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Estimating capacity utilization using meta-frontier free disposal hull frameworks

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Abstract: *Measuring capacity utilization is an important aspect among processes in order to determine overcapacity or undercapacity. Furthermore, in many situations, the convexity and homogeneity properties are not satisfied. Accordingly, in this paper, meta-frontier free disposal hull (FDH) frameworks are proposed to estimate output-oriented and input-oriented capacity utilization (CU) rates of firms under nonconvexity property and heterogeneity of technology. Also, the introduced technique is applied to assess capacity utilization of some Iranian hospitals. The findings show the presented approach is beneficial to measure capacity utilization rates of systems in the presence of nonconvexity and heterogeneity.*

Keywords*: Capacity utilization, Free disposal hull, Meta-frontier, DEA.*

1. Introduction

Evaluating how fully a firm employs its production capacity is a key element of managerial analysis and strategic planning. In economic theory, capacity output is commonly viewed as the maximum attainable production level when fixed inputs remain unchanged and only variable inputs can adjust. The input-oriented capacity measure assesses variable input values relative to the variable inputs consistent with a zero-output level [2]. Measuring capacity utilization (CU) is therefore important for identifying idle capacity and assessing how effectively production potential is used across industries.

A firm's performance depends on different factors such as its input mix, production technology, technical efficiency, and resource availability. Improvements in these areas can enhance effective capacity and overall productivity. Because CU indicates how intensively production assets are operated, it provides valuable insights for both operational decision-making and broader policy analysis.

The literature on CU covers a wide range of methodologies and applications. Singh et al. [14] emphasized the need for comprehensive surveys of CU measurement techniques across different sectors and national contexts. Data Envelopment Analysis (DEA), a widely used non-parametric approach, has been applied frequently due to its flexibility and intuitive structure. Early work by Färe et al. [6], rooted in Farrell-type efficiency measures and the capacity concept of Johansen [4], introduced DEA-based indicators for evaluating capacity and CU from a physical perspective. Building on these foundations, Färe and Grosskopf [5] employed directional distance functions within the DEA framework to assess capacity utilization. Subsequent contributions include the

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decomposition of CU into inefficiency and idleness components by Sahoo and Tone [6], the analysis of CU dynamics within Chinese manufacturing by Yang et al. [16], and the examination of competitive efficiency in the steel industry through unrestricted directional distance functions by Fukuyama et al. [7]. Shen et al. [13] incorporated weak disposability assumptions to evaluate short- and long-term CU measures, while Wang and Feng [15] proposed a meta-frontier DEA approach to accommodate heterogeneous technologies, considering Charnes, Cooper and Rhodes (CCR) and Banker, Charnes and Cooper (BCC) models. Cesaroni et al. [2] introduced a new concept of plant capacity from an input-oriented perspective. Despite these advances, research on CU measurement under nonconvex and non-homogeneous technologies remains limited.

For many real-world systems—particularly those involving diverse operational units—production technologies are heterogeneous and often nonconvex. Reliable CU estimation in such environments is vital for identifying performance gaps, diagnosing bottlenecks, and guiding resource allocation decisions.

Motivated by these gaps, this study develops a radial CU measurement approach designed specifically for heterogeneous processes that exhibit nonconvex characteristics. The framework applies the free disposal hull (FDH) model to estimate CU under both group-specific and meta-frontier technologies, enabling consistent assessment when decision-making units (DMUs) operate under distinct technological conditions and variable returns to scale. The usefulness of the proposed method is illustrated through an empirical application in the hospital sector, highlighting its ability to analyze CU and performance in complex, nonconvex settings.

The major contributions of this study are threefold:

- An analysis of performance and CU in heterogeneous systems under non-convexity property.
- The introduction of a meta-frontier FDH technique for assessing CUs.
- Estimating output-oriented and input-oriented CU rates using the proposed technique.
- An illustration of the proposed technique through a real-world case study conducted in the Iranian hospital sector.

