Supply chain optimization policy for a supplier selection problem: a mathematical programming approach

J. Gheidar-Kheljani[1], S.H. Ghodsypour[,1] and S.M.T. Fatemi Ghomi[1]

Most supplier selection models consider the buyer's viewpoint and maximize only the buyer's profit. This does not necessarily lead to an optimal situation for all the members of a supply chain. Coordination models have been developed to optimize the entire supply chain and align the decisions between its entities. Little research has been done on the application of these models in the supplier selection problem. Here, a combination of a supplier selection model and a co-ordination model in a centralized supply chain is considered. In the proposed model, the buyer selects the right supplier and order an appropriate quantity. The suppliers split the buyer ordered quantities into small lot sizes and deliver them over multiple periods. The objective function is to minimize the total cost of the supply chain. A nonlinear mathematical model is proposed for the problem. The total cost of the supply chain includes the costs of the buyer and the suppliers. The nonlinear model is transformed into a concave minimization problem and solved by considering the specific characteristics of the problem. Finally, the proposed model is illustrated by a numerical example.

1. Introduction

A key issue in Supply Chain Management (SCM) is development of mechanisms to align the objectives of independent supply chain participants and coordinating their decisions and activities in order to optimize the system performance. The cost of a product includes not only the manufacturing cost but also the marketing cost, the cost of the services and the delivery cost. Consequently, in order to reduce the global cost, firms have to plan all the activities of the supply chain in a coordinated manner. Manufacturers have to cooperate or interact with suppliers to maximize the productivity at the smallest cost while satisfying customer requirements Jain et al [26].

Narasimhan and Carter [36] showed that a well-integrated supply chain requires coordinating the flow of materials and information between suppliers, manufacturers, retailers and other component of the Supply Chain (SC). Thomas and Griffin [48] stated that global planning and coordination among all entities in a supply chain is needed to achieve effective supply chain management. Several models such as quantity discounts, credit option and buy back/return policies have been applied to align and coordinate the decisions between members of a supply chain. Most of these models have been developed to improve the effectiveness of an existing SC, but coordination and alignment of decisions between entities are also of great importance to the design and set-up of a new SC. Supplier selection and order allocation problems are two main factors to consider in the SC design process. Most supplier selection models consider and optimizing only buyer's objectives with no respect to the impact of this policy on suppliers. In these models,

suppliers offer their conditions such as production constraints, sale prices and discounts, and the buyer selects the right suppliers (from buyer's point of view) and allocates orders to them. Here, the buyer's bargaining power impels suppliers to accept and follow its decisions. Here we develop a supplier selection and order allocation model to minimize the annual total cost incurred in the whole SC.

The remainder of this paper is organized as follows the previous past studies is reviewed in the next section. Section 3 describes the problem and the notation. Section 4 presents components of the annual cost of SC. In Section 5, a single-objective integer nonlinear model is developed to minimize the annual total cost of the SC, and the model is transformed into a concave minimization problem. Then, a near optimal solution procedure is proposed with respect to some specific characteristics of the problem. In Section 6, the proposed model is illustrated by a numerical example. Finally, conclusions and directions of future research are discussed in Section 7.

2. Literature review

Ghodsypour and O'Brien [14] noted that supplier selection models could be broken down into single source and multiple source models. In single source models, one supplier is able to respond to a buyer's demand. In multiple source models, the allocation problem is considered to be the same as the selection problem Sharafali [45]. Ranking techniques are usually applied to single source models, but in multiple source models mathematical programming models are developed. Table 1 shows classification of mathematical models with single objective functions. Chaudry et al. [8] considered the vendor selection problem under quality, delivery, and capacity constraints as well as price-break regimes. They formulated linear and mixed integer models for a single objective (cost) problem.

Rosenthal et al. [42] studied a purchasing problem where suppliers offered discounts when a "bundle" of products was bought, and when one needed to select suppliers for multiple products. Sarkis and Semple [44] discussed optimization of the total purchasing cost in the presence of business volume discounts. They considered only one period and thus did not take inventory costs and other time dependent parameters into account. Jayaraman et al. [27] formulated a mixed integer linear programming model that considered quality (in terms of proportion of defective products supplied), production capacity, lead-time, and storage capacity limits. Their model is an single period model in which a fixed cost is required to deal with a supplier (sourcing cost). Cakravastia et al. [5] developed a mixed integer programming model for the supplier selection process by designing SC networks. The constraints on the capacity of each potential supplier were considered in the process. The objective was to minimize the level of customer dissatisfaction, which was evaluated by two performance criteria: (i) price and (ii) delivery lead time.

Degraeve and Roodhooft [10] further developed a multi-period, multi-item, multi-vendor mixed-integer programming model based on the TCO, to determine an optimal ordering and inventory policy and jointly to decide on the best combination of suppliers. Their model covers the total cost incurred, including the purchasing cost, the ordering cost, the transportation costs and so forth. Ghodsypour and O'Brien [14] developed a decision support system that combined the analytical hierarchy process with linear programming. Ghodsypour and O'Brien [15] first presented a single objective mixed-integer nonlinear programming model to minimize total cost. In that model, they considered quality as a constraint, and then developed a multi-objective model with one of its objectives to maximize the orders quality. Tempelmeier [47] developed a heuristic to solve a mixed-integer linear formulation of the vendor selection and order-sizing problem for a single item under dynamic demand in the absence of supplier capacity constraints.

