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Assigning independent resources to fire stations to minimize the influence of the shortage

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A new integer program is presented to model an independent resources assignment problem with resource shortages in the context of municipal fire service. When shortage in resources exists, a critical task for fire department's administrator in a city is to assign the available resources to the fire stations such that the effect of the shortage to cover (in providing service, in extinguishing fire and so on) is minimized. To solve the problem, we propose a polynomial time greedy algorithm.

Keywords*: Resource assignment problem, Integer programming, Fire stations, Shortage, Greedy algorithm.*

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1. Introduction

A fire department's administrator encounters many decision making problems (see [4,5,10]; also, [2,3,7,9]). One of these decision problems is to assign available resources (e.g., equipments or fire engines, etc.) to fire stations (see [9], page 524, for some references). It is possible not to have adequate resources to satisfy all the requirements of the fire stations in a real world situation. Thus, some possible events might not be covered (to extinguish or to service) in some of the fire stations (in fact, in their region). In our proposed model, to reduce the influences of the shortage for a type of resource, some penalties are considered. These penalties are computed based on the importance of the resources in each fire station, separately. In a fire station, to compute the importance of a required resource, we estimate the possibility of occurrence of each event (that needs the resource) in the fire station's region. Next, based on the predefined impact of the events and the estimated possibility for occurrence of the events, the corresponding penalties are considered (see Section 2). Our aim is to assign the available resources to fire stations such that the sum of the corresponding penalties is minimized. Throughout our work here, we assume that all types of resources are independent. In other words, when two or more types of resources are dependent, one can consider them as one type of resource (i.e., a package). Here, we discuss the problem in the context of municipal fire service but one can use the proposed model for other problems arising from emergency management (humanitarian relief, disaster relieves, etc.;see [1]). We give an integer linear formulation to model the problem. In addition, we propose a greedy algorithm to solve the problem. We will prove that the greedy algorithm gives the optimal solution. The remainder of the paper is organized as follows: In Section 2, we give an integer program to model the problem. The proposed algorithm is presented in Section 3. Numerical results are provided in Section 4. We conclude in Section 5.

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2. Mathematical model

Before introducing the integer program, we describe the procedure to determine the corresponding penalty of the resource's shortage in a fire station. Table 1 presents some parameters and the given data of the problem.

Table 1 Parameters and given data

m:	different types of resources.
T_i:	the availability of resource i, $i = 1, \dots, m$.
q:	number of possible events.
n:	number of fire stations.
f_{ik}:	required number of resource i to give service.
p_{kj}:	frequency of event k in the region corresponding to station j.
pr_{kj}:	possibility of event k occurring in region corresponding to station j (given by an expert).
w_k:	importance (weight) of event k.
β_j:	importance (weight) of fire station j.

Note: In Table1, the pr_{kj} and the w_k and the β_j are in [0,1].

Note that pr_{kj} is obtained from prior information. We gather the frequency of event k in region corresponding to the fire station j for a period of ten years (to implement the model in the city of Mashhad). But, each pr_{kj} is given by experts and it shows the possibility of occurrence of the event k in the region of fire station j. Consider a hospital recently founded in the region of the fire station j and suppose that there hasn't been any hospital in this region. Therefore, some events have rarely occurred there. But, the possibility of rare occurrence or no occurrence of events is increased. Thus, pr_{kj} and p_{kj} can be completely different. β_j imposes the manager's decision to assign the resource to the fire station j. As an example, consider that there are two fire stations with the same conditions needing the same type of equipment. Also, consider that one of them is in the center of the city and the other is on the outskirts. Suppose that there is one equipment. In this situation, it is usually preferred to assign this equipment to the fire station that is in the center of the city. Therefore, the manager can assign a higher weight to the fire station that is in the center of the city to impose her preferences in the model.

To compute the associated penalty, we first consider the relative frequency for event k in the region of the fire station j as follows:

$$u_{kj} = \frac{p_{kj}}{\sum_{j=1}^{n} p_{kj}} \tag{1}$$

Remark 1: Note that one can consider u_{kj} in (1) as an estimation of the probability of occurrence of the event k in the region of the fire station j.

