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Developing a Correlated Robust Optimization Model to Reduce the Price of Robustness in the Supply Chain Coordination Problem

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Coordination is a critical factor in optimizing supply chain performance. Given the pervasive uncertainties in supply chain management, it is essential to develop decisions that are robust against these uncertainties while preserving operational efficiency. This paper aims to determine an optimal supply chain policy that ensures the total system cost remains robust against correlated uncertainties in demand and lead time. To address the correlation among demand data and avoid overly conservative solutions, a novel robust optimization model is proposed based on a correlated polyhedral uncertainty set. This approach explicitly accounts for demand correlation, thereby reducing the price of robustness. Numerical results demonstrate that integrating coordination as a strategic decision and employing robust optimization as a tactical tool significantly enhances supply chain performance. Moreover, incorporating demand correlation in the proposed model leads to a substantial reduction in the price of robustness and, consequently, higher supply chain profitability. Extending this framework to more complex supply chain models with multiple sources of uncertainty holds great potential for further improving the robustness and practical applicability of supply chain decision-making.

Keywords: *robust optimization, correlated uncertainty, price of robustness, supply chain coordination.*

1. Introduction

1.1. Background and motivation

A supply chain consists of all members involved in making a product available to customers. Each member plays a unique role in this process. Supply chains are classified into three types—decentralized, centralized, and coordinated—based on how their members are interconnected. In a decentralized supply chain, members independently make decisions to minimize their own costs. In contrast, in a centralized supply chain, authority rests with a single leading member who makes all decisions concerning the entire supply chain's performance, with the aim of minimizing the total cost of the supply chain [22]. In recent years, supply chain coordination has attracted considerable scholarly attention due to the challenges associated with implementing centralized supply chain management [20,24]. Coordination of supply chain performance is typically achieved through

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mechanisms such as revenue sharing and quantity discounts, which enable the supply chain to operate similarly to a centralized system [13].

The foundation of supply chain coordination is established when members are highly motivated to cooperate in order to reach consensus [3,11]. Since members' objectives may conflict, researchers have sought to align these objectives by proposing various coordination mechanisms. Under coordinated conditions, the aggregate cost of all members should be lower than that in the decentralized case, thereby providing an incentive for members to shift from a decentralized to a centralized decision-making model [12].

Uncertainty in data is a major factor influencing real-world supply chain performance. In the context of supply chain coordination, demand and lead time are recognized as the most critical sources of such uncertainty. To hedge against data uncertainty in mathematical programming, researchers typically rely on either stochastic programming or robust optimization. Robust optimization yields a solution with guaranteed optimality under uncertainty. However, a well-known shortcoming of this approach is its inherent conservatism, which leads to a deterioration of the objective function value. To address this issue, Bertsimas and Sim [8] introduced the price of robustness as a metric to quantify the conservatism level of a robust model. Specifically, this indicator captures the extent to which objective function quality is sacrificed a priori to ensure solution robustness.

1.2. Literature Review

Literature reviews indicate that among uncertain parameters, most studies have predominantly concentrated on stochastic demand, while there remains considerable room for comprehensive modeling of uncertain lead time [4,6]. For instance, Li et al. [17] demonstrated that unstable lead time can directly increase inventory and environmental costs, thereby negatively affecting overall supply chain performance. Meanwhile, early analytical models such as Cobb [7] specifically examined the impact of uncertain lead time on two-echelon inventory policies. Thorsen and Yao [26] developed robust inventory models under demand and lead time uncertainty; Karimi Movahed and Zhang [16] examined robust (s,S) policies in multi-level supply chains; and Pamulety and Pillai [21] showed how sharing customer demand information mitigates the bullwhip effect. Together, these contributions underscore the need for broader modeling of uncertainty beyond demand, incorporating lead time variability and information disruptions to strengthen supply chain resilience.

Other studies have examined coordination and contract design under demand and lead time uncertainty, aiming to reduce the overall cost of the supply chain and improve service levels. For instance, Malik and Sarkar [18] showed that by modeling stochastic demand together with uncertain lead time, cost-effective frameworks can be developed for production planning and ordering. Moreover, more advanced approaches—such as the use of deep reinforcement learning for modeling multi-stage supply chains under uncertain demand and lead time—reflect the growing interest of the research community in integrating artificial intelligence methods with real-world uncertainties [2,15]. In addition, frameworks developed for uncertain lead time using machine learning have demonstrated their potential to mitigate the adverse effects of lead time and to optimize purchasing and production decisions [19,23]. Zhang [27] investigated robust optimization techniques for centralized supply chain planning under demand uncertainty, introducing target-attainment criteria to balance efficiency and resilience. Together, these studies underscore the growing convergence of classical optimization, robust frameworks, and artificial intelligence methods in tackling uncertainty in supply chain coordination.

Recent advancements in coordination management emphasize the integration of digital platforms and sophisticated optimization models to enhance supply chain performance. Research has conceptualized platform-driven frameworks that leverage digital integration and agility management to align stakeholders across production and distribution [1]. From an operational perspective, mixed-integer linear fractional programming (MILFP) has been successfully applied to optimize multi-facility vendor–buyer coordination, demonstrating superior profit margins through improved information sharing [9]. Furthermore, integrating raw material procurement with cyclic production scheduling in three-stage supply chains has proven effective in minimizing total costs, particularly within the fast-moving consumer goods sector [25].

Studies on correlated uncertainty are extremely rare, and this represents a significant gap in the literature. Garcia-Castro et al. [10] introduced the concept of correlated uncertainty as a method for modeling dependencies among uncertain parameters in supply chain network design, although in their case the parameters were energy prices and carbon prices rather than demand or lead time. Likewise, Disney et al. [8] investigated the bullwhip effect in supply chains and attempted to examine temporal dependencies between lead times and demand patterns, which may indirectly relate to the issue of correlated uncertainty.

