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Optimizing Hub-And-Spoke Networks Under Demand Uncertainty: A Stochastic Capacitated Single Allocation P-Hub Covering Model with Lagrangian Relaxation

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Traditional maximal p-hub covering problems focus on scenarios where network flow is constrained by resource limitations. However, many existing models rely on static parameters, overlooking the inherent randomness present in real-world logistics. This oversight can result in suboptimal network designs that are vulnerable to congestion and rising costs as demand varies. To address this issue, we propose a novel mathematical model for the capacitated single allocation maximal p-hub covering problem that takes into account stochastic variations in origin-destination flows. Although solving this model poses computational challenges, we utilize a Lagrangian relaxation algorithm to enhance efficiency. Computational experiments using the CAB dataset highlight the effectiveness of our approach in achieving optimal solutions while reducing computation time. This framework offers valuable insights for designing robust hub-and-spoke networks in the face of demand uncertainty.

***Keywords**: Maximal hub covering, Demand uncertainty, Lagrangian Relaxation Method, Capacitated hubs.*

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1. Introduction

Hub networks are a fundamental infrastructure in logistics and telecommunications, facilitating efficient flow movement between origin and destination points. These networks rely on strategically located hub facilities that provide crucial services like switching, sorting, and consolidating flows. By optimizing network connections and minimizing costs, hub networks play a vital role in various industries [7].

Hub location is a strategic decision-making problem. At the strategic level, long-term decisions are made that are usually difficult to change and require significant amounts of time and cost to implement. Hub location and network design are typically based on the forecast of future demand, which is inherently stochastic. Therefore, this demand uncertainty cannot be ignored in hub network design problems [31]. Parameters such as customer demand, cost, and travel time naturally involve uncertainty and cannot be accurately estimated with deterministic data. Some researchers have examined how to incorporate various aspects of uncertainty in hub network design.

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Marianov and Serra [18] Provided the first research on uncertainty in hub location within air transport networks, modeling hubs as M/D/C queue systems. Their mathematical model is based on the probability of customer presence in the system, which later forms the probabilistic capacity constraints of the hub. They used a tabu search heuristic for problem-solving due to the computational complexity of their model. Sim et al. [28] presented the probabilistic single allocation p-hub center problem and used a chance-constrained mathematical model to ensure minimum service levels, with travel times assumed to follow a normal distribution. A heuristic algorithm was proposed for solving the model. Yang [29, 30] developed a probabilistic programming model for fixed-cost hub location with single allocation, considering seasonal demand variations as different scenarios. The model included direct connections between non-hub nodes and used real data from Taiwan and China airlines, solved with GAMS and OSL solvers. Bashiri et al. [5] proposed a probabilistic single allocation p-hub center problem with travel times modeled as normally distributed random variables. The objective was to maximize the minimum service level for the maximum travel time, using a genetic algorithm for solution. Contreras et al. [9] formulated a two-stage stochastic integer programming model for fixed-cost, multiple allocation hub location, considering uncertain demand and flow costs. They introduced three different probabilistic models and employed Monte Carlo simulation-based algorithms and Benders decomposition for solving. Mohammadi et al. [20] extended Marianov and Serra's model for container transport, considering random transport times and truck arrival rates. They used a combined genetic and imperialist competitive algorithm for solving. Zhai et al. [32] proposed a two-stage stochastic hub location model with risk minimization criteria and uncertain demand represented by a random vector. They showed that the two-stage programming is equivalent to a single-stage p-model, solved using branch and bound methods. Alumur et al. [2] presented three models for uncertain fixed-cost hub location problems with both single and multiple allocations. They used various scenarios for uncertain costs and demands, analyzed using CAB data and CPLEX software. Mohammadi et al. [21] developed a multi-objective stochastic model for complete single allocation hub covering problems under uncertainty, including risk factors for transport times. They compared their results using multi-objective imperialist competitive algorithm with NSGA-II and PAES algorithms. Chen et al. [8] provided a two-stage stochastic programming model for single allocation hub center location with budget constraints, aiming to minimize the longest expected path, particularly for disaster response facility location. Hult et al. [15] developed a probabilistic single allocation hub center location model with stochastic travel times, aiming to minimize the maximum travel time given a minimum service level. They proposed exact solution methods based on variable reduction and decomposition algorithms. Sadeghi et al. [25] offered a probabilistic complete coverage hub location model with random capacity paths, aiming to minimize total transportation and hub establishment costs. They used differential evolution and standard genetic algorithms for comparison. Adibi and Razmi [1] proposed a two-stage stochastic programming model for fixed-cost, multiple allocation hub location, considering uncertain demand and transportation costs, analyzed using Iranian air data with GAMS and CPLEX. Ebrahimizade et al. [11] developed a bi-objective probabilistic hub covering model with uncertain transport, maximizing flow and reliability on the weakest network path, using fuzzy multi-objective linear programming for solving. Yang et al. [31] proposed a two-stage stochastic hub network design model with fixed costs, considering seasonal demand variations as discrete distributions with multiple scenarios. Zhalechian et al. [33] proposed a multi-objective mixed integer nonlinear mathematical model for hub location with probabilistic-possibilistic uncertainty, considering various transport modes and independent travel times. Shang et al. [26] formulated a stochastic multi-commodity hub location problem with direct link strategy and multiple capacity levels, using expected value and chance-constrained programming techniques. Hu et al. [14] developed a stochastic single allocation hub location model with capacity constraints and independent normally distributed random demands, approximated using piecewise tangent and linear approximations. Rostami et al. [24] offered a fixed-cost, single allocation hub location model under demand uncertainty, optimized using a custom branch-and-cut algorithm. Ghaffari-Nasab [12] formulated a stochastic hub location problem with Bernoulli demands, providing both single and multiple allocation models, solved using Benders decomposition and Lagrange relaxation techniques. Rahmati et al. [23] proposed a profit-maximizing hub location model considering carbon emission control and population density, using advanced sampling and Benders decomposition algorithms. Bayram et al. [6] formulated a hub network design problem considering congestion, capacity, and stochastic demand, extending classical hub location problems. Demand was defined as random based on different scenarios, with a Benders decomposition and column generation algorithm for solving. Janschekowitz et al. [16] proposed an optimization-simulation iterative approach for hub network design under single and multiple allocation uncertainty, considering uncertain demand, transportation costs, and fixed costs for establishing hubs and connections. The approach integrated scenario-based optimization and simulation techniques, incorporating flow-dependent scale economies in the simulation phase. Andaryan et al. [3] formulated a single allocation hub location problem with capacity constraints, considering Bernoulli demand distribution. The problem was examined under facility and customer outsourcing policies, with two-stage stochastic programming formulations and a Tabu search-based algorithm for large-scale instances. Guillot et al. [13] present a novel approach to designing reconfigurable park-and-ride systems by integrating hub location and fleet assignment decisions. Their two-stage stochastic model effectively captures the dynamic nature of travel demand and traffic conditions, leading to more resilient and efficient system designs. The authors' development of advanced solution methodologies, including L-shaped and metaheuristic approaches, is commendable. By applying their model to real-world data from Lyon, France, the study provides valuable insights into the practical implementation of reconfigurable park-and-ride systems.

