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Pricing in a dual channel closed-loop supply chain: A game theory approach

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In this paper the pricing of reverse products in a two-level closed-loop supply chain is considered and a game theory approach is used to solve it. Pricing is a sensitive and vital issue for businesses. In the market of reverse products, this issue will be much more difficult and complex due to difficulties associated with collecting and re-manufacturing processes. On the other hand, the use of the Internet and direct channels for collecting products from customers alongside traditional retailers is an important issue that requires management and coordination. The proposed price for buying second-hand and defective products from customers should be high enough to convince them that returning the products has more benefits than discarding or keeping them at home. At the same time, the price should be low enough to make it economically viable for producers to carry out the repair and re-manufacturing operations and resell them in the direct supply chain for the producer. The use of game theory, where the decisions of one player affect the decisions and outcomes of other players as well as their outcomes, is a suitable method for solving the problem of pricing reverse products in a two-level closed-loop supply chain. In this study, an attempt has been made to encourage the attention of business owners to invest in recycling.

Keywords: *Pricing, closed loop supply chain, dual channel, game theory*

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1. Introduction

According to the United Nations report in 2020, the trend of using electronic devices is on the rise. Therefore, electronic waste management has become a serious issue in today's technology-dependent world. 54.7% of households worldwide have access to the internet as of 2021. It can be seen that policies and regulations regarding e-waste play an important role in the performance of stakeholders who are involved with these wastes in News [23]. It has been estimated since 2005 that the production of electronic and electrical waste in the world will increase annually by 5%, which is three times higher than the production rate of other types of waste in Rahmani et al [12]. Studies have shown that electronic waste such as televisions and monitors releases 718,000 tons of lead and 287 tons of mercury per year when buried in landfills in Santos et al [15]. These heavy metals penetrate the soil, rivers, and groundwater, causing pollution. Also, burning these wastes releases hazardous gases such as carbon monoxide, sulfur dioxide, and dioxins, which are very harmful to health if inhaled. Cao et al [3]. Despite the mentioned drawbacks, these wastes are highly valuable, and their recycling can have a significant impact on the economy of countries. According to a study conducted in 2017, e-waste contains rich reserves of gold, silver, copper, platinum, palladium, and other high-value recyclable materials, with an estimated total value of $55 billion, which is more than the gross domestic product of most countries in the world in News [23]. With the rapid advancement of the internet and the popularity of e-

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commerce, large companies and websites have launched e-waste collection programs. However, due to the lack of advertising campaigns and the neglect of governments, they have not been widely used 4 Online markets make it easier for customers to access prices, resulting in reduced production and distribution costs. Therefore, considering the extraordinary ease that the internet and e-commerce have brought about, this issue can be seen as an opportunity to encourage customers to return used products and help with recycling and re-manufacturing. Regarding the inexperience of managers in pricing in online markets, determining the price of products and services is much more sensitive and crucial in Kotler and Keller [9]. Therefore, there is a strong need for a novel approach to pricing that considers the overall benefits of the supply chain. This paper aims to determine the price of buying products from customers in both retail and online markets (manufacturer's website), under the following scenario based on game theory. The price of returned products is investigated under non-cooperative equilibrium conditions in a scenario where the manufacturer and retailer have equal decision-making power, and strive to optimize their profits simultaneously and independently.

Closed-loop supply chains are a type of supply chain that includes a closed loop for collecting and recycling products. If both forward and reverse flows are considered simultaneously in a model, it is called a closed-loop supply chain network. With the increasing importance of issues such as reducing the use of raw materials and protecting the environment, manufacturers have a greater tendency to collect their used product waste. One of the advantages of these supply chains is cost savings in production which results in reducing the production costs. From a logistics perspective, reuse leads to material flow from the consumer back to the manufacturer. Closed-loop supply chain solutions provide the best strategy for reducing consumption and reusing materials in Atasuet al [1].

Refurbished products refer to products that require cleaning, a new layer of paint, new parts, and repairs, such as refurbished smartphones that come with a warranty. Refurbished products, which are mostly used in the electronics and electrical industry, are products that have been returned to the seller or manufacturer for various reasons such as being defective or not being sold in the market due to introducing a new product and are inspected for functionality and to be flawless before being resold by the original manufacturer. In the process of remanufacturing recycled components, usable product components, and modules are reused in the production process, such as recycled electronic and electrical equipment. Recycling is done at the level of raw materials, in which no part of the product is preserved, but the raw materials used to make the product are recycled in Van Engeland et al [20].

As shown in Figure 1, reuse, repair, and refurbishment are done on the final product; remanufacturing, production, and recycling are done on the components, parts, and modules, and final refining and disposal are done on the raw materials.

