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Utilizing Fuzzy Linear Programming for Addressing Ecological Decision-Making Under Uncertainty

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This article examines and analyzes fuzzy linear programming models and techniques. Since its emergence in the 1970s, fuzzy linear programming has addressed the growing complexity of decision-making problems in uncertain and dynamic environments. This study proposes a solution method based on Yager's linear ranking function for solving fuzzy linear programming problems with inequality constraints. Quantitative results from solving a production planning case study show that the proposed method achieves an optimal total profit, where only the production of the third product is economically justified. The sensitivity analysis performed on the model's fuzzy parameters indicates that the optimal solution remains stable within a range of possible values for profit and machine availability. The scope of this method encompasses decision-making problems under uncertainty with trapezoidal or triangular fuzzy parameters and can be applied in various fields, including supply chain management, production planning, and natural ecosystems. This research is a step towards developing efficient and practical methods for decision-making in complex and imprecise environments.

Keywords: *Fuzzy optimization, Fuzzy constraints, Fuzzy linear programming, Fuzzy number.*

1. Introduction

Optimization under uncertainty represents a highly compelling area of contemporary research, as real-world scenarios are inherently characterized by varying degrees of uncertainty. This uncertainty can be examined from multiple perspectives and addressed through a diverse array of methodological approaches. This study focuses on the Fuzzy Linear Programming (FLP) model pioneered by [64], which developed a methodology for solving Linear Programming (LP) problems involving fuzzy constraints. This seminal work profoundly influenced subsequent research and facilitated the integration of fuzzy reasoning into optimization, making it imperative to acknowledge associated fuzzy mathematical models and methods. Following Zimmermann's groundbreaking publications in 1977, a substantial body of related literature emerged. Numerous scholars have contributed to the evolution of Fuzzy Mathematical Programming (FMP), building upon the foundational principles established by [12], who created a coherent framework for fuzzy Decision-Making (DM).
The literature features several notable reviews on FLP. For instance, Baykasoglu et al. [10] introduced 15 distinct types of FMP models categorized by their fuzzy components, while Schryen et al. [53] conducted a survey on duality in fuzzy linear programming, identifying 31 potential classes of FLP. More focused reviews include the work of [20], which examined various FLP solution methods based on the classification proposed by [55] who provided a concise overview of several FLP models and suggested only three categories. Furthermore, [25] recently published a comprehensive review of models and methods for solving fuzzy linear programming, classifying them into five groups based

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on Fuzzy Parameters (FPs) and variables. In contrast, [59] offered a critical analysis highlighting certain limitations and mathematical inaccuracies in fuzzy operations research methods, including FLPs.

The principal contribution of this study, in contrast to the aforementioned reviews, is the establishment of a classification framework for fuzzy linear programming based on two key criteria: the number of incorporated Fuzzy Parameters (FPs) and the nature of the resulting solution. This research provides an analysis of the influence exerted by the most frequently cited scholarly articles, evaluating their impact through two metrics: the number of non-self-citations and the year of publication. Additionally, it delineates the connections between fuzzy linear programming and other methodological approaches while also visualizing emerging trends, future perspectives, and novel solution techniques.

The paper is structured as follows: Section 1 provides an introduction; Section 2 covers fundamental concepts related to FLPs; Section 3 offers a review of the existing literature; Section 4 presents a classification based on modeling and solution techniques; Section 5 highlights emerging trends, future research directions, and suggested topics in FLP; Section 6 is devoted to the proposed approach, which utilizes Yager's Linear Ranking Functions (LRFs); and finally, Section 7 concludes the paper.

2. Fuzzy Linear Programming

In this section, we begin by explaining the concepts of Fuzzy Numbers (FNs), Trapezoidal Fuzzy Numbers (TrFN), and LR-type FNs. To keep the paper concise, we will not include the arithmetic operations for FNs. A real FN $\tilde{A}$ is defined as any fuzzy subset of the real line $\mathbb{R}$, characterized by its membership function $\mu_{\tilde{A}}$, which is:

- A continuous function that maps from the real numbers $\mathbb{R}$ to the closed interval
- [0, 1],
- constant on $(-\infty, e]: \mu_{\tilde{A}}(x) = 0, \forall x \in (-\infty, e]$,
- increasing consistently on $[e, f]$,
- constant on $[f, g]: \mu_{\tilde{A}}(x) = 1, \forall x \in [f, g]$,
- consistently declining on $[g, h]$, and
- constant on $[h, +\infty]: \mu_{\tilde{A}}(x) = 0, \forall x \in [h, +\infty]$;

$e \leq f \leq g \leq h$ are real numbers. Furthermore, we might have $e = -\infty$ or $f = g$ or $e = f$ or $g = h$ or $h = +\infty$. We represent the collection of all FNs as $F(\mathbb{R})$.

A specific kind of FN is the TrFN, which is defined by its membership function as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - e)/(f - e) & e \leq x \leq f \\ 1 & f \leq x \leq g \\ (h - x)/(h - g) & g \leq x \leq h \\ 0 & otherwise \end{cases}$$

and is represented by $\tilde{A} = (e, f, g, h)$. It is evident that when $f = g$, a TrFN simplifies to a triangular FN (TFN). In [19] presented the LR-type representation of fuzzy numbers by examining the left and right reference functions, denoted as L and R, respectively. L is considered a reference function if it satisfies a condition

(1)$L(x)=L(-x)$

(2)$L(0)=1$

(3)L is nonincreasing on $[0,+\infty)$.

