Section 5. The conclusion and some suggestion are given in Section 6.

1. Preliminaries

In this section, we provide some preliminaries. The notations and results in this section are taken from Mahdavi-Amiri and Nasseri (2007), Nasseri (2010) and Nasseri and Khabiri (2017).

Definition 2.1. A fuzzy number $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ is said to be a trapezoidal fuzzy number, if $a^L \leq a^U$ and $a^\alpha, a^\beta > 0$ and

$$\tilde{a}(x) = \begin{cases} \dfrac{x}{a^\alpha} + \dfrac{a^\alpha - a^L}{a^\alpha}, & x \in [a^L - a^\alpha, a^L] \\ 1, & x \in [a^L, a^U] \\ \dfrac{-x}{a^\beta} + \dfrac{a^U + a^\beta}{a^\beta}, & x \in [a^U, a^U + a^\beta] \end{cases} \tag{2.1}$$

The set of all trapezoidal fuzzy numbers denote by F(ℝ). A trapezoidal fuzzy number $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ is said to be non-negative if and only if $a^L - a^\alpha \geq 0$. Also, $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ is said to be symmetric if $a^\alpha = a^\beta$.

Assume that $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ and $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$ are two trapezoidal fuzzy numbers, then

i) $\tilde{a} \oplus \tilde{b} = (a^L + b^L, a^U + b^U, a^\alpha + b^\alpha, a^\beta + b^\beta)$, (2.2)

ii) $\tilde{a} \ominus \tilde{b} = (a^L - b^U, a^U - b^L, a^\alpha + b^\beta, a^\beta + b^\alpha)$, (2.3)

iii) If $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ and $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$ are two non-negative trapezoidal fuzzy numbers, then

$$\tilde{a} \otimes \tilde{b} = \big(a^L b^L, a^U b^U, a^L b^L - (a^L - a^\alpha)(b^L - b^\beta), (a^U + a^\beta)(b^U + b^\beta) - a^U b^U\big). \tag{2.4}$$

Assume that $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ and $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$ are two trapezoidal fuzzy numbers, then

$$\tilde{a} \approx \tilde{b} \Leftrightarrow a^L = b^L, a^U = b^U, a^\alpha = b^\alpha, a^\beta = b^\beta \tag{2.5}$$

Now, we review some definitions and results which are established by Nasseri (2010). Let $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$, be an arbitrary trapezoidal fuzzy number. Define

$$\underline{a} = a^m - \frac{1}{2} h_{\underline{a}}, \quad \overline{a} = a^m + \frac{1}{2} h_{\overline{a}}, \tag{2.6}$$

where $a^m = \frac{a^L + a^U}{2}$, and $h_{\underline{a}} = \frac{a^\alpha}{a^\alpha + a^\beta}$, $h_{\overline{a}} = \frac{a^\beta}{a^\alpha + a^\beta}$.

Now assume that $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ and $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$ are two trapezoidal fuzzy numbers. Let

$$\overline{R}(\tilde{a},\tilde{b}) = \overline{a} - \overline{b}, \tag{2.7}$$
$$\underline{R}(\tilde{a},\tilde{b}) = \underline{a} - \underline{b}, \tag{2.8}$$

then, we have

$$\overline{R}(\tilde{a},\tilde{b}) = -\overline{R}(\tilde{b},\tilde{a}) = \overline{R}(-\tilde{b},-\tilde{a})$$
$$\underline{R}(\tilde{a},\tilde{b}) = -\underline{R}(\tilde{b},\tilde{a}) = \underline{R}(-\tilde{b},-\tilde{a})$$

Definition 2.2. Assume that $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ and $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$ are two trapezoidal fuzzy numbers and $\underline{R}(\tilde{b},\tilde{a}) \geq 0$. Define the relations $\prec$ and $\approx$ on F(ℝ) as given below:

i) $\tilde{a} \approx \tilde{b}$ if and only if $\underline{R}(\tilde{b},\tilde{a}) = \overline{R}(\tilde{a},\tilde{b})$,
ii) $\tilde{a} \prec \tilde{b}$ if and only if $\underline{R}(\tilde{b},\tilde{a}) > \overline{R}(\tilde{a},\tilde{b})$.