A review of the maximal hub covering problem literature reveals that most models assume static future decision parameters and are developed in a certain environment. By assuming parameter certainty, designing and solving a hub network becomes simpler compared to models that consider dynamic, ambiguous, or stochastic conditions. However, these static models often lack practicality. For instance, in air transport, seasonal changes in travel volume and climate can affect key decision-making parameters such as flow and travel time. A hub network designed with a fixed flow volume may face congestion, rising costs, and poor service delivery if actual flows exceed hub capacity. Addressing flow changes can mitigate such issues, leading to a more stable network. For example, Ball, Barnhart [4] estimated the total effect of flight delays in the United States in 2007 at about $30 billion, accounting for lost revenue, flight crew overtime, passenger re-accommodation costs, and other welfare losses.

In this study, we tackle stochastic variations in origin-destination (O/D) flows by developing a mathematical formulation for the Stochastic Capacitated Single Allocation Maximal Hub Covering Problem (SCSMHCP). We model uncertainty in O/D flows using a finite set of scenarios, each assigned a specific probability. The proposed model operates in two stages: In the first stage, we focus on hub location, assuming it remains stable across scenarios due to its strategic nature and immunity to random parameter fluctuations. In the second stage, we adapt the flow transfers from origins to destinations in response to the revealed uncertainties, while accounting for coverage and capacity constraints. This approach enables flexible routing of flows, minimizing the risk of network congestion.

Due to the computational complexity of the proposed model, we utilize the Lagrangian relaxation algorithm to obtain feasible solutions. We assess the model and solution methods by solving a range of problems. Remarkably, to the best of our knowledge, our study is the first to incorporate flow uncertainty and capacity constraints into maximal hub covering problems.

The paper is organized as follows: Section 2 proposes the mathematical model for the capacitated single allocation maximal hub covering problem. Section 3 presents the Lagrangian relaxation method. Section 4 shows the experimental results. Finally, the conclusion is presented in Section 5.

2. Mathematical Formulation

2.1. Assumptions

The Stochastic Capacitated Single Allocation Maximal Hub Covering Problem (SCSMHCP) model incorporates the following assumptions:

- *Discrete Flow Distribution*: Demand uncertainty is represented by a discrete probability distribution for origin-destination (O/D) flows. This means there are a finite number of possible flow realizations, also referred to as scenarios.
- *Known Scenario Information*: In each scenario, the O/D flow values are known and interdependent. Additionally, the probability of occurrence for each scenario is also known.
- *Hub-Encouraged Transportation*: Transportation via hubs is assumed to be incentivized due to cost and distance savings.
- *Symmetric Travel*: Direct travel time (or cost) is considered equal for both directions between two nodes.
- *Limited Direct Connections*: Direct transportation between non-hub nodes is not permitted. All O/D flows must be routed through hubs.
- *Complete Inter-Hub Network*: The network formed by the hubs themselves is considered a complete graph, and the O/D route passes at least one and at most two hubs.
- *Single/Double Hub Routing*: O/D flows must pass through at least one and at most two hubs on their route.
- *Predefined Hub Count*: The number of hubs to be located is predetermined before solving the model.
- *Hub Node Selection Flexibility*: Any node in the network can be chosen as a hub location.
- *Single Hub Allocation*: Each non-hub node can be assigned to only one hub for flow routing.
- *Hub Capacity Constraints*: A capacity constraint is imposed on the incoming flow at each hub, representing the maximum flow a hub can handle.
- *Known Hub Capacities*: The capacities of all hub nodes are assumed to be known beforehand.

2.2. Notation

In this section, we introduce the mathematical notation employed throughout the analysis. We define the sets, indices, parameters, and decision variables used in the model. This notation provides a foundation for understanding the subsequent development and results presented in the following sections.

Notations and Parameters:

N: Set of nodes	$N=\{1,\dots,n\}$
S: Set of scenarios	$S=\{1,\dots,m\}$
i,j: Indices of origin (destination) nodes	$i,j = 1,\dots,n$
k,l: Indices of hubs	$k,l = 1,\dots,n$

s: Indices of scenarios $s = 1, \dots, m$
P: Number of hubs
D_{ij}: Travel time of direct path from node i to node j $i, j = 1, \dots, n$
W_{ij}^{s}: The amount of demand flow from origin node i to destination node j under scenario s. $i, j = 1, \dots, n$ $s = 1, \dots, m$
p_s: The probability of occurrence of scenario s ($\sum_{s\in S} p_s = 1$) $s = 1, \dots, m$
α: The discount factor for transferring flow between two hub nodes ($0<\alpha\leq 1$)
β: Coverage radius (allowable travel time/cost between O/D nodes)
Γ_k: The capacity of hub k $k = 1, \dots, n$

Decision variables:

$y_{ijkl}^{s} = 1$, if the flow from node i is transported to node j through hubs k and l under scenario s, otherwise 0. $i, j, k, l = 1, \dots, n$ $s = 1, \dots, m$

$x_{ik}^{s} = 1$, if node i is allocated to hub k under scenario s, otherwise 0. $i, k = 1, \dots, n$ & $i \neq k$ $s = 1, \dots, m$

$v_k = 1$, if a hub is established in node k under scenario s, otherwise 0 $i, k = 1, \dots, n$