Degraeve and Roodhooft [10] used the TCO concept in a service purchasing context. They proposed a model to select vendors of a multiple item service and simultaneously determined market shares of the selected suppliers. Murthy et al. [35] addressed buyer's selection problem for make-to-order items where the goal was to minimize sourcing and purchasing costs in the presence

of fixed costs, shared capacity constraints, and volume-based discounts for bundles of items. They introduced a mixed-integer linear programming model and solved it by Lagrangian relaxation. Hong et al. [24] developed a mixed integer programming model to select right suppliers and maximize revenue while satisfying the customer needs. They considered changes in suppliers' capabilities and customer requirements over the horizon of the problem. In their model, the suppliers which satisfy many parts of the ideal procurement condition are selected more often than other suppliers. Basnet and Leung [3] developed a model to combine lot-sizing with supplier selection problem. They considered a multi-period inventory lot-sizing scenario where multiple products could be sourced from a set of selected suppliers in each cycle. The objective function consists of purchasing price, inventory holding cost and transaction cost for minimization and an enumerative search algorithm was proposed to solve the problem.

In addition to the supplier selection problem, we also address the issue of coordination between the suppliers and buyer. The idea of a joint optimization between buyer and supplier was initiated by Goyal [16] and later reinforced by Banerjee [2]. Banerjee [[2] introduced a joint economic lot size (JELS) model with a single vendor and a single buyer to minimize the joint total relevant cost. Lu [34] developed an identical delivery quantity (IDQ) policy and proposed an optimal solution for the single buyer–single vendor model. Weng [50] showed that when both parties coordinate, the order quantity and joint profit will increase and the selling price will decrease. Hill [21], Goyal and Nebebe [18] discussed production-delivery policies in a single manufacturer and single retailer environment, while Goyal [17] and Lu [34] proposed production-delivery policies in a single manufacturer and multiple retailers environment. Li and Wang [33] provided a review of SC coordination models based on centralized and decentralized decision structures. They concluded that an important number of papers had been written on centralized SC systems because the coordination of decentralized SC systems was more difficult. These works mainly considered only single supplier-single buyer or one supplier-multiple buyer cases.

Recently some researchers have developed joint decision making in multiple supplier-single buyer SCs. Kim et al. [29] developed a production-delivery policy in an SC consisting of a single manufacturer with multiple plants in parallel, a single warehouse, and a single retailer. They built a model to determine an optimal production cycle length (or interchangeably production lot size) for the manufacturer, a delivery policy (i.e. frequency and quantity) for the retailer, and a production allocation scheme for multiple plants so as to minimize the average total cost incurred at both the manufacturer and the retailer. Yung et al. [52] presented heuristic algorithms for a joint decision problem with single as well as multiple products in a production distribution network system including multiple suppliers and multiple destinations. The joint decision problem addressed in their work was to simultaneously determine the annual production quantity and lot size to be assigned to the suppliers, and the annual shipment amounts and order quantities from each supplier to individual destinations to meet their respective total demands at a minimum total cost within the network. Park et al. [39] developed a mathematical model in which the retailer placed orders based on the EOQ policy and allocated them to the multiple manufacturers. In their model, production allocation ratios and the shipment frequencies at the manufacturers as well as the purchasing cycle length at the retailer were formulated to minimize the average total cost at the manufacturers and retailer.

The available literature on the published work in the context of supplier selection problem does not consider the whole SC optimization in formulating the problem. Most researchers have considered only buyer's objectives in the supplier selection problem. Coordination models can be used to take into account the suppliers' objectives as well as buyer's objectives. As far as we know, Herer et al. [20] are the first to discuss the application of coordination models to a supplier selection problem. The major advantages of the proposed model over their work are:

- The limited annual production rate is added to each supplier.
- The number of deliveries per inventory cycle at each supplier can be greater than 1.
- The inventory holding cost of each supplier is taken into account.

Here we develop a supplier selection model in combination with the coordination concept in an SC that includes multiple suppliers and a single buyer. The global optimization of the SC is handled in a centralized decision making system (DMS) framework.

3. Problem statement

In this section, a supplier selection model is developed based on the production allocation and shipment polices model introduced by Park et al. [39]. The advantages of the proposed model over their work are:

- Setup times, transportation costs, and unit production costs are added at each supplier.
- The buyer purchasing quantity per year at each supplier is assumed to be unknown.
- The suppliers' inventory holding costs are different from each other.
- It is allowed that the time between two deliveries at each supplier to be different from other suppliers.

By considering the mentioned advantages, the proposed model also becomes a generalized version of the single-supplier single-buyer joint decision model introduced by Kim and Ha [29].

3.1. Problem definition

We consider a set of m suppliers and one buyer in a centralized supply network as illustrated in Figure 1. The objective of the buyer is to select the right suppliers and assign order quantity to each so that the total cost of the SC is minimized. The horizon of the problem is a year. The buyer has a definite annual demand denoted by D. Each vendor has a finite annual production rate denoted by P_i and the available annual production capacity (in hours) denoted by Z_i. In other words, it is assumed that when the vendor starts to produce each production lot size, its production rate is P_i. So, it is not the production capacity of the vendor.

