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Cost and revenue efficiency in DEA with production trade-offs

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In this paper, in order to apply the decision-maker (DM)'s preferred information in the process of measuring the economic efficiency of the decision-making units (DMUs), we use the production trade-offs method in data envelopment analysis (DEA). We propose the cost and revenue efficiency measurement models to evaluate DMUs based on the DM's opinion. In this regard, the importance of inputs and outputs relative to each other is considered in the performance evaluation process. The cost (price) of each input (output) is different for different DMUs. An application of the presented models is provided in the banking sector. In order to sensitivity analysis of the results related to cost and revenue efficiency measurement models, we change production trade-offs matrices. We have shown that by changing the matrixes of production trade-offs, the cost and revenue efficiency score of the banks and their corresponding targets also change. For each bank, total minimum cost and the minimum level of inputs were also provided. The presented models can provide the appropriate targets for the performance of DMUs based on the manager's opinion from an economic point of view.

Keywords: *Data envelopment analysis; Cost efficiency; Revenue efficiency; Production trade-offs.*

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1. Introduction

One of the techniques to evaluate the performance of a set of DMUs is DEA. This method based on mathematical programming models was initially presented by Charnes et al. [1]. This technique obtains the performance of DEA based on inputs and outputs. This model calculates the efficiency score of the DMUs in such a way that the efficiency score of the DMU under evaluation is maximized. DEA for evaluation of DMUs does not consider any priority on input and output components. One of the models for measuring the efficiency of DMUs is the envelope model. This model measures the efficiency of each DMU by constructing a set called the production possibility set (PPS) based on the inputs and outputs of the observed data. This model obtains the radial improvement of the input and output components by projecting these DMUs onto the frontier of a set, which is called the efficiency frontier. This set is formed by accepting a series of axioms in the production process (Geramiet al. [2, 3]). The DMUs that are on the frontier of this set are efficient, and other DMUs are inefficient. Banker et al. [4]. In order to apply the superior information of the DM in the process of performance evaluation, different methods have been presented in DEA. For example, there are weight restrictions methods (Podinovski [5, 6]), production trade-offs methods (Podinovski and Bouzdine-Chameeva [7]), interactive methods in multi-objective programming (Tavana et al. [8]), Gerami et al. [9]), cross-efficiency methods (Lin et al. [10]), and value efficiency methods (Gerami et al. [11]).

The traditional DEA models obtain the technical efficiency of DMUs. However, measuring the efficiency of a group of DMUs is also important from an economic point of view. (Afriat, [12]). To calculate efficiency from an economic perspective, we require price information of inputs and

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outputs. When pricing information is available, we can obtain useful results based on the DEA technique in the evaluation. In this way, the two concepts of cost and revenue efficiency are defined in DEA. Farrell [13] developed a cost-efficiency model in the presence of input prices. This model offers the ability of a DMU to generate its current output at the lowest cost. Next, Färe, Grosskopf, and Lovell [14] presented a cost efficiency model for cost efficiency estimation in the form of a linear programming model. Tonne [15] proved that the cost efficiency evaluation model introduced by Färe, Grosskopf, and Lovell [16] has several weaknesses and drawbacks. In such a way that if two DMUs have the same input and output values and different input price values, then the two DMUs have the same cost and technical efficiency scores. He proposed the cost-based PPS and developed a new model for cost efficiency evaluation to overcome the mentioned drawbacks.

In the following, other studies were presented to evaluate the cost and revenue efficiency in the presence of certain and uncertain prices of inputs and outputs. For example, see the following papers. Tone [15]; Kuosmanen and Post [17], Tone and Sahoo [18, 19], Jahanshahloo, Soleimani-Damaneh, and Mostafaee [20], Mostafaee and Saljooghi [22], Sahoo, Kerstens, and Tone [23]. The application of evaluation models of cost and revenue is also mentioned in the following studies. Banks (Paradi and Zhu [23]), insurance (Tone & Sahoo, [18]), power plants (Hiebert, [24]), agriculture (Rungsuriyawiboon & Hockmann, [25]).

The cost-efficiency evaluation models can choose the price corresponding to the inputs differently. These studies were developed based on the model provided by Tone [15]. Kuosmanen and Post [17] proposed a new model for measuring economic efficiency. They considered incomplete price information and applied their approaches to assess the performance of European commercial banks. Camanho and Dyson [26] used the cost efficiency evaluation model to evaluate the performance of bank branches when prices are uncertain. Tone and Sahoo [18] proposed a cost-efficiency evaluation model in the Indian life insurance industry. Mostafaee and Saljooghi [21] proposed a new approach based on the directional distance function model for measuring cost, revenue, and profit efficiency in DEA. Rungsuriyawiboon and Hockmann [25] developed a dynamic efficiency approach to measure the cost efficiency of the Polish agricultural industry. An et al. [27] developed a fixed cost allocation model for network structure in DEA. They investigated the relation primal and dual in the cost allocation models. An et al. [28] proposed the fixed cost allocation in DEA formwork. They considered two scenarios, including cooperative and no cooperative games between DMUs. Dai et al. [29] proposed a DEA-based model for allocating cost and revenue. They applied a two-step incentive allocation method. Zhu et al. [30] applied DEA models to evaluate the environmental efficiency of EU countries; they proposed the allocation of fixed environmental costs and three different decision objectives for member states. Nguyen and Donnell [31] describe how DEA estimators can be used in order to estimate cost, technical, and allocative efficiency. They used their models to Australian data on hospital and health service providers. Barbero and Zofío [32] developed an open-source Julia package to measure economic efficiency and decompose economic efficiency in technical and allocative efficiencies and consider Farrell’s measures, Russell, weighted additive, and directional functions. Merkert [33] the impact of engine standardization on the cost efficiency of airlines. He showed that both airframe and engine commonality impact airline cost efficiency.