We denote $\tilde{a} \preccurlyeq \tilde{b}$ if and only if $\tilde{a} \approx \tilde{b}$ or $\tilde{a} \prec \tilde{b}$. Therefore, $\tilde{a} \preccurlyeq \tilde{b}$ if and only if $\underline{R}(\tilde{b},\tilde{a}) \geq \overline{R}(\tilde{a},\tilde{b})$. Also $\tilde{b} \succ \tilde{a}$ if and only if $\tilde{a} \prec \tilde{b}$. We set $\tilde{0} \coloneqq (0,0,0,0)$ as a zero trapezoidal fuzzy numbers.

Lemma 2.1. Assume $\tilde{a} \prec \tilde{b}$, then $-\tilde{a} \succ -\tilde{b}$.
Proof. The proof is straightforward by Definition 2.2.

Nasseri (2010) showed that $\approx$ is an equivalence relation (reflexive, symmetric, and transitive) and $\preccurlyeq$ is a partial order on F(ℝ). Note that the relation $\preccurlyeq$ is a linear order on F(ℝ) too, because any two elements in F(ℝ) are comparable by this relation.

The following definitions and results are discussed by Nasseri and Khabiri (2017).
The following definition of ranking function $\mathcal{R}: F(\mathbb{R}) \to \mathbb{R}$ is corresponding to Hadi ranking, which maps each trapezoidal fuzzy number into the real line.

Definition 2.3. Assume that $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ is an arbitrary trapezoidal fuzzy number. Define:

$$\mathcal{R}(\tilde{a}) = a^L + a^U + \frac{1}{2}\left(\frac{a^\beta - a^\alpha}{a^\alpha + a^\beta}\right). \tag{2.9}$$

Note that for the symmetric trapezoidal fuzzy number, the above formula can be defined as $\mathcal{R}(\tilde{a}) = a^L + a^U$.

Theorem 2.1. Assume that $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ and $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$ are two trapezoidal fuzzy numbers and $\underline{R}(\tilde{b},\tilde{a}) \geq 0$.
i) $\tilde{a} \approx \tilde{b}$ if and only if $\mathcal{R}(\tilde{a}) = \mathcal{R}(\tilde{b})$,
ii) $\tilde{a} \prec \tilde{b}$ if and only if $\mathcal{R}(\tilde{a}) < \mathcal{R}(\tilde{b})$.

Similar to the results of Theorem 3.1, we have

$\tilde{a} \preccurlyeq \tilde{b}$ if and only if $\mathcal{R}(\tilde{a}) \leq \mathcal{R}(\tilde{b})$. (2.10)

In this paper, we deal to a special class of trapezoidal fuzzy numbers.

Definition 2.4. A trapezoidal fuzzy number $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ is said to be an k-scale trapezoidal fuzzy number if $a^\beta = ka^\alpha$ where $k \in \mathbb{N}$. The class of k-scale trapezoidal fuzzy numbers is denoted by $F_k(\mathbb{R})$.

It is clear that $F_1(\mathbb{R})$ is the class of symmetric trapezoidal fuzzy numbers. $F_1(\mathbb{R})$ is considered in many researches; see e.g. [Ganesan and Veeramani (2006)] and the references therein. Therefore, our results covered a wider class of fuzzy optimization problems.

Theorem 2.2. If $\left\{\tilde{a}_i = \left(a_i^L, a_i^U, a_i^\alpha, a_i^\beta\right)\right\}_{i=1}^m \subseteq F_k(\mathbb{R})$, then

$$\sum_{i=1}^m \mathcal{R}(\tilde{a}_i) = \mathcal{R}\left(\sum_{i=1}^m \tilde{a}_i\right) + \frac{m-1}{2}\left(\frac{k-1}{k+1}\right) \tag{2.11}$$

Remark 2.1. Now we generalize the relation (2.11) for two cases.
i) Assume that $\left\{\tilde{a}_{ij} = \left(a_{ij}^L, a_{ij}^U, a_{ij}^\alpha, a_{ij}^\beta\right)\right\}_{\substack{i=1,\dots,m\\ j=1,\dots,n}} \subseteq F_k(\mathbb{R})$, we have

$$\mathcal{R}\left(\sum_{i=1}^m\sum_{j=1}^n \tilde{a}_{ij}\right) = \mathcal{R}\left(\sum_{i=1}^m\sum_{j=1}^n a_{ij}^L, \sum_{i=1}^m\sum_{j=1}^n a_{ij}^U, \sum_{i=1}^m\sum_{j=1}^n a_{ij}^\alpha, \sum_{i=1}^m\sum_{j=1}^n a_{ij}^\beta\right)$$