The buyer places an order to each supplier, the supplier splits the ordered quantities into small lot sizes and delivers them over multiple elementary periods in order to meet the buyer's demand. The supplier needs to hold the inventory throughout the production of each lot size. In our model, the buyer is assumed to pay transportation costs in order to facilitate frequent deliveries.
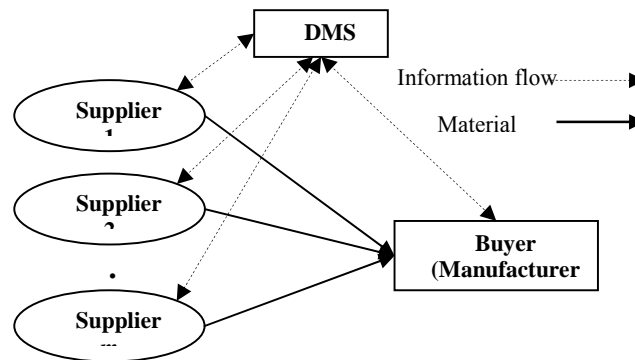

Figure 1- A Centralized supply chain with one buyer and m suppliers

Notations
D : Annual demand of the buyer
m : Number of suppliers
D_i : The part of the annual buyer's demand assigned to supplier i

P_i: The annual production rate of supplier i

Z_i: The annual production capacity of supplier i (in hours)

A_b: Fixed ordering cost paid by the buyer for each order that is salaries and expenses of processing an order such as any approval steps, the cost to process the receipt, incoming inspection, invoice processing and vendor payment, regardless of the order quantity,

S_i: Set up cost paid by supplier i for each delivery

Q_i: The production lot size at supplier i (unit)

h_i: Holding cost which is the cost incurred by supplier i to keep in stock one unit of product during one year

u_i: Time required by supplier i to manufacture one unit of product

N_i: Number of deliveries per inventory cycle at supplier i (integer value)

q_i: Delivery size per trip at supplier i, $Q_i = N_i q_i$

h_b: The annual holding cost at buyer ($/unit),

F_i: Fixed transportation cost per delivery from supplier i paid by the buyer

r_i: The unit production cost at supplier i ($/unit)

D_i, Q_i, N_i and q_i are decision variables and other parameters are constant.

3.2. Assumptions
1) No inventory shortage is allowed for buyer and suppliers.
2) No overstock is permitted. Thus, inventory cannot be carried from previous period to the next period.
3) The first delivery at supplier i is carried out as soon as the inventory level reaches into q_i.
4) The multiple deliveries of each supplier are to be arranged in such a way that each succeeding delivery arrives at the time that all inventories from previous delivery of that supplier have just been depleted.

In order to get a feeling, a sample inventory trajectories for a single supplier and a single buyer are shown in Figure 2 that is taken from Kim and Ha [29].

4. Annual total cost of SC

Our aim is to determine D_i, Q_i, N_i and q_i in order to minimize the annual total cost of SC that includes buyer and suppliers annual total costs. Next, we deal with the calculations of buyer and suppliers total costs.

In the remainder of the paper, the set of suppliers that are selected by the buyer is denoted by E. Note that if a supplier does not belong to E, the corresponding parameters are set equal to zero for that supplier.

4.1. Annual total cost of buyer (TC_b)

The buyer's annual total cost is composed of ordering cost, inventory holding cost, and transportation cost. These costs are as follows.
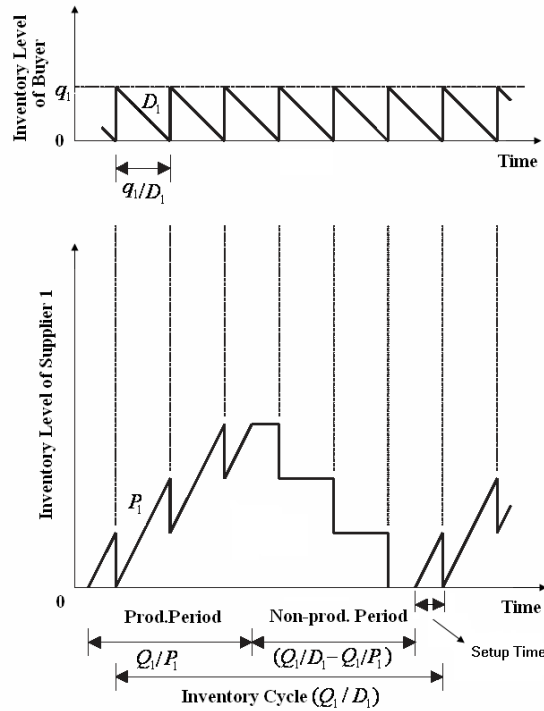
Figure 2 -Inventory levels of one buyer and one supplier

Buyer's annual ordering cost: The number of orders to supplier $i \in E$ is $\dfrac{D_i}{Q_i}$ per year, thus, the annual ordering cost paid by the buyer for orders assigned to supplier $i \in E$ is $A_b \dfrac{D_i}{Q_i}$. Then, the buyer's annual ordering cost is $\sum\limits_{i \in E} A_b \dfrac{D_i}{Q_i}$.

Buyer's annual inventory holding cost: The buyer's maximum inventory on-hand from supplier $i \in E$ is q_i and its minimum value is 0 in each period, so and average inventory becomes $\dfrac{q_i}{2}$ if a regular consumption is assumed. Hence, the buyer's inventory holding cost per year purchased from supplier $i \in E$ is $h_b \dfrac{q_i}{2}$. Then, the buyer's annual inventory holding cost is $h_b \sum\limits_{i \in E} \dfrac{q_i}{2}$.

