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ANN-DEA Integrated Approach for Sensitivity Analysis in Efficiency Models

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Here, we examine the capability of artificial neural networks (ANNs) in sensitivity analysis of the parameters of efficiency analysis model, namely data envelopment analysis (DEA). We are mainly interested to observe the required change of a group of parameters when another group goes under a managerial change, maintaining the score of the efficiency. In other words, this methodology provides a platform for simulating the level of some parameters against the remaining parameters for generating different scenarios, as being in demand for managers.

Keywords: *Data envelopment analysis (DEA), Artificial neural networks (ANN), Sensitivity analysis.*

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1. Introduction

Management and control systems require surveying the alteration in some factors of Decision Making Units (DMUs) and monitoring their impact on the other factors when the score of the efficiency is preserved. Among the exiting methods, DEA and ANN have shown to be appropriate tools for this analysis. Using ANN, it is possible to alter one (several) parameter(s) and estimate the level of other parameters. Therefore, DEA and ANN are integrated to solve this kind of problems.

The change in the output parameters is largely influenced by the input parameters and they are closely related to one another, that is, when the level of input parameter is changed in a particular DMU, we expect that the output level to after as well. The inability of DEA in estimating this correlation has motivated us to propose a new algorithm. In other words, the classical sensitivity analysis in mathematical programming, in general, and in DEA, in particular, studies the radius of stability for the parameters when the efficiency score remains unchanged. Therefore, DEA is not able to address our purpose in changing the parameters and at the same time estimating the value of other parameters when the efficiency is fixed.

For the following reasons, the classical sensitivity analysis models in DEA are not appropriate, and thus ANN is utilized in this context.

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1. DEA has no ability to estimate the outputs according to the input parameters and efficiency scores when they are correlated.
2. In dealing with sensitivity analysis of DEA, in order to preserve the classification of efficient and inefficient *DMUs* by metric approaches, a radius of stability is produced. If we change the input parameters of the considered radius, it could not be possible to preserve the outputs of the model to remain in the same radius. Since alteration of the outputs is not directly under the control of managers, in most of sensitivity analysis models, alteration in the input and output parameters are often considered simultaneously, where as explicit variation in the output parameters is seldom possible.

Henceforth, we survey the related work in the literature:

*On sensitivity analysis of efficient DMU*s*:* Charnes and Neralic [1] studied sensitivity analysis of the additive model in simultaneous change of all inputs and all outputs of an efficient decision making unit, preserving efficiency. Zhu [11] introduced a method of sensitivity analysis for upward variations of inputs and for downward variations of outputs of an extremely efficient DMU that remains efficient. Seiford and Zhu [8] developed a procedure for performing a sensitivity analysis of the efficient DMUs within CCR[1] model of DEA, which yields an exact input stability region and output stability region where the efficiency of a specific efficient DMU remains unchanged. Their study gave both necessary and sufficient conditions for an efficient DMU to remain efficient. Jahanshahloo et al. [5] introduced a sensitivity analysis approach for CCR, BCC[2] and additive models, where variations in the data were considered for a specific efficient DMU and the data for remaining DMUs were assumed to be fixed.

On sensitivity analysis of inefficient DMUs*:* Jahanshahloo et al. [6] suggested a procedure for performing a sensitivity analysis of the inefficient DMUs yielding an exact necessary change region in which the efficiency score of specific inefficient DMU changes to a defined efficiency score.

On stability of DMUs classifications: Cooper et al. [2] surveyed the developed analytical methods for studying the sensitivity of DEA results to variations in the data. They focused on the stability of classification of DMUs into efficient and inefficient performers. Takeda and Nishino [9] have proposed a technique for assessing the sensitivity of efficiency classifications, where efficient and/or inefficient remained unchanged under perturbations of the data. Zhu [12] studied an approach for the sensitivity analysis of DEA models by using various supper-efficiency DEA models. This approach simultaneously considered data perturbations in all DMUs and developed necessary and sufficient conditions for preserving efficiency when data changes are made for all DMUs. Using super-efficient DEA models, the sensitivity analysis of DEA efficiency was achieved. Jahanshahloo et al. [4] found radius of stability for all *DMU*s with interval data. In their work, classification remains unchanged under perturbation of the interval data. Gholam Abri [3] proposed a method of calculating the stability radius, in which the data variations neither change the class of efficiency units nor the class of return to scale.