Definition 1: Let

$$v_{kj} = \max\{u_{kj}, pr_{kj}\}. \tag{2}$$

where pr_{kj} is given by the manager and u_{kj} is calculated by (1). We define $v_{kj}\, w_k$ as the impact of the event k on the station j, for $j = 1, \dots, n$ and $k = 1, \dots, q$.

Definition 2: We define possible events in the fire station j (or its region) as follows:

$$\theta_j = \{k |\ v_{kj} > 0\} \tag{3}$$

where v_{kj} is defined by (2).

Now, we can compute the maximum number of required resource of type i in station j as follows:

$$b_{ij} = \max_{k \in \theta_j} f_{ik}. \tag{4}$$

It is evident that if we assign b_{ij} units or more of the resource i to the station j, then we do not have any shortage; otherwise, we have a shortage.
Let c_{ij} be an estimation of the importance of resource i in the station j. We propose the following formula to compute c_{ij} :

$$c_{ij} = \sum_{k \in \theta_j \& f_{ik} > 0} \beta_j w_k v_{kj}, \tag{5}$$

where β_j, θ_j and v_{kj} are defined in Table 1, and the relation (3) and (2), respectively. We now introduce the decision variables in Table 2.

Table 2 Decision variables

x_{ij}: number of resource of type i that are assigned to station j.
S_{ij}^-: shortage of resource i in station j.
S_{ij}^+: surplus of resource i in station j.

2.1 The Model

Motivated by goal programming models (see [8]), the proposed integer program is given next.

$$\min z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} S_{ij}^- \tag{6}$$

$$s.t.$$

$$\sum_{j=1}^{n} x_{ij} \le T_i, \qquad i = 1, \dots, m, \tag{7}$$

$$x_{ij} + S_{ij}^- - S_{ij}^+ = b_{ij}, \qquad j = 1, \dots, n, i = 1, \dots, m, \tag{8}$$

$$x_{ij}, S_{ij}^-, S_{ij}^+ \in Z^+ \cup \{0\}, \qquad j = 1, \dots, n, i = 1, \dots, m.$$

The set of constraints in (7) show that the number of assigned resource i must be less than T_i. To explain constraints (8), consider that we assign x_{ij} units of resource i to station j. Suppose that $x_{ij} < b_{ij}$. Then, S_{ij}^- takes the shortage for servicing some of the possible events in station j and the corresponding penalty is added to the objective function's value. In the other case, if $x_{ij} > b_{ij}$, based on the objective function, S_{ij}^{-} takes to be zero and then the corresponding penalty is zero. In this situation, there is a surplus that can be useful in sensitivity analysis. Note that in the objective function (6), S_{ij}^{-} shows the sum of the corresponding penalties in all fire stations for each type of resources.

3. A greedy algorithm

In this section, we give a greedy algorithm to solve the problem. At first, all the x_{ij} are zero. Then, for resource i, in an iterative manner, we select the fire station that has the largest c_{ij} among the stations not being chosen. We then assign the maximum amount of available resource i to that station. The detail of our algorithm is given in Algorithm 1.

Step 1 **{Initialization}** Let $x_{ij} \leftarrow 0$, for $j = 1, \dots, n$ and $i = 1, \dots, m$.

Step 2 **{Main Loop}**

For $i = 1$ to m **do**

- Let $U \leftarrow T_i$ and $N_i = \{j | b_{ij} > 0\}$.
- **While** $U > 0$ and $N_i \neq \emptyset$:
  - Let $j = argmax\{c_{i\bar{j}}^{-} | \bar{j} \in N_i\}$.
  - Let $x_{ij} \leftarrow \min\{b_{ij}, U\}$.
  - Let $U \leftarrow U - x_{ij}^{*}$ and $N_i \leftarrow N_i - \{j\}$.

endwhile

endfor

Step 3 **Return** $x^{*} = [x_{ij}^{*}]_{m \times n}$ as the optimal solution.

Algorithm 1: Greedy Algorithm.

Proposition 1. Algorithm 1 has $O(mn \ln n)$ complexity.

Proof To select j in each iteration of the while loop, prior to the loop, we sort the fire stations according to their importance, c_{ij}, in $O(n \ln n)$. Thus, in each iteration of the while loop, the selection is performed in $O(1)$. Therefore, Algorithm 1 has $O(mn \ln n)$ complexity.