1.3. Research Gap and contribution

A review of the literature reveals that, in the context of supply chain coordination, demand and lead time constitute the most critical sources of uncertainty. The vast majority of existing studies, however, assume these uncertain parameters to be uncorrelated. In practice, multiple uncertain parameters often share a common underlying driver, leading to interdependence—for instance, correlated patterns in demand data across different products.

In the presence of correlated uncertain parameters, conventional robust optimization models tend to yield overly conservative solutions, as they indiscriminately hedge against both correlated and uncorrelated disturbances. Restricting robustness to only correlated disturbances alleviates this issue and reduces unnecessary conservatism. For example, in a supply chain with correlated product demands, any disruption affects all products in the same direction, thereby eliminating the need for robustness against uncorrelated variations.

Given the practical importance and increasing real-world complexity of modern supply chains, research that explicitly models and analyzes correlations among uncertain data—such as demand or lead time—remains scarce. To fill this gap, the present study proposes a correlation-aware robust optimization framework for supply chain coordination. Unlike previous research, the contribution of this paper lies in introducing a novel robust optimization model that, in the presence of correlated uncertainty, explicitly incorporates correlations among uncertain data to avoid overly conservative solutions and thereby reduce the price of robustness. In the absence of correlation, the proposed framework behaves consistently with conventional robust optimization approaches.

1.4. Paper organization

The remainder of this paper is organized as follows. Section 2 describes the methodology adopted in this study. Section 3 defines the problem, along with the assumptions and symbols used throughout the paper. Section 4 presents the mathematical formulation, including the deterministic decentralized (DD) model, the deterministic centralized (DC) model, the robust centralized (RC) model, and the proposed correlated robust centralized (CRC) model. Section 5 provides the experimental results and sensitivity analysis, covering a sample problem, a comparison between the conventional robust model and deterministic models under uncertainty, and an evaluation of robust solutions against correlated uncertainty. Section 6 concludes the paper with a discussion of the findings and concluding remarks.

2. Methodology

The present research was carried out in the following steps:

- Conducting an initial literature review on JELS problems, supply chain coordination under uncertainty, and robust optimization.
- Investigating uncertainty approaches in supply chain management and selecting the appropriate approach for the problem domain.
- Developing a mathematical model and extracting the required data for the problem.
- Presenting the model output and performing sensitivity analysis on the key parameters using appropriate software.

In this section, we first discuss the problem definition, assumptions, parameters, and variables used in the mathematical model. Then, by identifying and defining the costs of the supply chain members, we model both decentralized and centralized inventory models, as well as the steady-state conditions for each.

3. Problem Definition

In this research, a two-level supply chain consisting of a retailer (buyer) and a supplier handling several product types is modeled. The retailer faces uncertain demand for all products, denoted as D_j. The retailer operates a continuous inventory system: when the inventory level reaches a predetermined reorder point, an order is placed with the supplier. The retailer's order quantity for each product is D_jT, which is delivered after a probabilistic lead time following an exponential distribution.

Lead time—also referred to as the supplier's delivery time—is the duration between the supplier dispatching the order D_jT and the shipment arriving at the retailer's location. For all products, the supplier produces a consolidated batch n_jD_jT at a constant production rate P_j and delivers it to the retailer in n shipments over the same time interval T. The retailer's lead time is characterized by an exponential distribution with both a deterministic and an uncertain component. Shortage cost is significant for the retailer, as it assumes a backlog of demand per unit of shortage. This shortage cost, which is assumed to be identical across all product types, is also taken into account in this study.

The objective of this problem is to determine the optimal values of the reorder point and the number of shipments for each product while minimizing system costs, including ordering, holding, and shortage costs. Furthermore, given that the problem is subject to demand and lead time uncertainty, it is essential that the optimal solution be protected against such uncertainty. Lead time fluctuations are treated as exponentially probabilistic, while demand variations are considered uncertain.

Another important assumption that distinguishes this study from previous research is that demand uncertainties for all products are correlated. This assumption is highly consistent with reality, particularly when the products in question share similar sales markets. For example, in the dairy products market, demand for different items is generally correlated. When the products under consideration are dairy items, the retailer faces correlated uncertain parameters that may influence the optimal solution. Accordingly, the aim of this study is to identify the best solution under these correlated conditions.

3.1. Assumptions and Symbols

The main assumptions of the model are as follows:

1. A supplier and a retailer constitute the two-level, multi-product supply chain.
2. Demand is uncertain, and the lead time for replenishing the retailer's inventory is probabilistic, following an exponential distribution with parameter λ.
3. The supplier is a manufacturer whose production rate exceeds the demand rate, and products are shipped simultaneously from multiple production lines over the same periods.
4. Inventory levels are controlled using a continuous review system.
5. The supplier can backorder unmet demand (i.e., backlogged orders are allowed).

Symbols used in the problem formulation:

Decision variables

r_j	The reorder point of the j^{th} product for the retailer
n_j	The number of shipments of the j^{th} product sent by the supplier
Z_0, P_{0j}	The dual variable from the robust model of the retailer's problem
Z_1, P_{1j}	The dual variable from the robust model of the supply chain problem
T	Time period

Parameters

D_j	Annual demand for the j^{th} product

P_j Production rate of the j[th] product by the manufacturer
SC_j Setup cost per unit of product j per unit of time for the supplier
O_r Retailer's ordering cost for all identical products
h_{sj} Holding cost per unit of product j per unit of time for the supplier
h_{rj} Holding cost per unit of product j per unit of time for the retailer
π_j Shortage cost per unit of product j for the retailer
λ_j The exponential distribution parameter for the lead time random variable of the j[th] product

4. Mathematical Formulation

In this section, we first present two supply chain inventory models under the assumptions of centralized and decentralized management. We then develop and introduce two different robust optimization models to address uncertainty. Subsequently, to evaluate the performance of the developed models, a sample problem is presented, and the models are subjected to sensitivity analysis under uncertainty.