Pourmahmoud [34] proposed a new Fuzzy DEA to evaluating the cost efficiency. He developed a fully fuzzy model with triangular fuzzy input-output data along with triangular fuzzy input prices.
Antunes et al. [35] proposed an innovative DEA model to estimate the cost efficiency of Chinese banks. They developed a stochastic structural relationship programming network model to evaluate the interrelationships between efficiency and other bank-specific variables. Gerami et al. [36] proposed the cost and revenue efficiency evaluation models in DEA in the presence of fuzzy inputs,

outputs. They also consider prices corresponding to inputs and outputs as fuzzy. They applied the proposed approach in the energy sector of the oil industry.

In order to apply value judgement in DEA models, additional restrictions on components on input or output weights can be used in multiplier models of DEA. Applying weight restrictions in DEA models gives results an improved ability to differentiate unit efficiencies (Allen et al. [37]; Cook and Zhu [38]). Podinovski [5] proved that applying weight restrictions in multiplier DEA models creates an additional term in envelope DEA models. They considered this additional term as a production trade-off. They show the relationship between weight restrictions and production trade-offs in the envelope and multiplier DEA models, respectively. They investigated that the application of weight restrictions in multiplier DEA models is equivalent to the presence of production trade-offs in envelope DEA models. The integration of production trade-offs in technologies leads to their expansion. Then it leads to improved discriminating performance of the resulting DEA models. For incorporating value or expert judgements in the production technology, we can use specifying production trade-offs. The production trade-offs were developed by Podinovski [5] originally as the dual forms of weight restrictions in the multiplier DEA models. The production trade-offs are interpretable as simultaneous changes to the inputs or outputs that are supposed to be technologically possible for any of the DMU in the production technology. The production trade-offs expand the production technology, leading to potentially lower efficiency scores and then improved discriminatory power. Podinovski [6] developed a three-stage procedure for obtaining efficient targets in DEA models with production trade-offs and weight restrictions. Podinovski and Bouzdine-Chameeva [7] illustrated that the application of weight restrictions in multiplier DEA models may lead to the infeasibility of these models. They showed that there are several drawbacks when we apply weight restrictions in multiplier DEA models and production trade-offs in envelope DEA models. They proved that in the presence of production trade-offs, the existence of free or unlimited production of outputs leads to problems. The multiplier model becomes infeasible, and the envelopment model has an unbounded optimal solution. They proposed the necessary and sufficient conditions to solve the above problems. They developed analytical criteria and computational methods to identify problematic situations and test free and unlimited production. Podinovski and Bouzdine-Chameeva [39] proposed consistent weight restrictions in DEA. Podinovski [40] shows that the optimal solutions of DEA models in the presence of production trade-offs are optimal among all DMUs in the PPS and are not optimal only among the observed DMUs. Podinovski [41] proposed a single-stage DEA model with weight restrictions for obtaining efficient targets. Podinovski et al. [42] developed DEA models with production trade-offs in the presence of ratio data and used their approach to evaluate the performance of schools in England.

Kraidi et al. [43] proposed a DEA model based on the weight restriction method under constant returns to scales. They applied their approach for measuring efficiency of internet banking in Turkey. Li et al. [44] proposed a new robust two-stage DEA model for analyzing bank efficiency. They considered the structure and uncertainty of nonperforming loans. They utilized their model to assess Chinese commercial banks' performance. They consider weight restriction method onto inputs and outputs. Zibaei Vishghaei et al. [45] proposed a chance-constrained inverse DEA approach under managerial and natural disposability. They consider undesirable outputs for the perturbation of managerial disposable random inputs while the stochastic efficiency is maintained. Moradi and Abbaszadeh [46] proposed a decision-making model for supplier selection based on DEA. They evaluated the technical and scale efficiency of 15 suppliers within a production unit over a three-year period using DEA.

The main contribution of this paper is that we derive cost and revenue efficiency measurement models in DEA for the presence of production trade-offs from inputs and outputs. We analyses the sensitivity of cost and revenue efficiency scores from DMUs to the change of trade-off matrix. We show that in the presence of production trade-offs, we can obtain efficient cost and revenue targets for inefficient DMUs. By applying production trade-offs in cost and revenue efficiency measurement models, we can apply the DM's opinion in the efficiency evaluation process. To demonstrate the applicability of the proposed approach in this paper, we apply it to evaluate a set of commercial banks.

The continuation of this paper is as follows: Section 2 presents cost efficiency measurement models in the presence of different input and output prices. In Section 3, we bring cost and revenue efficiency measurement models in the presence of production trade-offs. In Section 4, we use the proposed approach in this paper to evaluate the performance of a set of commercial banks operating in a competitive market. Section 5 brings the results of the paper.

2. The cost and revenue efficiency in DEA

In this section, we introduce cost and revenue efficiency concepts in DEA.