An LR-type fuzzy number is represented symbolically $\tilde{A}=(f,g,\alpha,\beta)_{LR}$, and its membership function is expressed in a specific form

$$\mu_{\tilde{A}}(x)=\begin{cases} L((f-x)/\alpha & x\le f \\ R((x-g)/\beta & x\ge g \\ 1 & otherwise \end{cases}$$

In this context, α and β represent the left and right spreads of the variable $\tilde{A}$, respectively. If $L(x)=R(x)=\max\{0,1-|x|\}$, in that case, the LR-type FN $\tilde{A}$ simplifies to a TrFN $\tilde{A}=(e,f,g,h)$, where $e=f-\alpha$ and $h=g+\beta$. We can also express it as $\tilde{A}=(e,f,h)$ when mentioning a TFN, and $\tilde{A}=(f,\alpha,\beta)_{LR}$ R in the context of LR-type fuzzy numbers without a flat section. Figure 1 illustrates the graphical representation of an LR-type fuzzy number and a triangular fuzzy number.

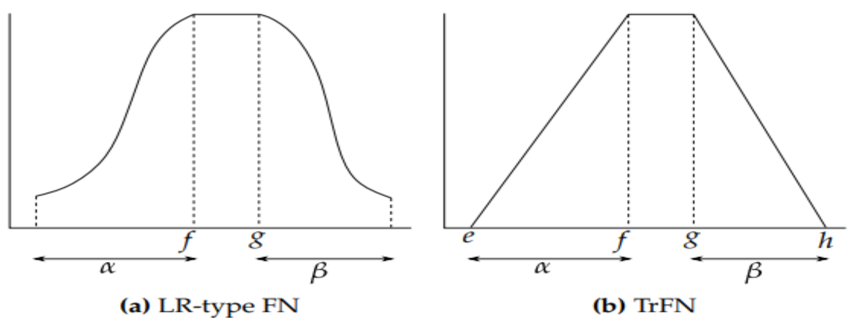

Figure 1. Visual depiction of a LR-type FN and a TrFN

3. Review of existing literature

The approach employed to construct a timeline and conduct a review of Fuzzy Linear Programs (FLPs) involves the retrieval of published scholarly articles characterized by the following criteria:
- Search Terms: fuzzy, linear, and programming
- Relevant Fields: article titles, abstracts, and keywords
- Subject Areas: theoretical frameworks, applications, and associated challenges

Numerous advancements in the theory and applications of Fuzzy Sets (FSs) emerged following Zadeh's introduction of FSs as a mathematical approach for analyzing uncertainty. We categorized FMP problems into two distinct groups: fuzzy linear constrained models and fuzzy ranked models.

The completely FLP issue discussed in this paper is: $\tilde{z}=Max\{\tilde{c}'x:\tilde{A}x\lesssim\tilde{b},x\in\mathbb{R}^{+}\}$.

where $\tilde{c} \in F\left(\mathbb{R}^n\right)$, $\tilde{A} \in F\left(\mathbb{R}^{mn}\right)$, $\tilde{b} \in F\left(\mathbb{R}^m\right)$ and $\lesssim$ represent the fuzzy max-order binary relation.

3.1. Chronology of Fuzzy ranked models and Fuzzy general models

Fuzzy ranked models: encompass the applications and theory of the fuzzy linear programming as outlined. This involves utilizing ranking measures like $r(\cdot)$ to determine a set of precise parameters (A, b, c) and subsequently address a crisp linear programming problem based on the provided ranked values:

$$\left(\tilde{A},\tilde{b},\tilde{c}\right)\xrightarrow{r(.)}\left(A,b,c\right)\rightarrow z^*$$

Fuzzy general models: This category encompasses both the theoretical frameworks and the practical implementations of transformations, along with the relevant methodologies associated with them to fuzzy logic programming, including functions like h(.), chance, possibilistic, interval, and others:

$$\left(\tilde{A},\tilde{b},\tilde{c}\right)\xrightarrow{h(.)}\left(A,b,c\right)\rightarrow z^*$$

A range of hybrid methodologies combining random, stochastic, and fuzzy elements have been developed to address diverse forms of uncertainty. Luhandjula [18] pioneered the field of fuzzy-random linear programming, a framework that was subsequently expanded by other researchers. Building on this, fuzzy chance-constrained models were introduced in [38]. The same author [18] also advanced models based on fuzzy-random sets, which were later formalized into a comprehensive fuzzy stochastic framework by [40], enabling the application of stochastic optimization techniques. Further extending the spectrum of uncertainty modeling, Luhandjula [39] proposed fuzzy-possibilistic linear programming. Angelov [4] contributed to the field by formulating optimization problems using Intuitionistic Fuzzy Sets (IFS) and later developing IFS-based LP models. More recent innovations include Neutrosophic linear programming models [1], which incorporate degrees of truth and belief beyond traditional FSs, and Pythagorean fuzzy linear programming models introduced in [16].

The application of fuzzy methodologies to classical problems is also well-established. The work in [13] applied fuzzy number ranking to solve crisp Linear Programming problems and introduced a technique for possibilistic LPs. Shih et al. [22] employed fuzzy sets to resolve a multi-level programming problem. Research in [41] addressed LPs involving fuzzy variables and uncertainty, applying fuzzy models to establish priorities in analytic hierarchy processes. Furthermore, [28] suggested a perceptual approach for challenges in fuzzy Data Envelopment Analysis (DEA), while [36] proposed the novel concept of possibilistic DEA.

A comprehensive analysis of the reliability of soft computing methodologies, including fuzzy optimization, was provided in [24]. The work in [61] introduced an interactive technique for solving fuzzy multiple attribute group decision-making problems. Interactive possibilistic programming models were employed by [37] to tackle multi-objective supply chain planning challenges. For multi-attribute group decision-making, developed a model utilizing interval and Intuitionistic Fuzzy Sets (IFSs). Additionally, Kumar et al. [33] suggested a lexicographic approach for solving general fuzzy models incorporating fuzzy equality constraints.

