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A New Model for Transportation Problem with Qualitative Data

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In today's highly competitive market, the pressure on organizations to find a better way to create and deliver value to customers is mounting. The decision involves many quantitative and qualitative factors that may be conflicting in nature. Here, we present a new model for transportation problem with consideration of quantitative and qualitative data. In the model, we quantify the qualitative data by using the weight assessment technique in the fuzzy analytic hierarchy process. Then, a preemptive fuzzy goal programming model is formulated to solve the proposed model. The software package LINGO is used for solving the fuzzy goal programming model. Finally, a numerical example is given to illustrate that the proposed model may lead to a more appropriate solution.

Keywords: *Multi-objective transportation problem, Qualitative data, Fuzzy goal programming, Fuzzy analytic hierarchy process.*

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1. Introduction

In general, distribution of product from depot to customer is called "transportation problem" (TP). The transportation problem has wide practical applications, not only in transportation systems, but also in various other systems. There are many problems which are not exactly a transportation problem but may well be modeled as such. For example, transportation models play important role in logistics and supply chain management for reducing costs and improving services. In 1941, transportation problems were first developed by Hitchcock [9]. The aim is usually to minimize the total transportation cost. The classical transportation problem model, the Hitchcock transportation problem, may have limitations in dealing with real world problems, because it has only a single objective where for certain practical problems, multi-objective models turn to be relevant.

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For example, the objectives may be minimization of the total cost, the total time, consumption of energy, or total deterioration of goods during the transportation.

In most investigations, the entire existing objectives in both single and multiple transportation problem (MOTP) are considered by quantitative information. For real-world problems, however, there exists a variety of important qualitative information such as public health, safety, climate change, comfort and security. Consideration of qualitative information in an MOTP is scarce in the literature. In the work of Korpela et al. [8], the total customer's preference value in a warehouse network and supply chain design objective is maximized by using the Analytic Hierarchy Process (AHP).

The proposed approaches enable the inclusion of both qualitative and quantitative customer service elements in designing the logistics network. A multi-objective transportation model with the consideration of both the depot to customer and customer to customer relationships is proposed by Nunkaew and Phruksaphanrat [11].

There are several methods to solve the MOTP problem. In 1975, Diaz [5] proposed procedures to generate all the non-dominated solutions to linear MOTP problem. Abd El-Wahed and Lee [1] proposed an interactive fuzzy goal programming approach to determine the preferred compromise solution for the MOTP problem. Gao and Liu [6] developed a two phase fuzzy goal programming technique for MOTP problem. Zangiabadi and Maleki [16] presented a fuzzy goal programming approach to determine an optimal compromise solution for the MOTP problem that focuses on minimizing the negative deviation variables from 1.

Here, we present a multi-objective transportation model with the consideration of quantitative and qualitative information. For solving the proposed model, we use the preemptive priority structure of the fuzzy goal programming approach of Zangiabadi and Maleki [16].

The reminder of our work is organized as follows. In Section 2, we describe the conventional transportation problem and its corresponding mathematical model. We also study the fuzzy analytic hierarchy process. A detailed discussion of incorporating qualitative data in MOTP is presented in Section 3, followed by the model formulation. There, we use a fuzzy goal programming approach for solving the proposed model. In Section 4, a practical example is worked out. Finally, the conclusions are provided in Section 5.

2. Preliminaries

In this section, we briefly review the multi-objective transportation problem (MOTP). Then, we describe the fuzzy AHP method, focusing on what is needed for our work here.

2.1. Transportation Problem

The classical single objective transportation problem is a special case of linear programming. The problem is concerned with the distribution of goods (products) from several sources (supply points) to several destinations (demand points) at a minimal total transportation cost. In the real word, however, all transportation problems are not single objective ones. The multi-objective transportation problem (MOTP), on the other hand, deals with the distribution of goods with the consideration of several objectives, such as transportation cost, delivery time and quantity of goods delivered, simultaneously. Consider m sources $S_1,\ S_2, \dots, S_m$, n destinations $D_1,\ D_2, \dots, D_n$ and r objectives $Z_1,\ Z_2, \dots, Z_r$. Without lose of generality, we assume that all r objectives are to be minimized. Suppose that the source S_i has a given available supply a_i $(i = 1, 2, \dots, m)$ and the destination D_j has a given required level of demand b_j $(j = 1, 2, \dots, n)$. For each objective Z_r, a penalty c_{ij}^r is associated with transportation of a unit of a goods from source S_i to destination D_j. Let x_{ij} represent the unknown quantity of goods to be transported from source S_i to destination D_j, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$. It is usual to assume that the balancing condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ holds, i.e., the total demand is equal to the total supply, because any imbalance can be corrected by introduction of a "fictitious" source or destination. With this assumption, the MOTP can be formulated as follows:

$$
\begin{aligned}
&\min\ Z_r\left(x_{ij}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r x_{ij}, \qquad r = 1, 2, \dots, l,\\
&s.t.\\
&\sum_{j=1}^{n} x_{ij} \le a_i, \qquad i = 1, 2, \dots, m,\\
&\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, \dots, n,\\
&x_{ij} \ge 0, \qquad \text{for all } i, j,
\end{aligned}
\tag{1}
$$

where $a_i > 0$, for all i, $b_j > 0$, for all j, $c_{ij}^r > 0$, for all (i, j).

Note that the balancing condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is both necessary and sufficient for the existence of a feasible solution for MOTP.

Owing to lack of qualitative information, model (1) may not admit practical optimal solutions, and so we need to develop the multi-objective transportation problem to consider qualitative information.

2.2. Using Fuzzy AHP for Quantifying Qualitative Information

In qualitative data, the relationship between a consequence (outcome) and the decision variable is unknown (i.e., is not quantified). To treat such a problem by means of a linear decision model, we have somehow to quantify this relationship. The weight assessment technique used in the AHP provides an excellent and systematic way of controlling estimation errors quantifying a qualitative relationship. The AHP is a theory of relative measurement with absolute scales of both tangible and intangible criteria based on the judgment of knowledgeable and expert people [12]. In AHP, a person (an expert or a judge) is asked to give ratios a_{ij}, for the relative importance of two criteria C_i and C_j. The relative importance is related using a scale with the values $1, 3, 5, 7$ and 9, where 1 refers to equally important, 3 denotes slightly more important, 5 equals strongly more important, 7 represents demonstrably more important and 9 denotes absolutely more important. In the conventional AHP, the pairwise comparison is made using a ratio scale. Even though the discrete scale of $1-9$ has the advantages of simplicity and easiness of use, it does not take into account the uncertainly associated with the mapping of one's perception (or judgment) to a number. So, the crisp pairwise comparison in the conventional AHP seems to be insufficient. In order to model this kind of uncertainly in human preference, fuzzy sets could be incorporated with the pairwise comparison as an extension of AHP. In the fuzzy AHP used here, the relative importance of each criterion in the same hierarchy level is identified by using triangular fuzzy number via pairwise comparisons. Fuzzy AHP approach allows a more accurate description of the decision-making process. The computation procedure of this methodology for each hierarchy in summarized in steps 1-6 below.

Step 1. {Comparing the performance score}

Since each number in the pairwise comparison matrix represents the subjective opinion of the decision maker and is thus ambiguous, fuzzy numbers appear to be appropriate to consolidate fragmented expert opinions. So, we make use of triangular fuzzy numbers to indicate the relative strength of each pair of elements in the same hierarchy. The triangular fuzzy numbers $\tilde{u}_{ij}$ are considered as follows:

$$\tilde{u}_{ij} = (l_{ij}, m_{ij}, u_{ij}), \qquad l_{ij} \leq m_{ij} \leq u_{ij},$$

$$l_{ij}, m_{ij}, u_{ij} \in \left[\frac{1}{9}, 1\right] \cup [1,9],$$

$$l_{ij} = min(B_{ijk}),$$

$$m_{ij} = \sqrt[n]{\prod_{k=1}^{n} B_{ijk}},$$

$$u_{ijk} = max(B_{ijk}),$$

where the parameters l_{ij}, m_{ij} and u_{ij}, respectively denote the smallest possible value, the most possible value and the largest possible value to describe a fuzzy event. Each B_{ijk} represents a judgment of expert k for the relative importance of two criteria C_i and C_j.