Theorem 1 *Algorithm 1 gives the optimal solution.*

Proof The main idea of the proof is inspired by the proof of the correctness of Dantzig's algorithm for solving the fractional knapsack problem (see [6]). Without loss of generality, assume that there is only one type of resource (one can repeat the argument m times to complete the proof). Therefore, throughout the proof, we remove the subscript i. It is obvious that if $\sum_{j=1}^{n} b_j \leq T$, then Alorithm 1 gives the optimal solution. So, we consider $\sum_{j=1}^{n} b_j > T$.

Suppose that $x = (x_1, \dots, x_n)^T$ is given by Algorithm 1 and $x^* = (x_1^*, \dots, x_n^*)^T$is an optimal solution. Since $\sum_{j=1}^{n} b_j > T$, we have $\sum_{j=1}^{n} x_j = \sum_{j=1}^{n} x_j^* = T$. Also, without loss of generality, from Proposition 1, assume that $c_1 \geq c_2 \geq \cdots \geq c_n$. If $x = x^*$, then the proof is trivial. So, we continue the proof with $x \neq x^*$. Note that S_{ij}^- is $\max\{b_j - x_j,\ 0\}$ and S_{ij}^{*-} is $\max\{b_j - x_j,\ 0\}$. Now, consider t as the first index such that $S_t^- \neq 0$. Thus, $S^- = (0, \dots, 0, b_t - x_t, b_{t+1}, \dots, b_n)^T$, $x = (b_1, \dots, b_{t-1}, x_t, 0, \dots, 0)^T$ and $\sum_{j=1}^{t} x_j = T$. Also, consider l to be the first index such that $x_l \neq x_l^*$. We first claim that $S_l^- < S_l^{*-}$. Consider three cases:

- Case $l = t$. From the structure of x, we have $\sum_{j=1}^{t-1} x_j + x_t = \sum_{j=1}^{t-1} x_j^* + x_t^* + \sum_{j=t+1}^{n} x_j^* = T$. But, $x_j = x_j^*$, for $j = 1, \dots, t-1$ and also $x_t \neq x_t^*$. Therefore, it is evident that $x_t > x_t^*$, and thus $S_t^- < S_t^{*-}$.
- Case $l < t$. Here, $x_l = b_l$ and $S_l^- = 0$. Since $x_l \neq x_l^*$, it is obvious that $S_l^- < S_l^{*-}$.
- Case $l > t$. This case is impossible because we here have $\sum_{j=1}^{t} x_j = \sum_{j=1}^{t} x_j^* = T$. Thus, it is impossible that $(0 =) x_l \neq x_l^*$, where $l > t$.

Therefore, the proof of the claim is complete. Now, we construct $\bar{x}$ from x^*. Since, $S_i^- < S_i^{*-}$ or equivalently, $x_i > x_i^*$, we can let $\bar{x}_j = x_j^* = x_j$, for $j = 1, \dots, l-1$, $\bar{x}_i = x_i$ and decrease the value of x_j^*, for $j = l+1, \dots, n$, such that $\bar{x}$ becomes a feasible solution, i.e., $\sum_{j=1}^{n} \bar{x}_j = T$. Now, we have

$$\sum_{j=1}^{n} c_j \left(S_j^{*-} - \bar{S}_j^-\right) = \sum_{j=1}^{l-1} c_j \left(S_j^{*-} - \bar{S}_j^-\right) + c_l\left(S_j^{*-} - \bar{S}_j^-\right) + \sum_{j=l+1}^{n} c_j \left(S_j^{*-} - \bar{S}_j^-\right),$$

but $S_j^{*-} = \bar{S}_j^-$, for $j = 1, \dots, l-1$, and so

$$\sum_{j=1}^{n} c_j \left(S_j^{*-} - \bar{S}_j^-\right) = c_l\left(S_j^{*-} - \bar{S}_j^-\right) + \sum_{j=l+1}^{n} c_j \left(S_j^{*-} - \bar{S}_j^-\right).$$

From the construction of $\bar{x}$ from x^*, we know that $S_j^{*-} \leq \bar{S}_j^-$, for $j = l+1, \dots, n$, and also $c_i \geq c_j$, for $j = l+1, \dots, n$. Thus,

$$c_l\left(S_j^{*-} - \bar{S}_j^-\right) + \sum_{j=i+1}^{n} c_j \left(S_j^{*-} - \bar{S}_j^-\right) \geq c_l \sum_{j=1}^{n} \left(S_j^{*-} - \bar{S}_j^-\right).$$