3.2. Chronology of the use of FLPs

The chronological development of Fuzzy Linear Programming (FLP) applications reflects its expanding scope. Narasimhan [43] provided an early discussion of fuzzy goal programming and proposed an initial solution method. Hannan [29] addressed the issue of fuzzy costs through the application of α-cuts. An interactive fuzzy satisfaction method for multi-objective linear programming, building on Zimmermann's concepts, was developed in [52]. Research in [15] examined the concept of optimality in transportation and focused on process planning under fuzzy uncertainty, while Roy and Maiti [51] explored a fuzzy Economic Order Quantity (EOQ) model that accounted for demand-dependent costs and storage capacity constraints.

A seminal paper on possibilistic linear programming models for portfolio selection was introduced by [30], who also applied genetic algorithms to multi-objective fuzzy job shop scheduling. The application of FLP in supply chain management is evident in [34] and [35], both of which utilized fuzzy goal programming for vendor selection. Possibilistic linear programming was employed by [60] to address aggregate production planning issues. Supply chain planning under uncertainties in supplies, demand, and processes was tackled using fuzzy methods in [54]. The integration of fuzzy SWOT analysis with FLP for supplier selection and allocation was demonstrated in [3]. More recently, research has focused on sustainable supply chains, with [56] applying fuzzy multi-objective models for low-carbon supplier selection and addressing multi-criteria supplier selection through multi-objective programming.

3.3. Fuzzy Linear Assignment Problem

The Linear Assignment (LA) problem is a combinatorial optimization challenge where a DM aims to allocate n resources to n tasks in a one-to-one manner while minimizing costs. With n! possible assignments, exhaustive enumeration methods become highly inefficient. Kuhn [31] introduced a primal-dual algorithm called the Hungarian Method. This method enabled a more efficient approach to solving high-dimensional LA problems for the first time.

Definition 1. Let $\tilde{c}_1=(c_1^l,c_1^c,c_1^u)$ and $\tilde{c}_2=(c_2^l,c_2^c,c_2^u)$ there be two triangular fuzzy numbers. The operations of addition (⊕) and multiplication (⊗) for the TFNs $\tilde{c}_1$ and $\tilde{c}_2$ are defined in the following manner: $\tilde{c}_1\oplus\tilde{c}_2=\left(c_1^l+c_2^l,c_1^c+c_2^c,c_1^u+c_2^u\right)$

If $\tilde{c}_1$ is not subject to any restrictions and $\tilde{c}_2$ is constrained to non-negative values, then

$$\tilde{c}_1\otimes\tilde{c}_2=\begin{cases}\left(c_1^lc_2^l,c_1^cc_2^c,c_1^uc_2^u\right) & c_1^l\geq 0\\ \left(c_1^lc_2^u,c_1^cc_2^c,c_1^uc_2^u\right) & c_1^l<0 \text{ and } c_1^u\geq 0\\ \left(c_1^lc_2^u,c_1^cc_2^c,c_1^uc_2^l\right) & c_1^u<0\end{cases}$$

In practical DM scenarios, the coefficients of the LA problem are typically derived from the subjective assessments of decision-makers and experts regarding potential assignments, where the costs are only roughly estimated. Within the context of FSs theory, these cost coefficients can be represented as fuzzy numbers. Therefore, it is logical to develop and address a fuzzy variant of the traditional LA problem. As a result, let S_n represent the collection of I={1,2,…,n} and $\tilde{C}=\left[\tilde{c}_{ij}\right]_{n\times n}, \tilde{c}_{ij}\in F\left(\mathbb{R}\right)$, which is a matrix containing the FNs associated with potential assignments. The goal is to identify a permutation $\hat{\varphi}\in S_n$ that minimizes

$\sum_{i=1}^{n}\tilde{c}_{i\varphi(i)}=\tilde{c}_{1\varphi(1)}\oplus\tilde{c}_{2\varphi(2)}\oplus\ldots\oplus\tilde{c}_{n\varphi(n)}$ the given expression. Therefore, the FLA problem can be expressed as shown in Equation (1).

$$\min_{\varphi\in S_n}\sum_{i=1}^{n}\tilde{c}_{i\varphi(i)} \tag{1}$$

The fuzzy linear programming formulation for the FLA problem is presented in Equation (2).

$$\begin{aligned}&\min\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}\\ &s.t.\ \sum_{j=1}^{n}x_{ij}=1\ for\ \ \mathrm{i}\in\mathrm{I}\\ &\quad\sum_{i=1}^{n}x_{ij}=1\ \text{for j}\in\mathrm{I}\\ &\quad x_{ij}\in\{0,1\}\ \text{for i,j}\in I\end{aligned} \tag{2}$$

3.3.1. Kumar and Gupta's Approach

In [32] converted the FLA problem into a precise one by applying a LRF. For a given TrFN $\tilde{A}=(e,f,g,h)$, this function takes the form $R\left(\tilde{A}\right)=\left(e+f+g+h\right)/4$. Consequently, their approach leads to the formulation of the LA problem (3).

$$\begin{aligned}&\min\sum_{i=1}^{n}\sum_{j=1}^{n}R(\tilde{c}_{ij})x_{ij}\\ &s.t.\ \sum_{j=1}^{n}x_{ij}=1\ for\ \ \mathrm{i}\in\mathrm{I}\\ &\quad\sum_{i=1}^{n}x_{ij}=1\ \text{for j}\in\mathrm{I}\\ &\quad x_{ij}\in\{0,1\}\ \text{for i,j}\in I\end{aligned} \tag{3}$$

The solution to the LA problem (3) can be achieved using any of the traditional assignment techniques available.

3.3.2. Baykaso glu et al.'s Approach

While the method developed by [11] was initially designed to address FLA issues involving triangular fuzzy coefficients, it can be easily adapted for trapezoidal fuzzy coefficients as well.