Buyer's annual transportation cost: The number of deliveries per year from supplier $i \in E$ is $N_i \dfrac{D_i}{Q_i}$. So, buyer's annual transportation cost is $\sum\limits_{i \in E} F_i N_i \dfrac{D_i}{Q_i}$. Thus, according to the above costs and the equation $Q_i = N_i q_i$, the buyer's annual total cost per year is:

$$TC_b = \sum\limits_{i \in E} \left[A_b \frac{D_i}{Q_i} + h_b \frac{Q_i}{2N_i} + F_i N_i \frac{D_i}{Q_i} \right]. \tag{1}$$

4.2. Annual total cost of suppliers (TC_s)

The annual total cost of each supplier is the sum of setup cost, inventory holding cost and unit production cost. The annual total cost for supplier $i \in E$ (TC_{si}) is detailed hereafter.

Annual setup cost of supplier $i \in E$: It is easy to see that the cost is $S_i \dfrac{D_i}{Q_i}$.

Annual production cost of supplier $i \in E$ is $r_i D_i$.

Annual inventory holding cost of supplier $i \in E$: The average inventory level at supplier $i \in E$ is $\dfrac{Q_i}{2N_i}\left\{\dfrac{D_i(2-N_i)}{P_i}+(N_i-1)\right\}$, as derived by Joglekar [28], and so the annual inventory holding cost of supplier $i \in E$ is:

$$h_i \frac{Q_i}{2N_i}\left\{\frac{D_i(2-N_i)}{P_i}+(N_i-1)\right\}. \tag{2}$$

As a result, adding the above three cost components for each supplier, the annual total cost of suppliers (TC_s) is obtained:

$$TC_s = \sum_{i \in E}\left[r_i D_i + S_i \frac{D_i}{Q_i} + h_i \frac{Q_i}{2N_i}\left\{\frac{D_i(2-N_i)}{P_i}+(N_i-1)\right\}\right]. \tag{3}$$

4.3. Annual total cost of supply chain (TC)

Adding Eqs. (1) and (3) yields the annual total cost of the SC as follows:

$$TC(E) = \sum_{i \in E}\left[r_i D_i + S_i \frac{D_i}{Q_i} + h_i \frac{Q_i}{2N_i}\left\{\frac{D_i(2-N_i)}{P_i}+(N_i-1)\right\}\right]$$
$$+ \sum_{i \in E}\left[A_b \frac{D_i}{Q_i} + h_b \frac{Q_i}{2N_i} + F_i N_i \frac{D_i}{Q_i}\right]. \tag{4}$$

5. Mathematical modeling and solution procedure

In this section, a single-objective mixed-integer nonlinear problem is presented and then a solution procedure is proposed for its solution.

5.1. Mathematical model

The developed model can be formulated mathematically as a single-objective mixed-integer nonlinear optimization problem to be shown as Problem A. Note that, when writing problem A, it is assumed that the set E of selected suppliers is known. Constraint (5) ensures that the buyer's demand is met while constraints (6) ensure that the production capacity of each supplier is enough to meet the demand.

Problem A :

$$Min \ TC(E) = \sum_{i \in E} \left[r_i D_i + S_i \frac{D_i}{Q_i} + h_i \frac{Q_i}{2N_i} \left\{ \frac{D_i(2-N_i)}{P_i} + (N_i - 1) \right\} \right]$$

$$+ \sum_{i \in E} \left[A_b \frac{D_i}{Q_i} + h_b \frac{Q_i}{2N_i} + F_i N_i \frac{D_i}{Q_i} \right]$$

s.t. $$\sum_{i \in E} D_i = D \tag{5}$$

$$D_i \le \frac{z_i}{u_i}, \ i \in E) \tag{6}$$

$$D_i \ge 0, \ i \in E$$

$$Q_i \ge 0, \ i \in E$$

$$N_i \ge 0 \ \text{and integer}, \ i \in E.$$

The objective function is a non convex function with respect to N_i, Q_i and D_i, $i \in E$. So, it is not easy to obtain the optimal values of N_i, D_i, Q_i, $i \in E$, simultaneously. Therefore, Problem A is transformed into a simplified equivalent problem with only the decision variables D_i, $i \in E$. Then, a method to solve the transformed problem is proposed.

5.2. Simplified problem

The objective function is the sum of $K = card \ (E)$ separate functions (that are separable) and each one has three decision variables; for example, the decision variables of the function related to index k are N_k, D_k and Q_k. For simplicity, but without loss of generality, assume that the objective function is continuous with regards to N_i, $i \in E$. So, for fixed D_i, $i \in E$, it can be easily shown that the Hessian matrix of the function corresponding to index i is positive definite. This ensures that it is strictly convex with respect to N_i and Q_i. Thus, by taking the first derivatives of the objective function with respect to N_i and Q_i, equating them equal to zero, and solving for N_i and Q_i simultaneously, the following formulas for $i \in E$ are obtained:

$$N_i^* = \sqrt{\frac{(A_b + S_i)\{P_i(h_b - h_i) + 2D_i h_i\}}{F_i(P_i - D_i)h_i}},$$

$$Q_i^* = \sqrt{\frac{2D_i(A_b + S_i)}{h_i(1 - \frac{D_i}{P_i})}}. \tag{7}$$

Substituting Q_i^* and N_i^*, for $i \in E$, into the objective function of problem A, the minimum annual total cost is obtained to be:

$$TC_{Q^*,N^*} = \sum_{i \in E} \left\{ \sqrt{2D_i h_i (A_b + S_i)(1 - \frac{D_i}{P_i})} + \sqrt{2D_i F_i (h_b - h_i + 2h_i \frac{D_i}{P_i})} + r_i D_i \right\}. \quad (8)$$