2.3. Mathematical Formulation of SCSMHCP

Building upon the provided notation and parameters, here's the mathematical formulation of the Stochastic Capacitated Single Allocation Maximal Hub Covering Problem (SCSMHCP):

$$\text{SCSMHCP: } Max \quad \sum_{s=1}^{m} p(s) \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} W_{ij}^{s} \; y_{ijkl}^{s} \tag{1}$$

s.t.

$$\sum_{k=1,\ k\neq i}^{n} x_{ik}^{s} = 1 - v_i \qquad i = 1, \dots, n, s = 1, \dots, m \tag{2}$$

$$x_{ik}^{s} \leq v_k \qquad i, k = 1, \dots, n, i \neq k, s = 1, \dots, m \tag{3}$$

$$\sum_{k=1}^{n} v_k = P \tag{4}$$

$$2\, y_{ijkl}^{s} \leq x_{ik}^{s} + x_{jl}^{s} \qquad i, j, k, l = 1, \dots, n, i \neq k, j \neq l, s = 1, \dots, m \tag{5}$$

$$2\, y_{kjkl}^{s} \leq v_k + x_{jl}^{s} \qquad j, k, l = 1, \dots, n, j \neq l, s = 1, \dots, m \tag{6}$$

$$2\, y_{ilkl}^{s} \leq x_{ik}^{s} + v_l \qquad i, k, l = 1, \dots, n, i \neq k, s = 1, \dots, m \tag{7}$$

$$2\, y_{klkl}^{s} \leq v_k + v_l \qquad k, l = 1, \dots, n, s = 1, \dots, m \tag{8}$$

$$\left(D_{ik} + \alpha D_{kl} + D_{lj}\right) \times \; y_{ijkl}^{s} \leq \beta \qquad i, j, k, l = 1, \dots, n, s = 1, \dots, m \tag{9}$$

$$\sum_{k=1}^{n}\sum_{l=1}^{n} y_{ijkl}^{s} \leq 1 \qquad i,j = 1,\dots,n\,, s = 1,\dots,m \qquad (10)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}^{s}\left[\sum_{l=1}^{n}\left(y_{ijkl}^{s} + y_{ijlk}^{s}\right) - y_{ijkk}^{s}\right] \leq \Gamma_{k} \qquad k = 1,\dots,n\,, s = 1,\dots,m \qquad (11)$$

$$y_{ijkl}^{s} \in \{0,1\} \qquad i,j,k,l = 1,\dots,n\,, s = 1,\dots,m \qquad (12)$$

$$x_{ik}^{s} \in \{0,1\} \qquad i,k = 1,\dots,n\,, i \neq k\,, s = 1,\dots,m \qquad (13)$$

$$v_{k} \in \{0,1\} \qquad k = 1,\dots,n \qquad (14)$$

Our model aims to maximize the total expected flow in a network with hubs and non-hub nodes. We achieve this through the following formulation:

- *Objective Function* (1): This function maximizes the summation of expected flows across all origin-destination pairs.
- *Constraints*:
  - *Hub Assignment* (2): These constraints ensure that each non-hub node can be assigned to at most one hub, and simultaneously prevent hubs from being assigned to other nodes.
  - *Hub Connection Eligibility* (3): This set restricts flow assignments under each scenario. Node *i* can only be assigned to node *k* if *k* is a designated hub.
  - *Number of Hubs* (4): This constraint guarantees that exactly *P* hubs are established within the network.
  - *Flow Establishment Conditions* (5-8): These constraints define the conditions under which flow is established between origin and destination pairs based on hub connections and selections.
    - *Constraint* (5): Flow is established if origin *i* connects to hub *k* and destination *j* connects to hub *l* (standard case).
    - *Constraints* (6) & (7): Flow may also occur if *i* is the hub or *j* is the hub, respectively.
    - *Constraint* (8): Flow is additionally possible if both *i* and *j* are hubs.
  - *Travel Time Restriction* (9): This constraint ensures that the path $i \rightarrow k \rightarrow l \rightarrow j$ can only be established if the travel time is less than the coverage radius β.
  - *Unique Routes* (10): This constraint guarantees that at most one route exists between each origin-destination pair.
  - *Hub Capacity* (11): This constraint limits the total entering flow to each hub under each scenario, ensuring it doesn't exceed the hub's capacity.
  - *Decision Variable Types* (12)-(14): These constraints define the type (binary, continuous, etc.) of each decision variable used in the model.

Following this formulation, we delve into the solution method for this proposed model in the subsequent section.

3. Solution Approach: Lagrangian Relaxation

The Stochastic Capacitated Single Allocation Maximal Hub Covering Problem (SCSMHCP) inherits the NP-hardness of the basic Maximal Hub Covering Problem (Kara and Tansel [17]).

Additionally, the introduction of scenarios and capacity constraints in SCSMHCP significantly increases the number of variables and constraints compared to simpler MHCP models. This renders directly solving the formulated model computationally intractable for large-scale problems.

To address this challenge and achieve efficient solutions, we employ the Lagrangian Relaxation (LR) method. This technique is particularly well-suited for problems with complex constraints, such as those relating hub location decisions variables (*v*,*x*) and the flow routing variables (*y*) in constraints (5) to (8) of the SCSMHCP model.

Core Idea of Lagrangian Relaxation:

1. *Constraint Relaxation:* The LR method relaxes a subset of complicating constraints by removing them from the original problem.
2. *Lagrangian Function:* A penalty term, representing the violation of the relaxed constraints, is added to the objective function. This term is multiplied by a Lagrange multiplier, which acts as a weighted penalty for violating the constraints.
3. *Sub-Problems:* By relaxing the constraints, the original problem is decomposed into two easier-to-solve sub-problems:
    - A relaxed master problem that optimizes the objective function considering the penalty terms.
    - A set of Lagrangian sub-problems, one for each relaxed constraint. These sub-problems minimize the violation of the relaxed constraints for the current values of the Lagrange multipliers.
4. *Iterative Improvement:* The Lagrange multipliers and solutions from the sub-problems are used iteratively to improve the solution of the master problem and ultimately obtain an optimal solution for the original problem.

By employing the LR method, we can decompose the complex SCSMHCP model into smaller, more manageable sub-problems. This allows for efficient solution techniques to be applied to each sub-problem, leading to faster computation of near-optimal solutions for large-scale instances of the SCSMHCP.