2.1 The cost efficiency

Suppose we have n DMUs as $DMU_j = \left(X_j, Y_j\right),\ j = 1, \dots, n$ such that each DMU_j consume the input vector $X_j = \left(x_{1j}, \dots, x_{mj}\right)$ to produce the output vector $Y_j = \left(y_{1j}, \dots, y_{sj}\right)$. Let that $x_{ij} > 0$, $y_{rj} > 0,\ i = 1, \dots, m,\ r = 1, \dots, s,\ j = 1, \dots, n$.

Traditional DEA models measure the technical efficiency of DMUs based on input and output data. In some situations, we have prices or priorities that can be considered for the inputs and outputs. When relative weights or prices of inputs and outputs are available, we can evaluate the performance of the DMU under evaluation in more detail and reduce overall production costs. Suppose that the vector $\mathrm{C} = (c_1, c_2, \dots, c_m) \in R_+^m$ is a vector corresponding to the price of the inputs. The production cost of $DMU_o = (X_o, Y_o)$ as under evaluation DMU can be calculated as $C^t X_o = \sum_{i=1}^m c_i x_{io}$. For measuring the cost efficiency score of DMU_o, we solve model (1). Suppose that X^* is the optimal solution corresponding to the model (1) (Tone [15]). This model is called DEA cost efficiency model.

$$
\begin{aligned}
&\min \sum_{i=1}^m c_i x_i \\
&s.t.\quad \sum_{j=1}^n \lambda_j x_{ij} \le x_i, && i = 1, \dots, m, \\
&\qquad\ \ \sum_{j=1}^n \lambda_j y_{rj} \ge y_{ro}, && r = 1, \dots, s \\
&\qquad\ \ \sum_{j=1}^n \lambda_j = 1\ ,\ \lambda_j \ge 0, && j = 1, \dots, n.
\end{aligned}
\tag{1}
$$

We obtain the minimum production cost of DMU_o under variable returns to scale (VRS) technology as follows.

$$
CE_{VRS}^o = \frac{C^t X^*}{C^t X_o} = \frac{\sum_{i=1}^m c_i x_i^*}{\sum_{i=1}^m c_i x_{io}} \tag{2}
$$

Definition 2.1 The cost efficiency score corresponding to $DMU_o = (X_o, Y_o)$ is defined as the ratio of minimum cost to the actual cost namely $CE_{VRS}^o = \frac{C^t X^*}{C^t X_o} = \frac{\sum_{i=1}^m c_i x_i^*}{\sum_{i=1}^m c_i x_{io}}$. If $CE_{VRS}^o = 1$ then DMU_o is called DEA cost efficient. Otherwise we call this DMU as DEA cost inefficient.

2.2 The revenue efficiency

Let vector $P = (p_1, p_2, \dots, p_m) \in R_+^s$ is a vector corresponding to the price of the outputs. The production revenue of $DMU_o = (X_o, Y_o)$ as under evaluation DMU can be calculated as $P^t Y_o = \sum_{r=1}^{s} P_r y_{ro}$. For measuring the revenue efficiency score of DMU_o, we solve model (3). Assume that Y^* is the optimal solution corresponding to the model (3). This model is called DEA revenue efficiency model.

$$\begin{aligned} &\max \sum_{r=1}^{s} p_r y_r \\ &s.t. \ \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, \qquad i = 1, \dots, m, \\ &\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_r, \qquad r = 1, \dots, s \\ &\qquad \sum_{j=1}^{n} \lambda_j = 1 \ , \lambda_j \geq 0, \qquad j = 1, \dots, n. \end{aligned} \tag{3}$$

We calculate the maximum production revenue corresponding to DMU_o under VRS technology as follows.

$$RE_{VRS}^{o} = \frac{P^t Y_o}{P^t Y^*} = \frac{\sum_{r=1}^{s} P_r y_{ro}}{\sum_{r=1}^{s} p_r y_r^*} \tag{4}$$

Definition 2.2 The revenue efficiency score of $DMU_o = (X_o, Y_o)$ is defined as the ratio of the actual revenue to the maximum production revenue namely $RE_{VRS}^{o} = \frac{P^t Y_o}{P^t Y^*} = \frac{\sum_{r=1}^{s} P_r y_{ro}}{\sum_{r=1}^{s} p_r y_r^*}$. If $RE_{VRS}^{o} = 1$ then DMU_o is called DEA revenue efficient. Otherwise we call this DMU as DEA revenue inefficient.

3. The cost and revenue efficiency with production trade offs

Now, we propose cost and revenue efficiency in presence of production trade-offs. Suppose, we have K judgements specifying production trade-offs in the following form:

(Φ_l, Ψ_l), $l = 1, \dots, K$. The vectors $\Phi_l \in R^m, \Psi_l \in R^s$ modify the components of inputs, and outputs of production unit respectively. These vectors can be positive, negative or zero. Assume $V \in R^m$, $U \in R^s$ show the weight vectors correspond to the components of inputs and output respectively. The corresponding weight restrictions of these production trade-offs can be expressed as follows.

$$U^T \Psi_l - V^T \Phi_l \leq 0, l = 1, \dots, K. \tag{5}$$

These weight restrictions with zero on the right-hand side refer as homogeneous. By considering the vectors Φ_l, Ψ_l non-zero, weight restrictions are called linked (Podinovski [5]).