4.1. Deterministic Decentralized inventory model (DD)

In the decentralized case, the total cost of the supply chain is the sum of the costs incurred by the supplier and the retailer.

$$TC = Min\, TC_b(r_1, r_2, \dots, r_N, T) + Min\, TC_s(n_1, n_2, \dots, n_N) \tag{1}$$

$$TC_r(r_1, r_2, \dots, r_N, T) = \frac{O_b}{T} + \sum_{j=1}^{N} hb_j \left(r_j + \frac{D_j T}{2} - \frac{D_j}{\lambda_j}\right) + \frac{D^2(\pi_j + hb_j)}{\lambda_j^2 Q}\left(e^{\frac{-r_j\lambda_j}{D_j}} - e^{\frac{-(r_j + D_j T)\lambda_j}{D_j}}\right) \tag{2}$$

$$TC_s(n_1, n_2, \dots, n_N) = \sum_{j=1}^{N} \frac{SC_j}{Tn_j} + hs_j \frac{D_j T}{2}\left(\left(n_j - 1\right)\left(1 - \frac{D_j}{P}\right) + \frac{D_j}{P}\right) \tag{3}$$

4.2. Deterministic Centralized inventory model (DC)

The optimization problem aims to minimize the total supply chain cost, including both retailer and supplier costs.

$$Min\, TC_{joint}(r_1, r_2, \dots, r_N, T, n_1, n_2, \dots, n_N) = \frac{1}{T}\left(O_b + \sum_{j=1}^{N} \frac{SC_j}{n_j}\right) + \sum_{j=1}^{N} hb_j \left(r_j + \frac{D_j T}{2} - \frac{D_j}{\lambda_j}\right) + \frac{D^2(\pi_j + hb_j)}{\lambda_j^2 Q}\left(e^{\frac{-r_j\lambda_j}{D_j}} - e^{\frac{-(r_j + D_j T)\lambda_j}{D_j}}\right) + hs_j \frac{D_j T}{2}\left(\left(n_j - 1\right)\left(1 - \frac{D_j}{P}\right) + \frac{D_j}{P}\right) \tag{4}$$

4.3. Robust Centralized inventory model (RC)

The centralized supply chain model in the steady state is derived as follows, incorporating not only the nominal values of the objective function but also an expression for the uncertain component—namely, product demand—into the primary model.

$$Min\ TC_{joint}(r_1, r_2, \dots, r_N, T, n_1, n_2, \dots, n_N) = \frac{1}{T}\left(O_b + \sum_{j=1}^{N} \frac{SC_j}{n_j}\right) + \sum_{j=1}^{N} hb_j \left(r_j + \frac{D_j T}{2} - \frac{D_j}{\lambda_j}\right) + \frac{D^2(\pi_j + hb_j)}{\lambda_j^2 Q}\left(e^{\frac{-r_j\lambda_j}{D_j}} - e^{\frac{-(r_j + D_j T)\lambda_j}{D_j}}\right) + hs_j \frac{D_j T}{2}\left((n_j - 1)\left(1 - \frac{D_j}{P}\right) + \frac{D_j}{P}\right) + \max_{\{\xi_j | \sum \xi_j \leq \Gamma. |\xi_j| \leq 0\}} \left\{\sum_{j\epsilon J_0}\left[hb_j\left(\frac{\hat{D}_j T}{2} - \frac{\hat{D}_j}{\lambda_j}\right) + \frac{\hat{D}^2(\pi_j + hb_j)}{\lambda_j^2 Q}\left(e^{\frac{-r_j\lambda_j}{\hat{D}_j}} - e^{\frac{-(r_j + \hat{D}_j T)\lambda_j}{\hat{D}_j}}\right) + hs_j \frac{\hat{D}_j T}{2}\left((n_j - 1)\left(1 - \frac{\hat{D}_j}{P}\right) + \frac{\hat{D}_j}{P}\right)\right] + (\Gamma - \lfloor\Gamma\rfloor)\left[hb_j\left(\frac{\hat{D}_j T}{2} - \frac{\hat{D}_j}{\lambda_j}\right) + \frac{\hat{D}^2(\pi_j + hb_j)}{\lambda_j^2 Q}\left(e^{\frac{-r_j\lambda_j}{\hat{D}_j}} - e^{\frac{-(r_j + \hat{D}_j T)\lambda_j}{\hat{D}_j}}\right) + hs_j \frac{\hat{D}_j T}{2}\left((n_j - 1)\left(1 - \frac{\hat{D}_j}{P}\right) + \frac{\hat{D}_j}{P}\right)\right]\right\} \quad (5)$$

In the objective function of the problem, for a predetermined integer Γ, the following operation is performed.

$$\max_{\{\xi_j | \sum \xi_j \leq \Gamma. |\xi_j| \leq 0\}} \left\{\sum_{j\epsilon J_0}\left[hb_j\left(\frac{\hat{D}_j T}{2} - \frac{\hat{D}_j}{\lambda_j}\right) + \frac{\hat{D}^2(\pi_j + hb_j)}{\lambda_j^2 Q}\left(e^{\frac{-r_j\lambda_j}{\hat{D}_j}} - e^{\frac{-(r_j + \hat{D}_j T)\lambda_j}{\hat{D}_j}}\right) + hs_j \frac{\hat{D}_j T}{2}\left((n_j - 1)\left(1 - \frac{\hat{D}_j}{P}\right) + \frac{\hat{D}_j}{P}\right)\right]\right\} = \gamma(x, \Gamma) \quad (6)$$