Step 2. {Constructing the fuzzy comparison matrix}

By using triangular fuzzy numbers, via pairwise comparisons, the fuzzy matrix $\tilde{\boldsymbol{A}}$ is constructed as:

$$\tilde{A} = [\tilde{a}_{ij}] = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 \ \cdots & & C_n \end{array} \\ \begin{pmatrix} 1 & (l_{12}, m_{12}, u_{12}) & \cdots & (l_{1n}, m_{1n}, u_{1n}) \\ (l_{21}, m_{21}, u_{21}) & 1 & \cdots & (l_{2n}, m_{2n}, u_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (l_{n1}, m_{n1}, u_{n1}) & (l_{n2}, m_{n2}, u_{n2}) & \cdots & 1 \end{pmatrix} \end{array},$$

where $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$ denotes a triangular fuzzy number for the relative importance of two criteria C_i and C_j. Also, $l_{ij} = \frac{1}{u_{ji}}$, $m_{ij} = \frac{1}{m_{ji}}$, $u_{ij} = \frac{1}{l_{ji}}$, for all $i, j = 1, \cdots, n,\ j \neq i$.

Step 3. **{**Defuzzification}

Among various defuzzification methods, the method used here is due to the method in [2]. This method expresses fuzzy perceptions as follows:

$$(a_{ij}^{\alpha})^{\lambda} = [\lambda l_{ij}^{\alpha} + (1-\lambda) u_{ij}^{\alpha}], \qquad 0 \le \lambda \le 1,\ 0 \le \alpha \le 1, \tag{2}$$

where $l_{ij}^{\alpha} = (m_{ij} - l_{ij})\alpha + l_{ij}$ and $u_{ij}^{\alpha} = u_{ij} - (u_{ij} - m_{ij})\alpha$ respectively represent the left-end value and right-end value of α-cut for c_{ij}, and

$$(a_{ji}^{\alpha})^{\lambda} = \frac{1}{(a_{ij}^{\alpha})^{\lambda}}, \qquad 0 \le \lambda \le 1,\ 0 \le \alpha \le 1,\ i > j.$$

Notably, α can be viewed as a stable or fluctuating condition. The range of uncertainly is the greatest when $\alpha = 0$. Meanwhile, the decision-making environment stabilizes by increasing α, while, simultaneously, the variance for decision-making decreases. Additionally, α can be any number between 0 and 1, and is usually set as the following 10 numbers, $0.1, 0.2, \cdots, 1$, for uncertainly emulation. Note that λ can be viewed as the degree of a decision maker's pessimism. When λ is 0, the decision maker is more optimistic and thus, the expert consensus is the upper-bound value U_{ij} of the triangular fuzzy number. Conversely, when $\lambda = 1$, the decision maker is pessimistic; however, five number $0.1, 0.3, 0.5,\ 0.7$ and 0.9, are used to emulate the state of mind of the decision.

The crisp pairwise comparison matrix is expressed as follows:

$$(A)^{\lambda} = [(a_{ij})^{\lambda}] \Rightarrow (A^{\alpha})^{\lambda} = ([(a_{ij}^{\alpha})^{\lambda}]. \tag{3}$$

Step 4. **{**Computing eigenvalue and eigenvector}

Assume $\bar{\lambda}_{max}$ to be the maximal eigenvalue of the crisp pairwise comparison matrix (3). Compute w by

$$(A^{\alpha})^{\lambda} w = \bar{\lambda}_{max} w \Rightarrow (A^{\alpha})^{\lambda} w - \bar{\lambda}_{max} w = 0,$$

where w denotes the eigenvector of the matrix $(A^{\alpha})^{\lambda}$.

Comparing A and $(A^{\alpha})^{\lambda}$, the traditional AHP methods only use a specific figure geometric mean to represent the expert opinions for the pairwise comparison matrix. However, the triangular fuzzy numbers are used to present the fuzzy opinions and expert consensus. Meanwhile, both approaches use the eigenvector method for weight calculation.

Step 5. **{**Consistency test}

The essential idea of the AHP is that a matrix A of rank n is only consistent, if it has one positive eigenvalue $n = \lambda_{max}$, while all other eigenvalues are zero. Furthermore, Saaty [13] developed the consistency index (CI) to measure the deviation from a consistent matrix: $CI = (\lambda_{max} - n)/(n-1)$.

The consistency ratio (CR) is introduced to aid the decision on revising the matrix or not. It is defined as the ratio of CI to the so-called random index (RI) which is CI of randomly generated matrices:

$$CR = CI/RI.$$

For $n = 3$, the required consistency ratio (CR^{Goal}) should be less than 0.05, for $n = 4$, it should be less than 0.08, and for $n \geq 5$, it should be less than 0.10 to get a sufficiently consistent matrix. Otherwise, the matrix should be revised [14].