To solve the SCSAMHCP model using the LR, the model's rigid constraints are first identified and added to the objective function. At first glance, it is clear from the proposed model that constraints set (5)-(8) establishes the relationship between the binary variables *v*, *x* and *y*. If the desired constraints are relaxed, the created Lagrangian problem can be solved more quickly. Let $e_{ijkl}^{s} \geq 0$ be the Lagrange multiplier for constraint (5), $f_{kjl}^{s} \geq 0$ be the Lagrange multiplier for constraint (6), $g_{ikl}^{s} \geq 0$ be the Lagrange multiplier for constraint (7), and $h_{kl}^{s} \geq 0$ be the Lagrange multiplier for constraint (8). By doing so, we obtain the following relaxed model:

$$
\begin{aligned}
LR\!:\ Max\ & \sum_{s=1}^{S} p(s) \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} W_{ij}^{s}\ y_{ijkl}^{s} \\
&+ \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \sum_{k=1,\neq i}^{N} \sum_{l=1,\neq j}^{N} e_{ijkl}^{s}\left(-2\, y_{ijkl}^{s} + x_{ik}^{s} + x_{jl}^{s}\right) \\
&+ \sum_{s=1}^{S} \sum_{k=1}^{N} \sum_{j=1,\neq l}^{N} \sum_{l=1}^{N} f_{kjl}^{s}\left(-2\, y_{kjkl}^{s} + v_{k} + x_{jl}^{s}\right) \\
&+ \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{k=1,\neq i}^{N} \sum_{l=1}^{N} g_{ikl}^{s}\left(-2\, y_{ilkl}^{s} + x_{ik}^{s} + v_{l}\right) \\
&+ \sum_{s=1}^{S} \sum_{k=1}^{N} \sum_{l=1}^{N} h_{kl}^{s}\left(-2\, y_{klkl}^{s} + v_{k} + v_{l}\right) \\
&\text{s.t. } (2)\text{-}(4), (9)\text{-}(14)
\end{aligned}
\tag{15}
$$

By expanding equation (15), and considering the constraints, the LR model can be separated into two location-allocation (LR_x) and routing (LR_y) sub-problems as follows.

$$
\begin{aligned}
LR_{x}\!:\ Max\ & \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \sum_{k=1,\neq i}^{N} \sum_{l=1,\neq j}^{N} e_{ijkl}^{s}\left(x_{ik}^{s} + x_{jl}^{s}\right) \\
&+ \sum_{s=1}^{S} \sum_{k=1}^{N} \sum_{j=1,\neq l}^{N} \sum_{l=1}^{N} f_{kjl}^{s}\left(v_{k} + x_{jl}^{s}\right) + \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{k=1,\neq i}^{N} \sum_{l=1}^{N} g_{ikl}^{s}\left(x_{ik}^{s} + v_{l}\right) \\
&+ \sum_{s=1}^{S} \sum_{k=1}^{N} \sum_{l=1}^{N} h_{kl}^{s}\left(v_{k} + v_{l}\right) \\
&\text{s.t. } (2)\text{-}(4), (13),(14)
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
LR_{y}\!:\ Max\ & \sum_{s=1}^{S} p(s) \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} W_{ij}^{s}\ y_{ijkl}^{s} \\
&- \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \sum_{k=1,\neq i}^{N} \sum_{l=1,\neq j}^{N} 2e_{ijkl}^{s}\, y_{ijkl}^{s} - \sum_{s=1}^{S} \sum_{k=1}^{N} \sum_{j=1,\neq l}^{N} \sum_{l=1}^{N} 2f_{kjl}^{s}\, y_{kjkl}^{s} \\
&- \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{k=1,\neq i}^{N} \sum_{l=1}^{N} 2g_{ikl}^{s}\, y_{ilkl}^{s} - \sum_{s=1}^{S} \sum_{k=1}^{N} \sum_{l=1}^{N} 2\, h_{kl}^{s}\, y_{klkl}^{s} \\
&\text{S.t. } (9)\text{-}(12)
\end{aligned}
\tag{17}
$$

The necessary parameters for applying the LR method are as follows:

$\boldsymbol{UB_x^r}$: The objective function value of $\boldsymbol{LR_x}$ in the rth iteration

$\boldsymbol{UB_y^r}$: The objective function value of LR_y in the *r*th iteration
$\boldsymbol{\varepsilon_{Max}}$: Maximum allowed iteration for Lagrangian relaxation algorithm
$\boldsymbol{T_{Max}}$: Maximum runtime for Lagrangian relaxation algorithm
$\boldsymbol{LB^r}$: The lower bound obtained for the original problem in the *r*th iteration
$\boldsymbol{UB^r}$: The upper bound obtained for the original problem in the *r*th iteration
$\boldsymbol{v_k^{*r}}$: The optimal value of the variable *v* in the rth iteration
$\boldsymbol{x_{iks}^{*r}}$: The optimal value of the variable *x* in the rth iteration
$\boldsymbol{y_{ijkls}^{*r}}$: The optimal value of the variable *y* in the rth iteration
$\boldsymbol{{e_{ijkl}^s}^r}$: The Lagrangian multiplier corresponds to the constraint (5) in the *r*th iteration
$\boldsymbol{{ge_{ijkl}^s}^r}$: The subgradients correspond to the constraint (5) in the *r*th iteration
$\boldsymbol{{f_{kjl}^s}^r}$: The Lagrangian multiplier corresponds to the constraint (6) in the *r*th iteration
$\boldsymbol{{gf_{kjl}^s}^r}$: The subgradients correspond to the constraint (6) in the *r*th iteration
$\boldsymbol{{g_{ikl}^s}^r}$: The Lagrangian multiplier corresponds to the constraint (7) in the *r*th iteration
$\boldsymbol{{gg_{ikl}^s}^r}$: The subgradients correspond to the constraint (7) in the *r*th iteration
$\boldsymbol{{h_{kl}^s}^r}$: The Lagrangian multiplier corresponds to the constraint (8) in the *r*th iteration
$\boldsymbol{{gh_{kl}^s}^r}$: The subgradients correspond to the constraint (8) in the *r*th iteration
$\boldsymbol{\theta_r}$: improvement Step Size of the Lagrangian multiplier in *r*th iteration
$\boldsymbol{\lambda_r}$: The variable coefficient for calculating θ_r , it has a value between zero and 2.
Limit: Maximum number of no improvement in the upper bound of the objective function in order to change the coefficient
No imp: Counter of no improvement in the upper bound
$\boldsymbol{\varphi}$: The number of necessary changes in the step size after reaching the Limit parameter
$\boldsymbol{\varepsilon psilon}$: Maximum Percentage gap between the upper and lower bounds for stopping algorithm

To obtain a lower-bound value in each iteration (LB^r), we set the v_k^{*r} values in the original problem and solve it to achieve the values of *y* and the objective function.