Let $\tilde{C}=\left[\tilde{c}_{ij}\right]_{n\times n}$ represent the fuzzy cost matrix for the FLA problem, where each $\tilde{c}_{ij}=(e_{ij},f_{ij},g_{ij},h_{ij})$ is a TrFN for $i,j\in I$. Employing the method developed in [11], the FLA issue is converted into a four-objective crisp LA problem, as represented in problem (4), with $C_e = \left[e_{ij}\right]_{n\times n'}, C_f = [f_{ij}]_{n\times n'}, C_g = [g_{ij}]_{n\times n}$ and $C_h = \left[h_{ij}\right]_{n\times n}$ representing the coefficient matrices associated with each of the objective functions. Compromise programming [63], utilizing a designated metric, is employed to address the four-objective crisp linear algebra problem.

$$\min\left(\sum_{i=1}^{n}\sum_{j=1}^{n} e_{ij}x_{ij}, \sum_{i=1}^{n}\sum_{j=1}^{n} f_{ij}x_{ij}, \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}x_{ij}, \sum_{i=1}^{n}\sum_{j=1}^{n} h_{ij}x_{ij},\right)$$

$$s.t. \sum_{j=1}^{n} x_{ij} = 1 \, for \quad \mathrm{i} \in \mathrm{I} \tag{4}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for j} \in \mathrm{I}$$

$$x_{ij} \in \{0,1\} \text{ for i,j} \in I$$

Definition 2. [27] Consider an arbitrary fuzzy number $\tilde{A}$ characterized by p defining parameters. Let f_k represent p linear functions of the parameters of $\tilde{A}$. It is assumed that the coefficient matrix associated with these p linear functions is non-singular. Additionally, let $\le_{lex}$ denote the lexicographic order relation on $\mathbb{R}^p$. For any two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ that possess the same type of membership functions, the strict inequality $\tilde{A} \prec \tilde{B}$ is satisfied if and only if $\left(f_k\left(\tilde{A}\right)\right)_{k=1,\dots,p} <_{lex} \left(f_k\left(\tilde{B}\right)\right)_{k=1,\dots,p}$.

The weak inequality $\tilde{A} \preceq \tilde{B}$ is satisfied if and only if $\left(f_k\left(\tilde{A}\right)\right)_{k=1,\dots,p} <_{lex} \left(f_k\left(\tilde{B}\right)\right)_{k=1,\dots,p}$ or $\left(f_k\left(\tilde{A}\right)\right)_{k=1,\dots,p} = \left(f_k\left(\tilde{B}\right)\right)_{k=1,\dots,p}$.

The lexicographic order relation 4 meets the criteria for a total order.

3.3.3. Lexicographic Approach

Definition 3. Let $\le_{lex}$ represent the lexicographic order relation in $\Re^3$ and $\tilde{c}_1 = (c_1^l, c_1^c, c_1^u)$ and $\tilde{c}_2 = (c_2^l, c_2^c, c_2^u)$ consider two arbitrary triangular fuzzy numbers. We define that is relatively less $\tilde{c}_1$ than $\tilde{c}_2$, denoted as $\tilde{c}_1 \prec \tilde{c}_2$, if and only if $\left(c_1^c, c_1^l - c_1^u, c_1^l + c_1^u\right) <_{lex} \left(c_2^c, c_2^l - c_2^u, c_2^l + c_2^u\right)$ certain conditions are met. Additionally, we state that $\tilde{c}_1$ is relatively less than or equal to $\tilde{c}_2$, denoted as , if and only if specific criteria are satisfied $\left(c_1^c, c_1^l - c_1^u, c_1^l + c_1^u\right) \le_{lex} \left(c_2^c, c_2^l - c_2^u, c_2^l + c_2^u\right)$.

Remark 1. $\tilde{c}_1 = \tilde{c}_2$ if and only if $c_1^c = c_2^c, c_1^l - c_1^u = c_2^l - c_2^u$ and $c_1^l + c_1^u = c_2^l + c_2^u$.

Since lexicographic ranking criteria offer superior discriminative abilities compared to LRFs, it makes sense to apply Definition 1 to convert the FLA problem (1) into a lexicographic linear assignment (LLA) problem (5).

$$lex \min_{\varphi \in S_n} \sum_{i=1}^{n} \left(f_k\left(\tilde{C}_{i\varphi(i)}\right)\right)_{k=1,\dots,4} \tag{5}$$

Figure 2 illustrates the membership functions associated with the total fuzzy assignment costs derived from the methodologies proposed by Kumar and Gupta [32], Baykasoğlu et al. [11], and the lexicographic approach [45].

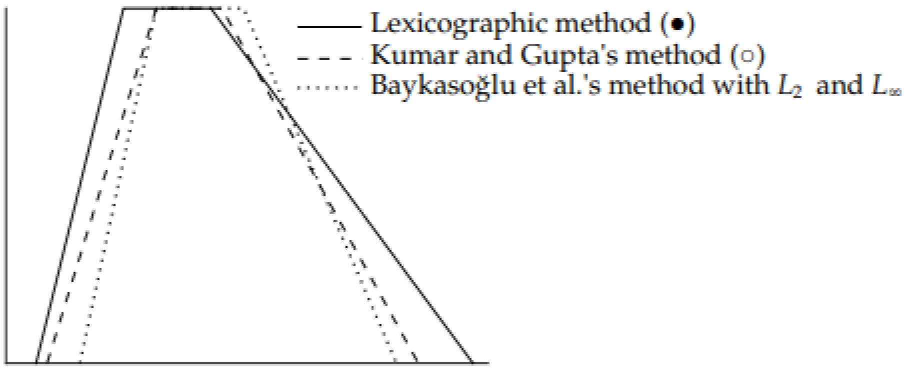

Figure 2. Charts depicting the membership functions for the overall fuzzy assignment costs.