$$\begin{aligned}
&= \sum_{i=1}^m\sum_{j=1}^n a_{ij}^L + \sum_{i=1}^m\sum_{j=1}^n a_{ij}^U + \frac{1}{2}\left(\frac{\sum_{i=1}^m\sum_{j=1}^n a_{ij}^\beta - \sum_{i=1}^m\sum_{j=1}^n a_{ij}^\alpha}{\sum_{i=1}^m\sum_{j=1}^n a_{ij}^\beta + \sum_{i=1}^m\sum_{j=1}^n a_{ij}^\alpha}\right)\\
&= \sum_{i=1}^m\sum_{j=1}^n a_{ij}^L + \sum_{i=1}^m\sum_{j=1}^n a_{ij}^U + \frac{1}{2}\left(\frac{\sum_{i=1}^m\sum_{j=1}^n ka_{ij}^\alpha - \sum_{i=1}^m\sum_{j=1}^n a_{ij}^\alpha}{\sum_{i=1}^m\sum_{j=1}^n ka_{ij}^\alpha + \sum_{i=1}^m\sum_{j=1}^n a_{ij}^\alpha}\right)\\
&= \sum_{i=1}^m\sum_{j=1}^n a_{ij}^L + \sum_{i=1}^m\sum_{j=1}^n a_{ij}^U + \frac{1}{2}\left(\frac{(k-1)\sum_{i=1}^m\sum_{j=1}^n a_{ij}^\alpha}{(k+1)\sum_{i=1}^m\sum_{j=1}^n a_{ij}^\alpha}\right)\\
&= \sum_{i=1}^m\sum_{j=1}^n a_{ij}^L + \sum_{i=1}^m\sum_{j=1}^n a_{ij}^U + \frac{1}{2}\left(\frac{k-1}{k+1}\right).
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\sum_{i=1}^m\sum_{j=1}^n \mathcal{R}(\tilde{a}_{ij}) &= \sum_{i=1}^m\sum_{j=1}^n a_{ij}^L + \sum_{i=1}^m\sum_{j=1}^n a_{ij}^U + \sum_{i=1}^m\sum_{j=1}^n \frac{1}{2}\left(\frac{a_i^\beta - a_i^\alpha}{a_i^\beta + a_i^\alpha}\right)\\
&= \sum_{i=1}^m\sum_{j=1}^n a_{ij}^L + \sum_{i=1}^m\sum_{j=1}^n a_{ij}^U + \sum_{i=1}^m\sum_{j=1}^n \frac{1}{2}\left(\frac{k-1}{k+1}\right)
\end{aligned}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{L} + \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{U} + \frac{mn}{2}\left(\frac{k-1}{k+1}\right)$$
$$= \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{L} + \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{U} + \frac{1}{2}\left(\frac{k-1}{k+1}\right) + \frac{mn-1}{2}\left(\frac{k-1}{k+1}\right)$$
$$= \mathcal{R}\left(\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{a}_{ij}\right) + \frac{mn-1}{2}\left(\frac{k-1}{k+1}\right).$$

Therefore

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \mathcal{R}(\tilde{a}_{ij}) = \mathcal{R}\left(\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{a}_{ij}\right) + \frac{mn-1}{2}\left(\frac{k-1}{k+1}\right). \tag{2.12}$$

ii) If $\left\{\tilde{a}_{ijl} = \left(a_{ijl}^{L}, a_{ijl}^{U}, a_{ijl}^{\alpha}, a_{ijl}^{\beta}\right)\right\}_{\substack{i=1,\dots,m\\ j=1,\dots,n\\ l=1,\dots,p}} \subseteq F_k(\mathbb{R})$, with a similar way we have

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{p} \mathcal{R}(\tilde{a}_{ijl}) = \mathcal{R}\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{p} \tilde{a}_{ijl}\right) + \frac{mnp-1}{2}\left(\frac{k-1}{k+1}\right). \tag{2.13}$$

Note that for $\tilde{a}, \tilde{b} \in F_k(\mathbb{R})$, in generally $\tilde{a} \otimes \tilde{b}$ does not belong to $F_k(\mathbb{R})$, even for k=1. To overcome this limitation for k=1, Ganesan and Veeramani (2006) have proposed a new product $\odot_{GV}$ for symmetric trapezoidal fuzzy numbers. Nasseri and Khabiri (2017). generalize their definition for k-scale trapezoidal fuzzy numbers.