[1] Charnes, Cooper & Rhodes
[2] Banker, Charnes & Cooper

In the proposed algorithm, DEA is to be used for efficiency analysis. So, all the DMUs' efficiencies according to the present technology is evaluated and the correlation among efficiency, input and output parameters is gained implicitly. In order to obtain the implicit correlation for these factors, ANN is used. For all DMUs, input parameters and the efficiency score obtained via DEA, constitute the input layer and the output parameters are expected to be estimated at the output layer. This methodology enables the manager to simulate the system prior to any action and establish validity of the scenarios.

In Section2, data envelopment analysis and artificial neural networks are described. In Section3, the proposed integrated algorithm is presented. Section 4 provides a case study for the algorithm. In Section5, we give our concluding remarks.

2. Data Envelopment Analysis and Artificial Neural Networks

2.1. Data Envelopment Analysis (DEA)

Since large groups of real life problems are in the context of variable returns to scale, we use the BCC model. The BCC model in the input-oriented mode with variable returns to scale, evaluates the relative efficiency of n DMUs each consuming m inputs and producing s outputs denoted by $x_{1j}, x_{2j}, \ldots, x_{mj}$ and $y_{1j}, y_{2j}, \ldots, y_{sj}$, respectively, by minimizing inputs when the outputs are fixed to be constant. The input-oriented BCC model is defined to be:

$$
\begin{aligned}
& Min \quad \theta \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \qquad i=1,\ldots,m, \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \qquad r=1,\ldots,s, \\
& \qquad \sum_{j=1}^{n} \lambda_j = 1, \\
& \qquad \lambda_j \geq 0, \qquad\qquad j=1,\ldots,n.
\end{aligned}
\tag{1}
$$

The calculations provide an efficiency score, θ, using linear optimization for each DMU with respect to the closest observation on the frontier based on orientation of the model.

2.2. Artificial Neural Networks (ANN)

When the relationships among the variables are not explicitly known, ANN plays an important role in dealing with such situations. Moreover, ANN has the capability to correlate a set of independent variables with more than one dependent variable. Therefore, ANN can practically correspond to DEA models when multiple inputs (parameters) are correlated to multiple outputs (parameters).

Here, we are looking for the relations among efficiency, input and output parameters. Since ANN is a nonparametric approach and does not make any assumption on the functional forms between inputs and outputs, it is used in order to approximate the function that maps input parameters and efficiency score into the output parameters. Among different networks, the feed forward neural network is used. From the existing learning rules, backpropagation (BP), which includes searching weights by minimizing the cost function, is utilized. The BP learning rule is a supervised learning algorithm proposed by Werbos [10] and later developed by Rumelhart and McClelland [7].

3. The Proposed Algorithm

In this section, our proposed methodology is introduced in details. It is assumed that the efficiency scores are initially evaluated by DEA. Then, the manager desires to alter a subset of the parameters. Since the parameters are practically correlated, we are interested to observe the impact of change in some parameters on the remaining parameters. Furthermore, we assume that the efficiency score remains unchanged. Therefore, ANN is the most appropriate tool to estimate the correlation of inputs, outputs and efficiency.

The efficiency scores of all the DMUs are obtained by the DEA model. Regularly managers are interested to change input parameters by assuming constant efficiency and would like to estimate the resulting variations in the output parameters.

In Fig. 1, the steps of the algorithm are shown.

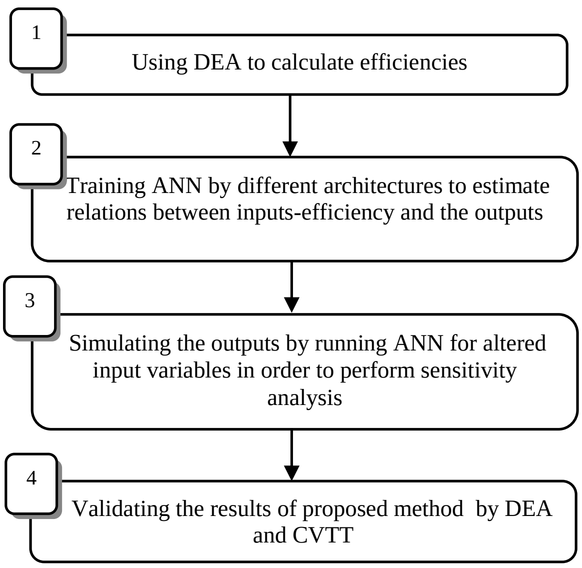

Figure 1. An integrated DEA-ANN algorithm for sensitivity analysis

3.1. Step 1: Using DEA to Calculate Efficiencies

Since for each DMU, the efficiency score is varying by the alteration in the inputs and outputs, it is necessary to have accurate specifications in all cases. In this step, the BCC model (1) is utilized. By applying input and output parameters and running the BCC model, efficiency score for each DMU is obtained.