4. Solution method Fuzzy linear programming (FFLP)

The standard representation of the FFLP problem, which includes arbitrary triangular fuzzy parameters and non-negative triangular fuzzy decision variables, can be expressed in the following way.

$$\begin{aligned} &\max \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j \\ &s.t \quad \sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \ (\preceq or = or \succeq) \tilde{b}_i \\ &i = 1,2,...,m \qquad j=1,2,...,n \end{aligned} \tag{5}$$

In the sections that follow, we will outline the procedure for addressing the FFLP problem (5).
Solution steps:
First stage. Let $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i, \tilde{x}_j$ Consequently, the FFLP problem (5) can be expressed as follows:

$$\begin{aligned} &\max \sum_{j=1}^{n} \left(c_j^l, c_j^c, c_j^u\right) \otimes \left(x_j^l, x_j^c, x_j^u\right) \\ &s.t \quad \sum_{j=1}^{n} \left(a_{ij}^l, a_{ij}^c, a_{ij}^u\right) \otimes \left(x_j^l, x_j^c, x_j^u\right) (\preceq or = or \succeq) \left(b_i^l, b_i^c, b_i^u\right) \\ &i = 1,2,...,m \qquad j=1,2,...,n \end{aligned} \tag{6}$$

5. Classification based on solutions

Section 3 categorizes research papers according to their mathematical representation models. However, it is common for researchers and practitioners to seek specific types of solutions to either investigate or choose a particular method. In this context, the previously mentioned categories fuzzy linear constrained models, fuzzy ranked models, and fuzzy general models—can be associated with two primary types of solutions: solutions based on fuzzy sets and deterministic solutions.

(a) Solutions based on fuzzy sets: The purpose of utilizing fuzzy sets to address uncertainty is to incorporate human-like perceptions into optimization models.

Definition 4. Any specific function referred to as the optimal solution $z^*: \mathbb{R}^{mn} \to \mathbb{R}$ generates a function $z^*: F_1(\mathbb{R}^{mn}) \to F_1(\mathbb{R})$ that is characterized by $\mu_{\tilde{z}(\tilde{A},\tilde{b},\tilde{c})}(z^*) = \sup_{z^* \in z^{-1}(x^*)} \{\tilde{A}', \tilde{b}', \tilde{c}'\}$.

The symbol $'$ denotes the set of parameters associated with the Optimal Solution (OS) and the constraints.

A function z, based on FPs $\tilde{A}, \tilde{b}, \tilde{c}$ and an OS x^*, generates a fuzzy set $\tilde{z}$ of OP that retains all fuzzy information. However, this process demands significantly more computational resources, making it impractical for large-scale, nonlinear problems.

(b) The computational challenges inherent in solving Fuzzy Linear Programming (FLP) problems, primarily stemming from the NP-hard nature of evaluating functions via the fuzzy extension principle, have led researchers to adopt simplified deterministic approaches. A prevalent strategy involves the initial ranking or defuzzification of fuzzy parameters (FPs) using appropriate measures, such as penalty-based ranking functions. This transformation allows the resulting deterministic problem to be solved using conventional mathematical programming techniques, including linear programming, nonlinear optimization, Lagrange multipliers, and gradient-based methods.

By applying such ranking mechanisms, the original FLP problem is converted into a crisp counterpart, the solution of which is regarded as a deterministic instance or one possible resolution—of the broader fuzzy problem. It is important to note that the selection of an appropriate methodology for handling FLPs is highly context-dependent. The chosen approach typically varies according to the source of uncertainty, its mathematical representation, and the decision-maker's (DM) preference regarding the type of solution desired. A structured overview of these methodological alternatives is presented in Fig. 3.

Ranking functions are a widely used method for managing FNs to create a meaningful ranking. These functions facilitate the comparison and arrangement of FNs, which is crucial in DM scenarios that involve uncertainty. The RF is denoted as F(R), where $R: F(R) \to R$, and F(R) represents the collection of FNs defined along a real line, where a natural order exists. Numerous types of ranking functions have been proposed in research, each providing unique advantages and practical applications. These functions are intended to provide a systematic way to compare and organize FNs, taking into account both their membership and non-membership values.

In the realm of linear programming involving FPs, ranking functions are essential for transforming fuzzy constraints or objectives into precise values, thereby enabling conventional applications [17, 50] When discussing LP methods, let $\tilde{A}$ and $\tilde{B}$ denote two triangular FNs. The ranking function $F(R)$ adheres to specific rules:

$$i)\tilde{A} \geq \tilde{B} \Rightarrow R(\tilde{A}) \geq R(\tilde{B})$$
$$ii)\tilde{A} > \tilde{B} \Rightarrow R(\tilde{A}) > R(\tilde{B})$$
$$iii)\tilde{A} = \tilde{B} \Rightarrow R(\tilde{A}) = R(\tilde{B})$$

Lemma 1: The following statements are true for any ranking function:

$$i)\tilde{A} \geq \tilde{B} \Leftrightarrow \tilde{A} - \tilde{B} \geq 0 \Leftrightarrow -\tilde{B} \geq -\tilde{A}$$
$$ii)\tilde{A} > \tilde{B} \text{ and } \tilde{C} \geq \tilde{D} \Rightarrow \tilde{A} + \tilde{C} \geq \tilde{B} + \tilde{D}$$

Yager Ranking Method: Yager ranking function is a technique for ranking FNs developed by Ronald R. Yager, a key contributor to fuzzy logic and DM. This method is part of a series of ranking functions suggested by Yager. It ranks FNs according to their centroid values, which indicate the "average" value of the FN.