And the optimal value of the equivalent mathematical model is expressed as follows:

$$\gamma(x, \Gamma) = Max\ \sum_{j\epsilon J_0} \xi_j \left[hb_j\left(\frac{\hat{D}_j T}{2} - \frac{\hat{D}_j}{\lambda_j}\right) + \frac{\hat{D}^2(\pi_j + hb_j)}{\lambda_j^2 Q}\left(e^{\frac{-r_j\lambda_j}{\hat{D}_j}} - e^{\frac{-(r_j + \hat{D}_j T)\lambda_j}{\hat{D}_j}}\right) + hs_j \frac{\hat{D}_j T}{2}\left((n_j - 1)\left(1 - \frac{\hat{D}_j}{P}\right) + \frac{\hat{D}_j}{P}\right)\right] \quad (7)$$

$s.t.$

$$\sum_{j\epsilon J_i} \xi_j \leq \Gamma \quad (8)$$

$$0 \leq \xi_j \leq 1, \quad \forall j \in J \quad (9)$$

4.4. Correlated Robust Centralized inventory model (CRC)

In the robust model presented in the previous section, it is assumed that the random parameters take values completely independently of one another. That is, their fluctuations are independent. In many practical situations, however, this assumption is violated. For instance, with respect to uncertain demand, a specific external factor may have similar effects on the demand for certain goods. For example, an unforeseen rise in temperature can lead to a correlated increase in demand for a range of summer essentials, such as summer clothing, cooling devices, beverages, and so on. As another example, an increase in government tariffs on imports from a particular country can lead to a correlated increase in the prices of goods produced in that country.

Jalilvand-Nejad et al. [14] proposed a robust optimization model based on a novel uncertainty set called the correlated polyhedral uncertainty set, which can significantly reduce the price of robustness when correlated uncertainty occurs. Accordingly, in this section, we develop the robust inventory model under correlated uncertainty based on their framework. Following the robust counterpart they provided, the function $\gamma(x,\Gamma_c)$ is rewritten as follows.

$$\gamma(x,\Gamma_c) = Max\ \sum_{j\epsilon J_0} \xi_j \left[hb_j\left(\frac{\widehat{D}_jT}{2}-\frac{\widehat{D}_j}{\lambda_j}\right)+\frac{\widehat{D}^2(\pi_j+hb_j)}{\lambda_j^2 Q}\left(e^{\frac{-r_j\lambda_j}{\widehat{D}_j}}-e^{\frac{-(r_j+\widehat{D}_jT)\lambda_j}{\widehat{D}_j}}\right)+ hs_j\frac{\widehat{D}_jT}{2}\left((n_j-1)\left(1-\frac{\widehat{D}_j}{P}\right)+\frac{\widehat{D}_j}{P}\right)\right] \tag{10}$$

$s.t.$

$$\xi_j+\sum_{k\neq j}\left[\left(1-\left(\frac{n-\Gamma_c}{n-1}\right)|\hat{\rho}_{jk}|\right)\xi_j\right]\le \Gamma_c,\quad \forall j\in J \tag{11}$$

$$0\le \xi_j\le 1,\quad \forall j\in J \tag{12}$$

Therefore, the final model will be as follows:

$$Min\ TC_{joint}(r_1,r_2,\dots,r_N,T,n_1,n_2,\dots,n_N)=\frac{1}{T}\left(O_b+\sum_{j=1}^{N}\frac{SC_j}{n_j}\right)+\sum_{j=1}^{N}hb_j\left(r_j+\frac{D_jT}{2}-\frac{D_j}{\lambda_j}\right)+ \frac{D^2(\pi_j+hb_j)}{\lambda_j^2Q}\left(e^{\frac{-r_j\lambda_j}{D_j}}-e^{\frac{-(r_j+D_jT)\lambda_j}{D_j}}\right)+hs_j\frac{D_jT}{2}\left((n_j-1)\left(1-\frac{D_j}{P}\right)+\frac{D_j}{P}\right)+\sum_{j=1}^{N}(z_j\Gamma_c+p_j) \tag{13}$$

$s.t.$

$$z_j+\sum_{k\neq j}\left[\left(1-\left(\frac{n-\Gamma_c}{n-1}\right)|\hat{\rho}_{jk}|\right)\right]z_j+p_j\ge hb_j\left(\frac{\widehat{D}_jT}{2}-\frac{\widehat{D}_j}{\lambda_j}\right)+\frac{\widehat{D}^2(\pi_j+hb_j)}{\lambda_j^2Q}\left(e^{\frac{-r_j\lambda_j}{\widehat{D}_j}}-e^{\frac{-(r_j+\widehat{D}_jT)\lambda_j}{\widehat{D}_j}}\right)+ hs_j\frac{\widehat{D}_jT}{2}\left((n_j-1)\left(1-\frac{\widehat{D}_j}{P}\right)+\frac{\widehat{D}_j}{P}\right),\quad \forall j\in J \tag{14}$$

$$z_j\ge 0,\quad \forall j\in J \tag{15}$$

$$p_j\ge 0,\quad \forall j\in J \tag{16}$$

5. Experimental Results and Sensitivity Analysis

The objective function values of the four aforementioned models must be calculated. The proposed correlated robust model is nonlinear, with uncertain parameters appearing in both the objective function and the constraints, which makes it computationally complex. The solutions for all four models are obtained using the Generalized Reduced Gradient (GRG) approach.

All optimization experiments were implemented in Python 3.10. The computational framework relied on the SciPy and Pyomo libraries for numerical optimization and model formulation, while NumPy and pandas were used for data handling and preprocessing. Python was selected due to its open-source nature, extensive ecosystem of scientific libraries, and scalability for large datasets.

The experiments were executed on a workstation running Windows 11 equipped with an Intel Core i5 processor (3.2 GHz) and 12 GB RAM. This configuration ensured efficient handling of stochastic optimization problems and reproducibility of results.