The Algorithm 1 shows the pseudo-code of the Lagrangian relaxation algorithm used to solve the SCSAMHCP problem. First, the input parameters of the algorithm are set. Then, LR_x and LR_y problems are solved, and a new upper bound value is obtained. If the new upper bound value is better than the current best upper bound value, it is replaced, and the counter of no improvement sets to zero (No imp=0). The original problem for determining the lower bound is solved by considering the hubs' locations obtained from the LR_x sub-problem. If a better upper bound is not obtained, one unit is added to the non-improvement counter, and if the counter reaches its upper limit, the λ_n value becomes smaller to make a more accurate search, and the counter will return to zero. After calculating the lower bound of the objective function, it is checked whether a better bound has been created or not. In the following, the algorithm's stop conditions are examined, including reaching the maximum iteration, reaching the maximum runtime, or meeting the conditions of maximum percentage gap between the upper and lower bounds. The new subgradients' values and the Lagrangian multipliers are calculated if the stop conditions are not met. The optimization condition of the current solution is then examined. The current solution is optimal if, for all non-zero subgradients' values, their multiplication with corresponding Lagrangian multipliers equals zero. In this case, the current solution will be the optimal solution to the original problem. If the stop conditions are not met, the previous steps are repeated.

Step 0: Set parameters as follows:

$r = 1, \quad Best\ LB = -\infty, \quad Best\ UB = +\infty$

$Limit = 5, \quad No\ imp = 0,$

$\lambda_r = 2, \varphi = 2, \varepsilon_{Max} = 100$

$T_{Max} = 3600, epsilon = 0.01$

Initialize Lagrangian multipliers:

$e_{ijkl}^{s} = 0, for\ i,j,k,l = 1,\dots,n, i \neq k, j \neq l,\ s = 1,\dots,m$

$f_{kjl}^{s} = 0, for\ j,k,l = 1,\dots,n, j \neq l,\ s = 1,\dots,m$

$g_{ikl}^{s} = 0, for\ i,k,l = 1,\dots,n, i \neq k,\ s = 1,\dots,m$

$h_{kl}^{s} = 0, for\ k,l = 1,\dots,n,\ s = 1,\dots,m$

Step 1: Solve LR_x and LR_y. Calculate $UB^r = UB_x^r + UB_y^r$

If $UB^r < Best\ UB$ then $Best\ UB = UB^r$ and $No\ imp = 0$

else

$No\ imp = No\ imp + 1$

If $No\ imp > Limit$ Then $\lambda_r = \lambda_r/\varphi$ and $No\ imp = 0$

Step 2: set $\{v_k^r = v_k^{*r}, for\ k = 1,\dots,n\}$ and solve *SCSMHCP* model.

If $LB^r > Best\ LB$ then $Best\ LB = LB^r$

Step 3: If $r > \varepsilon_{Max}$ or $T > T_{Max}$ STOP

Else update subgradients as follows:

$ge_{ijkl}^{s\ \ r} = -2y_{ijkl}^{s*\ \ r} + x_{ik}^{s*r} + x_{jl}^{s*r} \quad , for\ i,j,k,l = 1,\dots,n, i \neq k, j \neq l, \quad s = 1,\dots,m$

$gf_{kjl}^{s\ \ r} = -2y_{kjkl}^{s*\ \ r} + v_k^{*r} + x_{jl}^{s*r} \quad , for\ j,k,l = 1,\dots,n, j \neq l, \quad s = 1,\dots,m$

$gg_{ikl}^{s\ \ r} = -2y_{ilkl}^{s*\ \ r} + x_{ik}^{s*r} + v_l^{*r} \quad , for\ i,k,l = 1,\dots,n, i \neq k, \quad s = 1,\dots,m$

$gh_{kl}^{s\ \ r} = -2y_{klkl}^{s*\ \ r} + v_k^{*r} + v_l^{*r} \quad , for\ k,l = 1,\dots,n, \quad s = 1,\dots,m$

$$\theta_r = \lambda_r \left(\frac{UB^r - LB^r}{\sum_{i,j,k,l,s}(ge_{ijkl}^{s\ \ r})^2 + \sum_{j,k,l,s}(gf_{kjl}^{s\ \ r})^2 + \sum_{i,k,l,s}(gg_{ikl}^{s\ \ r})^2 + \sum_{k,l,s}(gh_{kl}^{s\ \ r})^2} \right)$$

$e_{ijkl}^{s\ \ r+1} = max\ \{0, e_{ijkl}^{s\ \ r} + \theta_n ge_{ijkl}^{s\ \ r}\}, for\ i,j,k,l = 1,\dots,n, i \neq k, j \neq l, s = 1,\dots,m$

$f_{kjl}^{s\ \ r+1} = max\ \{0, f_{kjl}^{s\ \ r} + \theta_n\ gf_{kjl}^{s\ \ r}\}\ , for\ j,k,l = 1,\dots,n, j \neq l, \quad s = 1,\dots,m$

$g_{ikl}^{s\ \ r+1} = max\ \{0, g_{ikl}^{s\ \ r} + \theta_n gg_{ikl}^{s\ \ r}\}, for\ i,k,l = 1,\dots,n, i \neq k, \quad s = 1,\dots,m$

$h_{kl}^{s\ \ r+1} = max\ \{0,\ h_{kl}^{s\ \ r} + \theta_n\ gh_{kl}^{s\ \ r}\}\ , for\ k,l = 1,\dots,n, \quad s = 1,\dots,m$

Step 4: If $\{(ge_{ijkl}^{s\ \ r} \geq 0\ and\ ge_{ijkl}^{s\ \ r} \times e_{ijkl}^{s\ \ r} = 0)$ and $(gf_{kjl}^{s\ \ r} \geq 0\ and\ gf_{kjl}^{s\ \ r} \times f_{kjl}^{s\ \ r} = 0)$ and $(gg_{ikl}^{s\ \ r} \geq 0\ and\ gg_{ikl}^{s\ \ r} \times g_{ikl}^{s\ \ r} = 0)$ and $(gh_{kl}^{s\ \ r} \geq 0\ and\ gh_{kl}^{s\ \ r} \times h_{kl}^{s\ \ r} = 0)$ for $i,j,k,l = 1,\dots,n, i \neq k, j \neq l, s = 1,\dots,m\}$STOP, the optimal solution is achieved

Else *r=r+1* and Go to Step 1.