Let $\tilde{A} = (a^l, a^u, \alpha, \beta)$ denote a trapezoidal FN. The ranking function is defined as follows [62]:

$$R(\tilde{A}) = \frac{\left(\int_0^1 \left(a^l - \alpha L^{-1}(\lambda)\right) d\lambda + \int_0^1 \left(a^u + \beta R^{-1}(\lambda)\right) d\lambda\right)}{2}$$

This simplifies to:

$$R(\tilde{A}) = \frac{\left(a^l + a^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right)}{2}$$

When using the Yager ranking function, the linear programming problem takes on a particular structure.

$$Max\ or\ Min = \sum_{j=1}^{n} \frac{1}{2}\left(c_j^l + c_j^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right) x_j$$
$$subject \text{ to} \sum_{j=1}^{n} \frac{1}{2}\left(a_{ij}^l + a_{ij}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right) x_j \leq \frac{1}{2}\left(b_i^l + b_i^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right)$$
$$i = 1, 2, ..., m x_j \geq 0$$

After formulating a fuzzy linear programming (FLP) problem, the next step involves selecting appropriate solution methods capable of handling the inherent uncertainties in the parameters. These may include fuzzy optimization techniques or specially designed ranking functions (RFs) for fuzzy linear programming.

It is essential to critically evaluate whether a given methodology constitutes a genuine fuzzy optimization model, regardless of its citation count or popularity. Studies classified as type (a) preserve fuzzy information throughout the solution process for instance, through α-cuts or by deriving the fuzzy set of optimal values and are thus regarded as true fuzzy programming models. In contrast, approaches falling under type (b) rely solely on ranking functions to convert the fuzzy problem into a crisp equivalent, thereby losing all fuzzy information and obscuring the semantic origin of the fuzzy sets in pursuit of a deterministic solution.

Meanwhile, interval linear programming (ILP) has made significant advances in developing coherent solution frameworks. For instance, Ashayerinasab et al. [7] proposed an exact method for obtaining the optimal solution set of an ILP problem, and [9] introduced a methodology for identifying efficient solutions in multi-objective ILPs.

While the simplest way to address FLPs is to tackle an instance of its FPs using traditional LP, this approach results in the loss of valuable insights from experts that are encapsulated in fuzzy sets. Therefore, it is essential to solve FLPs in a manner that retains fuzzy information, even if the final solution is expressed as a single value for the entire problem. There is a need to expand their findings to encompass other forms of uncertainty and to create a method for identifying OS while safeguarding fuzzy information.

Figure 3.a and 3.b categorizes fuzzy linear programming into two categories: (a) fuzzy solutions, and (b) deterministic-based models.

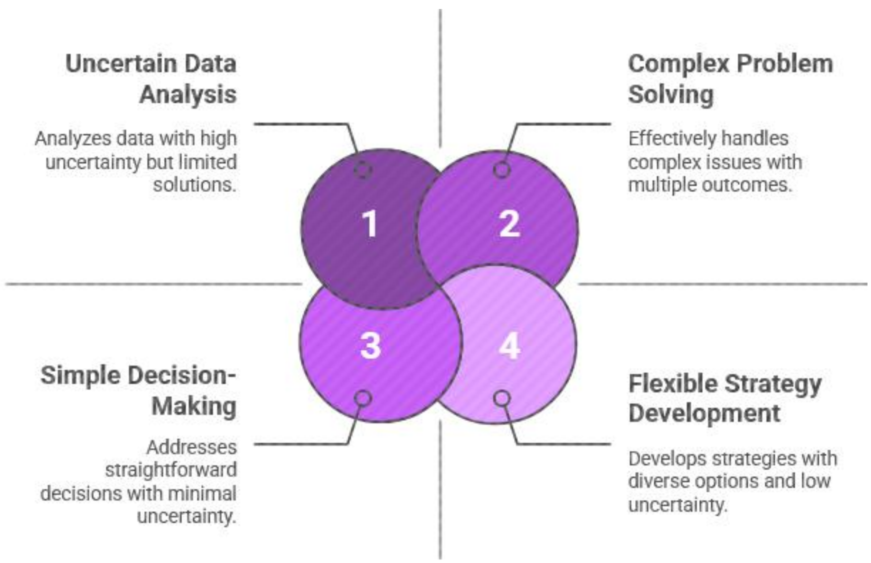

Figure 3.a Categorization of FLPs according to the solutions obtained.

Figure 3.b Categorization of FLPs according to the solutions obtained.

In contrast, a distinct stream of research has focused on solving Facility Location Problems (FLPs) using metaheuristic algorithms, particularly for cases that involve high complexity or large scale. Many of these studies adopt hybrid strategies that merge evolutionary metaheuristics such as Particle Swarm Optimization [8], genetic algorithms, harmony search, and scatter search to enhance solution quality and computational efficiency. Some publications also provide comparative analyses of multiple metaheuristics [26] to evaluate their performance. The central aim of this body of work is to refine search mechanisms and identify high-quality, practical solutions for complex FLP instances.

6. Algorithm for Solving the FLP Problem Using a Ranking Function

The steps detailed in the suggested method are as follows:

Step 1: In the context of the FLP problem with inequality constraints, where the coefficients $\tilde{c}_j, \tilde{b}_1$ and $\tilde{a}_{ij}$ are represented as FNs, we incorporate these values into the problem formulation: $\tilde{C}^T = \left[\tilde{C}_j\right]_{1\times n}, \tilde{A} = \left[\tilde{a}_{ij}\right]_{m\times n}$ and , $\tilde{B} = \left[\tilde{b}_i\right]_{m\times 1}$ as outlined.

Step 2: Use the ranking function described in Yager ranking function along with the FLP framework presented

$$Max\ or\ Min = \sum_{j=1}^{n} \frac{1}{2}\left(c_j^l + c_j^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right)x_j$$

$$subject\ \text{to} \sum_{j=1}^{n} \frac{1}{2}\left(a_{ij}^l + a_{ij}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right)x_j \le \frac{1}{2}\left(b_i^l + b_i^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right)$$

$$i = 1, 2, \ldots, m x_j \ge 0$$

to transform the issue into a clear LP problem.