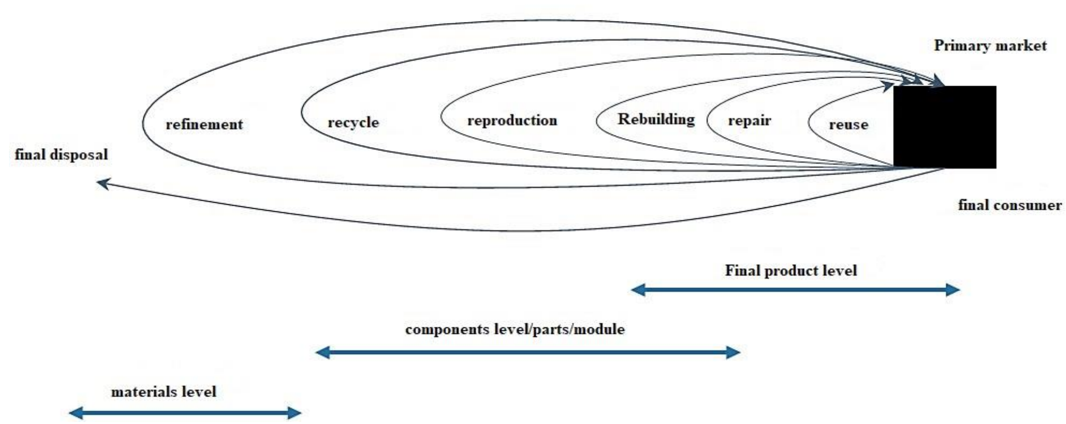

Figure 1: Types of returns in a closed-loop supply chain

Since the electronics industry is a high-tech industry, it undergoes frequent changes and as a result, the products lifecycle in this industry is very short. Also, industrial controllers are no exception to this rule, and old controllers will lose their functionality over time. However, by replacing some of the old parts of the controllers and replacing them with new parts or performing refurbishment operations, they can be resold. On the other hand, these products consist of many electronic components; therefore, a defective controller has numerous usable parts that can be reused in the production of new controllers after inspection and ensuring that there are no operational issues. Therefore, in the case of this research, products are returned to the manufacturer for refurbishment.

Considering the return rate as a dependent variable on price is a suitable method for comparing the performance of retail and online channels, which has been considered in this paper. As mentioned in the problem statement, many well-known electronic equipment manufacturing companies have realized that by collecting electronic waste and using it in the process of remanufacturing, they can not only help the environment and reduce the harmful effects of this waste but also greatly reduce their extra costs. This research aims to demonstrate the necessity of using e-commerce infrastructure in collecting defective products from customers by simultaneously employing both retail and online collection channels and comparing the return rates of these two channels. In addition to using the online collection channel, this research intends to encourage retailers to collect as many defective products as possible from customers and collaborate more with the manufacturer by employing game theory and especially cost-sharing contracts. It should be noted that most articles that have examined cost-sharing contracts have used them in the green supply chain to share the cost of greening the product, which is the responsibility of the manufacturer.

The confrontation between economic entities in various aspects of the supply chain leads to the emergence of conflicting goals among the members of the chain. Issues such as those in which two or more decision-makers have conflicting goals can be resolved by using game theory. Game theory deals with situations that involve both conflict and cooperation. In general, situations in which there is mutual dependence between the actions of players are called games. In this way, each player's action leads to a positive or negative reaction from the other player. In other words, in a game, the consequences for players (profit, income, desirability, etc.) are a function of the actions of other players. Three conditions are necessary for a game to take shape in Haugen and Nilsen [6]:

1. There must be at least two players in the game.
2. Players in the game have conflicting interests.
3. Each player strives to gain more desirability for him(her)self, but a player's victory or defeat is not only a function of his(her) own efforts, but also depends on the opponent's actions.

Game theory attempts to model behaviors and rules governing a game mathematically to ultimately achieve the final goal of finding an equilibrium strategy for the players. The rest of the paper is organized as follows. Section 2 is devoted to reviewing literature and identifying research gaps and innovation. In Section 3, the problem modeling as a game theory and calculating the best solutions and executing games are presented. In Section 4 the sensitivity analysis of the models is investigated. The last section ends the paper with conclusion and future research directions.

2. Review and research background

Although the discussion of the closed-loop supply chain has been developed for a long time, theoretical research in this area is rapidly increasing. Until 2022, a large number of studies have focused on examining closed-loop supply chain models, pricing, and contract design in Wu et al [22]. Table 1 shows researches which have investigated the pricing issue in the supply chain.

Table 1. A comprehensive review of the existing literature in the area of supply chain pricing

row	Validation method	Pricing method	The type of return channel	levels	Supply chain type	Decision variables	reference
1	Numerical examples	Game theory	Unknown	2	Closed loop	Price and optimal decisions	Taleizadeh et al [19]
2	Numerical examples	Game theory	retailer	3	Closed loop	Price, product quality, Amount of fundraising efforts	Taleizadeh et al [17]
3	Numerical examples	Game theory	×	2	Green	Price and carbon release level	Taleizadeh et al [16]
4	Numerical examples	Game theory	retailer	2	Closed loop	price and service level	Xie et al [24]
5	Numerical examples	Game theory	×	2	Green	Price, level of greenness, level of retailer effort	Ranjan and Jha[14]
6	Numerical examples	Dynamic game theory	Producer, retailer	3	Closed loop	price and rate of return	Atasu et al [1]
7	Numerical examples	Game theory	×	2	Closed loop	price and rate of return	Liu and Xiao[10]
8	Case study	Game theory	×	2	Green	price	Parsaeifar et al[11]
9	Numerical examples	Game theory	Dual	2	Closed loop	price and rate of return	Yang et al [25]
10	Experiment design	Game theory	Producer	2	Closed loop	price and rate of return	Wen et al [21]
11	Experiment design	Game theory	Producer	2	Closed loop	price and rate of return	Govindan and Nicoleta[4]
12	Numerical examples	Game theory	Dual	3	×	Profit, Demand of product, Quality	Jabarzare and Rasti-Barzoki[7]