Step 6. **{**Computing the overall hierarchy weight}

After the weights for various hierarchy and elements are computed, compile the computation results for the overall hierarchy weights.

3. Quantify a Qualitative Data in Transportation Problem

3.1. A Qualitative Relationship Evaluation

Qualitative data in a MOTP can be described by a linear objective function as follows:

$$z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} z_{kij} = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{kij}x_{ij}, k = 1,2,\dots,t, \tag{4}$$

where

$$z_{kij} = a_{kij}x_{ij}, \qquad i = 1,2,\dots,m, \quad j = 1,2,\dots,n.$$

Because of the linearity of the model, it is possible to evaluate each coefficient a_{kij} in row k independently of the others. However, the lack of natural quantitative scale for decision variable x_{ij} or an outcome variable z_k, among other things, may make it difficult to specify the linear objective function.

To estimate a_{kij}, we have presented each relationship by the following form:

$$\delta z_{kij} = a_{kij}\delta x_{ij}, \qquad i = 1,2,\cdots,m, \quad j = 1,2,\cdots,n, \quad k = 1,2,\cdots,t. \tag{5}$$

To begin with, we need to fix the difference in each x_{ij} being considered. It is not necessary that there exist a numerical scale for x_{ij}. However, it is possible to describe the change in x_{ij} using soft expressions such as: “a little”, “a lot”, “much”, “some”, etc. Next, one needs to estimate the corresponding change in z_{kij}, regardless of whether z_k having a natural numerical scale or not.

Now, the change δz_k in row k can be written as follows:

$$\delta z_k = z_k(\delta x) = s_k w_k \delta x, \qquad k = 1, 2, \dots t,$$

where s_k $(k = 1, 2, \dots, t)$ is an (unspecified) scaling factor for the coefficients rows k, $k = 1, 2, \dots, t$. Thus, we have

$$a_{kij} = s_k w_{kij}, \qquad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n.$$

In order to specify the linear objective function (4), we should find a vector w_k and a scaling factor s_k. By using the fuzzy AHP described in Section 2.2 for row k $(k = 1,2,\dots,t)$, we can easily find a vector $w_k = \{w_{kij}, i = 1, 2, \dots, m, j = 1, 2, \dots, n\}$, $\sum_{i=1}^{m}\sum_{j=1}^{n} w_{kij} = 1$, to describe the relative effect of the change (δx_{ij}) of each decision variable on row k. Recall from Korhonen and Wallenius [7] that we can use one of the following principles to find the scaling factor s_k:

(1) $s_k = 1$ or any other constant, for all $k = 1, 2, \dots, t$.

(2) $s_k = \frac{1}{\max_{ij} w_{kij}}$, for all $k = 1, 2, \dots, t$.

(3) s_k is calibrated by the decision maker (DM), e.g., on the base of a one-unit change in each x_{ij}, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$.

(4) s_k is calibrated with respect to an ideal value of a consequence.

The first principal is appropriate, if the scale of consequence z_k is not very important and the DM is only interested in how the current value is related to the range of z_k . If each decision variable is allowed to change by one unit, then the change in the value of z_k is equal to one.

The second principle is suitable, when the maximum value/unit has a special meaning for the DM. In a maximization problem, this principle implies that a one-unit change in the value of the decision variable with the largest coefficient changes the value of the consequence $z_k$by one unit.

When there exists a natural scale for some of the rows, we could calibrate the corresponding outcome variable z_k onto this scale. We may ask the DM to evaluate how large of a change a one-unit change in each decision variable will cause in the outcome variable. This proved us with the following pairs $(\delta z_{kij}, \delta x_{ij})$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, in which $\delta x_{ij} = 1$. We have assumed that $\delta z_{kij} = s_k w_{kij} \delta x_{ij}$. The scaling factor can now simply be found through $\sum_{i=1}^{m}\sum_{j=1}^{n} \delta z_{kij} = s_k \sum_{i=1}^{m}\sum_{j=1}^{n} w_{kij}\delta x_{ij} = s_k \sum_{i=1}^{m}\sum_{j=1}^{n} w_{kij} = s_k$.

The forth principle refers to an idea, in which the DM is asked to specify the ideal values (not all zeros) for the decision variables, and to specify the value of the corresponding outcome. This idea may work for problem in which the best value for each decision variable is, for example, one and the DM can easily specify the impact of the sum of the variables.