The structure of this paper is as follows: In Section 2, we discuss the fundamental concept of capacity utilization, alongside FDH approach and the meta-frontier framework. Section 3 introduces the methodology utilizing meta-frontier FDH for estimating CUs. Section 4 presents a practical example from the Iranian hospital sector to illustrate the proposed methodology. Finally, Section 5 offers conclusions and recommendations.

2. Provisions

2.1. Capacity utilization

Capacity utilization is a cornerstone concept in DEA, critically shaping the efficiency assessment of processes. Suboptimal resource employment by a process can mask its true operational potential, leading to its classification as inefficient. Conversely, DMUs efficiently utilizing all resources to generate maximum outputs operate at their full capacity. DEA commonly measures this through the ratio of actual to potential output, given available resources. Johansen [8] established capacity output as the maximum production achievable with existing assets over a period, assuming variable inputs are non-limiting. Fare et al. [6] initially integrated this potential output concept into DEA, introducing minor revisions. Cesaroni et al. [2] rendered a novel notion of the input-oriented capacity measure as evaluating capacity by contrasting the minimum variable inputs for an observation at a given output level with those for a translated observation producing zero output. Understanding capacity utilization in DEA is key to identifying factors contributing to resource under- or overutilization, thus guiding targeted improvements in performance.

2.2. FDH framework

FDH was initially introduced by [3]. Let $DMU_j(j=1,\ldots,n)$ produce output vector $y_j \in \mathbb{R}_+^s$ using input vector $x_j \in \mathbb{R}_+^m$.

The FDH production possibility set (P^{FDH}) is defined as:

$$P^{FDH} = \left\{(x,y) \in \mathbb{R}_+^m \times \mathbb{R}_+^s \mid \exists j \text{ s.t. } x \geq x_j, y \leq y_j\right\}$$

This set is formed by taking all observed DMUs (x_j, y_j) and including all points "weakly dominated" by them (i.e., points that require at least as much input for at least as much output). This set is essentially the union of all input-output combinations dominated by observed DMUs [1, 3]. Key Axioms follows:

- Free Disposal: For any $(x,y) \in P^{FDH}$, if $x' \geq x$ and $y' \leq y$, then $(x',y') \in P^{FDH}$. This implies that having more of an input or less of an output is always feasible and costless to achieve.
- Non-Convexity: FDH works with the non-convex hull.
- The efficient frontier is the boundary of P^{FDH}. DMUs lying on this boundary are considered technically efficient under the FDH assumptions

2.3. Non-homogeneous DMUs

The inherent non-homogeneity among firms signifies the diverse attributes and differentials operative within an industry or market. These divergences, attributable to technological advancements, scale of operations, geographical location, and environmental contexts, impede the uniform comparison of DMU efficiencies using a singular production frontier. In certain situations, DMUs may represent distinct technological regimes. The meta-frontier is posited as a function that envelops discrete group frontiers, each characterized by its unique technology and environmental determinants. Accordingly, a production unit's distance from the meta-frontier is a function of its position relative to its group frontier and the latter's stance relative to the meta-frontier. These combined positional metrics are fundamental to meta technical efficiency, a comprehensive indicator reflecting the group frontier's properties [9, 10]. Nevertheless, empirical studies investigating capacity utilization within heterogeneous DEA models, particularly under conditions of non-convexity, are notably lacking. To address this, the subsequent section will introduce FDH models for assessing CU in the presence of both meta and group technologies, specifically when the convexity property is contravened."