3.2. Step 2: Training ANN by Different Architectures to Estimate Relation Between the inputs-efficiency and the Outputs

In this step, the input parameters and efficiency score obtained from DEA are used as input layer and the output parameters of DEA as the output layer for training the ANN. Therefore, by training the ANN, we can find the relations among these factors. We consider output parameters as the targets and in the training phase, ANN is designed to close up the outputs to the targets. Finally, in the process of training, the relations are estimated.

Here, in order to improve the neural network's generality, early stopping method is used. In this method, the data set is divided into three subsets:

1. *Training set:* training set is utilized to compute the gradient and update the network weights and biases.
2. *Validation set:* in the training process, the error of the validation set is used for supervision of training phase. Validation set's error should be decreased in the training process. When the network is experiencing too much of an adaptation to data, the validation error begins to rise. When the validation error increases for a specified number of iterations, the training is stopped and the weights and biases at the minimal validation error are returned.
3. *Test set:* The test set error is not used during the training, but it is used to compare different networks.

In order to find a suitable architecture, the transfer function for each layer, the arbitrary number of neurons in the hidden layer, which is determined by trial and error, and the learning algorithm are specified. Also, the performance function for calculating the error in each step of the algorithm between actual and estimated outputs is chosen.

3.3. Step 3: Simulating the Outputs by Running ANN for Altered Input Variables in Order to Perform Sensitivity Analysis

Now, it is possible to estimate the outputs by preferred alteration of the input parameters and efficiency scores, using the ANN obtained from the previous step. This method helps managers to decide if the input sources are consumed according to the desired efficiencies, then how much output may be obtained.

3.4. Step 4: Validating the obtained Results by DEA and CVTT[1]

To evaluate the validity of the proposed method, the inputs and estimated outputs of ANN are used to run a DEA model to obtain the efficiencies. Cross validation test technique is performed to study the quality of the results of the method. In order to survey the efficiency deviations as compared to the efficiency obtained in the previous step, the MAE[2] measure is applied.

4. The Case Study

The proposed hybrid algorithm was used for 914 branches of a large banking industry for sensitivity analysis by altering the input parameters. The details for all the steps are provided below.

4.1. Using DEA to Calculate Efficiency Scores

Inputs and outputs of bank branches in this study are illustrated in Table 1. The input oriented BCC model is used for evaluating the efficiency of DMUs and based on the efficiency scores, the efficient and inefficient units are identified.
The efficiency scores obtained in this step and the input-output specifications are used for the next step.

4.2. Training ANN by Different Architectures to Estimate Relations between Inputs-efficiency and the Outputs

From manager's point of view, it is sometimes important to investigate the case when a fixed efficiency is used by some special inputs, for the amount to be produced. So, in order to learn this relation, we use the input parameters and efficiency as the inputs of the ANN and the output

Table1. Inputs and outputs of bank branches

Inputs	**Outputs**
I_1: Personnel scores	**O_1:** Facilities
I_2: Paid profits	**O_2:** Sum of deposits
I_3: Pending demands	**O_3:** Received profits
	O_4: Received wages
	O_5: Other resources

[1] Cross Validation Test Technique
[2] Mean Absolute Error

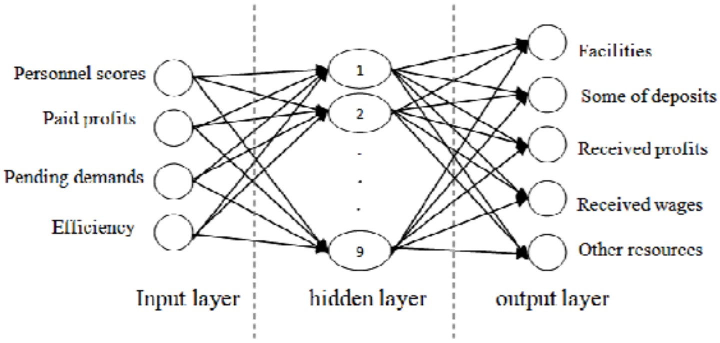

Figure 2. Topology of the proposed network

parameters as the outputs of ANN. Therefore, in the architecture of ANN, the input vector for the input parameters and efficiency has four elements and the output layer has five neurons. A topology of the network is illustrated in Fig. 2.