But, from the construction of $\bar{x}$, we have $\sum_{j=1}^{n}\left(S_j^{*-} - \bar{S}_j^-\right) = 0$. Therefore, we have

$$\sum_{j=1}^{n}\left(S_j^{*-} - \bar{S}_j^-\right) \geq 0.$$

Since x^* is an optimal solution, we conclude that $\bar{x}$ is an optimal solution. Now, if $x = \bar{x}$, then the proof is complete; otherwise, the number of the leading equal components of $\bar{x}$ (with respect to x^*) and $\bar{x}$ is increased at least by 1. Now, we repeat the argument by replacing x^* with $\bar{x}$. Applying above procedure at most n times, one can prove that x is equal to an optimal solution. This ends the proof.

4. Numerical result

In this section, we look at the implementation results of mathematical model and the greedy algorithm for assigning independent resources to fire stations. We solved the mathematical model by Cplex Studio IDE 12.6.1 and ran the greedy algorithm using Matlab R2017 b on a system with

8GB RAM and \$2GHz core i5 CPU. Our first instance is a fire department with $n = 10$ stations, $m = 5$ resources and $q = 5$ events.

$$T = [3 \quad 3 \quad 8 \quad 2 \quad 5]$$

$$f = \begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 5 & 4 & 4 & 3 & 2 \\ 4 & 0 & 4 & 5 & 4 \\ 3 & 0 & 4 & 3 & 0 \\ 2 & 3 & 0 & 4 & 0 \end{bmatrix}$$

$$p = \begin{bmatrix} 17 & 40 & 29 & 10 & 96 & 42 & 30 & 67 & 3 & 37 \\ 39 & 53 & 43 & 37 & 92 & 55 & 70 & 17 & 56 & 46 \\ 83 & 42 & 1 & 20 & 5 & 95 & 67 & 12 & 89 & 99 \\ 81 & 66 & 99 & 49 & 74 & 42 & 54 & 100 & 67 & 15 \\ 6 & 63 & 16 & 34 & 27 & 99 & 70 & 17 & 19 & 86 \end{bmatrix}$$

$$pr = \begin{bmatrix} 0.6447 & 0.1206 & 0.2518 & 0.9827 & 0.9063 & 0.0225 & 0.4229 & 0.6999 & 0.5309 & 0.9686 \\ 0.3763 & 0.5895 & 0.2904 & 0.7302 & 0.8797 & 0.4253 & 0.0942 & 0.6385 & 0.6544 & 0.5313 \\ 0.1909 & 0.2262 & 0.6171 & 0.3439 & 0.8178 & 0.3127 & 0.5985 & 0.0336 & 0.4076 & 0.3251 \\ 0.4283 & 0.3846 & 0.2653 & 0.5841 & 0.2607 & 0.1615 & 0.4709 & 0.0688 & 0.8200 & 0.1056 \\ 0.4820 & 0.5830 & 0.8244 & 0.1078 & 0.5944 & 0.1788 & 0.6959 & 0.3196 & 0.7184 & 0.6110 \end{bmatrix}$$

The greedy algorithm solves the instance in 0.017 seconds and the obtained solution is:

$$x = \begin{bmatrix} 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Where x_{ij} gives the number of resource i assigned to station j.

The Cplex solver solves the instance in 1.96 seconds and the solution is exactly equal to the one obtained by the greedy algorithm.

To compare the performance of the greedy algorithm and the Cplex solver in larger scale, we executed the programs for a fire departments with $n = 100$ stations, $m = 50$ resources and $q = 20$ events. The greedy algorithm solved this instance in 0.77 seconds, while the Cplex solver solved it in 2.55 seconds.

5. Conclusions

A new integer program was presented to model an independent resource assignment problem with resource shortages in the context of municipal fire service. When shortages exist, a critical task for fire department's administrator in a city is to assign the available resources to the fire stations such that the effect of the shortage to cover (in providing service, in extinguishing fires and so on) is minimized. To solve the model, we proposed a polynomial time greedy algorithm to compute the optimal solution.

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