13	Case study	Game theory	×	×	Closed loop	price	Kazi and FaruqueHasan[8]
14	Numerical experment	Game theory	Retailer, Dual	2	×	price	Zhao and Li [26]
15	Case study and sensitivity analysis	Game theory	Dual	2	Closed loop	price and rate of return	present research

In the study given blow, a game theory approach was employed to dynamically solve the model. The lower-level optimal values (pertaining to retailers and suppliers) were computed based on the higher-level values (pertaining to the manufacturer), effectively transforming the multi-level model into a single-level model for the calculation of higher-level optimal values in Beiranvand and Davoodi [2]. The research introduced a government-backed agri-food supply chain model (GBASM) among the farmers and the agricultural enterprises. The government provides subsidies In GBASM, to encourage collaboration among supply chain members on a shared economic platform. A game-theoretic perspective has been established among the farmers and the agricultural enterprises to assess their participation in the new model for economic benefits. A Nash game is proposed, assuming rational actors (the farmers and the agricultural enterprises) cooperate on market pricing within GBASM in Hamidoglu [5]. Investigating pricing decisions in closed-loop supply chains (CLSCs) considering supply disruptions, the study has focused on two CLSCs. One involved a retailer and two internal and external suppliers to address supply disruptions, while the other benefited from a single integral supplier and a retailer. Decision variables were derived through a game-theoretic approach in two interdependent levels of competition. The first level involved two Stackelberg games and one simultaneous moving game between retailers, while the second level utilized one Stackelberg game for each supply chain, involving competition between retailers and suppliers in Rajabzadeh et al[13].

Research studies can be classified based on a variety of criteria. The mentioned research, concentrating on pricing returned products in a closed-loop supply chain within the electrical and electronic product manufacturing sector, in terms of its objective, is considered applied research. Given its focus on pricing and the use of modeling as an analytical tool, it is categorized as quantitative research in terms of data type.

3. Mathematical Model Design

The supply chain includes a producer and a retailer through two retail channels (conventional channel) and a direct channel (producer's website) to collect used electronic devices or industrial controllers. Customers return the products for various reasons such as being defective, end of life, and obsolescence. A game between these two channels takes place regarding the return of products from customers. The proposed price for returned products not only affects the return rate of the same channel but also affects the return rate of the other channel. Products that meet the necessary standards from the perspective of the retailer and the producer are purchased from the customer and are sold again after undergoing the necessary production processes. In this case, the retailer and the producer have equal power in decision-making, and the conditions for non-cooperative equilibrium between the players are established. Figure 2 shows the general formulation of the problem along with the desired parameters and variables.

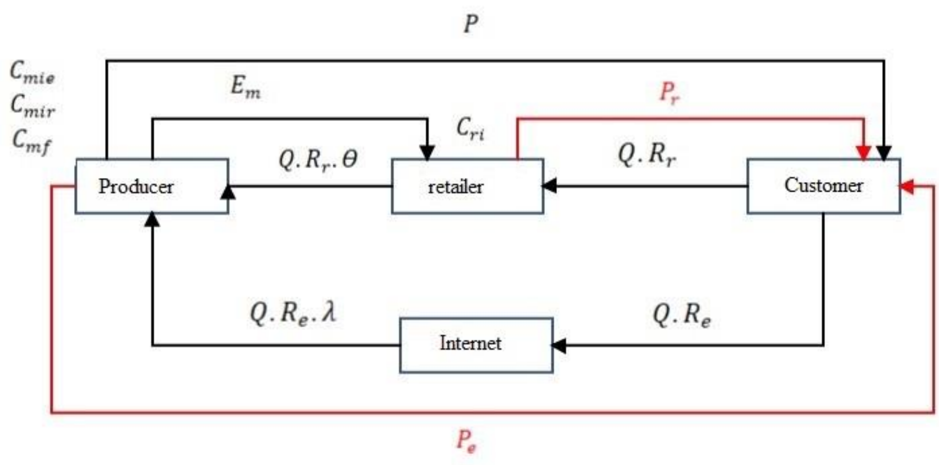

Figure2: Schematic view of the supply chain under study along with the variables and parameters considered.

Table 2 shows the symbols used in this study.

Table 2: Research Model Symbolism

row	symbols	function/ variable/ parameter	Description
1	E_m	Variable	The cost paid by the manufacturer to the retailer for each unit of product returned
2	C_{ri}	Variable	Inspection cost at the retail location per unit of returned product
3	θ	Variable	Acceptance rate in the traditional channel
4	λ	Variable	Acceptance rate in the Internet channel
5	C_{mie}	Variable	Manufacturer's inspection cost for each product unit returned through the Internet channel.
6	C_{mir}	Variable	Manufacturer's inspection cost for each product unit returned through the traditional channel.
7	Q	Variable	Number of new products sold directly in the supply chain after remanufacturing.
8	P	Variable	The selling price of each unit of products that are remanufactured and sold by the manufacturer.
9	C_{mf}	Variable	The cost of reproducing each product unit for the manufacturer
10	P_i^{max}	Variable	The maximum price offered to customers for which all