Now, we are ready to present the proposed model for the multi-objective transportation problem with consideration of qualitative data.

3.2. Model Formulation

The proposed model for the multi-objective transportation problem with consideration of qualitative data can be written as

$$
\begin{aligned}
&\min z_r\left(x_{ij}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij}, \qquad r = 1, 2, \dots, l,\\
&\min z_k\left(x_{ij}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} w_{kij} s_k\, x_{ij}, \qquad k = l+1, l+2, \dots, l+t,\\
&s.t.\\
&\qquad \sum_{j=1}^{n} x_{ij} \le a_i, \qquad i=1, 2, \dots, m,\\
&\qquad \sum_{i=1}^{m} x_{ij} = b_j, \qquad j=1, 2, \dots, n,\\
&\qquad x_{ij} \ge 0, \quad \text{for all } i, j .
\end{aligned}
\tag{6}
$$

Here, it will be assumed that the first l quantitative objectives are significantly more important than the qualitative objectives. Therefore, we use the preemptive fuzzy goal programming approach to solve the proposed model.

3.3. Preemptive Fuzzy Goal Programming Approach

Goal programming (GP) is one of the most popular method for solving multi-objective linear programming problems, first introduced by Charnes and Cooper [3]. The idea of goal programming is to establish a goal level of achievement for each criterion. However, it is difficult for the decision-maker to determine precisely the goal value of each objective, since possibly only some partial information is known. An application of fuzzy set theory to GP was made by Narasimhan in 1980 [10].

In the proposed model, we supposed that the first l quantitative objective functions are significantly more important than the qualitative objectives, and so we use the preemptive fuzzy goal programming to solve model (6). We apply the following linear membership function corresponding to the rth goal:

$$\mu\big(Z_r(x)\big) = \begin{cases} 1, & if\ Z_r \le L_r, \\ 1 - \dfrac{Z_r - L_r}{U_r - L_r}, & if\ L_r < Z_r < U_r, \\ 0, & if\ \ Z_r \ge L_r, \end{cases}$$

where, $\mu(z_r)$ is the membership function of rth goal and L_r and U_r are the best and the worst values for the rth objective function, respectively.

To compute L_r and $U_r,$ we solve the multi-objective transportation problem as a single objective transportation problem and compute the objective function value, taking each time only one objective as the objective function and ignore all the others. Each time for each objective we find min (L_r) and max (U_r) values for an rth objective function. Using the preemptive priority structure of the fuzzy goal programming approach presented by Zangiabadi and Maleki [16] for the model (6), we have the following problem:

$$\begin{aligned}
&\text{lex min } [\phi_1, \phi_2] \\
&\quad s.t. \\
&\qquad \mu(z_r) + d_r^- - d_r^+ = 1, \quad r = 1,2,\dots,l+t, \\
&\qquad \phi_1 \ge d_r^-, \qquad\qquad r = 1,2,\dots,l, \\
&\qquad \phi_2 \ge d_r^-, \qquad\qquad r = l+1,2,\dots,l+t, \\
&\qquad d_r^+ d_r^- = 0, \qquad\qquad r = 1,2,\dots,l+t, \qquad\qquad (7) \\
&\qquad \sum_{j=1}^{n} x_{ij} = a_i\,, \quad i = 1,2,\dots,m, \\
&\qquad \sum_{i=1}^{m} x_{ij} = b_j\,, \quad j = 1,2,\dots,n, \\
&\qquad \phi_1 \le 1, \quad \phi_2 \le 1, \\
&\qquad \phi_1 \ge 0, \quad \phi_2 \ge 0, \ x_{ij} \ge 0, \quad \text{for all } i,j\,.
\end{aligned}$$

In this model, we minimize the negative deviation variables from 1 to obtain a compromise solution for the multi-objective transportation problem. We use the Linear Interactive General Optimization LINGO 11.0 software package to solve the model (7).

4. Application Example

As mentioned previously, to alleviate the shortcomings of the conventional transportation model, we presented a multi-objective transportation model with consideration of both quantitative and qualitative data. In the proposed model, we supposed that the quantitative objective functions are significantly more important than the qualitative objectives. Thus, we use the preemptive fuzzy goal programming to solve the model. To illustrate the proposed approach, a case study of the distribution problem is presented. The proposed model considers both the depot to customer and customer to customer relationships by determining the lowest total transportation cost and the nearest vicinity of customers.