3. CU estimation using meta-frontier FDH

In this section, a DEA-based procedure is provided to assess the CU of heterogeneous processes, $DMU_j(j=1,...,n)$ under non-convexity property. The inputs and outputs are shown by

$x_{ij}(i=1,...,m)$ and $y_{rj}(r=1,...,s)$, respectively. Due to the difference in production technologies, processes are categorized into $\overline{n}>1$ groups that $\sum_{e=1}^{\overline{n}} c_e = n, \tau=\{1,...,\overline{n}\}$. The number of units in the group e is shown by c_e. Furthermore, inputs classified into fixed inputs $x_{ij}^{f}(i=1,...,I)$ and variable inputs $x_{ij}^{v}(i=1,...,V)$ that $m=I+V$. Fixed inputs are factors that cannot be easily changed. To estimate technical CU of processes under meta and group technologies, free disposal hull (FDH) models are presented under variable returns to scale assumption. DMU_o shows the process under consideration. The output-oriented FDH model (1), which operates under the assumption of variable returns to scale, is presented to evaluate the group efficiency of the heterogeneous firms in question, DMU_o, encompassing both fixed and variable inputs and considering non-convexity property.

Output-oriented CU

$$\begin{aligned}
\varphi_g^o = & Max \ \ \varphi \\
s.t. \quad & \sum_{j=1}^{c_e} \lambda_j x_{ij} \le x_{io}, i=1,...,m, \\
& \sum_{j=1}^{c_e} \lambda_j y_{rj} \ge \varphi y_{ro}, r=1,...,s, \\
& \sum_{j=1}^{c_e} \lambda_j = 1, \\
& \lambda_j \in \{0,1\}, \forall j.
\end{aligned} \tag{1}$$

To analyze the group efficiency including fixed inputs, the following FDH procedure (2) is provided:

$$\begin{aligned}
\varphi_g^F = & Max \ \ \varphi \\
s.t. \quad & \sum_{j=1}^{c_e} \lambda_j x_{ij}^f \le x_{io}^f, i=1,...,I, \\
& \sum_{j=1}^{c_e} \lambda_j y_{rj} \ge \varphi y_{ro}, r=1,...,s, \\
& \sum_{j=1}^{c_e} \lambda_j = 1, \\
& \lambda_j \in \{0,1\}, \forall j.
\end{aligned} \tag{2}$$

The CU measure under group technology can be determined by

$$CU_g^O = \frac{\varphi_g^o}{\varphi_g^F} \tag{3}$$

$CU_g^O = 1$ means that short-run capacity is fully and efficiently applied under the group technology and $CU_g^O < 1$ indicates that the organization does not utilize its resources rationally. Actually, it under-utilizes its resources under the group technology.

By considering all inputs, variable and fixed ones, the following meta-frontier FDH model (4) under the variable returns to scale property is provided to assess the meta efficiency of DMU_o.

$$
\begin{aligned}
\varphi_M^o &= Max \ \ \varphi \\
s.t. \quad & \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j x_{ij} \le x_{io}, i = 1, \ldots, m, \\
& \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j y_{rj} \ge \varphi y_{ro}, r = 1, \ldots, s, \\
& \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j = 1, \\
& \lambda_j \in \{0,1\}, \forall j.
\end{aligned}
\tag{4}
$$

By considering the meta technology, model (2) can be transformed into the following model that contains fixed inputs.

$$
\begin{aligned}
\varphi_M^F &= Max \ \ \varphi \\
s.t. \quad & \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j x_{ij}^f \le x_{io}^f, i = 1, \ldots, I, \\
& \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j y_{rj} \ge \varphi y_{ro}, r = 1, \ldots, s, \\
& \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j = 1, \\
& \lambda_j \in \{0,1\}, \forall j.
\end{aligned}
\tag{5}
$$

The capacity utilization measure within the context of meta technology can be assessed using formula (6):

$$
CU_M^O = \frac{\varphi_M^o}{\varphi_M^F} \tag{6}
$$

$CU_M^O = 1$ shows the effective utilization of short-run capacity, within the constraints of existing technology and fixed inputs, signals the organization's ability to reach its peak output potential using current variable inputs. Conversely, an indication of $CU_M^O < 1$ suggests that the prevailing variable inputs are insufficient to meet the demand for potential output. To address this, an increase in variable inputs is necessary to elevate output and ensure more rational allocation of organizational resources.