			customers return their defective products . $(i = r, e)$
11	$\alpha_1 = \frac{1}{P_r^{max}}$	Variable	Direct impact on the retail channel
12	α_2	Variable	The effect of P_r on the return rate of the Internet channel.
13	$\alpha_2' = \frac{\alpha_2}{P_r^{max}} = \alpha_1 \times \alpha_2$	Variable	Interaction effects of P_r on internet channel return rate
14	$\beta_1 = \frac{1}{P_e^{max}}$	Variable	Direct impact on the Internet channel
15	β_2	Variable	The effect that P_e has on the return rate of the retail channel.
16	$\beta_2' = \frac{\beta_2}{P_e^{max}} = \beta_1 \times \beta_2$	Variable	The interaction effects of P_e on retail channel return rate
17	P_r	Variable	The price that the retailer offers to customers for each unit of returned product. (Considering the cost of reproduction)
18	P_e	Variable	The price that the manufacturer offers to customers for each returned product unit. (Considering the cost of reproduction)
19	μ_1	Variable	The retailer's share of the cost-sharing contract
20	μ_2	Variable	Manufacturer's share in cost sharing contract
21	R_r	Function	The rate of return of the retail channel (it is a function of the prices offered to customers $0 \prec R_r \prec 1$ in both channels
22	R_e	Function	Return rate of the Internet channel (it is a function of the prices offered to customers , $0 \prec R_e \prec 1$in both channels)
23	π_{SC}	Function	Supply chain profit
24	π_R	Function	Retailer profit
25	π_M	Function	Producer profit

The model assumptions are as follows:

- The supply chain is a closed-loop, two-tier system consisting of a single producer and retailer that employs offline and online retail channels for product collection.
- Reverse pricing is the focus, thus a return rate concept is defined for both channels as a function of their offered prices. The return rate function is assumed to be linear.

- All games are analyzed under complete information access in static, dynamic, collaborative, and non-collaborative modes.
- Rational behavior is assumed for both the retailer and producer.
- The selling price is for remanufactured products sold directly to customers in the supply chain. It is assumed that the producer sells these products to the retailer or customer at the same price.

The investigated supply chain here is composed of only one retailer. The profit of this retailer (TR_R) is obtained from the difference between its revenue and costs (TC_R) according to equation 1:

$$\pi_R = TR_R - TC_R \tag{1}$$

The revenue function of the retailer is according to equation 2:

$$TR_R = R_r \cdot \theta \cdot Q \cdot E_m \tag{2}$$

,where E_m is the amount paid by the producer to the retailer for each unit of returned product, θ is the acceptance rate in the traditional channel, which represents the percentage of returned products that are suitable for re-entering the remanufacturing process from the perspective of the retailer, Q is the number of remanufactured products that are resold by the producer after the remanufacturing process, R_r is the return rate of the traditional channel, which shows the customers' willingness to return products through the retail channel.

The total retail cost function includes the following components, which are as equation (3)

$$TC_R = R_r \cdot \theta \cdot Q \cdot P_r + R_r \cdot Q \cdot C_{ri} = R_r \cdot \theta \cdot Q\left(P_r + \frac{C_{ri}}{\theta}\right) \tag{3}$$

,where P_r is the cost that the retailer offers to customers for each unit of returned product and C_{ir} is the inspection cost that the retailer incurs for each unit of returned product.

After differentiating these two functions, we obtain equation (4), which represents the derivative function of the sales function.

$$\pi_R = R_r \cdot \theta \cdot Q \cdot E_m - \left(R_r \cdot \theta \cdot Q\left(P_r + \frac{C_{ri}}{\theta}\right)\right)_{\rightarrow} \pi_R = R_r \cdot \theta \cdot Q\left[E_m - P_r - \frac{C_{ri}}{\theta}\right] \tag{4}$$

The noteworthy point is that R_r is a function of the proposed price of the retail channel P_r and the proposed price of its competitor P_e, which is the online channel. The return rate of the retail channel represents customers' willingness to return defective goods through this channel. The return rate function of the retail channel is defined according to equation (5)

$$R_r = \frac{P_r}{P_r^{max}} - \beta_2\left(\frac{P_e}{P_e^{max}}\right) = \alpha_1 \cdot P_r - \beta_2' \cdot P_e \tag{5}$$

$$\uparrow P_e^{max} \beta_1 = \underset{\rightarrow}{\frac{1}{P_e^{max}}} \downarrow \beta_1 \rightarrow \uparrow \beta_2 \beta_2' = \underset{\rightarrow}{\beta_1} \times \beta_2 ? \beta_2' \tag{6}$$

According to (6), increasing the maximum price offer to the customer decreases the internet channel's offer price and the return rate of its channel (i.e. β_1). On the other hand, this increase will raise the return rate of the competitor channel, i.e. the retail channel, β_2, so changing the parameter β_2' will be uncertain. If the effect of increasing β_2 is greater than the effect of decreasing β_1, then the effect of the Internet channel price on the return rate of the retail channel is greater than the effect of this price on the return rate of its own channel, and as a result, increasing the value of β_2 may decrease the return rate of the retail channel. In equation (7), α_1 represents the direct effect on the retail channel (the effect of the retailer's suggested price P_r on the return rate of the retail channel), β_2 represents the effect of P_e on the return rate of the competitor's channel (retail), β_2' also reflects the reciprocal effects of the price offered by the competitor channel (Internet) on the return rate of the retail channel, and it is obtained from dividing β_2 by P_e^{max} (the highest price offered to customers by the Internet channel, for which all customers of their defective products are returned).