In our study, 80 percent of the data is used for training, 10 percent for validation and 10 percent for testing. The neural network that is used here is a two layer neural network with a single hidden layer. Among different examined ANNs, we considered the following architecture:

- Early stopping method for improving the generalization
- Tansig transfer function in both layers
- Resilient backpropagation (Rprop) training algorithm
- 9 neurons in the hidden layer
- Msereg[1] performing function with perform ratio equal to 0.5 in order to compute the error in the training process and improve the generalization of the network

Fig. 3 illustrates the error in training, validation and the test data sets. Fig. 4 shows the correlations coefficient among outputs and targets in training, validation and test sets. In Fig. 5, the correlation between each output and its target is shown.

4.3. Simulating the Outputs by Running ANN for Altered input Variables in Order to Perform Sensitivity Analysis

In this step, by altering the input variables and considering the ideal efficiency, the output variables are estimated by ANN. Considering the variation and the ideal efficiency, preferred ANN has estimated the outputs. The variation of inputs by the manager and the estimated outputs by ANN are shown in Table 2 and Table 3, respectively.

[1] Mean Squared Error with Regularization

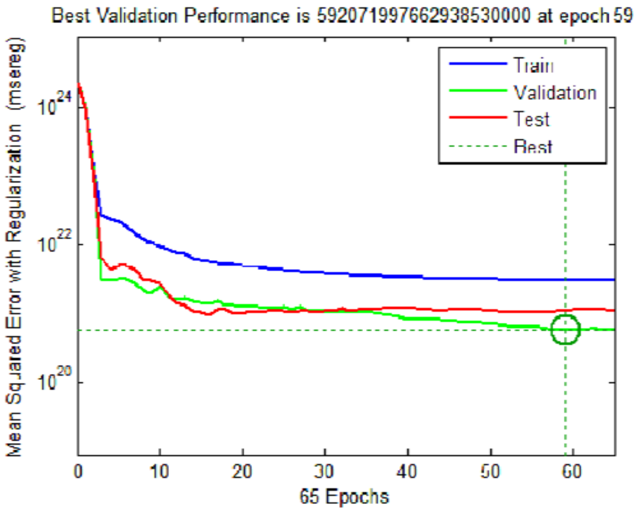

Figure 3. Error in the training, validation and test process

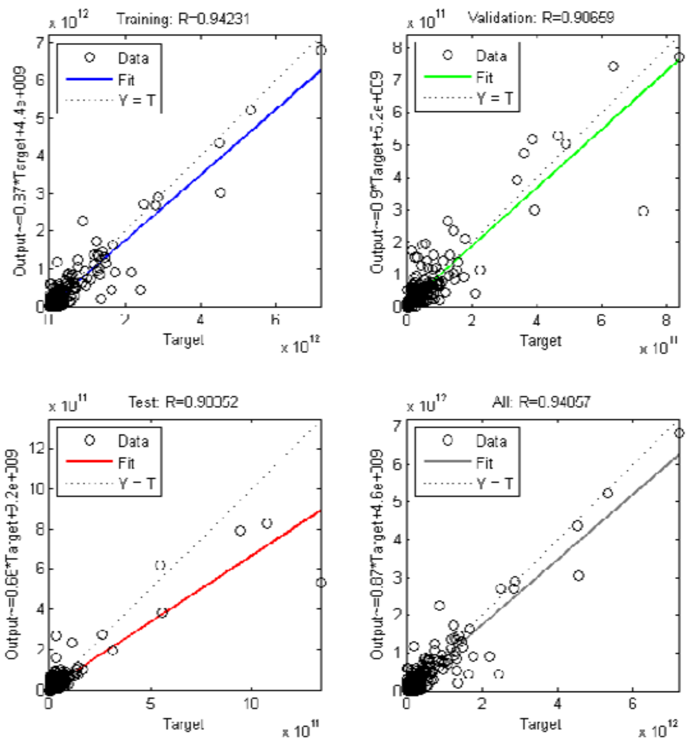

Figure 4. The R-values in training, validation and test set

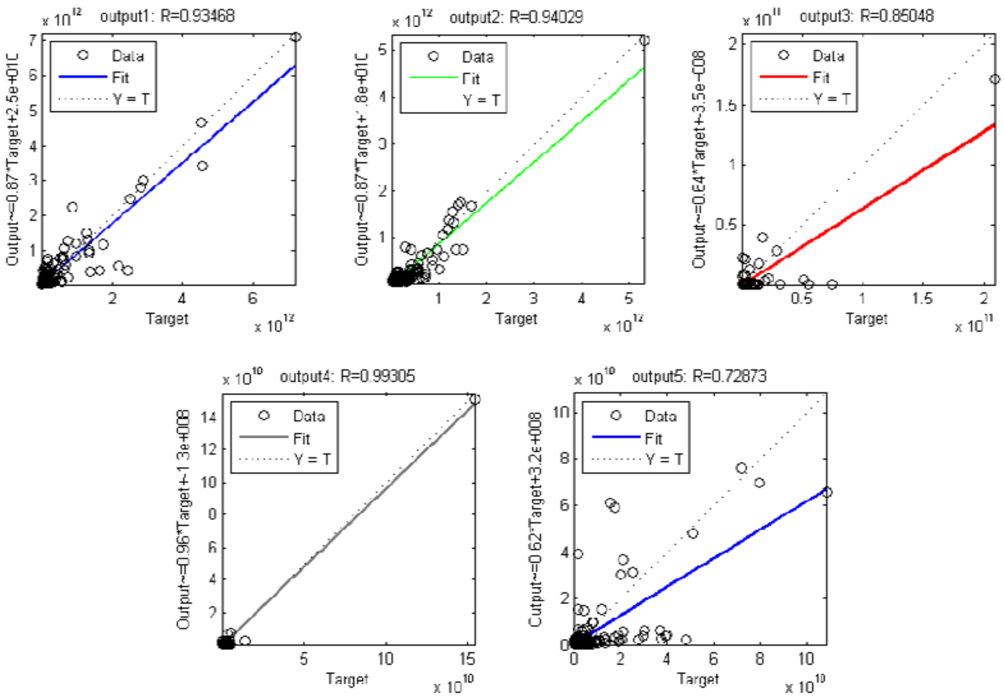

Figure 5. The R-values among outputs and targets