Step 3: Next, we find the best solution for the resulting crisp linear programming problem using established techniques.

Case study

The manufacturing process encompasses three distinct products: P1, P2, and P3, which are produced using four separate machines: M1, M2, M3, and M4. The time necessary for the production of a single unit of each product, along with the daily output capacity of the machines, is outlined in the following details:

Table 1: Data of time requred of each product and daily capacity of the machines.

Time Needed per unit (in minutes)												
Machines	**P1**				**P2**				**P3**			
M1	12	16	2	3	13	16	2	1	12	16	3	5
M2	14	20	2	2	....				13	18	2	3
M3	12	17	3	2	15	20	2	3	....			
M4	15	18	2	3	14	17	2	1	16	18	2	2
Ptrofit/Rs	13	15	2	2	12	14	3	2	15	18	3	2

Table 2: Daily capacity of the machines.

Machine Capasity (min/day)				
	Capasity			
M1	490	510	9	8
M2	470	490	10	6
M3	480	505	7	8
M4	388	425	5	8

Daily operational constraints, such as unexpected equipment downtime or variations in work shifts, can lead to fluctuations in available machine time. Simultaneously, product profit margins may shift in response to market-driven price changes. The objective is to determine optimal daily production quantities for each product that maximize total profit, under the assumption of full market absorption of all manufactured items.

Given the inherent uncertainties in both profitability and production capacity, determining output levels becomes a decision problem under uncertainty. Consequently, this study models the production planning challenge as a Fuzzy Linear Programming (FLP) problem, representing the imprecise parameters through Trapezoidal Fuzzy Numbers (TrFNs). The FLP formulation is constructed as follows:

x_1 : the daily production quantity of Product 1 (P1)

x_2 : the daily output amount of Product 2 (P2)

x_3 : the daily manufacturing volume of Product 3 (P3)

The goal is to optimize profit while taking into account the fluctuations in profit for each item.

$$
\begin{aligned}
&Max\ z = (13,15,2,2)x_1 + (12,14,3,2)x_2 + (15,18,3,2)x_3 \\
&s.t \quad (12,16,2,3)x_1 + (12,16,2,1)x_2 + (12,16,3,4)x_3 \le (490,510,9,8) \\
&\qquad (14,20,2,2)x_1 + (13,18,2,3)x_3 \le (470,490,10,6) \\
&\qquad (12,17,3,2)x_1 + (15,20,2,3)x_2 \le (480,505,7,8) \\
&\qquad (13,15,2,2)x_1 + (12,14,3,2)x_2 + (15,18,3,2)x_3 \le (388,425,5,8) \\
&\qquad x_i \ge 0 \quad i=1,2,3
\end{aligned}
$$

By applying ranking functions to transform FNs into precise values, you can address the resulting crisp linear programming problem to find an OS that takes into account the uncertainties inherent in the original fuzzy issue.

By utilizing the suggested approach, the previously mentioned FLP issue is transformed into a clear linear programming problem. Subsequently, the simplex method is employed to determine the optimal solution.

$$
\begin{aligned}
&Max\ z = 13.86\ x_1 + 12.46\ x_2 + 15.96\ x_3 \\
&s.t \quad 14.2\ x_1 + 14\ x_2 + 14.1\ x_3 \le 499.1 \\
&\qquad 16.9\ x_1 + 15.7\ x_3 \le 478 \\
&\qquad 14\ x_1 + 17.7\ x_2 \le 492.4 \\
&\qquad 16.7\ x_1 + 15\ x_2 + 16.9\ x_3 \le 407.2 \\
&\qquad x_i \ge 0 \quad i=1,2,3
\end{aligned}
$$

Using the simplex method, the best solution for the linear programming problem mentioned above is as follows: ($x_1 = 0, x_2 = 0, x_3 = 24.09, z = 384.7$).

According to what we have obtained the daily production quantity of Product 1 (P1) =0, the daily output amount of Product 2 (P2) =0, the daily manufacturing volume of Product 3 (P3) =24.9 and Maximum profit equals z=384.7.

7. Trends, viewpoints, and emerging pathways

Recent years (2018–2024) have witnessed a notable increase in publications on fuzzy linear programming (FLP), with several emerging contributions likely to influence new research directions. Recent work has extended lexicographic techniques to FLP, broadening optimality conditions for fuzzy optimization [47], while also advancing non-dominance theory in facility location problems [6]. New methods include an epsilon-constraint approach for fully fuzzy multi-objective linear programming [48] and the introduction of fuzzy Pareto solutions [5]. Additional developments include fuzzy relational linear programming [14], linear programming with fuzzy decision variables, and interval fuzzy linear models [49]. Furthermore, FLP has been extended to Pythagorean, spherical [57], neutrosophic [2], and bipolar [42] fuzzy sets, though solving models with fuzzy variables remains a challenging nonlinear problem requiring further research.

Promising future directions include intuitionistic fuzzy sets, type-2 fuzzy sets, and integer fuzzy linear programming. Recent applications have also emerged in facility location [57, 58, 44] and vehicle routing problems [46, 43], including multi-shift, chance-constrained, and time-window constrained models under fuzziness. Other evolving areas include uncertain programming which extends the λ-satisfaction degree within uncertainty theory and flexible optimization, integrating fuzzy and possibilistic programming into a unified framework.