$$\alpha_1 = \frac{1}{P_r^{max}} \left\{ \begin{aligned} \beta_2' &= \frac{\beta_2}{P_e^{max}} \\ \beta_1 &= \frac{1}{P_e^{max}} \end{aligned} \right\} \rightarrow \beta_2' = \beta_2 \times \beta_1 \tag{7}$$

Therefore, the general model of the retailer, taking into account the limitations mentioned in the model assumptions section, is as Model (8).

$$\begin{aligned} Max.\quad & \pi_R = R_r \cdot \theta \cdot Q \left[E_m - P_r - \frac{C_{ri}}{\theta} \right] \\ s.t.\quad & R_r = \alpha_1 \cdot P_r - \beta_2' \cdot P_e \\ & 0 \prec \beta_2' \prec \alpha_1 \\ & 0 \prec \theta \prec 1 \\ & 0 \prec R_r \prec 1 \\ & 0 \prec P_r \\ & 0 \prec P_e \end{aligned} \tag{8}$$

Also, the investigated supply chain consists of only one producer, whose outcome function is obtained from the difference between the producer's revenues TR_M and his costs TC_M as in equation (9):

$$\pi_M = TR_M - TC_M \tag{9}$$

Since the producer is involved in both traditional and direct channels, the income of both channels should be considered, i.e. producer's income in traditional channel, TR_{Mr} and producer's income in direct channel, TC_{Me}. The producer's income in both channels is from the resale of products that have been remanufactured after returning. P is the selling price of each unit of products which is remanufactured and sold again by the manufacturer. Equation (10) shows the producer's income in different channels:

$$\begin{aligned} TR_{Mr} &= R_r \cdot \theta \cdot Q \cdot P \\ TR_{Me} &= R_e \cdot \lambda \cdot Q \cdot P \end{aligned} \tag{10}$$

Therefore, the general function of the producer's income is obtained from the sum of the above two functions as equation (11)

$$\begin{aligned} TR_M &= TR_{Mr} + TR_{Me} \\ &= R_r \cdot \theta \cdot Q \cdot P + R_e \cdot \lambda \cdot Q \cdot P = Q \cdot P \cdot (R_r \cdot \theta + R_e \cdot \lambda) \end{aligned} \tag{11}$$

The producer in the traditional channel bears the following costs: The remanufacturing cost for each item C_{mf}, the cost paid to the retailer for each returned product unit E_m, the inspection fee for returned products at the retail location for acceptance or non-acceptance, C_{mir}. In the direct channel, the producer bears the following costs: The remanufacturing cost for each item C_{mf}, the cost paid to the customer for each unit of returned product P_e, the inspection fee for returned products through the manufacturer's website C_{mie}.

λ represents the acceptance rate of the direct channel, i.e. the percentage of the returned products which are in accordance with the standards available on the manufacturer's website, suitable for entering the remanufacturing process, Q is the number of remanufactured and sold products by the manufacturer after this process. Equation (12) shows the producer's costs in these two channels:

$$\begin{aligned} TC_{Mr} &= Q \cdot C_{mf} \cdot \theta \cdot R_r + Q \cdot E_m \cdot \theta \cdot R_r + Q \cdot C_{mir} \cdot \theta \cdot R_r TC_{Me} \\ &= Q \cdot C_{mf} \cdot \lambda \cdot R_e + Q \cdot P_e \cdot \lambda \cdot R_e + Q \cdot C_{mie} \cdot R_e \end{aligned} \tag{12}$$

The producer's total costs also consist of the total costs of the traditional and the direct channels. Equation (13) shows the total cost of the producer:

$$\begin{aligned} TC_M = \big(Q \cdot C_{mf} \cdot \theta \cdot R_r + Q \cdot E_m \cdot \theta \cdot R_r + Q \cdot C_{mir} \cdot \theta \cdot R_r\big) \\ + \big(Q \cdot C_{mf} \cdot \lambda \cdot R_e + Q \cdot P_e \cdot \lambda \cdot R_e + Q \cdot C_{mie} \cdot R_e\big) Q \cdot \theta \\ \cdot R_r\big(C_{mf} + E_m + C_{mir}\big) + Q \cdot \lambda \cdot R_e \left(C_{mf} + P_e + \frac{C_{mie}}{\lambda}\right) \end{aligned} \tag{13}$$

Differentiating these two functions, leads to equation (14), which represents the producer's outcome function:

$$\begin{aligned} \pi_M &= Q \cdot P \cdot (R_r \cdot \theta + R_e \cdot \lambda) \\ &\quad - \left[Q \cdot \theta \cdot R_r\big(C_{mf} + E_m + C_{mir}\big) + Q \cdot \lambda \cdot R_e \left(C_{mf} + P_e + \frac{C_{mie}}{\lambda}\right)\right] \to \pi_M \\ &= Q\left[\big(P - C_{mf} - E_m - C_{mir}\big) \cdot \theta . R_r + \left(P - C_{mf} - P_e - \frac{C_{mie}}{\lambda}\right) \cdot \lambda \cdot R_e\right] \end{aligned} \tag{14}$$

Internet channel return rate R_e indicates the desire of customers to return defective goods through the Internet channel and according to equation (15), it is a function of the channel's suggested price P_e and the rival channel's suggested price P_r.