Definition 3.1 Suppose the intensity vector $\lambda \in R_+^n$, vector $\eta_l \in R_+^K$ and slack vectors $\alpha \in R_+^m$ and $\beta \in R_+^s$ exist so that

$$\begin{aligned} X &= \sum_{j=}^{n} \lambda_j X_j + \sum_{l=1}^{K} \eta_l \Phi_l + \alpha, \\ Y &= \sum_{j=}^{n} \lambda_j Y_j + \sum_{l=1}^{K} \eta_l \Psi_l - \beta, \\ &\sum_{j=}^{n} \lambda_j = 1. \end{aligned} \tag{6}$$

Then PPS under VRS with production trade-offs includes all DMUs $(X, Y) \in R_+^{m+s}$ that satisfy in the equation 6.

In equation (6), the expressions $\sum_{j=}^{n} \lambda_j X_j$ and $\sum_{j=}^{n} \lambda_j Y_j$ show an arbitrary DMU in production technology under VRS. The expressions $\sum_{l=1}^{K} \eta_l \, \Phi_l$ and $\sum_{l=1}^{K} \eta_l \, \Psi_l$ modify this DMU according to production trade-offs (Φ_l, Ψ_l), $l = 1, \dots, K$ in some proportions $\eta_l \geq 0$. The resulting DMU changes by increasing its inputs with the vector α and decreasing its outputs with the vector β.

3.1 The cost efficiency with production trade offs

Now we calculus the cost efficiency score of DMU_o by considering production trade-offs on inputs and outputs. For this purpose, we solve model (7) as follows.

$$
\begin{aligned}
& min \ \sum_{i=1}^{m} c_i x_i \\
& s.t. \ \ \sum_{j=1}^{n} \lambda_j \, x_{ij} + \sum_{l=1}^{K} \eta_l \, \Phi_{il} + \alpha_i \leq \ x_i, \qquad i = 1, \dots, m, \\
& \qquad \sum_{j=1}^{n} \lambda_j \, y_{rj} + \sum_{l=1}^{K} \eta_l \, \Psi_{rl} - \beta_r \geq y_{ro}, \qquad r = 1, \dots, s \\
& \qquad \sum_{j=1}^{n} \lambda_j = 1, \ \ \lambda_j \geq 0, \qquad j = 1, \dots, n, \\
& \qquad \sum_{j=1}^{n} \lambda_j \, x_{ij} + \sum_{l=1}^{K} \eta_l \, \Phi_{il} + \alpha_i \geq 0 \ , \ \ i = 1, \dots, m, \\
& \qquad \qquad \alpha_i \geq \text{o}, \quad i = 1, \dots, m, \\
& \qquad \qquad \beta_r \geq 0, \quad r = 1, \dots, s, \\
& \qquad \qquad \eta_l \geq 0, \ l = 1, \dots, K.
\end{aligned}
\tag{7}
$$

Note that model (7) can be simplified. At any of its optimal solutions, vector β must be a zero vector. Therefore, model (7) can be restated as follows.

$$
\begin{aligned}
& min \ \sum_{i=1}^{m} c_i x_i \\
& s.t. \ \ \sum_{j=1}^{n} \lambda_j \, x_{ij} + \sum_{l=1}^{K} \eta_l \, \Phi_{il} + \alpha_i \leq \ x_i, \qquad i = 1, \dots, m, \\
& \qquad \sum_{j=1}^{n} \lambda_j \, y_{rj} + \sum_{l=1}^{K} \eta_l \, \Psi_{rl} \geq y_{ro}, \qquad r = 1, \dots, s \\
& \qquad \sum_{j=1}^{n} \lambda_j = 1, \ \ \lambda_j \geq 0, \qquad j = 1, \dots, n, \\
& \qquad \sum_{j=1}^{n} \lambda_j \, x_{ij} + \sum_{l=1}^{K} \eta_l \, \Phi_{il} + \alpha_i \geq 0 \ , \ \ i = 1, \dots, m, \\
& \qquad \qquad \alpha_i \geq \text{o}, \ i = 1, \dots, m, \\
& \qquad \qquad \eta_l \geq 0, \ l = 1, \dots, K.
\end{aligned}
\tag{8}
$$

Suppose that X' is the optimal solution corresponding to the model (8). We obtain the minimum production cost of DMU_o with production trade-offs as follows.

$$
CE_{TO}^{o} = \frac{C^t X^*}{C^t X_o} = \frac{\sum_{i=1}^{m} c_i x_i'}{\sum_{i=1}^{m} c_i x_{io}}
\tag{9}
$$

Definition 3.1.1 The cost efficiency score with production trade-offs corresponding to $DMU_o = (X_o, Y_o)$ is defined as the ratio of minimum cost to the actual cost namely $CE_{TO}^{o} = \frac{C^t X'}{C^t X_o} = \frac{\sum_{i=1}^{m} c_i x_i'}{\sum_{i=1}^{m} c_i x_{io}}$. If $CE_{TO}^{o} = 1$ then DMU_o is called DEA cost efficient with production trade-offs. Otherwise we call this DMU as DEA cost inefficient.

