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A new approach on fuzzy multiobjective multicommodity minimal cost flow problems

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In this work, we consider a multiobjective minimal cost flow (MMCF) problem where there are several commodities to transport from sources to destinations and there is more than one conveyance for those transporting. We also assume that in each conveyance, there are distinct capacities for each commodity. The obtained model is not necessary balanced and we introduced a method to solve this model without converting it to a balanced model. The advantages of the proposed method are also discussed.

Keywords: *Fuzzy multi-objective solid minimal cost flow problem, LR flat fuzzy number, multicommodity minimal cost flow problem.*

1. Introduction

Minimum Cost Flow (MCF) problems have many applications in almost all industries, such as agriculture, communications, education, energy, manufacturing, medicine, and transportation [1]. Generally, the MCF problem minimizes the cost of transporting some product that is available at some sources and required at some destinations. However, in the real word, there are few MCF problems with only a single objective. Therefore, in the recent years multiple objective minimum cost flow problems have been considered by many authors [14]. Another complexity which exists in the real problems is the impreciseness of values of coefficients of the variables in the objective functions, availability and demand of the products. The fuzzy set theory introduced by Zadeh [25] is a good alternative for this impreciseness. To the best of our knowledge, the first formulation of fuzzy multiobjective linear programming is proposed by Zimmermann [26]. Also, the first time, Shih and Lee [24] considered a fuzzy MCF problem. After that, this problem has been studied by many researchers from several viewpoints; see [2, 13, 17] and the references therein. Recently, Bavandi and Nasseri [3, 4] provided the model that manages unknown coefficients in fractional multi-commodity networks. In these problems, the coefficients of the objective function in the numerator of the fraction and the arc capacity are assumed to be fuzzy random variables and the coefficients of the objective function in the denominator of the fraction are assumed to be fuzzy variables. Since this problem is investigated simultaneously in both random and fuzzy environments, they used a probability-possibility approach to convert the problem to a deterministic form and then proposed solving process. Our motivation in this paper is recent works of Kaur and Kumar [16,17]. In [16], the authors consider fuzzy multiobjective transportation problems where there exists some nodes, called intermediate nodes, at which the product may be stored in case of the excess of the available product and later on the product may be supplied from these intermediate nodes to the destinations. As well as, they assumed that there are different types of conveyances such as trucks, cargo flights, trains, ships, etc., for transporting the products from sources to destinations. Such multiobjective transportation problems in which both the conveyances as well as intermediate nodes are used simultaneously are known as multi-objective Solid Minimal Cost Flow (SMCF) problems [16]. The SMCF problem with fuzzy data studied by several authors [5, 7, 16, 19, 21]. In some situations, we must transport more than one commodity from sources to destinations. These problems are called multicommodity flow problem. Ghatee and Hashemi [12] studied fuzzy multicommodity flow problem and Chakraborty and et al. [11], Dalman and et al. [10], Kundu and et al. [20], and Rani and et al. [24] considered multiobjective multi item solid transportation problem under uncertainty. In this paper, we consider a fuzzy multiobjective multicommodity minimal cost flow (FMMMCF) problem when there are some limitations on conveyances. In fact, in our model, a conveyance may be allowed to transport a certain amount of a commodity. To the best of our knowledge, there is not any research for this model even for deterministic data.

This paper is organized in 6 sections. In the next section some preliminaries of fuzzy numbers are reviewed. In Section 3, we describe our model and a formulation of FMMMCF problem is introduced. In Section 4 the new method is proposed and we illustrate this method by some numerical example in Section 5. The conclusion and some suggestion are given in Section 6.

2. Preliminaries

In this section we provide some preliminaries.

Definition 2.1. [23] A function $L: [0, \infty) \to [0,1]$ (or $R: [0, \infty) \to [0,1]$) is said to be a reference function of fuzzy numbers if and only if
(i) $L(0) = 1$ (or $R(0) = 1$)
(ii) L (R) is non-increasing on $[0, \infty)$.

Definition 2.2. [10] A fuzzy number $\tilde{a} = (m, n, \alpha, \beta)_{LR}$ is said to be LR flat fuzzy number if its membership function $\mu_{\tilde{a}}(x)$ is given by

$$\mu_{\tilde{a}} = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right), & for\ x \le m, \alpha > 0 \\ R\left(\dfrac{x-n}{\beta}\right), & for\ x \ge n, \beta > 0 \\ 1, & otherwise \end{cases}$$

Definition 2.3. [10] Two LR flat fuzzy numbers $\tilde{a}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{a}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ are said to be equal i.e., $\tilde{a}_1 = \tilde{a}_2$ if and only if $m_1 = m_2$, $n_1 = n_2$, $\alpha_1 = \alpha_2$, and $\beta_1 = \beta_2$.

Definition 2.4. [9] An LR flat fuzzy number $\tilde{a} = (m, n, \alpha, \beta)_{LR}$ is said to be non-negative LR flat fuzzy number if and only if $m - \alpha \geq 0$.

Let $\tilde{a}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\tilde{a}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be two LR flat fuzzy number. Then
(i) $\tilde{a}_1 \oplus \tilde{a}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)_{LR}$.
(ii) Let $\tilde{a}_1$ and $\tilde{a}_2$ be non-negative LR flat fuzzy numbers. Then

$$\tilde{a}_1 \otimes \tilde{a}_2 \simeq (m_1 m_2, n_1 n_2, (m_1 - \alpha_1)(m_2 - \alpha_2) - m_1 m_2, (n_1 + \beta_1)(n_2 + \beta_2) - n_1 n_2)_{LR}$$

(iii) Let λ be a real number. Then

$$\lambda\tilde{a}_1 = \begin{cases} (\lambda\, \mathrm{m}_1, \lambda\, \mathrm{n}_1, \lambda\, \alpha_1, \lambda\, \beta_1)_{\mathrm{LR}} & \lambda \geq 0 \\ (\lambda\, \mathrm{n}_1, \lambda\, \mathrm{m}_1, -\lambda\, \beta_1, -\lambda\, \alpha_1)_{\mathrm{LR}} & \lambda < 0 \end{cases}$$

In this paper we use of modified Liou and Wang's ranking [9] for the comparison of fuzzy numbers.
Assume, $\tilde{a} = (m, n, \alpha, \beta)_{LR}$.

$$\Re(\tilde{a}) = \gamma \left[\int_0^1 (m\lambda + n(1 - \lambda)) d\lambda \right] + (1 - \gamma) \left[\lambda \int_0^1 \left(m - \alpha\, L^{-1}(\rho)\right) d\rho + (1 - \lambda) \int_0^1 \left(n + \beta\, R^{-1}(\rho)\right) d\rho \right],$$

where $\gamma \in [0,1]$ and $\lambda \in [0,1]$.