Algorithm 1. pseudo-code of Lagrangian relaxation algorithm

4. Computational Results

This section presents the computational experiments conducted to evaluate the performance of the proposed SCSAMHCP model. We describe the data generation methods employed for simulating different demand scenarios and analyze the obtained results.

4.1. Data Generation

To assess the SCSAMHCP model's efficacy under varying demand conditions, three scenarios were considered: low demand, medium demand, and high demand.

- **Medium Demand:** Data for this scenario was obtained from the publicly available CAB dataset [22].
- **Low Demand:** In this scenario, the flow rate was reduced by 50% compared to the medium demand scenario, reflecting lower customer demand [29, 30].
- **High Demand:** Conversely, the flow rate was increased by 50% compared to the medium demand scenario, simulating high customer demand conditions [29, 30].

To further diversify the test environments, we employed varying values for the discount factor and radius of coverage, as suggested by Silva and Cunha [27]. Additionally, the capacity level for each node was calculated using the method proposed by Ebery, Krishnamoorthy [10]. All parameter values used in the data generation process are summarized in Table 1

Table 1- Parameter Values for Data Generation

Parameters	Values
n	10, 15, 20, 25
P	2, 3, 4, 5
m	3
$p(s)$	$\frac{1}{3} \quad s = 1,2,3$
w_{ij}^{s}	w_{ij}^{1} = CAB dataset $w_{ij}^{2} = 0.5w_{ij}^{1}$ $w_{ij}^{3} = 1.5w_{ij}^{1}$
Γ_k	$\Gamma_k = \left(\frac{n}{P}\right) \times \sum_{s=1}^{m} \sum_{j=1}^{n} p(s) \times W_{kj}^{s} \qquad k = 1, \dots, n$

4.2. Results Analysis

We employed GAMS software (version 24.9.1) and CPLEX solvers (version 12.7.1) to solve the proposed algorithms. The software ran on a computer equipped with a 3 GHz Intel processor and 4GB of RAM. We set a runtime limit of 3600 seconds for the implementation. The Lagrangian relaxation algorithm was implemented with specific settings: $\lambda_n = 2$, $\varphi = 2$, $\text{Limit} = 5$, $\text{εpsilon} = 0.01$, $\varepsilon_{Max} = 100, T_{Max} = 3600$. To illustrate the algorithm's performance, we generated sample problems with specific settings detailed in Table 2.

Table 2- Sample issues settings

Problem Number	Parameters settings: Number of Nodes	Number of Hubs	Discount Factor	Coverage Radius	Problem Number	Parameters settings: Number of Nodes	Number of Hubs	Discount Factor	Coverage Radius
1	10	2	0.2	1425	41	20	2	0.2	1851
2	10	3	0.2	1117	42	20	3	0.2	1549
3	10	4	0.2	811	43	20	4	0.2	1356
4	10	5	0.2	736	44	20	5	0.2	1162
5	10	2	0.4	1627	45	20	2	0.4	2067
6	10	3	0.4	1185	46	20	3	0.4	1744
7	10	4	0.4	970	47	20	4	0.4	1473
8	10	5	0.4	863	48	20	5	0.4	1386
9	10	2	0.6	1671	49	20	2	0.6	2255
10	10	3	0.6	1387	50	20	3	0.6	1996
11	10	4	0.6	1148	51	20	4	0.6	1835
12	10	5	0.6	1079	52	20	5	0.6	1663
13	10	2	0.8	1744	53	20	2	0.8	2493
14	10	3	0.8	1589	54	20	3	0.8	2264
15	10	4	0.8	1457	55	20	4	0.8	2154
16	10	5	0.8	1413	56	20	5	0.8	2118
17	10	2	1	1839	57	20	2	1	2611
18	10	3	1	1791	58	20	3	1	2605
19	10	4	1	1770	59	20	4	1	2601
20	10	5	1	1766	60	20	5	1	2600
21	15	2	0.2	2004	61	25	2	0.2	2136
22	15	3	0.2	1638	62	25	3	0.2	1913
23	15	4	0.2	1324	63	25	4	0.2	1617
24	15	5	0.2	1149	64	25	5	0.2	1346
25	15	2	0.4	2019	65	25	2	0.4	2401
26	15	3	0.4	1741	66	25	3	0.4	2099
27	15	4	0.4	1436	67	25	4	0.4	1881
28	15	5	0.4	1287	68	25	5	0.4	1597
29	15	2	0.6	2103	69	25	2	0.6	2557
30	15	3	0.6	1844	70	25	3	0.6	2336
31	15	4	0.6	1756	71	25	4	0.6	2184
32	15	5	0.6	1560	72	25	5	0.6	2002
33	15	2	0.8	2424	73	25	2	0.8	2713
34	15	3	0.8	2165	74	25	3	0.8	2552
35	15	4	0.8	2100	75	25	4	0.8	2457
36	15	5	0.8	2080	76	25	5	0.8	2307
37	15	2	1	2611	77	25	2	1	2806
38	15	3	1	2610	78	25	3	1	2762
39	15	4	1	2605	79	25	4	1	2726
40	15	5	1	2600	80	25	5	1	2725

4.2.1 Performance Evaluation of SCSAMHCP Model

To assess the effectiveness of the proposed stochastic model (SCSAMHCP), we evaluated two key metrics: Expected Value of Perfect Information (EVPI) and Value of the Stochastic Solution (VSS) [19]. These metrics are calculated by comparing the model's solution to the optimal solution of the Stochastic Problem (SP):

EVPI: This metric quantifies the value of knowing future conditions with certainty.

VSS: This metric represents the benefit of using probabilistic information about future outcomes instead of relying solely on deterministic data.