Furthermore, technology gap ratios (TRG) of firms in two cases, including all inputs and fixed inputs are computed in the following ways:

$$TGR_A^b = \frac{\frac{1}{\varphi_M^o}}{\frac{1}{\varphi_g^o}} = \frac{\varphi_g^o}{\varphi_M^o} \tag{7}$$

$$TGR_F^b = \frac{\frac{1}{\varphi_M^F}}{\frac{1}{\varphi_g^F}} = \frac{\varphi_g^F}{\varphi_M^F} \tag{8}$$

The TGR value serves as a measure of how closely a group production frontier aligns with the overarching meta-frontier. A lower ratio indicates a higher achievable operational efficiency.

Thereafter, by following Cesaroni et al. [2], the input-oriented FDH model (9), which operates under the assumption of variable returns to scale, is presented to evaluate the group efficiency of the heterogeneous firms, DMU_o, considering non-convexity property.

Input-oriented CU

$$\begin{aligned}
&\theta_g^o = Min \ \ \theta \\
&s.t. \quad \sum_{j=1}^{c_e} \lambda_j x_{ij}^f \le x_{io}^f, i = 1,\dots,I, \\
&\qquad \sum_{j=1}^{c_e} \lambda_j x_{ij}^v \le \theta x_{io}^v, i = 1,\dots,V, \\
&\qquad \sum_{j=1}^{c_e} \lambda_j y_{rj} \ge y_{ro}, r = 1,\dots,s, \\
&\qquad \sum_{j=1}^{c_e} \lambda_j = 1, \\
&\qquad \lambda_j \in \{0,1\}, \forall j.
\end{aligned} \tag{9}$$

To analyze the group efficiency, the following input-oriented FDH procedure (10) is provided:

$$
\begin{aligned}
\theta_g^F = & Min \ \ \theta \\
s.t. \quad & \sum_{j=1}^{c_e} \lambda_j x_{ij}^f \le x_{io}^f, i = 1,\ldots, I, \\
& \sum_{j=1}^{c_e} \lambda_j x_{ij}^v \le \theta x_{io}^v, i = 1,\ldots, V, \\
& \sum_{j=1}^{c_e} \lambda_j y_{rj} \ge 0, r = 1,\ldots, s, \\
& \sum_{j=1}^{c_e} \lambda_j = 1, \\
& \lambda_j \in \{0,1\}, \forall j.
\end{aligned} \tag{10}
$$

In which, the input-oriented FDH efficiency is measured by the reduction of variable inputs while the output level is considered zero.

The input-oriented CU measure under group technology can be determined by

$$
CU_g^I = \frac{\theta_g^o}{\theta_g^F} \tag{11}
$$

The following input-oriented meta-frontier FDH model (12) under the variable returns to scale property is provided to assess the meta efficiency of DMU_o.

$$
\begin{aligned}
\theta_M^o = & Min \ \ \theta \\
s.t. \quad & \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j x_{ij}^f \le x_{io}^f, i = 1,\ldots, I, \\
& \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j x_{ij}^v \le \theta x_{io}^v, i = 1,\ldots, V, \\
& \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j y_{rj} \ge y_{ro}, r = 1,\ldots, s, \\
& \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j = 1, \\
& \lambda_j \in \{0,1\}, \forall j.
\end{aligned} \tag{12}
$$

By considering the meta technology, model (10) can be transformed into the following model that the levels of outputs observed on the right side of the output constraints are adjusted to zero.

$$
\begin{aligned}
&\theta_M^F = Min \ \ \theta \\
&s.t. \quad \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j x_{ij}^f \leq x_{io}^f, i = 1,\ldots, I, \\
&\qquad \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j x_{ij}^v \leq \theta x_{io}^v, i = 1,\ldots, V, \\
&\qquad \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j y_{rj} \geq 0, r = 1,\ldots, s, \\
&\qquad \sum_{e=1}^{\bar{n}} \sum_{j=1}^{c_e} \lambda_j = 1, \\
&\qquad \lambda_j \in \{0,1\}, \forall j.
\end{aligned} \tag{13}
$$

The input-oriented short-run CU can be approximated in the following manner:

$$CU_M^I = \frac{\theta_M^o}{\theta_M^F} \tag{14}$$

It is evident that the numerator is greater than or equal to the denominator in expressions (11) and (14), resulting in $CU_g^I \geq 1$ and $CU_M^I \geq 1$. It should be noted that optimal short-term capacity utilization occurs when CU is equal to 1. This measure indicates the extent to which variable inputs can be adjusted (compared to zero production) to achieve the observed output quantity with the available fixed input resources.