Definition 2.5. Assume that $\tilde{a} = (a^L, a^U, a^\alpha, a^\beta)$ and $\tilde{b} = (b^L, b^U, b^\alpha, b^\beta)$ are two non-negative trapezoidal fuzzy numbers, we define

$$\tilde{a} \odot_{NK} \tilde{b} = \left[\left(\frac{a^L + a^U}{2}\right)\left(\frac{b^L + b^U}{2}\right) - \left(\frac{a^U b^U - a^L b^L}{2}\right), \left(\frac{a^L + a^U}{2}\right)\left(\frac{b^L + b^U}{2}\right) + \left(\frac{a^U b^U - a^L b^L}{2}\right), a^U b^\alpha + b^U a^\alpha, a^U b^\beta + b^U a^\beta\right]$$

From this definition, if $\tilde{a}, \tilde{b} \in F_k(\mathbb{R})$, then $\tilde{a} \odot_{NK} \tilde{b} \in F_k(\mathbb{R})$. It is clear that for $\lambda \geq 0$, $\lambda\tilde{a} = (\lambda a^L, \lambda a^U, \lambda a^\alpha, \lambda a^\beta)$.

2. Fully Fuzzy Solid Transportation Problem

In this section we introduce a fully fuzzy solid transportation problem. We are going to transport a homogeneous product from m sources to n destinations using several conveyances. In the following list, parameters of our model are introduced. Let $I = \{1, \dots, m\}$, $J = \{1, \dots, n\}$, and $P = \{1, \dots, p\}$.

- $\tilde{a}_i$: The fuzzy availability at ith source node.
- $\tilde{b}_j$:The fuzzy demand at jth destination node.
- $\tilde{e}_l$: The fuzzy capacity of the lth conveyance for transfer the product.
- $\tilde{c}_{ijp}$: The fuzzy penalty per unit of flow from ith source to jth destination by means of the pth conveyance.
- $\tilde{x}_{ijp}$: The fuzzy quantity of the product that should be transported from ith node to jth node by means of the pth conveyance in order to minimize the objective function.

We assume that $\tilde{a}_i$, $\tilde{b}_j$, $\tilde{c}_{ijp}$, and $\tilde{e}_p$ are non-negative k-scale trapezoidal fuzzy numbers. With these notations, an unbalanced FFSTP can be formulated into the following fuzzy linear programming problem:

$$\text{Minimum} \quad \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\left(\tilde{c}_{ijp} \odot_{NK} \tilde{x}_{ijp}\right)$$

$$\text{Subject to} \tag{2.14}$$

$$\sum_{j\in J}\sum_{p\in P}\tilde{x}_{ijp} \preccurlyeq \tilde{a}_i \qquad i \in I,$$

$$\sum_{i\in I}\sum_{p\in P}\tilde{x}_{ijp} \succcurlyeq \tilde{b}_j \qquad j \in J,$$

$$\sum_{i\in I}\sum_{j\in J}\tilde{x}_{ijp} \preccurlyeq \tilde{e}_l \qquad p \in P$$

$$\tilde{x}_{ijp} \succcurlyeq \tilde{0} \qquad i \in I, j \in J, p \in P$$

3. Proposed method

As we mentioned, we solve our model without convert it to a balanced model. In our model, there are some equalities and inequalities, and we will see, using proposed ranking function, we can convert it to a crisp linear programming.

Step 1: Assume that $\tilde{c}_{ijp} = \left(c_{ijp}^{L}, c_{ijp}^{U}, c_{ijp}^{\alpha}, \beta_{ijp}^{tl}\right)$, $\tilde{x}_{ijp} = \left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right)$, $\tilde{a}_i = \left(a_i^L, a_i^U, a_i^\alpha, a_i^\beta\right)$, $\tilde{b}_j = \left(b_j^L, b_j^U, b_j^\alpha, b_j^\beta\right)$, $\tilde{e}_p = \left(e_p^L, e_p^U, e_p^\alpha, e_p^\beta\right)$ .Therefore Problem (۲,۱4) can be written as:

$$\text{Minimum} \quad \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\left(c_{ijp}^{L}, c_{ijp}^{U}, c_{ijp}^{\alpha}, \beta_{ijp}^{tl}\right)\odot_{NK}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right)$$