Let $\tilde{a}$ and $\tilde{b}$ be two LR flat fuzzy numbers. Then $\tilde{a} \gtrapprox \tilde{b}$ ($\tilde{a} \lessapprox \tilde{b}$) if $\Re(\tilde{a}) \geq \Re(\tilde{b})$ $\left(\Re(\tilde{a}) \leq \Re(\tilde{b})\right)$.

3. Fully fuzzy multicommodity multiobjective model

In this section we introduce a fully fuzzy multicommodity multiobjective model for solid minimal cost flow problems which there are limitation on conveyances for transport the products. For example, assume that we want to transport coal and petroleum from a city to another one by train. For each commodity (coal and petroleum) we need a special tank. Therefore, we cannot allocate the all capacity of the train to a commodity. For another example, assume that we want to send grease and petroleum from a country to another country by ship. Assume that there are some rules for import grease or petroleum by ship in the destination country which do not allow to you to send more than certain value of these materials. Therefore, you cannot allocate the all capacity of the ship to a commodity. These examples show that we must design a new model to cover these problems.

Assume that $G=(N,A)$ is a given network where N is the set of nodes and A is the set of links. We describe our problem with a simple example. Consider a network with two nodes, shown in Figure 1. We want to send two commodity t_1 and t_2 from node 1 to node 2. There exist three conveyance between these two nodes and each conveyance has a total capacity e and furthermore each conveyance have a capacity for each commodity as e_{t_1} and e_{t_2}. Note that, we cannot send commodity more of the total capacity e, while we can have $e_{t_1}+e_{t_2}\geq e$.

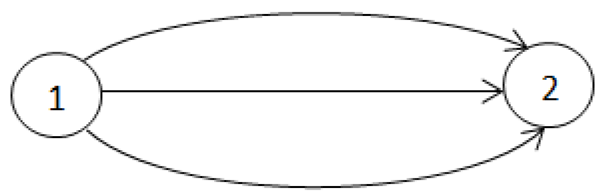

Figure 1. Network representing FMMMCF

Similar to [14,15], we categorize the nodes as follows:

Purely source node: Those nodes S which there exists some node S' such that the product may be supplied from S to S' while there does not exist any node S'' to transport product from S'' to S. The set of all such nodes is denoted by N_{PS}.

Purely destination node: Those nodes D which there does not exist any node D' such that the product may be supplied from D to D' while there exist some node D'' to transport product from D'' to D. The set of all such nodes is denoted by N_{PD}.

Intermediate node: The intermediate nodes are another part of a network. In the following we sort the types of these nodes:

(i) Those nodes S which have some quantity of the product for supplying to other nodes and also there exist some nodes such that some quantity of the product is transporting from those nodes to node S. Such nodes are called source nodes and the set of all such nodes is dented by N_S.

)ii) Those nodes D which require some quantity of the product and also there exist some nodes such that the product is supplying from node D to those nodes. Such nodes are called destination nodes and the set of all such nodes is dented by N_D.

(iii) Those nodes T which neither any quantity of the product is available at them to transship nor any quantity of the products is required, and all quantity of the product which are transferred from some nodes to node T is supplying from T to some other nodes. Such nodes are called transition nodes and the set of all such nodes is dented by N_T.

In the following, we list notations which we use them in the representation of our model.

- $\tilde{a}_i^t$: The fuzzy availability of the product t at ith purely source node.
- $\tilde{a}_i^{t\prime}$: The fuzzy availability of the product t at ith source node.
- $\tilde{b}_j^t$:The fuzzy demand of the product t at jth purely destination node.
- $\tilde{b}_j^{t\prime}$:The fuzzy demand of the product t at jth destination node.
- $\tilde{e}_k^t$: The fuzzy capacity of the kth conveyance for transfer the product t.
- $\tilde{e}_k$: The total fuzzy capacity of the kth conveyance.
- $\tilde{c}_{ijk}^{tl}$: The fuzzy penalty per unit of flow $t$from ith (purely) source to jth (purely) destination by means of the kth conveyance in the lth objective function.
- $\tilde{x}_{ijk}^t$: The fuzzy quantity of the product t that should be transported from ith node to jth node by means of the kth conveyance in order to minimize all objective functions.
- S_C: The set of all available conveyances.

We assume that $\tilde{a}_i^t$, $\tilde{a}_i^{t\prime}$,$\tilde{b}_j^t$, $\tilde{b}_j^{t\prime}$, $\tilde{e}_k^t$, $\tilde{e}_k$ are non-negative LR flat fuzzy numbers. We also assume that there are L objective function and Tcommodities. With these notations, a FMMMCF problem can be formulated into the following fuzzy multiobjective linear programming problem:

Minimum $\sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}(\tilde{c}_{ijk}^{tl}\otimes\tilde{x}_{ijk}^t)$ $\quad l=1,\ldots,L$		(1)
Subject to		
$\sum_{j:(i,j)\in A}\sum_{k\in S_C}\tilde{x}_{ijk}^t \;\tilde{\leqq}\; \tilde{a}_i^t$	$i\in N_{PS}, t=1,\ldots,T$	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C}\tilde{x}_{ijk}^t \;\tilde{\leqq}\; \sum_{j:(j,i)\in A}\sum_{k\in S_C}\tilde{x}_{jik}^t\oplus\tilde{a}_i^{t\prime}$	$i\in N_S, t=1,\ldots,T$	
$\sum_{i:(i,j)\in A}\sum_{k\in S_C}\tilde{x}_{ijk}^t \;\tilde{\geqq}\; \tilde{b}_j^t$	$j\in N_{PD}, t=1,\ldots,T$	
$\sum_{i:(i,j)\in A}\sum_{k\in S_C}\tilde{x}_{ijk}^t \;\tilde{\geqq}\; \sum_{i:(j,i)\in A}\sum_{k\in S_C}\tilde{x}_{jik}^t\oplus\tilde{b}_j^{t\prime}$	$j\in N_D, t=1,\ldots,T$	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C}\tilde{x}_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C}\tilde{x}_{jik}^t$	$i\in N_T, t=1,\ldots,T$	
$\sum_{t=1}^{T}\sum_{j:(i,j)\in A}\tilde{x}_{ijk}^t \;\tilde{\leqq}\; \tilde{e}_k$	$k\in S_C$	
$\tilde{x}_{ijk}^t \;\tilde{\leqq}\; \tilde{e}_k^t$	$k\in S_C, (i,j)\in A, t=1,\ldots,T$	

where $\tilde{x}_{ijk}^{t}$ is a non-negative LR flat fuzzy number for all $(i,j) \in A$ and $k \in S_C$.

4. Proposed method

Almost in all available algorithms for MCF problems, we must examine that the problem is balanced or unbalanced and with some modifications, convert an unbalanced one to balanced. This process may be so expensive and therefore it is better to solve our model without this assumption. In our model there are some equalities and inequalities, therefore with a generalization of the proposed algorithm in the Subsection 5.4.2 of [13] and using of existing methods [18] for solving of a multiobjective linear programming, we can obtain the optimal compromise solution for our model. We recall the definition of fuzzy efficient solution from the literature.

Definition 4.1. A fuzzy feasible solution $\tilde{x} = \{\tilde{x}_{ijk}^{t}\}$ is said to be a fuzzy efficient solution of the fully fuzzy multiobjective SMMMCF problem if there is no other fuzzy feasible solution $\tilde{x}' = \{\tilde{x}_{ijk}^{t\prime}\}$ such that

$$\sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}\Re(\tilde{c}_{ijk}^{tl}\otimes\tilde{x}_{ijk}^{t\prime}) \le \sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}\Re(\tilde{c}_{ijk}^{tl}\otimes\tilde{x}_{ijk}^{t})$$

for all $l \in \{1,\dots,L\}$, and

$$\sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}\Re(\tilde{c}_{ijk}^{tl}\otimes\tilde{x}_{ijk}^{t\prime}) < \sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}\Re(\tilde{c}_{ijk}^{tl}\otimes\tilde{x}_{ijk}^{t})$$

for at least one $l \in \{1,\dots,L\}$.

Note that, for real world problems, we do not need to obtain the set of all fuzzy efficient solutions. It is sufficient to compute a fuzzy optimal compromise solution. A fuzzy optimal compromise solution of the FMMMCF problem is a feasible solution which is preferred by the decision maker to all other solutions, taking into consideration all criteria contained in the multiobjective functions. We accept that a fuzzy optimal compromise solution has to be a fuzzy efficient solution.

Step 1: Assume that $\tilde{c}_{ijk}^{tl} = \left(p_{ijk}^{tl}, q_{ijk}^{tl}, \alpha_{ijk}^{tl}, \beta_{ijk}^{tl}\right)_{LR}$, $\tilde{x}_{ijk}^{t} = \left(y_{ijk}^{t}, z_{ijk}^{t}, \gamma_{ijk}^{t}, \delta_{ijk}^{t}\right)_{LR}$, $\tilde{a}_i^t = (r_i^t, s_i^t, \varepsilon_i^t, \zeta_i^t)_{LR}$, $\tilde{a}_i^{t\prime} = (r_i^{t\prime}, s_i^{t\prime}, \varepsilon_i^{t\prime}, \zeta_i^{t\prime})_{LR}$, $\tilde{b}_j^t = \left(v_j^t, w_j^t, \eta_j^t, \theta_j^t\right)_{LR}$, $\tilde{b}_j^{t\prime} = \left(v_j^{t\prime}, w_j^{t\prime}, \eta_j^{t\prime}, \theta_j^{t\prime}\right)_{LR}$, $\tilde{e}_k = (g_k, h_k, \lambda_k, \mu_k)_{LR}$, and $\tilde{e}_k^t = (g_k^t, h_k^t, \lambda_k^t, \mu_k^t)_{LR}$ .Therefore Problem (1) can be written as:

$$\text{Minimum} \quad \sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}\left(p_{ijk}^{tl}, q_{ijk}^{tl}, \alpha_{ijk}^{tl}, \beta_{ijk}^{tl}\right)_{LR} \otimes \left(y_{ijk}^{t}, z_{ijk}^{t}, \gamma_{ijk}^{t}, \delta_{ijk}^{t}\right)_{LR} \qquad l = 1,\dots,L \tag{2}$$

Subject to

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C}\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \ \tilde{\leqq}\ (r_i^t, s_i^t, \varepsilon_i^t, \zeta_i^t)_{LR} \qquad i\in N_{PS}, t=1,\dots,T$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C}\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \ \tilde{\leqq} \sum_{j:(j,i)\in A}\sum_{k\in S_C}\left(y_{jik}^t, z_{jik}^t, \gamma_{jik}^t, \delta_{jik}^t\right)_{LR} \oplus (r_i^{t'}, s_i^{t'}, \varepsilon_i^{t'}, \zeta_i^{t'})_{LR} \qquad i\in N_S, t=1,\dots,T$$

$$\sum_{i:(i,j)\in A}\sum_{k\in S_C}\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \ \tilde{\geqq}\ \left(v_j^t, w_j^t, \eta_j^t, \theta_j^t\right) \qquad j\in N_{PD}, t=1,\dots,T$$

$$\sum_{i:(i,j)\in A}\sum_{k\in S_C}\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \ \tilde{\geqq} \sum_{i:(j,i)\in A}\sum_{k\in S_C}\left(y_{jik}^t, z_{jik}^t, \gamma_{jik}^t, \delta_{jik}^t\right)_{LR} \oplus \left(v_j^{t'}, w_j^{t'}, \eta_j^{t'}, \theta_j^{t'}\right)_{LR} \qquad j\in N_D, t=1,\dots,T$$