3.2 The revenue efficiency with production trade offs

We measure the revenue efficiency score of DMU_o by considering production trade-offs on inputs and outputs. In this way, we solve model (10) as follows.

$$
\begin{aligned}
& max \sum_{r=1}^{s} p_r y_r \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + \sum_{l=1}^{K} \eta_l \Phi_{il} + \alpha_i \leq x_{io}, \qquad i = 1, \dots, m, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} + \sum_{l=1}^{K} \eta_l \Psi_{rl} - \beta_r \geq y_r, \qquad r = 1, \dots, s \\
& \qquad \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0, \qquad j = 1, \dots, n, \\
& \qquad \sum_{j=1}^{n} \lambda_j x_{ij} + \sum_{l=1}^{K} \eta_l \Phi_{il} + \alpha_i \geq 0, \ i = 1, \dots, m, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} + \sum_{l=1}^{K} \eta_l \Psi_{rl} - \beta_r \geq 0, \qquad r = 1, \dots, s \\
& \qquad \qquad \alpha_i \geq o, \quad i = 1, \dots, m, \\
& \qquad \qquad \beta_r \geq 0, \quad r = 1, \dots, s, \\
& \qquad \qquad \eta_l \geq 0, l = 1, \dots, K.
\end{aligned}
\tag{10}
$$

Assume that Y' is the optimal solution corresponding to the model (10). This model is called DEA revenue efficiency model with production trade-offs. We calculate the maximum production revenue corresponding to DMU_o with production trade-offs as follows.

$$
RE_{TO}^{o} = \frac{P^t Y_o}{P^t Y'} = \frac{\sum_{r=1}^{s} P_r y_{ro}}{\sum_{r=1}^{s} p_r y_r'} \tag{4}
$$

Definition 3.2.1 The revenue efficiency score with production trade-offs of $DMU_o = (X_o, Y_o)$ is defined as the ratio of the actual revenue to the maximum production revenue namely $RE_{TO}^{o} = \frac{P^t Y_o}{P^t Y'} = \frac{\sum_{r=1}^{s} P_r y_{ro}}{\sum_{r=1}^{s} p_r y_r'}$. If $RE_{VRS}^{o} = 1$ then DMU_o is called DEA revenue efficient with production trade-offs. Otherwise we call this DMU as DEA revenue inefficient.

4. Application of proposed approach in banking industry

In its modern description, a bank is a legal entity in the economy that accepts deposits and pays loans and may perform other ancillary services, such as money transfer, investment, etc., in addition to this task. By aggregating various micro-deposits and employing experienced and expert consultants in financial fields, banks finance the economy, increase its productivity, and make its future path smoother. They also credit and rate their customers, which affects their growth and development. They shape the money flows and payment routes and the rules that govern them, and for this reason, they act as money handlers in the economy. Banks are generally referred to as the pillars of the economy, and if they fail, the entire economy may collapse, and therefore, in critical situations, governments may take over them completely. Also, if there is a deposit refund crisis, the central bank may rush to help banks by printing power money and increasing the amount of money in the economy, even if it leads to an inflationary wave.

In this section, we use a practical example to demonstrate the application of the proposed approach in the paper. The banking industry is one of the most important sectors of the industry of any country that can affect the economy of that country. In this regard, it is important for managers to evaluate banks from an economic point of view. In this regard, it is important to identify banks that have a favorable performance from the point of view of cost and revenue. It is also important for senior bank managers to identify banks that do not perform well from the point of view of cost and revenue. Because we can provide a suitable model for them using the models presented in this paper. These inefficient banks can bring their input and output levels to the levels of their corresponding target banks in the future in order to become efficient. If cost reduction or increase in revenue is important for the bank's senior managers, DEA models based on cost and revenue efficiency can be used and appropriate goals can be achieved.

The banking industry is among the top 10 industries in the stock market and entered the stock market in the 1980s. Although little time has passed since this industry joined the stock market, it has a strong background and foundation. We use the affiliates of 17 banks in Iran. In this paper, in order to apply the opinion of senior bank managers in the process of evaluating the performance of banks, we use the method of production trade-offs in DEA. In this method, we consider the level of importance corresponding to the input and output components relative to each other in the cost and revenue efficiency evaluation model.

The method of determining inputs and outputs is determined based on the intermediary role of banks in the economy. Banks borrow and lend funds. In this evaluation, we consider two inputs for banks. These two inputs are more important from the point of view of bank management. These inputs include interest expenses and non-interest expenses.

Interest expenses are the expenses incurred by the bank for borrowed funds and represent the expenses payable for deposits and other borrowed funds. Therefore, interest expenses are associated with attracting and maintaining the depositor's funds.
Non-interest expenses are the operational expenses of the bank, the expenses of converting deposits into loans. Non-interest expenses include all operational and overhead expenses of the bank, such as employee salaries and benefits, professional and administrative services, equipment, and other expenses.
In order to determine the price of inputs in measuring cost efficiency, interest and non-interest costs were converted into deposit amounts and working hours. Assume interest costs on deposits and borrowed funds are 2%, or **0.02 $** per dollar deposited or borrowed. Also, we calculated employee salaries and benefits as a proxy for non-interest costs of **33$**, an input cost of **33$** by dividing the average weekly salary and benefits in the financial sector obtained from Iran Statistics by one week. It is a 40-hour job.
In this evaluation of banks, two outputs are used in the form of interest income and non-interest income.
Interest income is interest income related to lending. This includes interest on personal loans, business loans, mortgages, and government securities.
Noninterest income represents fees on transactions and deposits, including monthly checking account service fees, inactivity fees, checking and deposit fees, annual credit card fees, and other banking operating income.

The data set is given in Table 1.