In the above function, each term involves just a single variable D_i, so that the function is separable into a sum of functions of individual variables. It can easily be shown that in Eq. (8) the Hessian matrix of the function corresponding to index i is negative semi- definite. So, Eq. (8) is a concave function, because each term is concave with respect to D_i by assuming that $h_b \geq h_i$ for, $i = 1,...,m$. This is a reasonable assumption, since product value increases as it moves down the distribution chain and its related holding costs increases (Hill [21], Hill below and Omar, [22]). Hence, the original Problem A can be equivalently rewritten as Problem B.

Problem B :

$$Min\ TC_{Q^*,N^*} = \sum_{i \in E} \left\{ \sqrt{2D_i h_i (A_b + S_i)(1 - \frac{D_i}{P_i})} + \sqrt{2D_i F_i (h_b - h_i + 2h_i \frac{D_i}{P_i})} + r_i D_i \right\}$$

s.t.

$$\sum_{i \in E} D_i = D$$

$$D_i \leq \frac{z_i}{u_i}, i \in E$$

$$D_i \geq 0, i \in E.$$

The optimal values of the components of the vector D^* for Problem B are also optimal for Problem A. But, remember that this vector has only K components. Note that TC_{Q^*,N^*} is concave with respect to D_i, $i \in E$. This implies that problem B belongs to the class of concave minimization problems subject to convex constraints. Horest et al. [25] and Chauhan and Proth [7] showed that the optimal solution of this type of concave problem can be achieved at an extreme point of the feasible region defined by the constraints that called an extreme point property. This implies that, at least one optimal solution D^* exists such that

$$D_i^* = \frac{z_i}{u_i}\ \text{or}\ D_i^* = 0,\ \text{for}\ i \in E,\ \text{except may be for one}\ j \in E,\ \text{for which}\ 0 < D_j^* < \frac{z_j}{u_j}\ \text{to satisfy}$$

$$\sum_{i \in E} D_i^* = D.$$

Since the suppliers $i \notin E$ are not selected such that $D_i = 0$, it is possible to integrate these suppliers in the above property and write:

There exists an optimal solution $D^{**} = \left\{ D_1^{**}, \cdots, D_m^{**} \right\}$ such that $D_i^{**} = \dfrac{z_i}{u_i}$ or $D_i^{**} = 0$, for

$i = 1, \cdots, m,$ except may be for one $j \in \left\{ 1, \cdots, m, \right\}$ for which $0 \le D_j^{**} \le \dfrac{z_j}{u_j}$ to satisfy

$\sum\limits_{i=1}^{m} D_i^{**} = D$.

However, to find out the global optimal solution D^{**} based on this property, all the available

suppliers are classified into three sets Φ_1, Φ_2 and Φ_3 such that $\sum\limits_{i=1}^{m} D_i = D$.

$$\Phi_1 = \left\{ i \mid D_i = 0 \right\}, \quad \Phi_2 = \left\{ i \mid D_i = z_i / u_i \right\}, \quad \Phi_3 = \left\{ i \mid 0 \le D_i \le z_i / u_i \right\} \quad (9)$$

It is clear that $m2^{m-1}$ combinations exist to classify m suppliers into the mentioned three sets.

5.3. Solution Algorithm
In this section, a twofold solution algorithm is proposed: Problem B is solved first and based on it, the solution of Problem A is computed.

5.3.1. Solution Algorithm of Problem B
Step 1: According to expression (9), classify the suppliers into three sets $\Phi_1 and \Phi_2$ and Φ_3. If

there are m suppliers, then $m2^{m-1}$ combinations exist. Note that Φ_3 has only one member.

Step 2: For each combination, assign values 0 and z_i / u_i to members of the sets Φ_1, Φ_2,

respectively. Then, find the value D_i, $i \in \Phi_3$, to satisfy $\sum\limits_{i \in \Phi_2} D_i + D_i = D$. Note that

$0 \le D_i \le \dfrac{z_j}{u_j}, i \in \Phi_3$. So if the D_i value is negative or greater than z_i / u_i, this combination is

infeasible and has to the be deleted.

Note that $D_i = 0$, $ie \in \Phi_1 \bigcup \Phi_2$ implies that it is not economical to allocate the order quantity to the corresponding supplier.

Step 3: For each feasible combination, substitute D_i, $i = 1, ..., m$ in the objective function of Problem B and calculate the value of the objective function.

Step 4: Choose the minimum value from all the objective function values calculated in step 3 and set its corresponding D_i values as the optimal allocation vector D^* for problem B.

Although for m suppliers, the number of combinations is $m2^{m-1}$ and in the proposed algorithm all combinations are enumerated, but the solution should not require a long time because in most practical cases there are usually a maximum of 12 vendors (Chaudhry et al. [8]).