$$R_e = -\alpha_2 \left(\frac{P_r}{P_r^{max}}\right) + \frac{P_e}{P_e^{max}} = -\alpha_2' \cdot P_r + \beta_1 \cdot P_e \tag{15}$$

$$\uparrow P_r^{max} \alpha_1 = \frac{1}{\underset{\to}{P_r^{max}}} \downarrow \alpha_1 \Longrightarrow \uparrow \alpha_2 \alpha_2' = \underset{\to}{\alpha_1} \times \alpha_2 ?\, \alpha_2' \tag{16}$$

According to equation (16), the more the retail channel increases its maximum price offer to the customer, the impact of the retail channel's offer price on the return rate of its channel decreases. On the other hand, this increase will rise the effect of P_r on the return rate of the rival channel. Therefore, the state of change of α'_2 will be uncertain. If the effect of increasing α_2 is greater than the effect of decreasing α_1, it means that the effect of the price of the retail channel on the return rate of the Internet channel is greater than the effect of this price on the return rate of its own channel, and as a result, increasing the value of α_2 can cause a decrease in the return rate. β_1 is a coefficient that represents the direct effect on the Internet channel (the effect of the manufacturer's suggested price P_e on the return rate of the Internet channel) and is obtained according to equation (17).

$$\beta_1 = \frac{1}{P_e^{max}} \tag{17}$$

α_2 represents the effect of P_r on the return rate of the rival channel (Internet channel). Also, α'_2 reflects the reciprocal effects of the price offered by the competitor channel (retail) on the return rate of the Internet channel and is obtained from dividing α_2 by P_r^{max} (the highest price offered to customers by the retail channel for which all customers return defective products themselves), is obtained according to equation (18).

$$\left\{\begin{matrix} \alpha'_2 = \dfrac{\alpha_2}{P_r^{max}} \\ \alpha_1 = \dfrac{1}{P_r^{max}} \end{matrix}\right\} \Rightarrow \alpha'_2 = \alpha_2 \times \alpha_1 \tag{18}$$

Therefore, the general model of the producer is written as equation (19), taking into account the limitations.

$$\begin{aligned} Max. \quad & \pi_M = Q\left[(P - C_{mf} - E_m - C_{mir}) \cdot \theta . R_r + (P - C_{mf} - P_e - \frac{C_{mie}}{\lambda}) \cdot \lambda \cdot R_e\right] \\ s.t. \quad & R_e = -\alpha'_2 \cdot P_r + \beta_1 \cdot P_e \\ & R_r = \alpha_1 \cdot P_r - \beta'_2 \cdot P_e \\ & 0 \prec \beta'_2 \prec \alpha_1 \\ & 0 \prec \alpha'_2 \prec \beta_1 \\ & 0 \prec \alpha'_2 \prec \alpha_1 \\ & 0 \prec \beta'_2 \prec \beta_1 \\ & 0 \prec \theta \prec \lambda \prec 1 \\ & 0 \prec R_r \prec 1 \\ & 0 \prec R_e \prec 1 \\ & 0 \prec P_r \\ & 0 \prec P_e \end{aligned} \tag{19}$$

The outcome function of the entire supply chain, which is the sum of the profit of the retailer and the manufacturer, can be written as follows

$$\pi_{SC} = \pi_M + \pi_R \tag{20}$$

Now, we calculate the best response of the retailer, i.e. $P_r^*$and R_r^* in 3 steps. In the first step, instead of the rate of return function of the retailer, we place its equivalent, which is a function of P_r and R_e. In the second step, we take the derivative of the retailer's profit function (the output of the previous step) with respect to P_r (retailer's suggested price to customers for each returned product unit) and set it equal to zero. In the third step, we place the obtained value of P_r^* in R_r to obtain R_r^*.

$$R_r^* = \alpha_1 \cdot \left(\frac{E_m}{2} - \frac{C_{ri}}{2\theta} + \frac{\beta_2 \cdot \beta_1 \cdot P_e}{2\alpha_1}\right) - \beta'_2 \cdot P_e \tag{21}$$

In a similar way, the similar equations could be obtained for the manufacturer. The relevant relations for the best responses of the players in the supply chain are given in Table 3.

Table 3. The best responses of the players in the supply chain under review

player	Best ideal
Producer	$P_e^* = \frac{(\alpha_1 \cdot \alpha_2 \cdot \lambda \cdot P_r) - \beta_1 \cdot \lambda \cdot \left(C_{mf} - P + \frac{C_{mie}}{\lambda}\right) + \beta_1 \cdot \beta_2 \cdot \theta \cdot \left(C_{mf} + C_{mir} + E_m - P\right)}{2\beta_1 \cdot \lambda}$ $R_e^* = -\alpha_2' \cdot P_r + \frac{(\alpha_1 \cdot \alpha_2 \cdot \lambda \cdot P_r) - \beta_1 \cdot \lambda \cdot \left(C_{mf} - P + \frac{C_{mie}}{\lambda}\right) + \beta_1 \cdot \beta_2 \cdot \theta \cdot \left(C_{mf} + C_{mir} + E_m - P\right)}{2\lambda}$
retailer	$P_r^* = \frac{E_m}{2} - \frac{C_{ri}}{2 \cdot \theta} + \frac{\beta_2 \cdot \beta_1 \cdot P_e}{2 \cdot \alpha_1}, R_r^* = \alpha_1 \cdot \left(\frac{E_m}{2} - \frac{C_{ri}}{2 \cdot \theta} + \frac{\beta_2 \cdot \beta_1 \cdot P_e}{2 \cdot \alpha_1}\right) - \beta_2' \cdot P_e$