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C}\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} = \sum_{j:(j,i)\in A}\sum_{k\in S_C}\left(y_{jik}^t, z_{jik}^t, \gamma_{jik}^t, \delta_{jik}^t\right)_{LR} \qquad i\in N_T, t=1,\dots,T$$

$$\sum_{t=1}^{T}\sum_{j:(i,j)\in A}\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \ \tilde{\leqq}\ (g_k, h_k, \lambda_k, \mu_k)_{LR} \qquad k\in S_C$$

$$\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \ \tilde{\leqq}\ (g_k^t, h_k^t, \lambda_k^t, \mu_k^t)_{LR} \qquad k\in S_C, (i,j)\in A, t=1,\dots,T$$

where $\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR}$ is a non-negative LR flat fuzzy number for all $k\in S_C, (i,j)\in A, t=1,\dots,T$.

Step 2. Assume that

$$\left(p_{ijk}^{tl}, q_{ijk}^{tl}, \alpha_{ijk}^{tl}, \beta_{ijk}^{tl}\right)_{LR} \otimes \left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} = \left(o_{ijk}^{tl}, u_{ijk}^{tl}, \varphi_{ijk}^{tl}, \tau_{ijk}^{tl}\right)_{LR},$$

$$\left(y_{jik}^t, z_{jik}^t, \gamma_{jik}^t, \delta_{jik}^t\right)_{LR} \oplus (r_i^{t'}, s_i^{t'}, \varepsilon_i^{t'}, \zeta_i^{t'})_{LR} = \left(m_{jik}^t, n_{jik}^t, \pi_{jik}^t, \sigma_{jik}^t\right)_{LR},$$

and

$$\left(y_{jik}^t, z_{jik}^t, \gamma_{jik}^t, \delta_{jik}^t\right)_{LR} \oplus \left(v_j^{t'}, w_j^{t'}, \eta_j^{t'}, \theta_j^{t'}\right)_{LR} = \left(m_{jik}^{t'}, n_{jik}^{t'}, \pi_{jik}^{t'}, \sigma_{jik}^{t'}\right)_{LR}.$$

With these notations, Problem (2) can be written as

$$\text{Minimum} \quad \sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}\left(o_{ijk}^{tl}, u_{ijk}^{tl}, \varphi_{ijk}^{tl}, \tau_{ijk}^{tl}\right)_{LR} \qquad l=1,\dots,L \tag{3}$$

Subject to

$$\sum_{j:(i,j)\in A}\sum_{k\in S_C}\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \ \tilde{\leqq}\ (r_i^t, s_i^t, \varepsilon_i^t, \zeta_i^t)_{LR} \qquad i\in N_{PS}, t=1,\dots,T$$

$\sum_{j:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR} \precapprox (m_{jik}^t, n_{jik}^t, \pi_{jik}^t, \sigma_{jik}^t)_{LR} \quad i\in N_S, t = 1,\dots,T$	
$\sum_{i:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR} \succapprox (v_j^t, w_j^t, \eta_j^t, \theta_j^t) \quad j\in N_{PD}, t=1,\dots,T$	
$\sum_{i:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR} \succapprox \sum_{i:(j,i)\in A}\sum_{k\in S_C}(m_{jik}^{t\prime}, n_{jik}^{t\prime}, \pi_{jik}^{t\prime}, \sigma_{jik}^{t\prime})_{LR} \quad j\in N_D, t = 1,\dots,T$	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR} = \sum_{j:(j,i)\in A}\sum_{k\in S_C}(y_{jik}^t, z_{jik}^t, \gamma_{jik}^t, \delta_{jik}^t)_{LR} \quad i\in N_T, t = 1,\dots,T$	
$\sum_{t=1}^{T}\sum_{j:(i,j)\in A}(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR} \precapprox (g_k, h_k, \lambda_k, \mu_k)_{LR} \quad k\in S_C$	
$(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR} \precapprox (g_k^t, h_k^t, \lambda_k^t, \mu_k^t)_{LR} \quad k\in S_C, (i,j)\in A, t=1,\dots,T$	

where $(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR}$ is a non-negative LR flat fuzzy number for all $k\in S_C, (i,j)\in A, t=1,\dots,T$.

Step 3. Using rank function $\Re$, we solve the following problem:

Minimum $\Re\left(\sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}(o_{ijk}^{tl}, u_{ijk}^{tl}, \varphi_{ijk}^{tl}, \tau_{ijk}^{tl})_{LR}\right) \quad l=1,\dots,L$	(4)
Subject to	
$\Re\left(\sum_{j:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR}\right) \le \Re(r_i^t, s_i^t, \varepsilon_i^t, \zeta_i^t)_{LR} \quad i\in N_{PS}, t=1,\dots,T$	
$\Re\left(\sum_{j:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR}\right) \le \Re(m_{jik}^t, n_{jik}^t, \pi_{jik}^t, \sigma_{jik}^t)_{LR} \quad i\in N_S, t=1,\dots,T$	
$\Re\left(\sum_{i:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR}\right) \ge \Re(v_j^t, w_j^t, \eta_j^t, \theta_j^t) \quad j\in N_{PD}, t=1,\dots,T$	
$\Re\left(\sum_{i:(i,j)\in A}\sum_{k\in S_C}(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR}\right) \ge \Re\left(\sum_{i:(j,i)\in A}\sum_{k\in S_C}(m_{jik}^{t'}, n_{jik}^{t'}, \pi_{jik}^{t'}, \sigma_{jik}^{t'})_{LR}\right)$ $j\in N_D, t=1,\dots,T$	