The computational time of the proposed algorithm dependes on $m2^{m-1}$. The programming language C is used to code the algorithm Gheidar-Kheljani, [12]. The written program makes at most $(m2^{m-1}) \times (2m^2 + 15m)$ operations before halting. If m gets its maximum value, i.e. , $m = 12$, the maximum number of operations will be $11558508 \approx 10^7$. Knowing that usual personal

computers do 10^9 operations per second, it will take one second to find the optimal solution of Problem B.

5.3.2. Algorithm of Problem A

Step 1: Substitute the optimal D_i values of problem B obtained at step 4 of Algorthm 1, into problem A. Notice that if $D_i^* = 0$, then $N_i^* = 0$ and $Q_i^* = 0$, and so the corresponding terms of that supplier have to be deleted from Problem A. The new Problem A is a mixed-integer nonlinear problem.

Step 2: Use LINGO or other solver to solve this new problem and get the values of Q_i^* and N_i^*, for $i \in E$.

In general, the solution that is found by the software can be either local or global optimum. To make sure that the answer is a global one, it is necessary to show that the objective function is convex. It was stated in Section 5.2, for fixed $D_i, i = 1,...,m$, the objective function of Problem A is convex, and so the answer that we get from the software is the global optimum. LINGO uses the branch-and-bound algorithm to determine the best feasible answer to a mixed-integer nonlinear model.

5.3.3. Solution Analysis

In Problem A, for fixed integer $N_i, i \in E$, it can be easily shown that the objective function is strictly concave with respect to D_i and Q_i, $i \in E$. So as mentioned in Section 5.2., the optimal solution of this type of concave problem is achieved at an extreme point of the feasible region defined by the constraints. This states that the optimal values of D_i, $i \in E$, even for integer $N_i, i \in E$, is at least one combination that is obtained by classifying suppliers into three sets Φ_1, Φ_2 and Φ_3 such that $\sum_{i=1}^{m} D_i = D$. Therefore, from $m2^{m-1}$ combinations of the three sets Φ_1, Φ_2 and Φ_3, at least one is an optimal solution of Problem A and at least one is an optimal solution of Problem B.

Suppose that the vector D^* is the optimal solution for Problem B, and so it is optimal to Problem A, when assuming $N_i, i \in E$, to be continuous. Replace the vector D^* into Problem A. For known $D_i^*, i \in E$, Problem A is sum of m separable independent functions, and so the minimum value of its objective function is equal to the sum of minimum values of separable functions. If W_i^*, $i \in E$ is the, minimum value of the function i. Then, $MinTC(E) = \sum_{i \in E} W_i^*$. Let $H = \{i \mid N_i \text{ is not Integer}\}$, if $H = \phi$. Then D^* is also optimal for Problem A, by assuming $N_i, i \in E$ integer. Otherwise, the optimal integer value of N_i, $i \in H$, can be found individually by branch and bound algorithm to minimize each function i, $i \in H$, The new minimum value of function i, $i \in H$ is denoted by $\hat{W}_i^*$.

Hence $MinTC(E) = \sum_{\substack{i \in E \\ i \notin H}} W_i^* + \sum_{i \in H} \hat{W}_i^*$ and,

$$\hat{W}_i^* \geq W_i^*, i \in H \quad . \qquad (10)$$

Note that to find the optimal minimum objective function value of Problem A, it is needed to replace all feasible combinations of three sets Φ_1, Φ_2 and Φ_3 into Problem A, respectively, and then solve all new mixed-integer nonlinear problems. In other words, at least $m2^{m-1}$ mixed-integer nonlinear programming problems need to be solved. D^* can be used as a near optimal solution of Problem A, if $H \neq \phi$.

6. Numerical example

Suppose that a purchasing manager would like to buy a product from 5 suppliers. The annual demand is 300,000, annual holding cost per unit (h_b) is 14 and fixed order cost (A_b) is 7500. Table 1 gives the suppliers' information.According to the developed algorithm, at first Problem B is formed. The number of combinations to classify suppliers into three sets Φ_1, Φ_2 and Φ_3 is 160. Some combinations are infeasible. The second column of Table 2 shows the optimal values of Problem B. Then considering the optimal values of Problem B, Problem A should be generated. The LINGO is used to solve Problem A, and the near optimal values of N_i and Q_i, for, $i = 1, ..., m$, are displayed on columns 3 and 4 of Table 2.

Table 1. Suppliers' information

	u_i	z_i	r_i	S_i	P_i	h_i	F_i
Supplier 1	0.25	9000	55	800	60000	13.5	133
Supplier 2	0.25	15000	53.5	850	95000	13.2	298
Supplier 3	0.25	13000	53	820	90000	13	115
Supplier 4	0.25	21000	52	900	100000	13.6	525
Supplier 5	0.25	28000	54	950	120000	13.1	670

7. Conclusions and recommendation for future research

For an organization within an SC, effectively managing identities is very important. Indeed, the need to coordinate all organizations within the SC is becoming increasingly critical because of competition and market pressures. Some models have been developed to coordinate decisions between members of an SC and guarantee cooperative relationship among them, and therefore minimize the total operational costs of the chain as a whole. Nevertheless, little attention has been paid to developing these models for supplier selection problem.