By determining the variables, parameters and initial conditions of the players, the mentioned scenario is executed. In this research, the price of returnable products under Nash equilibrium is determined simultaneously, statically, with full information and in a decentralized mode. The relationships related to this scenario are only based on the best response of the players and can be relied upon discarding the objective function. In order to achieve the overall optimal solution, the Lingo software has been used by defining the virtual objective function and placing the best answers as the constraints of the problem.

$$P_e^* = \frac{(\alpha_1 \cdot \alpha_2 \cdot \lambda \cdot P_r) - \beta_1 \cdot \lambda \cdot \left(C_{mf} - P + \frac{C_{mie}}{\lambda}\right) + \beta_1 \cdot \beta_2 \cdot \theta \cdot \left(C_{mf} + C_{mir} + E_m - P\right)}{2\beta_1 \cdot \lambda} R_e^* = -\alpha_2' \cdot P_r + \frac{(\alpha_1 \cdot \alpha_2 \cdot \lambda \cdot P_r) - \beta_1 \cdot \lambda \cdot \left(C_{mf} - P + \frac{C_{mie}}{\lambda}\right) + \beta_1 \cdot \beta_2 \cdot \theta \cdot \left(C_{mf} + C_{mir} + E_m - P\right)}{2\lambda} P_r^* = \frac{E_m}{2} - \frac{C_{ri}}{2 \cdot \theta} + \frac{\beta_2 \cdot \beta_1 \cdot P_e}{2 \cdot \alpha_1} R_r^* = \alpha_1 \cdot \left(\frac{E_m}{2} - \frac{C_{ri}}{2 \cdot \theta} + \frac{\beta_2 \cdot \beta_1 \cdot P_e}{2 \cdot \alpha_1}\right) - \beta_2' \cdot P_e \tag{22}$$

4. Research findings

In order to compare and analyze all the designed models, at first the data related to the case studies given in Tables 4 and 5 are implemented in the designed model. The parameters of the problem are divided into two categories; the first group of parameters of the case studies are listed in Table 4. The second category are the parameters that express the acceptance rate of each of the channels (λ and Θ), and the influence coefficient of the offered price of each channel on the return rate of the competitor channel (α_2 and β_2); The value of this group of parameters which is selected based on similar researches in Govindan and Nicoleta [4] and Taleizadeh and Sadeghi [18], are given in Table 5. It should be noted that all financial parameters are in Tomans and the relevant data shows the performance of one season (three-month period) of the company under review.

Table 4. Values of model parameters regarding the case study

row	parameter	value
1	P	990,000
2	E_m	290,000
3	C_{mie}	148,000
4	C_{mir}	198,000
5	C_{mf}	257,000
6	C_{ri}	20,000
7	Q	1,500
8	P_r^{max}	300,000
9	P_e^{max}	360,000
10	$\alpha_1 = 1/P_r^{max}$	0.00000333
11	$\beta_1 = 1/P_e^{max}$	0.00000278
12	$\alpha_2' = \alpha_2/P_r^{max}$	120.00000133

13	$\beta_2' = \beta_2 / P_e^{max}$	130.00000056

Table5. The values of model parameters based on similar researches in [4] and [18].

row	parameter	value
1	λ	0.85
2	θ	0.7
3	α_2	0.4
4	β_2	0.2

In order to solve the best response equations of the investigated supply chain players, it is necessary to insert the values of the parameters of the case study into the equations of the best responses and then using the Lingo software, calculate the global optimal solution. Then, using the Excel software, the outcome functions of the players and the supply chain are calculated. The final solutions of each game are shown in Table 6.

Table 6. The response of the Nash equilibrium model of the designed game

row	parameter	Best ideal
1	P_r	151,999
2	P_e	302,019
3	R_r	0.350
4	R_e	0.500
5	R_{SC}	0.850
6	π_R	40,215,367
7	π_M	253,787,887
8	π_{SC}	294,003,254

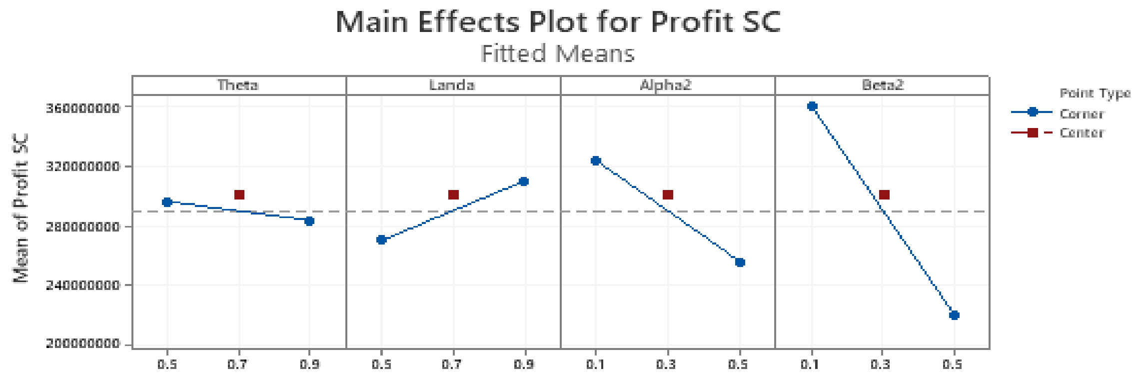

Figure 4. The direct effects of the parameters on the profit of the closed loop supply chain

According to Figure 4, the following results could be obtained.