Table 1. Data set of Iranian banks

Bank	I1	I2	O1	O2	Technical efficiency
B01	288.53	63.07	41.6	253.64	1
B02	6989.14	142.25	3163.1	3545.57	1
B03	4240.72	16942.76	3819.33	12696.67	1
B04	10949.39	18268.66	5059.34	18635.02	1
B05	126.08	27.19	18.05	107.41	0.9932
B06	2547.12	5369.35	2936.3	8998.12	0.528
B07	27578	24448	14178	34391	1
B08	509.75	673.4	463.39	890.33	0.7582
B09	14453.35	5119.73	4154.83	12335.67	1
B10	14425.17	5873.94	4773.5	12965.07	0.9863
B11	7797.03	4279.64	1362.13	8298.93	1

B12	10399.16	4125.31	3116.13	10608.52	0.8808
B13	4156	5075	3707	6322	0.8274
B14	6879	7998	5440	10478	0.8271
B15	25.1	4.43	1.07	17.04	1
B16	17.1	119.2	20	135.4	1
B17	39.45	6.98	6.31	38.65	0.7328
Average	6554.12	5796.29	3074.12	8277.47	0.91375

We put unit input prices of 0.02 and 33 for interest expenses and non-interest expenses, respectively. First, we obtain the technical efficiency scores of banks by using the BCC model under VRS technology (Banker et al. [4]). Due to the importance of inputs, we use models in the input-orientated evaluation. The results are given in the sixth column of Table 2. As can be seen, banks B01, B02, B03, B04, B07, B09, B11, B15, and B16 are efficient, and other banks are inefficient. In the following, we calculate the cost efficiency of banks under VRS technology based on model (1). The results are given in Table 2. The second and third columns of Table 2 show the optimal level of inputs based on the cost efficiency model corresponding to banks. The optimal input level indicates the amount of specific input to the units in order to reach the cost efficiency level of the banks. The third and fourth columns contain the total cost observed and the total minimum cost assigned to the bank in the cost efficiency evaluation process. The last column shows the cost efficiency scores. As can be seen, banks B02, B03, B07, B09, and B15 are efficient cost banks, and other banks are inefficient cost efficient.

Table 2. The results of cost efficiency model

Bank	Optimal input level		Total observed cost	Total minimum cost	Cost efficiency
B01	126.7165	183.9356	84.5332	63.2331	0.748
B02	3163.1	3545.57	1233.3	1233.3	1
B03	3819.33	12696.67	4266.288	4266.288	1
B04	6566.035	18390.59	6250.743	6200.215	0.9919
B05	49.2656	81.1919	35.8063	27.7786	0.7758
B06	1746.65	4619.723	3028.106	1559.442	0.515
B07	14178	34391	11632.59	11632.59	1
B08	295.618	675.0793	303.0767	228.6885	0.7546
B09	4154.83	12335.67	4153.868	4153.868	1
B10	4354.396	12807.36	4373.943	4313.517	0.9862
B11	3467.703	6889.414	2765.89	2342.861	0.8471
B12	3709.857	8697.562	3563.134	2944.393	0.8263
B13	2440.313	5230.858	2160.4	1774.99	0.8216
B14	3583.111	8687.333	3566.54	2938.482	0.8239
B15	1.07	17.04	5.6446	5.6446	1
B16	26.9416	102.9541	45.082	34.5137	0.7656
B17	7.8416	25.7284	12.8807	8.6472	0.6713
Average	3040.634	7610.452	2793.049	2572.262	0.8545

We consider unit output prices of **1.5$** and **4$** for interest income and non-interest income according to the opinion of the bank's senior managers, respectively. Now, we calculate the revenue efficiency of banks under VRS technology based on model (3). The results are given in Table 3. The second and third columns of Table 3 propose the optimal level of outputs based on the revenue efficiency model corresponding to banks. The optimal output level indicates the amount of specific output to the banks in order to reach the revenue efficiency level of the banks. The third and fourth columns contain the total revenue observed and the total maximum revenue assigned to the bank in the revenue efficiency evaluation process. The last column shows the revenue efficiency scores. Banks B03, B04, B07, B11, B15, and B16 are efficient revenue banks, and other banks have inefficient revenue.

Table 3. The results of revenue efficiency model

Bank	Optimal output level		Total observed revenue	Total maximum revenue	Revenue efficiency
B01	120.1312	192.4445	685.075	949.9747	0.7212
B02	1198.237	4718.085	11052.71	20669.7	0.5347
B03	4240.72	16942.76	74132.12	74132.12	1
B04	10949.39	18268.66	89498.73	89498.73	1
B05	26.1506	98.6118	297.88	433.6731	0.6869
B06	3011.056	12001.98	25298.08	52524.51	0.4816
B07	27578	24448	139159	139159	1
B08	315.4443	1171.031	3458.225	5157.292	0.6706
B09	4120.698	16460.51	42158.95	72023.09	0.5854
B10	4529.446	17035.61	45133.52	74936.63	0.6023
B11	7797.03	4279.64	28814.11	28814.11	1
B12	3858.512	13715.71	32099.98	60650.6	0.5293
B13	2121.322	8427.033	26534	36890.12	0.7193
B14	3503.075	13978.91	42310.5	61170.24	0.6917
B15	25.1	4.43	55.37	55.37	1
B16	17.1	119.2	502.45	502.45	1
B17	28.6935	30.0109	87.095	163.0836	0.5341
Average	4320.006	8934.86	33016.34	42219.45	0.7504

4.1 Efficiency of cost and revenue of banks based on the opinion of bank managers

In this section, we measure the cost and revenue efficiency of banks with the opinions of senior bank managers. For this purpose, we use the method of production trade-offs and consider the importance of inputs and outputs relative to each other in the model. At first, we consider two different weight restrictions to solve models (8) and (10). We select production trade-offs matrixes Φ_l, Ψ_l as follows.