5.1. Sample Problem

As a sample problem, we examine a scenario involving a single supplier and a retailer. The supplier provides 10 different products. The lead time for each product follows an exponential distribution with parameter λ, as reported in Table 1.

Demand values are subject to uncertainty, with the limits also presented in Table 1. In both scenarios, the production rates for all products are assumed to be constant, set at 5000 and 8000 units per time period, respectively, and exceed the demand rate.

Table 1. Parameters related to order items in the sample problem

j	D_j	λ_j	j	D_j	λ_j
1	[30,50]	20	6	[50,70]	18
2	[30,50]	17.75	7	[45,50]	27
3	[20,50]	16	8	[40,50]	24
4	[60,80]	20	8	[60,70]	17
5	[45,55]	26	10	[35,50]	22

5.2. Evaluating the performance of the conventional robust model compared to Deterministic models under uncertainty

In the first step, we aim to examine the necessity of employing robust optimization to handle demand uncertainty. To this end, the performance of the model developed based on conventional robust optimization methods—namely Model (5)—is compared with the deterministic models (1-3) and (4). Specifically, the optimal solutions of Models (1-3), (4), and (5) are tested under different levels of demand data perturbations. Table 2 presents the results of 1000 simulation runs of the models' solutions for varying percentages of deviation from the estimated demand.

Table 2. The performance of the deterministic and robust centralized model compared to the decentralized model under uncertainty

	DD	DC		RC (Γ=6)		
Optimal solution	5477.15	5037.5		5186.5		
%dev. from the estimated demand	Results after 1000 simulation replications					
	Avg. cost	Avg. cost	% Improvement	Avg. cost	% Improvement (over DC)	% Improvement (over DD)
0%	5477.15	5037.5	8.0 %	5181.5	-2.9%	5.4%
5%	5568.46	5121.35	8.0 %	5189.1	-1.3%	6.8%
10%	5709.73	5180.67	9.3%	5193.7	-0.3%	9.0%
15%	5763.25	5218.92	9.4%	5195.2	0.5%	9.9%
20%	5982.12	5386.17	10.0%	5199.4	3.5%	13.1%
25%	6067.98	5414.07	10.8%	5221.2	3.6%	14.0%
30%	6182.24	5492.88	11.2%	5216.9	5.0%	15.6%
35%	6318.46	5545.5	12.2%	5238.5	5.5%	17.1%
40%	6554.9	5690.73	13.2%	5257.8	7.6%	19.8%

Table 2 and Figure 1 illustrate that the centralized model typically results in lower supply chain costs compared to the decentralized model. Moreover, as demand variations intensify, the efficiency of the centralized approach improves relative to the decentralized form. This outcome is influenced by the supplier's reaction to demand variations. Furthermore, the table and figure show that while the robust model (RC) consistently outperforms the decentralized model (DD), its performance is inferior to that of the deterministic centralized model in the absence of demand fluctuations—which is expected. The efficiency gap between these two models reflects the price of robustness, which must be incurred if the robust model is selected and a guarantee of solution performance is required. However, even when demand data experience disturbances of up to 40% relative to their nominal values, the performance of the robust solution remains almost stable, whereas solutions obtained from deterministic models rapidly expose the supply chain to significantly higher costs.

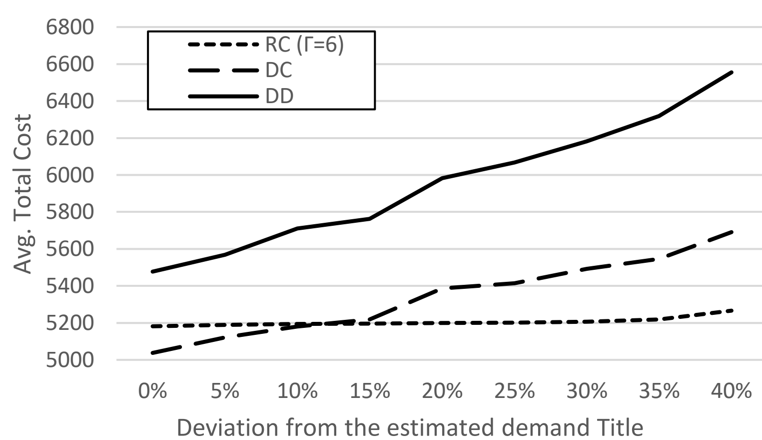

Figure 1. The performance of the robust centralized model compared to the deterministic and decentralized model under uncertainty

The level of protection of the robust model against uncertainty can be adjusted via the parameter Γ. This parameter represents the number of uncertain parameters that can vary across their entire predicted range while the model's solution remains robust. In other words, the higher the value of Γ, the more robust the solution is against fluctuations in a greater number of parameters. Since there are 10 uncertain demand parameters in the sample problem of Section 4.1, the maximum value of Γ is 10.

To evaluate the impact of this parameter on the model, Table 3 and Figure 2 compare the responses of the robust model at three different levels of Γ alongside the deterministic model responses across varying uncertainty levels. The parameter Γ reflects the decision maker's degree of conservatism. Thus, the RC model with Γ=6 is the most conservative, while the DC model is the most risk-taking. As shown in Table 3 and Figure 2, the more robust (i.e., more conservative) the model, the weaker its performance at zero or low uncertainty levels; however, it better maintains performance as demand fluctuations increase. Figure 2 also shows that the RC model with Γ = 2 loses its robustness beyond 20% deviation, and the model with Γ=4 loses it beyond 40%. Considering 10 uncertain parameters, a fluctuation of 20% in each corresponds to a total disturbance of 200% (i.e., Γ=2). Accordingly, the RC model with Γ=2 ensures solution robustness up to 20% disturbance in each parameter, while higher levels of uncertainty lead to increased costs. Selecting a larger value of Γ raises the robustness threshold of the solution against disturbances. Choosing the optimal value of Γ depends on the decision maker's assessment of perturbation probabilities and the extent of their risk aversion.