The following equations (Equations 22, 23, and 24) were employed to calculate EVPI and VSS [19].

$$EVPI = (WS - SP)/SP \tag{18}$$

$$VSS = (SP - EVS)/SP \tag{19}$$

$$EVS \leq SP \leq WS \tag{20}$$

4.2.2 Comparison of Modeling Methods

To determine the SCSAMHCP model's advantage, we compared it with two alternative solution approaches:

- Wait-and-See (WS) Solution: This approach involves solving the deterministic equivalent (DE) for each scenario independently. The final solution is then obtained by averaging the objective values across all scenarios, weighted by their respective probabilities.
- Expected Value Solution (EVS): This approach involves solving the deterministic model with the average flow parameter across all scenarios to determine hub locations. Subsequently, the optimal solution for each scenario is calculated individually using the established hubs. Finally, the EVS value is the average objective value obtained from all scenarios.

The results obtained for evaluating the SCSAMHCP model are presented in Table 3.

Table 3- Results of evaluating SCSAMHCP model

Problem Number	SP	EVPI	VSS	Problem Number	SP	EVPI	VSS
1	919,935	1.07%	0.00%	**11**	744,260	0.61%	0.05%
2	830,171	0.24%	0.00%	**12**	686,828	1.07%	1.50%
3	723,825	1.15%	0.00%	**13**	939,834	0.74%	3.51%
4	679,364	1.14%	1.11%	**14**	834,474	0.45%	0.00%
5	945,279	0.20%	0.00%	**15**	756,879	0.53%	0.00%
6	824,102	0.40%	0.00%	**16**	697,326	1.14%	0.00%
7	737,296	1.41%	0.48%	**17**	951,047	0.00%	4.21%
8	678,675	0.87%	1.01%	**18**	837,261	0.28%	0.00%
9	945,279	0.00%	0.00%	**19**	764,001	0.31%	0.00%
10	830,145	0.56%	0.00%	**20**	703,409	0.34%	0.00%
				Average	801,470	0.63%	0.59%

Analysis of Results:

The VSS values in Table 3 indicate that the SCSAMHCP model often provides superior solutions compared to the deterministic models (positive VSS values). This improvement arises from incorporating probabilistic data rather than relying solely on deterministic assumptions. Deterministic models, employing average data, have a solution space inherently limited compared to the probabilistic model. This limitation explains why they cannot outperform the SCSAMHCP model, which was confirmed by solving the problems.

Furthermore, solving problems with more than 15 nodes revealed a significant increase in execution time due to the model's NP-hard nature. This highlights the need for further exploration of computational efficiency techniques for larger-scale instances.

4.2.3 Evaluating the Performance of the Proposed Solution Method

This section evaluates the performance of the proposed Lagrangian Relaxation (LR) method for solving the Stochastic Capacitated Maximal Hub Covering Problem (SCSAMHCP).

Since finding the optimal solution for all problem instances can be computationally challenging, an upper bound for the SCSAMHCP problem was established (Equation (21)) to assess the error percentage of each method. These upper bound leverages the fact that the objective functions of the multiple allocation version of stochastic maximal hub covering problems inherently create an upper bound for single-allocation problems. To achieve this, constraints set (2) were removed from the SCSAMHCP model and re-solved.

$$Error = \frac{(Upperbound\_Value) - (Objective\_Value)}{(Upperbound\_Value)} \tag{21}$$

The detailed results of the performance evaluation are presented in Appendix. The LR method exhibits a consistently lower error rate (4.22%) compared to other method (26.82%), indicating superior solution quality. It also demonstrates a smaller average runtime, making it a more efficient approach for larger problem instances. Based on these findings, we can conclude that the Mixed-Integer Linear Programming (MILP) model remains a viable option for solving problems with smaller dimensions. However, for larger and more complex scenarios, the LR method emerges as a more efficient and accurate solution strategy.

5. Conclusions

This study tackles the challenge of designing robust hub-and-spoke networks amid demand uncertainty in logistics and telecommunications. We proposed a novel mathematical model known as the Stochastic Capacitated Single Allocation Maximal Hub Covering Problem (SCSMHCP), which accounts for variability in origin-destination flows. Our model represents a significant advancement over existing approaches that depend on static flow assumptions.

The Lagrangian Relaxation (LR) algorithm was employed to solve the computationally challenging SCSAMHCP model efficiently. The effectiveness of the proposed model and solution approach was evaluated through computational experiments using various problem instances. The results demonstrated that:

- The SCSAMHCP model provides superior solutions compared to deterministic models that neglect demand uncertainty. This is evident from the positive Value of the Stochastic Solution (VSS) values.

- The LR algorithm offers a good balance between solution quality and computational efficiency, making it suitable for solving large-scale problems.

Our findings indicate that the SCSAMHCP model, along with the Lagrangian Relaxation (LR) solution method, serves as an effective tool for designing robust and flexible hub-and-spoke networks capable of managing fluctuating demand conditions in real-world logistics and telecommunications contexts. This research paves the way for further exploration in various areas. Future work may include:

- Investigating the scalability of the proposed model and solution approach for even larger network instances.
- Developing more sophisticated methods for incorporating different types of demand uncertainties, such as temporal variations.
- Exploring the integration of the SCSAMHCP model with dynamic network optimization frameworks for real-time flow management in hub-and-spoke networks.

By focusing on these areas, we can further improve the practical applicability of the proposed model and solution method for creating robust and efficient hub-and-spoke networks in response to dynamic demand conditions.

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Appendix: Results obtained from different methods of solving the SCMAMHCP model