Production trade-offs 1: $\Phi_1 = \binom{2}{1}, \ \Psi_1 = \binom{1}{3}$.

Then $i = 2, r = 2, t = 1$.

The weight restriction corresponding to these matrixes on the components of input and output are as follows.

$$3u_2 + u_1 - 1v_2 - 2v_1 \leq 0.$$

where u_1, u_2, v_1 and v_2 are weights corresponding to output components and input components, respectively. In this weight restriction, the importance corresponding to the outputs and inputs is like this: the sum of three times the second output and one time the first output is less than or equal to the sum of one time the second input and two times the first input. In this way, the importance of inputs and outputs, according to the opinion of bank managers, is included in the cost and revenue evaluation models.

We consider again unit input prices of 0.02$ and 33$ for interest expenses and non-interest expenses, respectively. We solve model (8) for measuring cost efficiency with production trade-offs. The results of model (8) are given in Table 4. The second and third columns of Table 4 show the optimal level of inputs based on the cost efficiency model with production trade-offs 1. The third and fourth columns contain the total cost observed and the total minimum cost assigned to the bank in the cost efficiency evaluation process by selecting production trade-offs 1. The last column shows the cost-efficiency scores of banks. As can be seen, banks B02 and B15 are only efficient cost banks, and other banks are inefficient cost efficient.

Table 4. The results of cost efficiency model with production trade-offs 1

Bank	Optimal input level		Total minimum cost	Total observed cost	Cost efficiency
B01	148.3951	159.3582	55.5561	84.5332	0.6572
B02	3163.1	3545.57	1233.3	1233.3	1
B03	11293.29	5663.15	2094.705	4266.288	0.491
B04	14323.46	8539.679	3104.563	6250.743	0.4967
B05	57.6899	71.6411	24.7954	35.8063	0.6925
B06	3903.294	2174.7	795.7169	3028.106	0.2628
B07	44340.82	24134.43	8851.178	11632.59	0.7609
B08	563.1708	371.7503	133.941	303.0767	0.4419
B09	18091.52	11009.78	3995.058	4153.868	0.9618
B10	18035.16	10981.6	3984.631	4373.943	0.911
B11	5667.852	4637.136	1643.612	2765.89	0.5942
B12	9983.14	6955.59	2495.008	3563.134	0.7002
B13	4464.601	2935.89	1058.136	2160.4	0.4898
B14	7189.3	4790.507	1724.653	3566.54	0.4836
B15	1.07	17.04	5.6446	5.6446	1
B16	77.5833	55.2967	19.7996	45.082	0.4392
B17	8.7611	24.6859	8.3216	12.8807	0.6461
Average	8312.483	5062.812	1836.978	2793.049	0.6488

Again, we consider unit output prices of **1.5$** and **4$** for interest income and non-interest income according to the opinion of the bank's senior managers, respectively. Now, we calculate the revenue efficiency of banks with production trade-offs under VRS technology based on model (10). For solving model (10), we select with production trade-offs 1. The results of model (10) are given in Table 5. The second and third columns of Table 3 propose the optimal level of outputs based on the revenue efficiency model corresponding to banks with considering production trade-offs 1 on inputs and outputs. The third and fourth columns contain the total revenue observed and the total maximum revenue assigned to the bank in the revenue efficiency evaluation process with production trade-offs 1, based on the opinion of bank managers. The last column shows the revenue efficiency scores. Banks B03, B04, B11, B15, and B16 are efficient revenue banks, and other banks are inefficient revenue with production trade-offs.

Table 5. The results of revenue efficiency model with production trade-offs 1

Bank	Optimal output level		Total observed revenue	Total maximum revenue	Revenue efficiency
B01	120.1312	192.4445	685.075	949.9747	0.7212
B02	2023.173	6774.63	11052.71	30133.28	0.3668

B03	4240.72	16942.76	74132.12	74132.12	1
B04	10949.39	18268.66	89498.73	89498.73	1
B05	26.1506	98.6118	297.88	433.6731	0.6869
B06	3101.715	12227.99	25298.08	53564.53	0.4723
B07	15508.72	31946.65	139159	151049.7	0.9213
B08	393.7714	1366.299	3458.225	6055.852	0.5711
B09	4295.239	16895.64	42158.95	74025.41	0.5695
B10	4673.115	18156.8	45133.52	79636.86	0.5667
B11	7797.03	4279.64	28814.11	28814.11	1
B12	3858.512	13715.71	32099.98	60650.6	0.5293
B13	2831.455	10197.38	26534	45036.69	0.5892
B14	4402.397	16220.89	42310.5	71487.17	0.5919
B15	25.1	4.43	55.37	55.37	1
B16	17.1	119.2	502.45	502.45	1
B17	28.6935	30.0109	87.095	163.0836	0.5341
Average	3781.907	9849.279	33016.34	45069.97	0.713

As can be seen, the cost and revenue efficiency scores corresponding to banks are reduced by applying weight restrictions on the input and output components, and no improvement is achieved. For example, the cost efficiency average of banks without considering the weight restrictions is equal to 0.8545, while its corresponding score by considering the weight restrictions is equal to 0.6488.

In order to sensitivity analysis of the results related to cost and revenue efficiency measurement models namely models (8) and (9) to the change of production trade-offs matrices, we select these matrixes Φ_l, Ψ_l as follows.