Table 3. The performance of deterministic and robust centralized models with different protection levels under uncertainty

Deviation	Average total cost after 1000 simulation replications			
	DC	RC (Γ=2)	RC (Γ=4)	RC (Γ=6)
0%	5037.5	5076.9	5118.3	5181.5
5%	5121.35	5082.1	5129.3	5189.1
10%	5180.67	5089	5131.2	5193.7
15%	5218.92	5097.3	5149.8	5195.2
20%	5386.17	5113.4	5154.2	5199.4
25%	5414.07	5139	5160.2	5221.2
30%	5492.88	5184.3	5172.8	5216.9
35%	5545.5	5256.9	5183.2	5238.5
40%	5690.73	5341.3	5189.1	5257.8
45%	5748.1	5387.3	5246.3	5238.1
50%	5812.7	5474.1	5322.4	5251.3

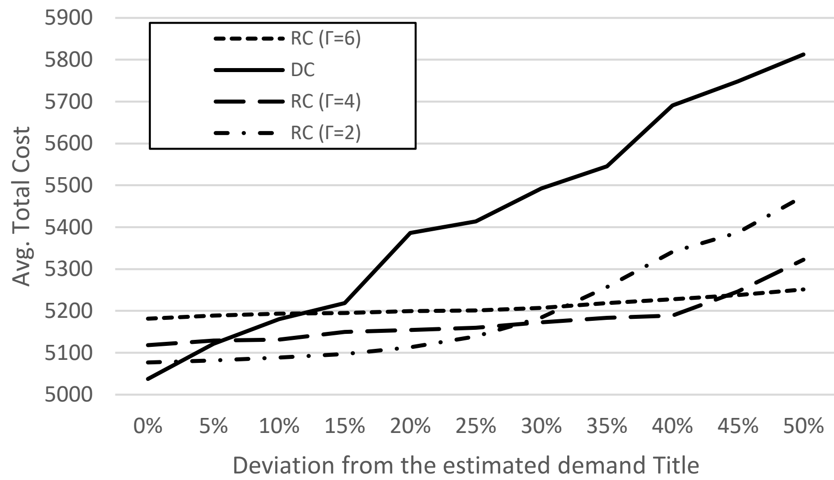

Figure 2. Effect of the protection level on the performance of the robust model under uncertainty

5.3. Evaluation of robust solutions against correlated uncertainty

In the previous sections on uncertainty modeling, it was assumed that deviations in uncertain parameters occur independently. However, a notable characteristic of uncertain parameters—such as demand—is that they often exhibit correlation. Common factors influencing the purchase of different products, such as air temperature, seasonal changes, upcoming holidays, or national and sports events, can have correlated effects on the demand for specific goods. Consequently, deviations in several or all demand parameters may occur in a correlated manner.

To address this issue, researchers have proposed various robust optimization models. Among these, Jalilvand-Nejad et al. [14] introduced a model based on a correlated polyhedral uncertainty set. The CRC robust model, which is suitable for this research, was developed in Section 3.3.4. In this model, the parameter Γ_c controls the protection level against correlated random perturbations. However, similar to the previous model, this parameter does not directly equal the maximum allowable number of perturbations (or protection level), represented by $\sum_{j\epsilon J_0} \xi_j$. Therefore, to determine the actual protection level in this model, the sum of the considered perturbations, $\sum_{j\epsilon J_0} \xi_j$, must be calculated. It is also important to note that these perturbations ξ_j are dual variables corresponding to the constraints of Model (7).

The CRC model depends on the degree of correlation among uncertain data, which is estimated by the parameter $\hat{\rho}$. Figure 3 shows the performance of this model under conditions where the actual correlation between the data is ρ=0.9. As previously mentioned, the distance between the robust model's solution and the deterministic model's solution (i.e., the gap between the two graphs at a deviation of 0%) is called the price of robustness. As seen in Figure 3, the CRC model imposes a lower price of robustness on the decision maker compared to the conventional model at the same protection level of 4. Furthermore, given that the actual correlation value between demand data is 0.9, the closer the estimated value of $\hat{\rho}$ is to its actual value, the lower the price of robustness and the overall model cost for different levels of demand data deviation, indicating greater efficiency of the obtained solutions.

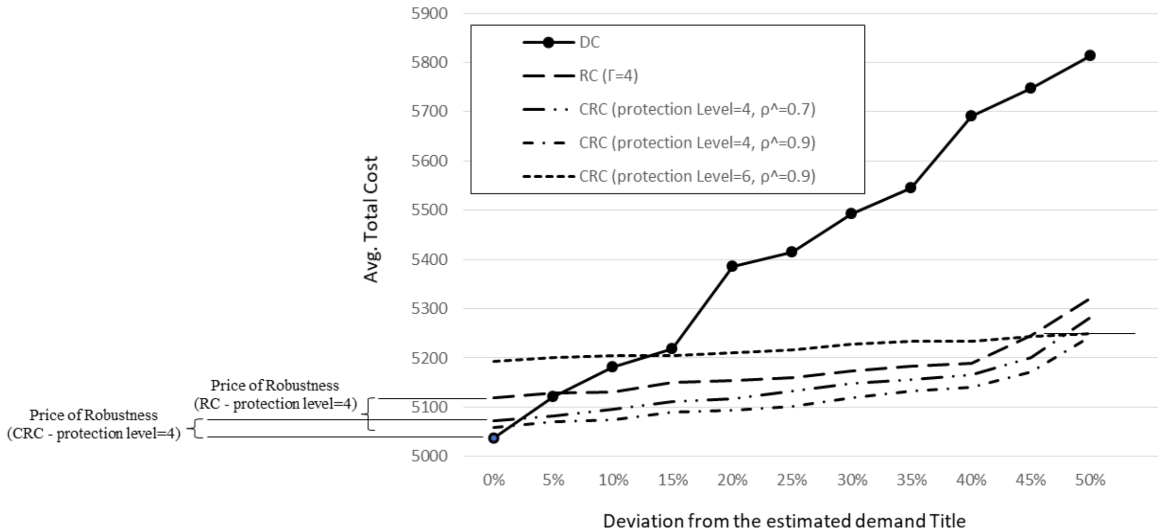

Figure 3. Comparison of the performance of robust RC and CRC models against correlated uncertainty