8. Research Gap and Novel Contribution of the Present Study

Despite significant advancements in the theory and application of FLP, important gaps persist in the existing literature. A review of previous studies reveals that a substantial number of works, particularly those categorized as Fuzzy Ranked Models, obliterate the intrinsic uncertainty information embedded within fuzzy numbers by converting the entire problem into a crisp model via a ranking function. While computationally simpler, this approach only solves a single instance of the original fuzzy problem and provides no insight into the solution's behavior under variations of the fuzzy parameters. On the other hand, methods based on fuzzy sets that preserve uncertainty information are often computationally complex and impractical for large-scale problems. Therefore, a clear research gap exists in developing a method that strikes an optimal balance between computational practicality and the preservation of fuzzy information for decision-making, one that not only produces an optimal solution but also quantifies the stability and sensitivity of that solution to changes in the input fuzzy parameters.

The significant novel contribution of this study is as follows:

An Integrated Solution Framework for Sensitivity Analysis in FLP: The primary innovation of this paper is not merely another ranking function, but the systematic integration of sensitivity analysis into the core of the solution method. Unlike most previous studies that treat sensitivity analysis as an optional and separate step, our proposed method inherently and simultaneously identifies and reports the *range* of fuzzy parameter values (specifically the left and right spread parameters) for which the optimal solution remains stable. This allows the decision-maker to be presented not just with a crisp number, but with a "confidence region" for their solution.

Emphasis on Solution Stability Beyond Mere Optimization: The focus of this paper extends beyond finding an optimal solution. Its novel contribution is the assessment of the reliability of that solution in a real-world setting full of uncertainty. By systematically testing variations in the fuzzy parameters, the method demonstrates how robust the proposed solution is to changes in the initial estimates (expressed as fuzzy numbers). This finding is far more valuable for practical decision-makers than providing an optimal value without knowledge of its sensitivity.

Practical Application in Ecological Decision-Making: Although the presented method is general, its application is demonstrated through a case study in ecological decision-making under uncertainty (e.g., production planning considering limited and variable resources). This application highlights the practical value of the method in tackling real-world problems where data is often qualitative, imprecise, and estimated, thereby contributing to bridging the gap between fuzzy theory and environmental management applications.

In summary, this study contributes to filling the gap between fully crisp methods (which ignore fuzzy information) and fully fuzzy methods (which are computationally heavy) by proposing a user-friendly and practical method that provides both an optimal solution and an understanding of its stability. This innovation makes decision-making under uncertainty more robust and reliable.

8.1. Managerial Implications

The findings of this study carry significant and practical implications for managers and decision-makers across various domains, particularly in operations and supply chain management under uncertainty:

1. Robust Decision-Making in the Face of Uncertainty: The proposed method empowers managers to receive not just a single optimal solution but also a stability range for that solution. For instance, in the production case study, managers not only know that producing 24.09 units of the third product is optimal but are also assured that this solution remains valid and stable even with fluctuations of up to approximately 10% in machine availability or product profits. This insight increases the confidence to make decisions in risky environments.
2. Prioritization of Resource Allocation: The quantitative results and sensitivity analysis clearly show that the system's sensitivity to changes in different parameters is not uniform. Managers can use this information to prioritize data collection accuracy and monitoring of critical resources. For example, if the sensitivity analysis reveals that the optimal solution is highly sensitive to changes in the availability of a specific machine, the manager can prioritize preventive maintenance strategies or secure alternative resources for that particular asset.
3. Facilitating Communication Between Technical and Executive Management: Presenting results simultaneously (an optimal value and a stability range) builds a bridge between technical precision and managerial understanding. Instead of presenting a complex technical number, senior executives can be told: "The optimal production plan guarantees a profit of approximately 385 units, and even in the most pessimistic scenarios (considering uncertainties), this profit will not fall below X units." This facilitates the adoption and implementation of quantitative model results at higher organizational levels.
4. Flexibility in Supply and Procurement Negotiations: In real-world environments, parameters like delivery time or costs are often not fixed and are determined through negotiations with suppliers and customers. A procurement manager can use this model to simulate different scenarios. For instance, if a supplier offers a price reduction in exchange for a longer delivery time, the manager can use the model's sensitivity analysis to quickly assess the impact of this trade-off on the production schedule and overall profit, thereby making the best decision.
5. Applicability in Sustainability and Ecosystem Domains: This method is particularly valuable for ecological and environmental problems where data is often imprecise and qualitative (e.g., estimating an ecosystem's recovery capacity or pollution levels). Environmental managers can use this framework to develop strategies for managing natural resources that are both optimal and resilient to the inherent uncertainties of the environment.

In summary, this research provides managers with a robust decision-making tool that enables them to manage uncertainty not as a threat, but as an integral part of the decision-making process, ultimately leading to more reliable and practical outcomes.

9. Conclusions

Mathematical and linear programming models designed to handle uncertainty have grown increasingly popular owing to their flexibility and ability to incorporate diverse sources of imprecision. In particular, linear programming models that employ fuzzy uncertainty have earned significant recognition for their capacity to integrate human expertise and linguistic interpretations into optimization frameworks via fuzzy sets specified by domain experts. Nevertheless, a large portion of the existing literature remains focused on solving specific instances of fuzzy linear programs using techniques such as centroid-based ranking or Yager’s index before applying conventional crisp or stochastic models. Relatively few studies delve into theoretical advances in optimization, whether within crisp or fuzzy environments. Given the current taxonomy of methods, future efforts should aim to strengthen the relationships among the fuzzy extension principle, α-cuts, λ-satisfaction measures, and interval-based techniques to efficiently determine the fuzzy set of optimal solutions ($O\tilde{S}z$). Moreover, there is considerable potential to extend existing models to support other representations of uncertainty, including intuitionistic fuzzy sets, type-2 fuzzy sets, Pythagorean fuzzy sets, spherical fuzzy sets, bipolar fuzzy sets, and neutrosophic sets.

Ultimately, future research should prioritize the development of standardized frameworks for solving linear programming problems affected by multiple forms of uncertainty. Progress in generalized uncertainty theory offers promising avenues for integrating various sources of imprecision within unified optimization paradigms.

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