Problem Number	Upper-Bound	SCSAMHCP			LR		
		Objective Value	Runtime (s)	Error	Objective Value	Runtime (s)	Error
1	919,935	919,935	36	0.00%	919,901	183	0.00%
2	830,171	830,171	45	0.00%	830,169	202	0.00%
3	723,825	723,825	25	0.00%	718,149	243	0.78%
4	679,364	679,364	1572	0.00%	678,837	1138	0.08%
5	945,279	945,279	23	0.00%	945,276	199	0.00%
6	824,100	824,100	67	0.00%	823,732	196	0.04%
7	737,296	737,296	39	0.00%	726,568	220	1.46%
8	678,675	678,675	30	0.00%	678,675	299	0.00%
9	945,279	945,279	20	0.00%	867,127	194	8.27%
10	830,145	830,145	83	0.00%	829,027	202	0.13%
11	744,260	744,260	56	0.00%	729,499	224	1.98%
12	686,828	686,828	51	0.00%	679,841	327	1.02%
13	939,834	939,834	78	0.00%	923,373	190	1.75%
14	834,474	834,474	261	0.00%	830,220	254	0.51%
15	756,879	756,879	100	0.00%	740,187	501	2.21%
16	697,326	697,326	95	0.00%	690,530	391	0.97%
17	951,047	951,047	29	0.00%	934,121	187	1.78%
18	837,261	837,261	135	0.00%	827,899	321	1.12%
19	764,001	764,001	1770	0.00%	752,591	497	1.49%
20	703,409	703,409	1163	0.00%	688,708	493	2.09%
21	2,293,477	2,293,477	572	0.00%	2,293,477	652	0.00%
22	2,193,061	2,193,061	754	0.00%	2,163,970	674	1.33%
23	2,089,862	2,089,862	872	0.00%	1,993,400	731	4.62%
24	2,082,010	1,981,617	3600	4.82%	1,981,610	1588	4.82%
25	2,293,477	2,293,477	809	0.00%	2,293,477	683	0.00%
26	2,175,294	2,175,294	2293	0.00%	2,173,330	730	0.09%
27	2,016,944	2,016,944	441	0.00%	1,882,820	771	6.65%
28	1,938,864	1,938,864	616	0.00%	1,873,240	971	3.38%
29	2,317,659	2,317,659	303	0.00%	2,317,659	678	0.00%
30	2,149,915	2,149,915	794	0.00%	2,102,090	685	2.22%
31	2,057,290	2,057,290	1581	0.00%	2,057,220	920	0.00%
32	2,234,163	1,941,342	3600	13.11%	1,880,920	1109	15.81%
33	2,334,862	2,334,862	512	0.00%	2,334,860	730	0.00%
34	2,199,318	2,199,318	1024	0.00%	2,113,370	979	3.91%
35	2,364,942	2,061,179	3600	12.84%	1,967,520	1198	16.80%
36	2,320,434	1,972,208	3600	15.01%	1,890,040	3600	18.55%

Problem Number	Upper-Bound	SCSAMHCP			LR		
		Objective Value	Runtime (s)	Error	Objective Value	Runtime (s)	Error
37	2,305,489	2,305,489	732	0.00%	2,239,610	679	2.86%
38	2,364,942	2,251,888	3600	4.78%	2,251,888	1103	4.78%
39	2,364,942	2,115,293	3600	10.56%	2,070,980	3600	12.43%
40	2,320,434	2,022,491	3600	12.84%	1,961,950	3600	15.45%
41	5,294,908	2,550,896	3600	51.82%	5,075,970	2064	4.13%
42	5,374,303	5,374,303	2997	0.00%	5,374,300	2786	0.00%
43	5,340,441	5,340,441	2941	0.00%	5,338,340	2255	0.04%
44	5,706,888	5,146,359	3600	9.82%	5,083,090	2313	10.93%
45	5,642,255	1,990,531	3600	64.72%	5,349,580	2151	5.19%
46	5,514,049	5,514,049	2812	0.00%	5,514,049	2241	0.00%
47	5,710,398	5,272,968	3600	7.66%	5,172,860	2537	9.41%
48	5,729,076	5,116,473	3600	10.69%	4,913,080	2476	14.24%
49	5,667,077	5,222,609	3600	7.84%	5,339,069	2749	5.79%
50	5,754,594	3,901,694	3600	32.20%	5,585,340	2190	2.94%
51	5,739,268	5,128,879	3600	10.64%	5,339,210	2466	6.97%
52	5,744,452	5,098,658	3600	11.24%	5,037,220	3513	12.31%
53	5,752,254	1,990,421	3600	65.40%	5,433,240	2991	5.55%
54	5,754,594	4,256,282	3600	26.04%	5,476,408	2240	4.83%
55	5,754,594	5,014,274	3600	12.86%	5,279,990	3600	8.25%
56	5,754,594	4,744,012	3600	17.56%	4,966,830	3600	13.69%
57	5,754,594	5,122,655	3600	10.98%	5,523,278	2580	4.02%
58	5,754,594	1,352,844	3600	76.49%	5,432,860	2381	5.59%
59	5,754,594	3,374,349	3600	41.36%	5,383,340	3600	6.45%
60	5,710,086	976,340	3600	82.90%	5,088,600	3600	10.88%
61	8,391,410	3,035,691	3600	63.82%	7,428,197	3600	11.48%
62	8,539,901	2,049,641	3600	76.00%	8,374,150	3600	1.94%
63	8,523,370	1,641,238	3600	80.74%	8,244,920	3600	3.27%
64	8,169,545	1,394,661	3600	82.93%	7,772,027	3600	4.87%
65	8,207,264	3,035,699	3600	63.01%	7,745,736	3600	5.62%
66	8,539,901	2,049,646	3600	76.00%	8,358,110	3600	2.13%
67	8,312,332	1,869,931	3600	77.50%	8,235,969	3600	0.92%
68	8,292,035	1,394,652	3600	83.18%	8,235,970	3600	0.68%
69	8,147,716	3,035,669	3600	62.74%	7,824,860	3600	3.96%
70	8,392,633	2,049,724	3600	75.58%	8,313,170	3600	0.95%
71	8,475,657	1,641,339	3600	80.63%	8,260,840	3600	2.53%
72	8,356,619	4,881,475	3600	41.59%	8,131,350	3600	2.70%
73	8,199,543	3,035,560	3600	62.98%	8,009,290	3600	2.32%
74	8,445,827	2,049,679	3600	75.73%	8,200,747	3600	2.90%
75	8,463,800	1,641,153	3600	80.61%	8,200,747	3600	3.11%

Problem Number	Upper-Bound	SCSAMHCP			LR		
		Objective Value	Runtime (s)	Error	Objective Value	Runtime (s)	Error
76	8,153,549	1,394,413	3600	82.90%	8,072,390	3600	1.00%
77	8,234,366	3,035,796	3600	63.13%	8,070,770	3600	1.99%
78	8,278,286	2,049,851	3600	75.24%	7,924,460	3600	4.27%
79	8,144,848	1,641,308	3600	79.85%	7,796,750	3600	4.27%
80	8,186,563	1,394,493	3600	82.97%	7,679,290	3600	6.20%