To better evaluate the conventional RC and the proposed CRC models based on the price of robustness index, we examined these models under different levels of demand perturbation, and the results are presented in Table 4. Specifically, the values reported in Table 4 represent the gap between the curves of the RC and CRC solutions in Figure 3 and the curve of the DC solutions. In this table, the second column corresponds to the solution of the RC model with a protection level of 4, while the third and fourth columns correspond to the solutions of the CRC model with the same protection level but with ρ=0.7 and ρ=0.9, respectively. Furthermore, the fifth column corresponds to the solution of the CRC model with a higher protection level, i.e., 6. As Table 4 illustrates, the CRC model solutions in the third and fourth columns achieve the same protection level as the RC model across all uncertainty levels while resulting in lower robustness costs. Moreover, the last column shows that when the protection level is increased, although the obtained solution yields a higher price of robustness for smaller deviations in demand, it again leads to lower prices under larger perturbations.

Table 4. Comparison of Robustness Costs between the Conventional RC Model and the Proposed CRC Models

Deviation	Price of Robustness			
	RC (Γ=4)	CRC (protection Level=4, $\hat{\rho}$=0.7)	CRC (protection Level=4, $\hat{\rho}$=0.9)	CRC (protection Level=6, $\hat{\rho}$=0.9)
0%	80.8	35.3	20.7	154.9
10%	-49.47	-84.47	-107.07	25.03
20%	-231.97	-269.87	-292.67	-175.47
30%	-320.08	-344.28	-373.28	-263.88
40%	-501.63	-525.43	-550.63	-457.13
50%	-490.3	-530.1	-569.1	-562.9

6. Discussion and Conclusion

This study investigated the supply chain coordination problem under demand and lead time uncertainty, with a particular emphasis on correlated uncertainties—an issue largely overlooked in the existing literature. We proposed a correlated robust centralized (CRC) inventory model and compared its performance against deterministic decentralized (DD), deterministic centralized (DC), and conventional robust centralized (RC) models.

6.1. Summary of Findings

The numerical results confirm that centralized decision-making consistently outperforms decentralized approaches, especially under high demand variability. The conventional robust model (RC) provides stability against uncertainty at the cost of efficiency loss under low uncertainty—the so-called price of robustness. More importantly, the proposed CRC model achieves the same protection level as the RC model while imposing a significantly lower price of robustness when demand parameters are correlated. As shown in Table 4 and Figure 3, when the actual correlation is high (e.g., $\rho = 0.9$) and the estimated $\hat{\rho}$ is close to the actual value, the CRC model reduces robustness costs across all uncertainty levels. Moreover, the CRC model with a higher protection level ($\Gamma = 6$) remains advantageous under large perturbations.

6.2. Managerial Insights

The findings offer several practical insights for supply chain decision-makers, including production and logistics managers responsible for inventory management.

First, centralized control reduces total chain costs but requires ownership alignment or information-sharing mechanisms. When such conditions are absent, decentralized models remain the only feasible option.

Second, managers facing demand uncertainty should first assess whether demand fluctuations across products are correlated. In many real-world settings—such as the dairy industry under new tariffs, or the automotive and apparel sectors during seasonal peaks—common external drivers cause demand for multiple products to move together. Under such correlated conditions, the CRC model provides a clear advantage by avoiding unnecessary conservatism, thereby increasing expected profitability while maintaining robustness.

Third, the price of robustness serves as a practical decision criterion. Our results show that the CRC model reduces this price when correlations exist. For a given protection level, accurate estimation of the correlation parameter ($\hat{\rho}$) enables managers to achieve the same risk protection at a lower operational cost.

Fourth, a practical decision rule for managers: if demand correlations are weak or absent, the conventional RC model suffices; if correlations are strong (e.g., $\rho \geq 0.7$) and estimable, the CRC model is superior; if the supply chain is highly risk-averse and correlations exist, the CRC model with a calibrated $\hat{\rho}$ remains advantageous under large disturbances.

Fifth, although the model was developed on a general two-echelon multi-product supply chain, its applicability extends to industries with high demand volatility and correlation, including automotive parts, apparel, dairy and food products, and electronics—wherever external shocks tend to simultaneously affect demand for multiple products.

6.3. Limitations and Future Research

This study has several limitations. The numerical experiments were conducted on a small-scale problem; large-scale validation remains for future work. Additionally, accurate estimation of the correlation parameter requires historical data that may not always be available. Future research should extend the model to multi-echelon settings, incorporate lead time correlation, and compare the CRC approach with alternative methods such as distributionally robust optimization.

6.4. Concluding Remarks

In conclusion, explicitly modeling correlations among uncertain parameters in supply chain coordination reduces the price of robustness without sacrificing solution guarantees. The proposed CRC model offers a practical tool for decision-makers operating in environments where demand for multiple products is driven by common external factors. As supply chains become increasingly interconnected, correlation-aware robust optimization represents a promising direction for both research and practice.

References

[1] Ahmadi, M. M., & Shayannia, S. A. (2025). Conceptualizing Next Generation Supply Chains: Embracing the Platform Paradigm. Iranian Journal of Operations Research, 16(1), 75-102.