Subject to (2.15)

$$\sum_{j\in J}\sum_{p\in P}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) \preccurlyeq \left(a_{i}^{L}, a_{i}^{U}, a_{i}^{\alpha}, a_{i}^{\beta}\right) \qquad i \in I$$

$$\sum_{i\in I}\sum_{p\in P}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) \succcurlyeq \left(b_{j}^{L}, b_{j}^{U}, b_{j}^{\alpha}, b_{j}^{\beta}\right) \qquad j \in J$$

$$\sum_{i\in I}\sum_{j\in J}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) \preccurlyeq \left(e_{p}^{L}, e_{p}^{U}, e_{p}^{\alpha}, e_{p}^{\beta}\right) \qquad p \in P$$

$$\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) \succcurlyeq (0,0,0,0) \qquad i \in I, j \in J, p \in P$$

Step 2. Assume that

$$\left(c_{ijp}^{L}, c_{ijp}^{U}, c_{ijp}^{\alpha}, c_{ijp}^{\beta}\right)\odot_{NK}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\beta}\right) = \left(d_{ijp}^{L}, d_{ijp}^{U}, d_{ijp}^{\alpha}, d_{ijp}^{\beta}\right).$$

With these notations, Problem (2.15) can be written as

$$\text{Minimum} \quad \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\left(d_{ijp}^{L}, d_{ijp}^{U}, d_{ijp}^{\alpha}, d_{ijp}^{\alpha}\right)$$

Subject to (2.16)

$$\sum_{j\in J}\sum_{p\in P}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) \preccurlyeq \left(a_{i}^{L}, a_{i}^{U}, a_{i}^{\alpha}, a_{i}^{\beta}\right) \qquad i \in I$$

$$\sum_{i\in I}\sum_{p\in P}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) \succcurlyeq \left(b_{j}^{L}, b_{j}^{U}, b_{j}^{\alpha}, b_{j}^{\beta}\right) \qquad j \in J$$

$$\sum_{i\in I}\sum_{j\in J}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) \preccurlyeq \left(e_{p}^{L}, e_{p}^{U}, e_{p}^{\alpha}, e_{p}^{\beta}\right) \qquad p \in P$$

$$\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) \succcurlyeq (0,0,0,0) \qquad i \in I, j \in J, p \in P$$

Step 3. Using proposed ranking function we solve the following linear programming:

$$\text{Minimum} \quad \mathcal{R}\left(\sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\left(d_{ijp}^{L}, d_{ijp}^{U}, d_{ijp}^{\alpha}, d_{ijp}^{\alpha}\right)\right)$$

Subject to (2.17)

$$\mathcal{R}\left(\sum_{j\in J}\sum_{p\in P}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right)\right) \leq \mathcal{R}\left(a_i^L, a_i^U, a_i^{\alpha}, a_i^{\beta}\right) \qquad i \in I$$

$$\mathcal{R}\left(\sum_{i\in I}\sum_{p\in P}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right)\right) \geq \mathcal{R}\left(b_j^L, b_j^U, b_j^{\alpha}, b_j^{\beta}\right) \qquad j \in J$$

$$\mathcal{R}\left(\sum_{i\in I}\sum_{j\in J}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right)\right) \leq \mathcal{R}\left(e_p^L, e_p^U, e_p^{\alpha}, e_p^{\beta}\right) \qquad p \in P$$

$$x_{ijp}^{L} - x_{ijp}^{\alpha} \geq 0 \qquad p \in P, i \in I, j \in J,$$

Step 4. Now, from (2.11), (2.12), and (2.13) we have:

$$\text{Minimum} \quad \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}\mathcal{R}\left(d_{ijp}^{L}, d_{ijp}^{U}, d_{ijp}^{\alpha}, d_{ijp}^{\alpha}\right) - \frac{mnp-1}{2}\left(\frac{k-1}{k+1}\right).$$