A mixed-integer nonlinear mathematical model was developed to select the appropriate suppliers with respect to the global SC optimization. The model was expressed as a concave minimization problem and a near optimal solution was generated by considering properties such as limitation of the number of suppliers and convexity of feasible solution area. In previous work by [13] it was shown that by considering coordination between buyer and suppliers in supplier selection problem it was shown, the total cost of the whole supply chain decreased. So, managers of a centralized supply chain can use our proposed model effectively. A manager can minimize the total cost of the

whole supply chain in supplier selection process, using the coordination concept. In other supplier selection models only buyer's cost is minimized without guaranteeing the total supply chain cost being minimized.

Most coordination models assume that an SC partner has complete information (including cost, demand, lead time, etc.) about the other partner. Coordination under limited information sharing is an important issue for future research. In addition, increasing the number of products, relaxing some assumptions, avoiding enumeration, comparing near optimal solution with optimal solution, solving the problem with heuristic methods considering other objectives such as quality can be recommended as new areas of future research.

Table 2. Near optimal solution

	D_i^*	Q_i^*	N_i^*
Supplier 1	0	0	0
Supplier 2	52000	11901.6	8
Supplier 3	52000	12622.28	15
Supplier 4	84000	25400.93	13
Supplier 5	112000	46356.97	19
Optimal value of objective function	16333600		

References

[1] Anthony, T.F. and Buffa, F.P. (1977), Strategic purchase scheduling. *Journal of Purchasing and Materials Management* 13, 27–31

[2] Banerjee, A. (1986), A joint economic-lot-size model for purchaser and vendor. *Decision Sciences* 17, 292–311

[3] Basnet, C. and Leung, J.M.Y. (2005), Inventory lot-sizing with supplier selection. *Computers and Operations Research* 32, 1–14

[4] Bender, P.S., Brown, R.W., Isaac, M.H. and Shapiro, J.F. (1985), Improving purchasing productivity at IBM with a normative decision support system. *Interfaces* 15(3), 106-115

[5] Cakravastia, A., Toha, I.S. and Nakamura, N. (2002), A two-stage model for the design of supply chain network. *International Journal of Production Economics* 80, 231–248

[6] Chappell, A.E. (1974), Linear programming cuts costs in production of animal feeds. *Operational Research Quarterly* 25, 19–26

[7] Chauhan, S.S. and Proth, J.M. (2003), The concave cost supply problem. *European Journal of Operational Research* 148, 374–383

[8] Chaudry, S.S., Frost, F.G., Zydiak, J.L. (1993), Vendor selection with price breaks. European *Journal of Operational Research* 70, 52–66

[9] Crama, Y., Pascual J.R., Torres A. (2004), Optimal procurement decisions in the presence of total quantity discounts and alternative product recipes. *European Journal of Operational Research* 159, 364–378

[10] Degraeve, Z. and Roodhooft, F. (2000), A mathematical programming approach for procurement using activity based costing. *Journal of Business Finance & Accounting* 27(1-2), 69–98.

[11] Gaballa, A.A. (1974), Minimum cost allocation of tenders. *Operational Research Quarterly* 25, 389-398

[12] Gheidar-Kheljani, J. (2006), A Quantitative Model to Select Suppliers in a Supply Chain, Multi-Supplier One-Buyer. Ph.D. Thesis, AmirKabir University of Technology, Tehran-Iran

[13]   Gheidar-Kheljani, J., Ghodsypour, S.H., O'Brien, C.O. (2009), Optimizing whole supply chain benefit versus buyer's benefit through supplier selection. *International Journal of Production Economics* 121(2), 482-493

[14]   Ghodsypour, S.H. and O'Brien, C.O. (1998), A decision support system for supplier selection using an integrated analytic hierarchy process and linear programming. *International Journal of Production Economics* 56-57(1-3), 199-212

[15]   Ghodsypour, S.H. and O'Brien, C.O. (2001), The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint. *International Journal of Production Economics* 73(1), 15-27

[16]   Goyal, S.K. and (1976), An integrated inventory model for a single supplier-single customer problem. *International Journal of Production Research* 15, 107-111

[17]   Goyal, S.K. (1995), A one-manufacturer multi-retailer integrated inventory model: A comment. *European Journal of Operational Research* 82(1), 209–210

[18]   Goyal, S.K. andNebebe F. (2000), Determination of economic production shipment policy for a single vendor single buyer system. *European Journal of Operational Research* 121(1), 175–178

[19]   Hassini, E. (2008), Order lot sizing with multiple capacitated suppliers offering leadtime-dependent capacity reservation and unit price discounts. *Production Planning & Control* 19(2), 142–149

[20]   Herer, Y.T., Rosenblatt, M.J. and Hefter I. (1996), Fast algorithms for single-sink fixed charge transportation problems with applications to manufacturing and transportation. *Transportation Science* 30(4), 276-290

[21]   Hill, R.M. (1997), The single-manufacturer single-retailer integrated production-inventory model with a generalized policy. *European Journal of Operational Research* 97, 493–499

[22]   Hill, R.M. and Omar, M. (2006), Another look at the single-vendor single-buyer integrated production-inventory problem. *International Journal of Production Research* 44(4), 791-800

[23]   Hong, J. and Hayya, J.C. (1992), Just-in-time purchasing: single or multiple sourcing? *International Journal of Production Economics* 27, 175–81