Production trade-offs 2: $\Phi_1 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \Psi_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

Then $i = 2, r = 2, t = 1$.

The weight restriction corresponding to these matrixes on the components of input and output are as follows.

$$2u_2 - u_1 + 2v_2 - 3v_1 \leq 0.$$

The results of models (8) and (9) by selecting production trade-offs 2 are given in Tables 6 and 7. According to the last column of Table 6, bank B07 is the only cost-efficient bank with production trade-offs 2. Also from the last column of Table 7, banks B03, B04, B07, B11, B15, and B16 are efficient revenue banks, and other banks are inefficient revenue with production trade-offs 2.

Table 6. The results of cost efficiency model with production trade-offs 2

Bank	Optimal input level		Total minimum cost	Total observed cost	Cost efficiency
B01	468.8202	0	9.3764	84.5332	0.1109
B02	16670.2	0	333.4039	1233.3	0.2703
B03	18184.19	0	363.6838	4266.288	0.0852
B04	64063.83	1133.78	1655.424	6250.743	0.2648
B05	204.6768	0	4.0935	35.8063	0.1143
B06	6950.244	0	139.0049	3028.106	0.0459
B07	14178	34391	11632.59	11632.59	1

B08	1068.303	0	21.3661	303.0767	0.0705
B09	53551.95	8141.7	3757.8	4153.868	0.9047
B10	53636.49	8085.34	3740.892	4373.943	0.8553
B11	20401.95	0	408.0391	2765.89	0.1475
B12	65714.52	33.32	1325.286	3563.134	0.3719
B13	8440.073	0	168.8015	2160.4	0.0781
B14	16161.45	0	323.2289	3566.54	0.0906
B15	40.4835	0	0.8097	5.6446	0.1434
B16	108.8017	0	2.176	45.082	0.0483
B17	63.8166	0	1.2763	12.8807	0.0991
Average	468.8202	0	9.3764	84.5332	0.1109

Table 7. The results of revenue efficiency model with production trade-offs 2

Bank	Optimal output level		Total observed revenue	Total maximum revenue	Revenue efficiency
B01	120.1312	192.4445	685.075	949.9747	0.7212
B02	1002.96	7440.866	11052.71	31267.91	0.3535
B03	4240.72	16942.76	74132.12	74132.12	1
B04	10949.39	18268.66	89498.73	89498.73	1
B05	26.1506	98.6118	297.88	433.6731	0.6869
B06	2989.596	12301.21	25298.08	53689.22	0.4712
B07	27578	24448	139159	139159	1
B08	296.9029	1429.557	3458.225	6163.584	0.5611
B09	4079.381	17036.6	42158.95	74265.47	0.5677
B10	4874.907	17645.62	45133.52	77894.85	0.5794
B11	7797.03	4279.64	28814.11	28814.11	1
B12	3858.512	13715.71	32099.98	60650.6	0.5293
B13	1953.22	10770.89	26534	46013.41	0.5767
B14	3290.189	16947.21	42310.5	72724.11	0.5818
B15	25.1	4.43	55.37	55.37	1
B16	17.1	119.2	502.45	502.45	1
B17	28.6935	30.0109	87.095	163.0836	0.5341
Average	4301.646	9510.084	33016.34	44492.8	0.7155

5. Research gap

Previous studies conducted to measure cost and revenue efficiency did not consider the opinion of the DM. This paper introduced value judgment in measuring cost and revenue efficiency based on the DEA. We used of production trade-offs method for incorporating a DM's a priori knowledge into the analysis. We analyses the sensitivity of cost and revenue efficiency scores from DMUs to the change of trade-off matrix. We show that in the presence of production trade-offs, we can obtain efficient cost and revenue targets for inefficient DMUs. By applying production trade-offs in cost and revenue efficiency measurement models, we can apply the DM's opinion in the efficiency evaluation process. We applied our approach to evaluate a set of commercial banks. By using the presented models, we can obtain the efficiency of banks from an economic point of view based on the opinion of the bank's senior managers. In this regard, the importance of inputs and outputs relative to each other is considered in the performance evaluation process. The presented models have a linear structure and can be easily solved with common optimization software such as GAMS.

6. Conclusion

In this paper, we presented cost and revenue efficiency evaluation models in the presence of production trade-offs from inputs and outputs. Production trade-offs in envelopment DEA models are equivalent to considering weight restrictions on input and output components in multiplier DEA models. To apply the opinion of the DM in the process of evaluating the cost and revenue efficiency, we can use the appropriate method of production trade-offs. What is important is the correct selection of the matrix of production trade-offs, because the inappropriate selection of these matrixes may cause the cost and revenue efficiency evaluation models to have unbounded optimal solutions. This issue is inappropriate from a computational point of view, and this result occurs when we face unlimited production of inputs and outputs in the cost and revenue efficiency evaluation models. In this paper, cost- and revenue-efficient targets corresponding to inefficient units were also presented. Inefficient units should bring their input and output level to the level of these cost and revenue-efficient units. We have shown that by changing the matrixes of production trade-offs, the cost and revenue efficiency score of the units and their corresponding targets also change. We presented an application of the models presented in this paper in the banking industry. As future work, we can also develop the approach presented in this paper to measure profit efficiency. We can also develop the models presented in this paper for the case where the data are imprecise numbers, such as fuzzy numbers.

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