Subject to (2.18)

$$\sum_{j\in J}\sum_{p\in P}\mathcal{R}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) - \frac{np-1}{2}\left(\frac{k-1}{k+1}\right) \leq \mathcal{R}\left(a_i^L, a_i^U, a_i^{\alpha}, a_i^{\beta}\right) \qquad i \in I$$

$$\sum_{i\in I}\sum_{p\in P}\mathcal{R}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) - \frac{mp-1}{2}\left(\frac{k-1}{k+1}\right) \geq \mathcal{R}\left(b_j^L, b_j^U, b_j^{\alpha}, b_j^{\beta}\right) \qquad j \in J$$

$$\sum_{i\in I}\sum_{j\in J}\mathcal{R}\left(x_{ijp}^{L}, x_{ijp}^{U}, x_{ijp}^{\alpha}, x_{ijp}^{\alpha}\right) - \frac{mn-1}{2}\left(\frac{k-1}{k+1}\right) \leq \mathcal{R}\left(e_p^L, e_p^U, e_p^{\alpha}, e_p^{\beta}\right) \qquad p \in P$$

$$x_{ijp}^{L} - x_{ijp}^{\alpha} \geq 0 \qquad p \in P, i \in I, j \in J,$$

Step 4. By solving the crisp programming problem (2.18), find the fuzzy optimal solution as: $x_{ijp}^{*} = \left(x_{ijp}^{L*}, x_{ijp}^{U*}, x_{ijp}^{\alpha*}, x_{ijp}^{\beta*}\right)$.

Step 5. Find the fuzzy optimal value of objective function by putting the values of $x^*_{ijp} = \left(x^{L*}_{ijp}, x^{U*}_{ijp}, x^{\alpha*}_{ijp}, x^{\beta*}_{ijp}\right)$ in $\sum_{i\in I}\sum_{j\in J}\sum_{p\in P}(\tilde{c}_{ijp} \otimes \tilde{x}_{ijp})$.

Remark 4.1. When FFSTP is balanced, i.e.

$$\text{Minimum} \quad \sum_{i\in I}\sum_{j\in J}\sum_{p\in P}(\tilde{c}_{ijp} \odot_{NK} \tilde{x}_{ijp})$$

Subject to (2.19)

$$\sum_{j\in J}\sum_{p\in P}\tilde{x}_{ijp} = \tilde{a}_i \qquad i \in I,$$

$$\sum_{i\in I}\sum_{p\in P}\tilde{x}_{ijp} = \tilde{b}_j \qquad j \in J,$$

$$\sum_{i\in I}\sum_{j\in J}\tilde{x}_{ijp} = \tilde{e}_l \qquad p \in P$$

$$\sum_{i\in I}\tilde{a}_i = \sum_{j\in J}\tilde{b}_j$$

$$\tilde{x}_{ijp} \succcurlyeq \tilde{0} \qquad i \in I, j \in J, p \in P$$

We can use a similar way to solve the problem.

4. Illustrative example

In this section, we illustrate our method with an example. In this example we consider a symmetric balanced fuzzy transportation problem introduced by Christi and Kumari (2015). In [Christi and Kumari (2015)], the authors assume that the fuzzy numbers are symmetric trapezoidal fuzzy number, however, some numbers are not such that. We correct data and solve the problem with our method.

Example 5.1. Consider following data:

	D_1	D_2	D_3	D_4	Supply
S_1	(1,2,3,3)	(1,3,4,4)	(9,11,12,12)	(5,7,8,8)	(1,6,7,7)
S_2	(0,1,2,2)	(-1,0,1,1)	(5,6,7,7)	(0,1,2,2)	(1,2,3,3)
S_3	(3,5,6,6)	(5,8,9,9)	(12,15,16,16)	(7,9,10,10)	(5,10,12,12)
Demand	(5,7,8,8)	(1,5,6,6)	(1,3,4,4)	(1,2,3,3)	

Christi and Kumari (2015) considered the following rank function (note that a symmetric trapezoidal fuzzy number is as $\tilde{a} = (a^L, a^U, c, c)$):

$$R(\tilde{a}) = \int_0^1 0.5(ct - (c - a^L), (a^U + c) - ct)dt.$$

Rank function $\mathcal{R}(\tilde{a})$ in Definition 2.3, for a symmetric trapezoidal fuzzy number as $\tilde{a} = (a^L, a^U, c, c)$ is $\mathcal{R}(\tilde{a}) = a^L + a^U$ which is easier to compute and also is a linear function. For these data we have

$$\mathcal{R}(1,6,7,7) + \mathcal{R}(1,2,3,3) + \mathcal{R}(5,10,12,12) = 25$$

and

$$\mathcal{R}(5,7,8,8) + \mathcal{R}(1,5,6,6) + \mathcal{R}(1,3,4,4) + \mathcal{R}(1,2,3,3) = 25$$