TRG of firms in two cases, including all inputs and fixed inputs are estimated in the subsequent ways:

$$TGR_A^c = \frac{\theta_M^o}{\theta_g^o} \tag{15}$$

$$TGR_F^c = \frac{\theta_M^F}{\theta_g^F} \tag{16}$$

The magnitude of the gap between a DMU's group frontier and the meta-frontier is inversely related to the meta-technology ratio. A ratio closer to unity indicates a minimal gap, whereas a ratio closer to zero denotes a significant divergence.

4. Application

The estimation of hospital capacity utilization is crucial for assessing the current deployment of critical resources, including beds, staffing levels, and medical equipment. Its significance lies in its role in optimizing patient flow, mitigating the risk of overcrowding, enhancing resource allocation efficiency, bolstering preparedness for emergent situations, and informing strategic planning. In this section, CU of eleven hospitals located in Tehran is addressed using the proposed framework. The hospitals are covered by Tehran University of Medical Sciences. The number of beds is considered as fixed input. The number of doctors is deemed as a variable input. The outputs encompass the number of patients admitted to hospitals and the number of individuals treated as outpatients. The data has been derived from Statistical Yearbook of Science and Medicine of Tehran [17] and related to the period 2021-2022. Tables 1 provides data. For grouping, the approach mentioned in Seiford and Zhu [12] and Ding et al. [4] has been used. By considering the threshold efficiency score 0.60, 11 hospitals are divided into two groups, A and B as presented in Table 1. To estimate, the output-oriented short-run CU of hospitals, statements (1)-(8) are considered. The results are presented in Table 2.

Table 1. Hospital data

#	Hospital	Group	Beds	Doctors	In-patients	Out-patients
1	Bharlo	A	322	548	8909	460544
2	Bahrami	B	142	268	3489	94920
3	Arash Women's General	A	126	107	7002	206015
4	Razi	A	75	133	216	398558
5	Sina	B	471	252	8258	200129
6	Shariati	B	637	667	14579	269886
7	Ziaian	A	159	487	5362	495660
8	Imam Khomeini	B	1172	1346	24975	707196
9	Amir- Alam	B	236	335	1784	254487
10	Yas	B	229	340	4911	166452
11	Children’s Medical Center	B	351	639	3104	267962

As can be seen, hospitals 2, 10 and 11 are inefficient by considering the meta technology and all inputs while hospitals 5 and 9 are added to these hospitals by considering only the fixed input, the number of beds. Under the group technology, all hospitals are determined as efficient in both cases, including all inputs and only the fixed input. Columns 7 and 8 shows the CU measure under meta and group technologies, respectively. As can be found, short-run capacity is fully and efficiently applied under the group technology, resulting from $CU_g^O = 1$. Furthermore, two hospitals 5 and 9 do not utilize its resources rationally under meta technology. Actually, the increase of the doctor number is necessary to improve the performance.

In this stage, the input-oriented CU of hospitals are measured using expressions (9)-(16). The consequences are provided in Table 3. As shown, hospitals 2, 10 and 11 are inefficient under meta technology and considering all inputs while only two hospitals 3 and 4 are determined as efficient under meta technology and including the fixed input. By considering all inputs, all hospitals are specified as efficient under the group technology. By considering the fixed input, the number of beds, only four hospitals, 2-5, are discovered as efficient. Two last columns show the CU ratios under meta and group frontiers, respectively. In two cases, hospitals 2, 3 and 4 utilize optimal short-term capacity. To more illustrate, hospitals, 2, 3, 4 and 10 use their capacity in an optimal way under meta frontier and hospitals 2, 3, 4 and 5 use optimal capacity under group technology.