$\sum_{j:(i,j)\in A}\sum_{k\in S_C} y_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} y_{jik}^t \qquad i\in N_T$	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C} z_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} z_{jik}^t \qquad i\in N_T$	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \gamma_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} \gamma_{jik}^t \qquad i\in N_T$	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \delta_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} \delta_{jik}^t \qquad i\in N_T$	
$\Re\left(\sum_{t=1}^{T}\sum_{j:(i,j)\in A}\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR}\right) \le \Re(g_k, h_k, \lambda_k, \mu_k)_{LR} \qquad k\in S_C$	
$\Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \le \Re(g_k^t, h_k^t, \lambda_k^t, \mu_k^t)_{LR} \qquad k\in S_C, (i,j)\in A, t=1,\dots,T$	
$y_{ijk}^t - \gamma_{ijk}^t, z_{ijk}^t - y_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t \ge 0 \qquad k\in S_C, (i,j)\in A, t=1,\dots,T$	

Step 4. With respect to linear property of rank function, (4) can be written as:

Minimum $\sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C} \Re\left(o_{ijk}^{tl}, u_{ijk}^{tl}, \varphi_{ijk}^{tl}, \tau_{ijk}^{tl}\right)_{LR} \qquad l=1,\dots,L$	(5)
Subject to	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C}\Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \le \Re(r_i^t, s_i^t, \varepsilon_i^t, \zeta_i^t)_{LR} \qquad i\in N_{PS}, t=1,\dots,T$	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C}\Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \le \Re\left(m_{jik}^t, n_{jik}^t, \pi_{jik}^t, \sigma_{jik}^t\right)_{LR} \qquad i\in N_S, t=1,\dots,T$	
$\sum_{i:(i,j)\in A}\sum_{k\in S_C}\Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \ge \Re\left(v_j^t, w_j^t, \eta_j^t, \theta_j^t\right) \qquad j\in N_{PD}, t=1,\dots,T$	
$\sum_{i:(i,j)\in A}\sum_{k\in S_C}\Re\left(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t\right)_{LR} \ge \sum_{i:(j,i)\in A}\sum_{k\in S_C}\Re\left(m_{jik}^{t'}, n_{jik}^{t'}, \pi_{jik}^{t'}, \sigma_{jik}^{t'}\right)_{LR}$ $j\in N_D, t=1,\dots,T$	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C} y_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} y_{jik}^t \qquad i\in N_T$	

$\sum_{j:(i,j)\in A}\sum_{k\in S_C} z_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} z_{jik}^t \qquad i\in N_T$	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \gamma_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} \gamma_{jik}^t \qquad i\in N_T$	
$\sum_{j:(i,j)\in A}\sum_{k\in S_C} \delta_{ijk}^t = \sum_{j:(j,i)\in A}\sum_{k\in S_C} \delta_{jik}^t \qquad i\in N_T$	
$\sum_{t=1}^{T}\sum_{j:(i,j)\in A} \Re(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR} \le \Re(g_k, h_k, \lambda_k, \mu_k)_{LR} \qquad k\in S_C$	
$\Re(y_{ijk}^t, z_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t)_{LR} \le \Re(g_k^t, h_k^t, \lambda_k^t, \mu_k^t)_{LR} \qquad k\in S_C, (i,j)\in A, t=1,\dots,T$	
$y_{ijk}^t - \gamma_{ijk}^t, z_{ijk}^t - y_{ijk}^t, \gamma_{ijk}^t, \delta_{ijk}^t \ge 0 \qquad k\in S_C, (i,j)\in A, t=1,\dots,T$	

Step 5. With solving the crisp programming problem (5), find the optimal compromise solution $x_{ijk}^{t*} = (y_{ijk}^{t*}, z_{ijk}^{t*}, \gamma_{ijk}^{t*}, \delta_{ijk}^{t*})_{LR}$.

Step 6. Find the fuzzy optimal value of each objective function by putting the values of $x_{ijk}^{t*} = (y_{ijk}^{t*}, z_{ijk}^{t*}, \gamma_{ijk}^{t*}, \delta_{ijk}^{t*})_{LR}$ in $\sum_{t=1}^{T}\sum_{(i,j)\in A}\sum_{k\in S_C}(\tilde{c}_{ijk}^{tl} \otimes \tilde{x}_{ijk}^{t})$.

5. Illustrative example

In this section, we illustrate our method with an example.

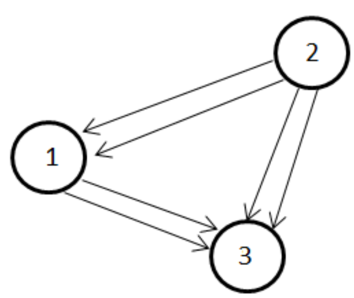

Figure ٢. Network representing Example 5.1.

Example 5.1. Consider the network Figure ٢ with the following data:

- Fuzzy penalty for 1st objective function to transport commodity 1st:

$$c_{131}^{11} = (3,4,2,2)_{LR}, \qquad c_{132}^{11} = (2,3,1,2)_{LR}$$
$$c_{211}^{11} = (4,5,3,3)_{LR}, \qquad c_{212}^{11} = (5,6,3,3)_{LR}$$
$$c_{231}^{11} = (5,7,4,3)_{LR}, \qquad c_{232}^{11} = (3,4,2,3)_{LR}$$

- Fuzzy penalty for 2nd objective function to transport commodity 1st:

$$c_{131}^{12} = (4,6,3,3)_{LR}, \qquad c_{132}^{12} = (3,4,2,2)_{LR}$$
$$c_{211}^{12} = (5,6,4,3)_{LR}, \qquad c_{212}^{12} = (6,7,4,4)_{LR}$$
$$c_{231}^{12} = (6,8,4,4)_{LR}, \qquad c_{232}^{12} = (4,5,2,3)_{LR}$$