Analysis of the effect of parameter $\boldsymbol{\theta}$

An increase in the acceptance rate of the retail channel θ in the range [0.5-0.9] in the Nash equilibrium will reduce the profit of the supply chain to a small extent. In the situation where the power of the players is equal (Nash equilibrium), the increase in the adoption rate of the retail channel not only does not help to improve the profit of the supply chain, but also reduces it.

Analysis of the effect of parameter $\boldsymbol{\lambda}$

An increase in the acceptance rate of the Internet channel λ in the range [0.5-0.9] will increase the profit of the supply chain.

Analysis of the effect of the parameter $\boldsymbol{\alpha_2}$

As the value of α_2 increases (the effect of the retail channel's suggested price on the Internet channel's return rate) in the range [0.1-0.5], the profit of the supply chain decreases. In non-cooperative conditions, we should seek to reduce the influence of the retail channel's suggested price on the internet channel's return rate. If this is not possible, by establishing the conditions of Nash equilibrium and creating equal power for the players, the profit of the supply chain will decrease to a lesser extent and better results could be created for the supply chain.

Analysis of the effect of the parameter $\boldsymbol{\beta_2}$

By increasing the value of β_2 (the effect of the internet channel's suggested price on the return rate of the retail channel) in the range [0.1-0.5], the profit of the supply chain decreases. In non-cooperative conditions, we should seek to reduce the effect of the Internet channel's suggested price on the return rate of the retail channel, if this is not possible, by applying the leadership conditions of the producer, the profit of the supply chain will be reduced to a smaller amount and better results can be created for the supply chain.

Figure 5 shows the simultaneous effects of four parameters on the profit of the supply chain in the Nash equilibrium state. The highest amount of the chain profit occurs in the upper vertices and the lowest amount of profit occurs in the lower vertices. The highest profit for the supply chain occurs in a state where θ and λ have

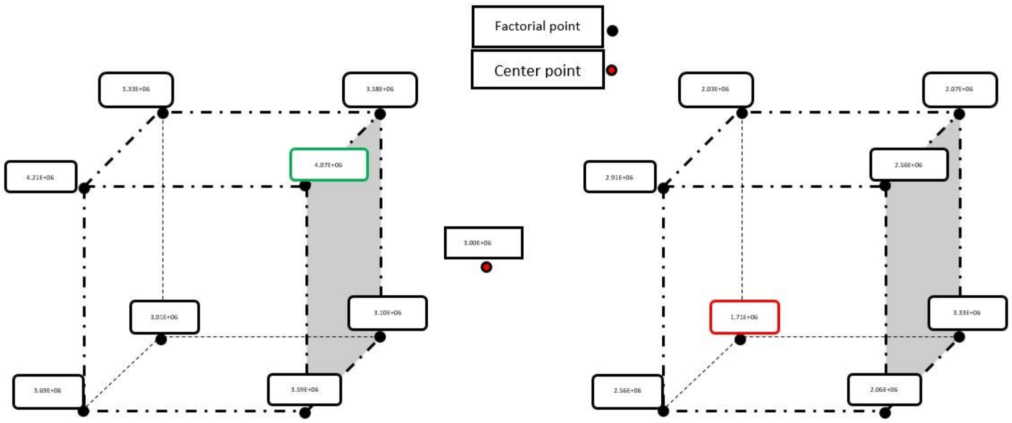

Figure 5. Simultaneous effects of four parameters on supply chain profit in Nash equilibrium

their maximum value, i.e. 0.9, and α_2 and β_2 have their minimum value, i.e. 0.1, which is marked in the figure with green color. The red color also indicates the lowest amount of the profit in the case of non-cooperative Nash equilibrium. the weakest state of the supply chain occurs, when θ and λ have their minimum values of 0.5 and α_2 and β_2 have their maximum values of 0.5.

5. Conclusion and summary

In this research, the pricing of the return products of a two-level closed-loop reverse supply chain, which collects defective products from customers through two traditional retail channels and the Internet channel, has been discussed. To consider the interests of all members of the supply chain, the game theory approach is used. Referring to the irreparable effects that return electrical and electronic equipment as electronic waste in nature, this research tries to encourage researchers to conduct more studies and business owners to invest in recycling, collection, and construction of return channels, and encourage double return and reproduction. The most important distinction of this research with previous researches is the simultaneous use of the dual return channel and modeling it as a problem in game theory, which solves the challenge related to the conflict between the retail channel and the direct channel in the product collection. In addition, the following executive suggestions are also proposed. It is necessary for the manufacturer and the retailer to make their maximum efforts in order to reduce the impact of their proposed prices on the cross-channel return rate so that these values reach their minimum level. The acceptance rate of each channel has a direct impact on the profit of each channel. Therefore, more efforts are needed to increase the acceptance rate in both channels. Since the direct channel avoids the doubling of the inspection cost, it is better to have a higher return rate compared to the retail channel. The desired model is formulated in certainty conditions; In order to obtain more realistic results, it is suggested that the number of products that are sold in the direct supply chain be considered fuzzy numbers. The development of the model in the form of a three-level supply chain with the use of service companies in the field of transportation, such as Snap, Alupik, etc., as a third company for collecting products from customers, can also be a considered as future research. Also, the use of inventory control models such as EPQ and EOQ in the direct supply chain can also be a field of research.

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