Example 1. Most existing research works on transportation problems only consider depot to customer relationship. However, the relationship between customer and customer is also critical, because, in fact, the vehicle route for each depot, is not comprised of merely movement from depot to customer, and back from customer to depot, as conveniently assumed in the conventional transportation model. However, the movement may more realistically be considered from depot to customer followed by movements to other customers. Moreover, suppose that two or more customers need to be served by the same depot. The conventional models appear to be improper as a result of lacking the customer to customer relationship as a qualitative data. In the following, a simple problem with two depots and ten customers is considered with the assumption that the demand of each customer must be served by only one depot. Moreover, a depot's capacity is sufficient to serve a customer. Fig. 1 depicts the location map, which we can presume the anticipated solution by quantitative data (the distance between depot and customer) with the depot to customer relationship consideration in which customers C_1, C_2, C_3 and C_4 should be served by depot D_1, and customers C_7, C_8, C_9 and C_{10} should be served by depot D_2, whereas customers C_5 and C_6 may be assigned by depot D_1 or D_2. But, we can clearly observe that customer C_5 and C_6 should be served by depot D_1, because of being in the vicinity of D_1. This means that customer to customer relationship consideration is also necessary for a transportation problem. So, we use the proposed approach in order to quantify the customer to customer relationship. The list of the basic data is shown in Table 1.

This problem considers two objective functions. The first objective function is to minimize the total transportation cost. The second objective function is to minimize the overall independence value between customer and customer. By using fuzzy AHP, we find a vector w_k to describe relative effects of change of each decision variable on the value of customer to customer relationship. Then, the relative effect on the independence value of customer to customer relationship can be calculated from $w_{max} - w_k$, where w_{max} is the maximum scale of the relative effect of decision variable on the value of customer relationship which is assigned to be 1.

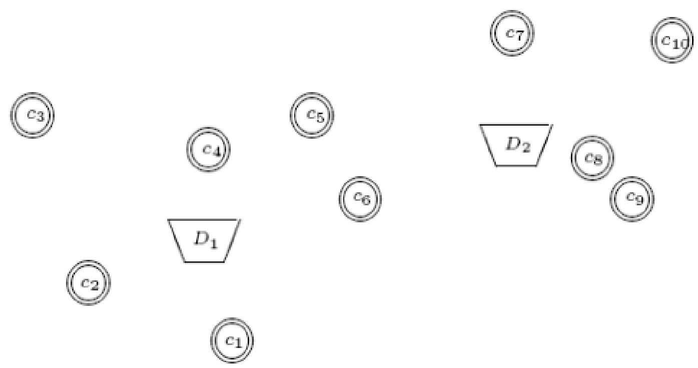

Figure 1: The location map

Table 1: Transportation cost per unit (in U.S. dollars) and customer's demand

Depot i			
Customer j	Transportation cost per unit C_{1j}	C_{2j}	b_j (Unit)
C_1	10	35	500
C_2	15	35	250
C_3	12.5	30	300
C_4	20	35	750
C_5	15	15	280
C_6	10	10	370
C_7	30	14	450
C_8	35	15	650
C_9	30	10	1000
C_{10}	40	15	250
Available supply a_j(unit)	3000	3000	

A zero-one integer programming is integrated into the proposed model while enforcing that each customer's demand can solely be served by only one depot. Let $s_k = 1$. Then, the mathematical model for this problem can be shown as follows:

$$\min \quad z_1(y_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}y_{ij}b_j$$

$$\min z_2\left(y_{ij}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} w_{1ij}y_{ij}$$

$$s.t.$$

$$\sum_{j=1}^{n} y_{ij}b_j \leq a_i, \qquad i = 1,2,$$

$$\sum_{i=1}^{m} y_{ij} = 1, \qquad j = 1,2,\ldots,n,$$

$$y_{ij} \geq 0, \qquad \text{for all } i,j,$$

where y_{ij} is 1, if customer j is served by the ith depot; otherwise, it is 0. According to Table 1, the model is specified as follows:

$$\begin{aligned}\min\ z_1 = {} & 500y_{11} + 3750y_{12} + 3750y_{13} + 15000y_{14} + 4200y_{15} + 3700y_{16} + 13500y_{17} \\ & + 22750y_{18} + 30000y_{19} + 10000y_{110} + 17500y_{21} + 8750y_{22} + 9000y_{23} \\ & + 26250y_{24} + 4200y_{25} + 3700y_{26} + 6300y_{27} + 9750y_{28} + 10000y_{29} \\ & + 3750y_{210}\end{aligned}$$

$$\begin{aligned}\min\ z_1 = {} & 0.924y_{11} + 0.8792y_{12} + 0.722y_{13} + 0.8742y_{14} + 0.9404y_{15} + 0.9404y_{16} \\ & + 0.94849y_{17} + 0.9898y_{18} + 0.9796y_{19} + 0.9941y_{110} + 0.9873y_{21} + 0.9911y_{22} \\ & + 0.9937y_{23} + 0.9788y_{24} + 0.954y_{25} + 0.954y_{26} + 0.9369y_{27} + 0.8295y_{28} \\ & + 0.9369y_{29} + 0.9492y_{210}\end{aligned}$$

$s.t.$

$$\begin{aligned}& 500y_{11} + 250y_{12} + 300y_{13} + 750y_{14} + 280y_{15} + 370y_{16} + 450y_{17} + 650y_{18} \\ & \quad + 1000y_{19} + 250y_{110} \leq 3000,\end{aligned}$$

$$\begin{aligned}& 500y_{21} + 250y_{22} + 300y_{23} + 750y_{24} + 280y_{25} + 370y_{26} + 450y_{27} + 650y_{28} + 1000y_{29} \\ & \quad + 250y_{210} \leq 3000,\end{aligned}$$

$$\sum_{i=1}^{m} y_{ij} = 1, \quad j = 1,2,\cdots,n,$$

$$y_{ij} \geq 0.$$

The solution corresponding to each single objective transportation problem is:

$$\begin{aligned} U_1 &= 145.650, & z_1 &= L_1 = 65.200, \\ U_1 &= 9.77089, & z_1 &= L_1 = 8.932700,\end{aligned}$$

Then, the proposed model for this problem can be shown as follows:

$$lex\ \min\ [\phi_1, \phi_2]$$

$$s.t.$$

$$1 - \frac{z_1 - 65.200}{80.450} + d_1^- - d_1^+ = 1,$$

$$1 - \frac{z_2 - 8.932700}{0.83819} + d_2^- - d_2^+ = 1,$$

$$\phi_1 \geq d_1^-,$$

$$\phi_2 \geq d_2^-,$$

$$d_1^+ d_1^- = 0,$$

$$d_2^+ d_2^- = 0,$$

$$\sum_{j=1}^{n} y_{ij} \le a_i\,, \qquad i = 1, 2, \dots, m,$$

$$\phi_1 \le 1, \phi_2 \le 1,$$

$$\phi_1 \ge 0, \phi_2 \ge 0,$$

$$y_{ij} = 0 \text{ or } 1, \qquad \text{for all } i, j.$$

This model was solved by using the LINGO software package and the results are:

$$y_{11}^* = y_{12}^* = y_{13}^* = y_{14}^* = y_{15}^* = y_{16}^* =, y_{27}^* = y_{28}^* = y_{29}^* = y_{210}^* = 1,$$

with all the other variables being zero. This is the best solution among all possible solutions which can be obtained by using our proposed model. In order to illustrate the effectiveness of the proposed model, we consider the case study given by Nunkaew et al. [11]. They solved a large scale transportation problem with consideration of the relationships among all customers. They concluded that the solution obtained by assigning customers to depots is different from the one obtained by the conventional approach. Moreover, the obtained delivery cost was reduced, as compared to the total delivery cost obtained by the conventional approach (for more details, see [11]). We expect that for large transportation problems with more than a thousand customers, consideration of customer to customer relationship may result in delivery costs for the transportation. Since customer to customer relationship is a qualitative feature of our proposed model, the model may be useful to arrive at better solutions for large-scale transportation problems.

5. Conclusions

Owing to the lack of qualitative data in transportation problems, a multi-objective transportation model with consideration of qualitative data was presented. The proposed model is more realistic than the conventional transportation model. Using a fuzzy goal programming technique, the decision maker may obtain a satisfactory solution. We made use of preemptive fuzzy goal programming to solve the proposed model. The proposed model can obtain a reasonable solution considering both the quantitative and the qualitative data.

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