therefore, the problem is balanced.
We must solve the following problem:

$$\begin{aligned}\text{Minimum }\ & (1,2,3,3)\odot_{NK}\tilde{x}_{11} + (1,3,4,4)\odot_{NK}\tilde{x}_{12} + (9,11,12,12)\odot_{NK}\tilde{x}_{13} \\ & + (5,7,8,8)\odot_{NK}\tilde{x}_{14} + (0,1,2,2)\odot_{NK}\tilde{x}_{21} + (-1,0,1,1)\odot_{NK}\tilde{x}_{22} \\ & + (5,6,7,7)\odot_{NK}\tilde{x}_{23} + (0,1,2,2)\odot_{NK}\tilde{x}_{24} + (3,5,6,6)\odot_{NK}\tilde{x}_{31} \\ & + (5,8,9,9)\odot_{NK}\tilde{x}_{32} + (12,15,16,16)\odot_{NK}\tilde{x}_{33} + (7,9,10,10)\odot_{NK}\tilde{x}_{34}\end{aligned}$$

Subject to

$$\sum_{j=1}^{4} \tilde{x}_{1j} = (1,6,7,7), \quad \sum_{j=1}^{4} \tilde{x}_{2j} = (1,2,3,3), \sum_{j=1}^{4} \tilde{x}_{3j} = (5,10,12,12)$$

$$\sum_{i=1}^{3} \tilde{x}_{i1} = (5,7,8,8), \quad \sum_{i=1}^{3} \tilde{x}_{i2} = (1,5,6,6), \quad \sum_{i=1}^{3} \tilde{x}_{i3} = (1,3,4,4), \quad \sum_{i=1}^{3} \tilde{x}_{i4} = (1,2,3,3)$$

$$\tilde{x}_{ij} \succcurlyeq \tilde{0} \qquad i = 1,2,3, j = 1, \dots ,4.$$

Assume that $\tilde{x}_{ij} = \left(x_{ij}^L, x_{ij}^U, c_{ij}, c_{ij}\right)$, then we have

$$\begin{aligned}\text{Minimum }\ & \left(\frac{5x_{11}^L}{4} - \frac{x_{11}^U}{4}, \frac{x_{11}^L}{4} + \frac{7x_{11}^U}{4}, 3x_{11}^U + 2c_{11}, 3x_{11}^U + 2c_{11}\right) \\ & + \left(\frac{3x_{12}^L}{2} - \frac{x_{12}^U}{2}, \frac{x_{12}^L}{2} + \frac{5x_{12}^U}{2}, 4x_{12}^U + 3c_{12}, 4x_{12}^U + 3x_{12}^{\beta}\right) \\ & + \left(\frac{19x_{13}^L}{2} - \frac{x_{13}^U}{2}, \frac{x_{13}^L}{2} + \frac{21x_{13}^U}{2}, 12x_{13}^U + 11c_{13}, 12x_{13}^U + 11c_{13}\right) \\ & + \left(\frac{11x_{14}^L}{2} - \frac{x_{14}^U}{2}, \frac{x_{14}^L}{2} + \frac{13x_{14}^U}{2}, 8x_{14}^U + 7c_{14}, 8x_{14}^U + 7c_{14}\right)\end{aligned}$$

$$+\left(\frac{x_{21}^L}{4}-\frac{x_{21}^U}{4},\frac{x_{21}^L}{4}+\frac{3x_{21}^U}{4},2x_{21}^U+c_{21},2x_{21}^U+c_{21}\right)+\left(-\frac{3x_{22}^L}{4}-\frac{x_{22}^U}{4},\frac{x_{22}^L}{4}-\frac{x_{22}^U}{4},x_{22}^U,x_{22}^U\right)$$
$$+\left(\frac{21x_{23}^L}{4}-\frac{x_{23}^U}{4},\frac{x_{23}^L}{4}+\frac{23x_{23}^U}{4},7x_{23}^U+6c_{23},7x_{23}^U+6c_{23}\right)$$
$$+\left(\frac{x_{24}^L}{4}-\frac{x_{24}^U}{4},\frac{x_{24}^L}{4}+\frac{3x_{24}^U}{4},2x_{24}^U+c_{24},2x_{24}^U+c_{24}\right)$$
$$+\left(\frac{7x_{31}^L}{2}-\frac{x_{31}^U}{2},\frac{x_{31}^L}{2}+\frac{9x_{31}^U}{2},6x_{31}^U+5c_{31},6x_{31}^U+5c_{31}\right)$$
$$+\left(\frac{23x_{32}^L}{4}-\frac{3x_{32}^U}{4},\frac{3x_{32}^L}{4}+\frac{29x_{32}^U}{4},9x_{32}^U+8c_{32},9x_{32}^U+8c_{32}\right)$$
$$+\left(\frac{51x_{33}^L}{4}-\frac{3x_{33}^U}{4},\frac{3x_{33}^L}{4}+\frac{57x_{33}^U}{4},16x_{33}^U+15c_{33},16x_{33}^U+15c_{33}\right)$$
$$+\left(\frac{15x_{34}^L}{2}-\frac{x_{34}^U}{2},\frac{x_{34}^L}{2}+\frac{17x_{34}^U}{2},10x_{34}^U+9c_{34},10x_{34}^U+9c_{34}\right)$$