- Fuzzy penalty for 1st objective function to transport commodity 2nd:
$$c_{131}^{21} = (2,3,1,1)_{LR}, \qquad c_{132}^{21} = (2,4,1,2)_{LR}$$
$$c_{211}^{21} = (4,5,3,2)_{LR}, \qquad c_{212}^{21} = (2,3,2,2)_{LR}$$
$$c_{231}^{21} = (4,6,3,3)_{LR}, \qquad c_{232}^{21} = (2,3,1,2)_{LR}$$
- Fuzzy penalty for 2nd objective function to transport commodity 2nd:
$$c_{131}^{22} = (3,4,2,1)_{LR}, \qquad c_{132}^{22} = (3,4,2,2)_{LR}$$
$$c_{211}^{22} = (5,7,4,3)_{LR}, \qquad c_{212}^{22} = (5,6,3,3)_{LR}$$
$$c_{231}^{22} = (5,6,4,3)_{LR}, \qquad c_{232}^{22} = (2,3,1,1)_{LR}$$
- Fuzzy availability of the commodity 1st at source node 1 and purely source node 2:
$$\mathrm{a}_1^{1\prime} = (30,40,20,10)_{\mathrm{LR}} \qquad \mathrm{a}_2^1 = (30,40,20,20)_{\mathrm{LR}}$$
- Fuzzy availability of the commodity 2nd at source node 1 and purely source node 2:
$$\mathrm{a}_1^{2\prime} = (40,60,20,30)_{\mathrm{LR}}, \qquad \mathrm{a}_2^2 = (40,50,30,30)_{\mathrm{LR}}$$
- Fuzzy demand of the commodity 1st at purely destination node 3:
$$b_3^1 = (30,50,20,30)_{LR}$$
- Fuzzy demand of the commodity 2nd at purely destination node 3:
$$b_3^2 = (40,50,30,30)_{LR}$$
- Fuzzy capacity of the 1st conveyance for transfer the commodities 1st and 2nd:
$$e_1^1 = (60,70,40,30)_{LR}, \qquad e_1^2 = (60,70,30,30)_{LR}$$
- Fuzzy capacity of the 2nd conveyance for transfer the commodities 1st and 2nd:
$$e_2^1 = (60,70,30,30)_{LR}, \qquad e_2^2 = (60,70,20,20)_{LR}$$
- Total fuzzy capacity of the 1st and 2nd conveyances:
$$e_1 = (70,70,30,30)_{LR}, \qquad e_2 = (70,80,20,20)_{LR}$$

We assume that $L(x) = R(x) = max\,\{0, 1 - x^4\}$. Therefore, for a fuzzy number $\tilde{a} = (m,n,\alpha,\beta)$, $\Re(\tilde{a}) = \frac{1}{2}(m+n) + \frac{4}{15}(\beta - \alpha)$ (see Remark 1 in [١٤]).

The model will be as:

$\text{Minimum } (3,4,2,2)_{\mathrm{LR}} \otimes \mathrm{x}_{131}^1 \oplus (2,3,1,2)_{\mathrm{LR}} \otimes \mathrm{x}_{132}^1 \oplus (4,5,3,3)_{\mathrm{LR}} \otimes \mathrm{x}_{211}^1 \oplus (5,6,3,3)_{LR} \otimes x_{212}^1 \oplus (5,7,4,3)_{\mathrm{LR}} \otimes \mathrm{x}_{231}^1 \oplus (3,4,2,3)_{\mathrm{LR}} \otimes \mathrm{x}_{232}^1 \oplus (2,3,1,1)_{LR} \otimes x_{131}^2 \oplus (2,4,1,2)_{\mathrm{LR}} \otimes \mathrm{x}_{132}^2 \oplus (4,5,3,2)_{\mathrm{LR}} \otimes \mathrm{x}_{211}^2 \oplus (2,3,2,2)_{LR} \otimes x_{212}^2 \oplus (4,6,3,3)_{\mathrm{LR}} \otimes \mathrm{x}_{231}^2 \oplus (2,3,1,2)_{\mathrm{LR}} \otimes \mathrm{x}_{232}^2$	(6)
$\text{Minimum } (4,6,3,3)_{\mathrm{LR}} \otimes \mathrm{x}_{131}^1 \oplus (3,4,2,2)_{\mathrm{LR}} \otimes \mathrm{x}_{132}^1 \oplus (5,6,4,3)_{\mathrm{LR}} \otimes \mathrm{x}_{211}^1 \oplus (6,7,4,4)_{LR} \otimes x_{212}^1 \oplus (6,8,4,4)_{\mathrm{LR}} \otimes \mathrm{x}_{231}^1 \oplus (4,5,2,3)_{\mathrm{LR}} \otimes \mathrm{x}_{232}^1 \oplus (3,4,2,1)_{LR} \otimes x_{131}^2 \oplus (3,4,2,2)_{\mathrm{LR}} \otimes \mathrm{x}_{132}^2 \oplus (5,7,4,3)_{\mathrm{LR}} \otimes \mathrm{x}_{211}^2 \oplus (5,6,3,3)_{LR} \otimes x_{212}^2 \oplus (5,6,4,3)_{\mathrm{LR}} \otimes \mathrm{x}_{231}^2 \oplus (2,3,1,1)_{\mathrm{LR}} \otimes \mathrm{x}_{232}^2$	
Subject to	