[24]   Hong, G.H., Park, S.C., Janga, D.S. and Rho, H.M. (2005), An effective supplier selection method for constructing a competitive supply-relationship. *Expert Systems with Applications* 28, 629–639

[25]   Horst, R., Pardalos, P.M. and Thoai, N.V. (1995), *Introduction to Global Optimization.* Kluwer Academic Publishers, Dordrecht

[26]   Jain, V., Wadhwa, S. and Deshmukh S.G. (2009), Select supplier-related issues in modelling a dynamic supply chain: potential, challenges and direction for future research. *International Journal of Production Research* 47(11), 3013-3039

[27]   Jayaraman, V., Srivastava, R. and Benton, W.C. (1999), Supplier selection and order quantity allocation: a comprehensive model. *The Journal of Supply Chain Management* 35, 50-58

[28]   Joglekar, P.N. (1988), Comments on 'A quantity discount pricing model to increase vendor profits'. *Management Science* 34(11), 1391–1398

[29]   Kim, S.L. and Ha, D. (2003), A JIT lot-splitting model for supply chain management: Enhancing buyer–supplier linkage. *International Journal of Production economics* 86, 1–10

[30]   Kim, T., Hong, Y. and Lee J. (2005), Joint economic production allocation and ordering policies in a supply chain consisting of multiple plants and a single retailer. *International Journal of Production Research* 43(17), 3619–3632

[31]   Kingsman, B.G. (1986), Purchasing raw materials with uncertain fluctuating prices. *European Journal of Operational Research* 25, 358–372

[32]   Kumar, M., Vrat, P. and Shankar, R. (2004), A fuzzy goal programming approach for vendor selection problem in a supply chain. *Computers & Industrial Engineering* 46, 69–85

[33]   Li, X. and Wang, Q. (2007), Coordination mechanisms of supply chain systems. *European Journal of Operational Research* 179, 1–16

[34]   Lu, L. (1995), A one vendor multi buyer integrated inventory model. *European Journal of Operational Research* 81(2), 312–323

[35]   Murthy, N.N., Soni, S. and Ghosh, S. (2004), A framework for facilitating sourcing and allocation decisions for make-to-order items. *Decision Sciences* 35(4), 609-637

[36]   Narasimhan, R. and Carter, J.R. (1998), Linking business unit and material sourcing strategies. *Journal of Business Logistics* 19(2), 155–171

[37]   Narasimhan, R. and Stoynoff, L.K. (1986), Optimizing aggregate procurement allocation decisions. *Journal of Purchasing and Materials Management* 22, 23–30

[38]   Pan, A.C. (1989), Allocation of order quantities among suppliers. *Journal of Purchasing and Materials Management* 25(2), 36-39

[39]   Park S.S., Kim T., Hong Y., 2006. Production allocation and shipment policies in a multiple-manufacturer–single-retailer supply chain. *International Journal of Systems Science* 37(3), 163–171

[40]   Pirkul, H. and Aras, O.A. (1985), Capacitated multiple item ordering problem with quantity discounts. *IIE Transactions* 17, 206–211

[41]   Rosenblatt, M.J., Herer, Y. T. and Hefter I. (1998), Note. An acquisition policy for a single item multi-supplier system. *Management Science* 44(11) Part 2 of 2, 96-100.

[42]   Rosenthal, E.C., Zydiak, J.L. and Chaudhry, S.S. (1995), Vendor selection with bundling. *Decision Sciences* 26, 35–48

[43]   Sadrian, A.A. and Yoon Y.S. (1994), A procurement decision support system in business volume discount environments. *Operations Research* 42(1), 14-23

[44]   Sarkis, J., Semple, J.H. (1999), Vendor selection with bundling: a comment. *Decision Sciences* 30(1), 265-271

[45]   Sharafali, M. and Henry, C.CO. (2000), Some models for understanding the cooperation between the supplier and the buyer. *International Journal of Production Research* 38(15), 3425-3449

[46]   Talluri, S. and Sarkis, J. (2002), A model for performance monitoring of suppliers. *International Journal of Production Research* 40(16), 4257-4269

[47]   Tempelmeier, H. (2002), A simple heuristic for dynamic order sizing and supplier selection with time varing data. *Production and Operations Management* 11(4), 499-515

[48]   Thomas, D.J. and Griffin, P.M. (1996), Coordinated supply chain management. *European Journal of Operational Research* 94(1), 1–15

[49]   Turner, I. (1988), An independent system for the evaluation of contract tenders. *Journal of the Operational Research Society* 39(6), 551-561

[50]   Weng, Z.K. (1999), The power of coordination decisions for short life cycles products in a manufacturing and distribution supply chain. *IIE Transactions* 31(11), 1037–1049

[51]   Xu, N. and Nozick, L. (2009), Modeling supplier selection and the use of option contracts for global supply chain design. *Computers & Operations Research* 36, 2786 – 2800

[52]   Yung, K.L., Tang, J., Ip, A.W.H. and Wang, D. (2006), Heuristics for Joint Decisions in Production, transportation, and order quantity. *Transportation Science* 40(1), 99-116

[53]   Zeng, A.Z. (1998), Single or multiple sourcing: an integrated optimization framework for sustaining time-based competitiveness. *Journal of Marketing Theory and Practice* 6(4), 10-25.