Subject to

$$\sum_{j=1}^{4}\tilde{x}_{1j}=(1,6,7,7),\quad \sum_{j=1}^{4}\tilde{x}_{2j}=(1,2,3,3),\sum_{j=1}^{4}\tilde{x}_{3j}=(5,10,12,12)$$
$$\sum_{i=1}^{3}\tilde{x}_{i1}=(5,7,8,8),\quad \sum_{i=1}^{3}\tilde{x}_{i2}=(1,5,6,6),\quad \sum_{i=1}^{3}\tilde{x}_{i3}=(1,3,4,4),\quad \sum_{i=1}^{3}\tilde{x}_{i4}=(1,2,3,3)$$

$$\tilde{x}_{ij}\succcurlyeq\tilde{0}\qquad i=1,2,3,j=1,\dots,4.$$

Now, using rank function $\mathcal{R}$, we solve the following problem

$$\text{Minimum}\quad \frac{3x_{11}^L}{2}+\frac{3x_{11}^U}{2}+2x_{12}^L+2x_{12}^U+10x_{13}^L+10x_{13}^U+6x_{14}^L+6x_{14}^U+\frac{x_{21}^L}{2}+\frac{x_{21}^U}{2}$$
$$-\frac{x_{22}^L}{2}-\frac{x_{22}^U}{2}+\frac{11x_{23}^L}{2}+\frac{11x_{23}^U}{2}+\frac{x_{24}^L}{2}+\frac{x_{24}^U}{2}+4x_{31}^L+4x_{31}^U+\frac{13x_{32}^L}{2}$$
$$+\frac{13x_{32}^U}{2}+\frac{27x_{33}^L}{2}+\frac{27x_{33}^U}{2}+8x_{34}^L+8x_{34}^U$$

Subject to

$$\sum_{j=1}^{4} x_{1j}^{L} + \sum_{j=1}^{4} x_{1j}^{U} = 7, \quad \sum_{j=1}^{4} x_{2j}^{L} + \sum_{j=1}^{4} x_{2j}^{U} = 3, \sum_{j=1}^{4} x_{3j}^{L} + \sum_{j=1}^{4} x_{3j}^{U} = 15$$
$$\sum_{i=1}^{3} x_{i1}^{L} + \sum_{i=1}^{3} x_{i1}^{U} = 12, \quad \sum_{i=1}^{3} x_{i2}^{L} + \sum_{i=1}^{3} x_{i2}^{U} = 6, \quad \sum_{i=1}^{3} x_{i3}^{L} + \sum_{i=1}^{3} x_{i3}^{U} = 4,$$
$$\sum_{i=1}^{3} x_{i4}^{L} + \sum_{i=1}^{3} x_{i4}^{U} = 3$$

$$x_{ij}^{L} - c_{ij} \geq 0 \qquad i = 1,2,3, j = 1, \ldots ,4.$$

After solving this problem, we have:
$$x_{12}^{L} = 6, x_{13}^{L} = 1, x_{23}^{L} = 3, x_{31}^{L} = 12, x_{34}^{L} = 3$$
and optimal value is 110.5.

5. Conclusion

In this paper, we considered a fully fuzzy solid transportation problem which there is more than one conveyance to transport goods from sources to destinations. We proposed a new method to solve k-scale fully fuzzy transportation problems. Our method also can solve symmetric fully fuzzy transportation problems. In this method we do not need to convert the problem into balanced one.

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