Table2. Variation in the input parameters by the manager

Input parameters		
I_1	**I_2**	**I_3**
+5%	+7%	-5%

Table3. Mean variation of estimated outputs

Output parameters				
O_1	**O_2**	**O_3**	**O_4**	**O_5**
+1%	+2%	+10%	+5%	-6%

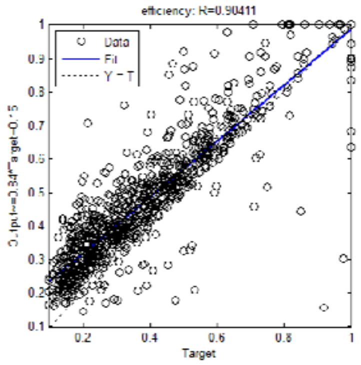

Figure 6. The R-value between DEA efficiencies and considered efficiency in the ANN training

4.4. Validating the Results of the Proposed Method by DEA and CVTT

For evaluating the validity of our method, the altered inputs and the estimated outputs of ANN are fed to the DEA model. In order to compare the obtained efficiency by DEA and the considered efficiency in the ANN training (Fig. 6), cross validation test is performed. The R-value between these two efficiencies is approximately 0.91, which being close to 1 shows a suitable correlation.
In order to examine the efficiency deviations compared to the obtained efficiency in the previous step, the MAE measure is applied. The average variation in efficiency scores is obtained to be 0.06, which shows the deviations to be appropriate.

5. Conclusion

Our proposed methodology provides facility to managers to check different variations of inputs and efficiency scores with respect to the outputs and generate different scenarios. The advantage of this process is to avoid solving a linear programming problem for each DMU when the DMU undergoes a drastic change, due to the resulting output being predicted by ANN. For a future study, one can design a decision support system in which any parameter (or an arbitrary set of parameters) may change and the system is automatically trained to predict the value of the remaining parameters.

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