$x_{211}^{1} \oplus x_{212}^{1} \oplus x_{231}^{1} \oplus x_{232}^{1} \tilde{\leqq} (30,40,20,20)_{LR}$	
$x_{211}^{2} \oplus x_{212}^{2} \oplus x_{231}^{1} \oplus x_{232}^{2} \tilde{\leqq} (40,50,30,30)_{LR}$	
$x_{131}^{1} \oplus x_{132}^{1} \tilde{\leqq} x_{211}^{1} \oplus x_{212}^{1} \oplus (30,40,20,10)_{LR}$	
$x_{131}^{2} \oplus x_{132}^{2} \tilde{\leqq} x_{211}^{2} \oplus x_{212}^{2} \oplus (40,60,20,30)_{LR}$	
$x_{131}^{1} \oplus x_{132}^{1} \oplus x_{231}^{1} \oplus x_{232}^{1} \tilde{\geqq} (30,50,20,30)_{LR}$	
$x_{131}^{2} \oplus x_{132}^{2} \oplus x_{231}^{2} \oplus x_{232}^{2} \tilde{\geqq} (40,50,30,30)_{LR}$	
$x_{211}^{1} \oplus x_{231}^{1} \oplus x_{131}^{1} \oplus x_{211}^{2} \oplus x_{231}^{2} \oplus x_{131}^{2} \tilde{\leqq} (70,70,30,30)_{LR}$	
$x_{211}^{1} \tilde{\leqq} (60,70,40,30)_{LR}$	
$x_{212}^{1} \tilde{\leqq} (60,70,30,30)_{LR}$	
$x_{231}^{1} \tilde{\leqq} (60,70,40,30)_{LR}$	
$x_{232}^{1} \tilde{\leqq} (60,70,30,30)_{LR}$	
$x_{211}^{2} \tilde{\leqq} (60,70,30,30)_{LR}$	
$x_{212}^{2} \tilde{\leqq} (60,70,20,20)_{LR}$	
$x_{231}^{2} \tilde{\leqq} (60,70,30,30)_{LR}$	
$x_{231}^{2} \tilde{\leqq} (60,70,20,20)_{LR}$	

and x_{ijk}^{t} $(i=j=1,2,3,k,t=1,2)$ is a non-negative LR flat fuzzy number. With respect to Steps 3 and 4 in Section 4, the fuzzy optimal solution can be obtained by solving the following problem:

$$\text{Minimum } \frac{1}{30}(61\, y_{131}^1 + 76\, z_{131}^1 + 8\, \gamma_{131}^1 + 48\, \delta_{131}^1 + 38\, y_{132}^1 + 61\, z_{132}^1 + 8\, \gamma_{132}^1 + 40\, \delta_{132}^1 + 84\, y_{211}^1 + 99\, z_{211}^1 + 8\, \gamma_{211}^1 + 64\, \delta_{211}^1 + 99\, y_{212}^1 + 114\, z_{212}^1 + 16\, \gamma_{212}^1 + 72\, \delta_{212}^1 + 107 y_{231}^1 + 129\, z_{231}^1 + 8\, \gamma_{231}^1 + 80\, \delta_{231}^1 + 61 y_{232}^1 + 84\, z_{232}^1 + 8\, \gamma_{232}^1 + 56\, \delta_{232}^1 + 38\, y_{131}^2 + 53\, z_{131}^2 + 8\, \gamma_{131}^2 + 32\, \delta_{131}^2 + 38\, y_{132}^2 + 76\, z_{132}^2 + 8\, \gamma_{132}^2 + 48\, \delta_{132}^2 + 84\, y_{211}^2 + 91\, z_{211}^2 + 8\, \gamma_{211}^2 + 56\, \delta_{211}^2 + 46 y_{212}^2 + 61\, z_{212}^2 + 0\, \gamma_{212}^2 + 40\, \delta_{212}^2 + 84 y_{231}^2 + 114\, z_{231}^2 + 8\, \gamma_{231}^2 + 72\, \delta_{231}^2 + 38 y_{232}^2 + 61\, z_{232}^2 + 8\, \gamma_{232}^2 + 40\, \delta_{232}^2) \tag{7}$$

$$\text{Minimum } \frac{1}{30}\Big(84\, y_{131}^1 + 114\, z_{131}^1 + 8\, \gamma_{131}^1 + 72\, \delta_{131}^1 + 61\, y_{132}^1 + 76\, z_{132}^1 + 8\, \gamma_{132}^1 + 48\, \delta_{132}^1 + 107\, y_{211}^1 + 114\, z_{211}^1 + 8\, \gamma_{211}^1 + 72\, \delta_{211}^1 + 122\, y_{212}^1 + 137\, z_{212}^1 + 16\, \gamma_{212}^1 + 88\, \delta_{212}^1 + 122 y_{231}^1 + 152\, z_{231}^1 + 16\, \gamma_{231}^1 + 96\, \delta_{231}^1 + 76 y_{232}^1 + 99\, z_{232}^1 + 16\, \gamma_{232}^1 + 64\, \delta_{232}^1 + 61\, y_{131}^2 + 68\, z_{131}^2 + 8\, \gamma_{131}^2 + 40\, \delta_{131}^2 + 61\, y_{132}^2 + 76\, z_{132}^2 + 8\, \gamma_{132}^2 + 48\, \delta_{132}^2 + 107\, y_{211}^2 + 129\, z_{211}^2 + 8\, \gamma_{211}^2 + 80\, \delta_{211}^2 + 99 y_{212}^2 + 114\, z_{212}^2 + 16\, \gamma_{212}^2 + 72\, \delta_{212}^2 + 107 y_{231}^2 + 114\, z_{231}^2 + 8\, \gamma_{231}^2 + 72\, \delta_{231}^2 + 38 y_{232}^2 + 53\, z_{232}^2 + 8\, \gamma_{232}^2 + 32\, \delta_{232}^2\Big)$$

Subject to

$$\frac{1}{2}(y_{211}^1 + y_{212}^1 + y_{231}^1 + y_{232}^1 + z_{211}^1 + z_{212}^1 + z_{231}^1 + z_{232}^1) + \frac{4}{15}(\delta_{211}^1 + \delta_{212}^1 + \delta_{231}^1 + \delta_{232}^1 - \gamma_{211}^1 - \gamma_{212}^1 - \gamma_{231}^1 - \gamma_{232}^1) \le 35$$

$$\frac{1}{2}(y_{211}^2 + y_{212}^2 + y_{231}^2 + y_{232}^2 + z_{211}^2 + z_{212}^2 + z_{231}^2 + z_{232}^2) + \frac{4}{15}(\delta_{211}^2 + \delta_{212}^2 + \delta_{231}^2 + \delta_{232}^2 - \gamma_{211}^2 - \gamma_{212}^2 - \gamma_{231}^2 - \gamma_{232}^2) \le 45$$