[2] Alves, J. C., & Mateus, G. R. (2022). Multi-echelon supply chains with uncertain seasonal demands and lead times using deep reinforcement learning. arXiv preprint arXiv:2201.04651.

[3] Antonio, F., Atayde, J., Yamzon, M. & Sy, C. (2022). An optimization model for the design of supply chains considering disruptions from pandemic uncertainty and infection trends. *Cleaner Engineering and Technology*, 11. https://doi.org/10.1016/j.clet.2022.100577.

[4] Ben-Ammar, O., Dolgui, A., Hnaien, F., & Ould-Louly, M. A. (2022). Supply planning and inventory control under lead time uncertainty: a literature review and future directions. IFAC-PapersOnLine, 55(10), 2749-2754.

[5] Bertsimas, D., Pachamanova, D. & Sim, M. (2004). Robust linear optimization under general norms. *Operations Research Letters*, 32(6), 510-516.

[6] Chen, L., Dong, T., Peng, J. & Ralescu, D. (2023). Uncertainty Analysis and Optimization Modeling with Application to Supply Chain Management: A Systematic Review. *Mathematics*, 11, 2530. https://doi.org/10.3390/math11112530.

[7] Cobb, B. R. (2016). Lead time uncertainty and supply chain coordination in lost sales inventory models. International Journal of Inventory Research, 3(1), 5-30.

[8] Disney, S. M., Michna, Z., & Nielsen, P. (2025). The bullwhip effect with correlated lead times and autocorrelated demand. International Journal of Production Research, 1-23.

[9] Forhad Uddin, M. (2014). Comparison and Supply Chain Optimization for Vendor-Buyer Coordination System. Iranian Journal of Operations Research, 5(1), 64-81.

[10] Garcia-Castro, F. L., Ruiz-Femenia, R., Salcedo-Diaz, R., & Caballero, J. A. (2023). Sustainable supply chain design under correlated uncertainty in energy and carbon prices. Journal of Cleaner Production, 414, 137612.

[11] Ghaffari-Hadigheh, A. & Mongeri, N. (2023). Supply Chain Coordination with Wholesale contract in an Uncertain Environment. *Journal of Industrial Engineering International*, 4(19), 65-79.

[12] Govindan, K., Popiuc, M. & Diabat, A. (2013). Overview of coordination contracts within forward and reverse supply chains. *Journal of Cleaner Production*, 47, 319-334.

[13] Heydari, J. (2014). Coordinating supplier's reorder point: A coordination mechanism for supply chains with long supplier lead time. *Computers & Operations Research*, 48, 89-101.

[14] Jalilvand-Nejad, A., Shafaei, R. & Shahriari, H. (2016). Robust optimization under correlated polyhedral uncertainty set. *Computers & Industrial Engineering*, 92, 82-94.

[15] Javadigargari, F., Amoozadkhalili, H., Tavakkoli-Moghaddam, R., (2021), Fuzzy Multi-Objective Scenario-based Stochastic Programming to Optimize Supply Chain, *International Journal of Operations Research*, 12(2): 54-72.

[16] Karimi Movahed, K., & Zhang, Z.-H. (2015). Robust design of (s, S) inventory policy parameters in supply chains with demand and lead time uncertainties. *International Journal of Systems Science, 46*(12), 2258–2268. https://doi.org/10.1080/00207721.2014.920653.

[17] Li, Z., Fei, W., Zhou, E., Gajpal, Y., & Chen, X. (2019). The impact of lead time uncertainty on supply chain performance considering carbon cost. Sustainability, 11(22), 6457.

[18] Malik, A. I., & Sarkar, B. (2020). Coordination supply chain management under flexible manufacturing, stochastic leadtime demand, and mixture of inventory. Mathematics, 8(6), 911.

[19] Meisheri, H., Nath, S., Baranwal, M., & Khadilkar, H. (2022). A Learning Based Framework for Handling Uncertain Lead Times in Multi-Product Inventory Management. arXiv preprint arXiv:2203.00885.

[20] Moradi, H., Abbasadeh, M., (2024), A Decision-Making Model for Supplier Selection Based on Data Envelopment Analysis, *International Journal of Operations Research*, 15(1): 28-40.

[21] Pamulety, T. C., & Pillai, V. M. (2016). Effect of customer demand information sharing on a four-stage serial supply chain performance: An experimental study. *Uncertain Supply Chain Management, 4*(1), 1–16. https://doi.org/10.5267/j.uscm.2015.9.001.

[22] Priyan, S. & Uthayakumar, R. (2015). Continuous review inventory model with controllable lead time, lost sales rate and order processing cost when the received quantity is uncertain. *Journal of Manufacturing Systems*, 34, 23-33.

[23] Rezaei, F., Pilevari, N., Radfar, R., (2024), Supply Chain Sustainability Assessment using Hybrid Simulation-Optimization Modeling, *International Journal of Operations Research*, 14(1): 202-224.

[24] Saha, S. & Goyal, S. (2015). Supply chain coordination contracts with inventory level and retail price dependent demand. *International Journal of Production Economics*, 161, 140-152.

[25] Shirvani, N., & Shadrokh, S. (2013). Coordination of a cyclic three-stage supply chain for fast moving consumer goods. Iranian Journal of Operations Research, 4(2), 175-190.

[26] Thorsen, A., & Yao, T. (2015). Robust inventory control under demand and lead time uncertainty. *Annals of Operations Research, 257*(1–2), 207–236. https://doi.org/10.1007/s10479-015-1902-7.

[27] Zhang, D. (2025). *Supply Chain Planning Using Robust Optimization*. PhD Thesis, University of New South Wales, Canberra. https://doi.org/10.26190/unsworks/31240.