$$\frac{1}{2}(y_{131}^1 + y_{132}^1 + z_{131}^1 + z_{132}^1) + \frac{4}{15}(\delta_{131}^1 + \delta_{132}^1 - \gamma_{131}^1 - \gamma_{132}^1) \le \frac{1}{2}(70 + y_{211}^1 + y_{212}^1 + z_{211}^1 + z_{212}^1) + \frac{4}{15}(\delta_{211}^1 + \delta_{212}^1 - \gamma_{211}^1 - \gamma_{212}^1 - 10)$$

$$\frac{1}{2}(y_{131}^2 + y_{132}^2 + z_{131}^2 + z_{132}^2) + \frac{4}{15}(\delta_{131}^2 + \delta_{132}^2 - \gamma_{131}^2 - \gamma_{132}^2) \le \frac{1}{2}(100 + y_{211}^2 + y_{212}^2 + z_{211}^2 + z_{212}^2) + \frac{4}{15}(\delta_{211}^2 + \delta_{212}^2 - \gamma_{211}^2 - \gamma_{212}^2 + 10)$$

$$\frac{1}{2}(y_{131}^1 + y_{132}^1 + y_{231}^1 + y_{232}^1 + z_{131}^1 + z_{132}^1 + z_{231}^1 + z_{232}^1) + \frac{4}{15}(\delta_{131}^1 + \delta_{132}^1 + \delta_{231}^1 + \delta_{232}^1 - \gamma_{131}^1 - \gamma_{132}^1 - \gamma_{231}^1 - \gamma_{232}^1) \ge \frac{128}{3}$$

$\frac{1}{2}(y^2_{131}+y^2_{132}+y^2_{231}+y^2_{232}+z^2_{131}+z^2_{132}+z^2_{231}+z^2_{232}) + \frac{4}{15}(\delta^2_{131}+\delta^2_{132}+\delta^2_{231}+\delta^2_{232}-\gamma^2_{131}-\gamma^2_{132}-\gamma^2_{231}-\gamma^2_{232}) \geq 45$	
$\frac{1}{2}(y^1_{211}+y^1_{231}+y^1_{131}+y^2_{211}+y^2_{231}+y^2_{131}+z^1_{211}+z^1_{231}+z^1_{131}+z^2_{211}+z^2_{231}+z^2_{131}) + \frac{4}{15}(\delta^1_{211}+\delta^1_{231}+\delta^1_{131}+\delta^2_{211}+\delta^2_{231}+\delta^2_{131}-\gamma^1_{211}-\gamma^1_{231}-\gamma^1_{131}-\gamma^2_{211}-\gamma^2_{231}-\gamma^2_{131}) \leq 70$	
$\frac{1}{2}(y^1_{211}+z^1_{211})+\frac{4}{15}(\delta^1_{211}-\gamma^1_{211}) \leq \frac{187}{3}$	
$\frac{1}{2}(y^1_{212}+z^1_{212})+\frac{4}{15}(\delta^1_{212}-\gamma^1_{212}) \leq 65$	
$\frac{1}{2}(y^1_{231}+z^1_{231})+\frac{4}{15}(\delta^1_{231}-\gamma^1_{231}) \leq \frac{187}{3}$	
$\frac{1}{2}(y^1_{232}+z^1_{232})+\frac{4}{15}(\delta^1_{232}-\gamma^1_{232}) \leq 65$	
$\frac{1}{2}(y^2_{211}+z^2_{211})+\frac{4}{15}(\delta^2_{211}-\gamma^2_{211}) \leq 65$	
$\frac{1}{2}(y^2_{212}+z^2_{212})+\frac{4}{15}(\delta^2_{212}-\gamma^2_{212}) \leq 65$	
$\frac{1}{2}(y^2_{231}+z^2_{231})+\frac{4}{15}(\delta^2_{231}-\gamma^2_{231}) \leq 65$	
$\frac{1}{2}(y^2_{232}+z^2_{232})+\frac{4}{15}(\delta^2_{232}-\gamma^2_{232}) \leq 65$	
$y^t_{ijk}-\gamma^t_{ijk}, z^t_{ijk}-y^t_{ijk}, \gamma^t_{ijk}, \delta^t_{ijk} \geq 0 \qquad k \in S_C, (i,j) \in A, t=1,\dots,T$	

With solving this problem using weighted sum method [18], we have,

y^1_{231}	0.0415	y^1_{232}	0.0103	y^1_{211}	0.0627
z^1_{231}	0.0198	z^1_{232}	0.0103	z^1_{211}	0.0979
γ^1_{231}	0	γ^1_{232}	0.6951	γ^1_{211}	0
δ^1_{231}	0	δ^1_{232}	0	δ^1_{211}	0

y_{212}^1	0.1259	y_{131}^1	0.0322	y_{132}^1	0.0481
z_{212}^1	0.1259	z_{331}^1	0.0322	z_{132}^1	0.2467
γ_{212}^1	0	γ_{131}^1	0	γ_{132}^1	0
δ_{212}^1	0	δ_{131}^1	0.1873	δ_{132}^1	0

y_{231}^2	0.0208	y_{232}^2	0.0437	y_{211}^2	0.0474
z_{231}^2	0.1824	z_{232}^2	0.0437	z_{211}^2	0.087
γ_{231}^2	0	γ_{232}^2	0	γ_{211}^2	0.0215
δ_{231}^2	0	δ_{232}^2	0	δ_{211}^2	0

y_{212}^2	-0.1491	y_{131}^2	-0.3316	y_{132}^2	-0.4967
z_{212}^2	-0.1491	z_{131}^2	-0.3316	z_{132}^2	-1.0268
γ_{212}^2	0	γ_{131}^2	0.3438	γ_{132}^2	0
δ_{212}^2	0	δ_{131}^2	0	δ_{132}^2	0

6. Conclusion

In this paper we introduced a new model for fully fuzzy multiobjective multicommodity minimal cost flow problems which there are several commodities to transport from sources to destinations and there is more than one conveyance for these transporting. We also assume that in each conveyance, there are distinct capacities for each commodity. We proposed a method for solving this problem without considering a balanced version of that. Our method can be also considered as a generalization of some methods for solving fuzzy multiobjective method in the presence of equalities and fuzzy inequalities.

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