TRG of hospitals achieved from the input-oriented (output-oriented) approach in two cases, including all inputs and fixed inputs are computed and presented in Table 4.

Table 2. Output-oriented CU

#	Hospital	Output-oriented framework									
		φ_M^o	φ_M^F	φ_g^o	φ_g^F	CU_M^O	CU_g^O	$1/\varphi_M^o$	$1/\varphi_M^F$	$1/\varphi_g^o$	$1/\varphi_g^F$
1	Bharlo	1	1	1	1	1	1	1	1	1	1
2	Bahrami	2.01	2.01	1	1	1	1	0.497512	0.497512	1	1
3	Arash Women's General	1	1	1	1	1	1	1	1	1	1
4	Razi	1	1	1	1	1	1	1	1	1	1
5	Sina	1	1.08	1	1	1.08	1	1	0.925926	1	1
6	Shariati	1	1	1	1	1	1	1	1	1	1
7	Ziaian	1	1	1	1	1	1	1	1	1	1
8	Imam Khomeini	1	1	1	1	1	1	1	1	1	1
9	Amir- Alam	1	1.95	1	1	1.95	1	1	0.512821	1	1
10	Yas	1.24	1.24	1	1	1	1	0.806452	0.806452	1	1
11	Children's Medical Center	1.73	1.73	1	1	1	1	0.578035	0.578035	1	1

Table 3. Input-oriented CU

#	Hospital	Input-oriented framework					
		θ_M^o	θ_M^F	θ_g^o	θ_g^F	CU_M^I	CU_g^I
1	Bharlo	1	0.2	1	0.2	5	5
2	Bahrami	0.4	0.4	1	1	1	1
3	Arash Women's General	1	1	1	1	1	1
4	Razi	1	1	1	1	1	1
5	Sina	1	0.42	1	1	2.380952	1
6	Shariati	1	0.16	1	0.38	6.25	2.631579
7	Ziaian	1	0.22	1	0.22	4.545455	4.545455
8	Imam Khomeini	1	0.08	1	0.19	12.5	5.263158
9	Amir- Alam	1	0.32	1	0.8	3.125	1.25
10	Yas	0.31	0.31	1	0.79	1	1.265823
11	Children's Medical Center	0.76	0.17	1	0.42	4.470588	2.380952

Table 4. Technology gap ratios

#	Hospital	Output-orientation		Input-orientation	
		TGR_A^b	TGR_F^b	TGR_A^c	TGR_F^c
1	Bharlo	1	1	1	1
2	Bahrami	0.497512	0.497512	0.4	0.4
3	Arash Women's General	1	1	1	1
4	Razi	1	1	1	1
5	Sina	1	0.925926	1	0.42

6	Shariati	1	1	1	0.421053
7	Ziaian	1	1	1	1
8	Imam Khomeini	1	1	1	0.421053
9	Amir- Alam	1	0.512821	1	0.4
10	Yas	0.806452	0.806452	0.31	0.392405
11	Children's Medical Center	0.578035	0.578035	0.76	0.404762

For enhanced clarity, consider the case of Hospital 9, Amir-Alam. This hospital's TGR_A is reported as 1.0 in two instances, applying to both input and output orientations (TGR_A^c and TGR_A^b). However, the TGR_F registers approximately 0.513 in the output-oriented context and 0.4 in the input-oriented context. The significance of the output-oriented TGR lies in its capacity to gauge how closely a group production frontier aligns with the overarching meta-frontier, representing best-practice performance. For input-oriented analyses, a TGR value approximating unity indicates a minimal technological gap, whereas a value approaching zero signifies a considerable divergence from the frontier.

5. Conclusions

In traditional DEA frameworks, the efficacy and capacity utilization of processes are evaluated based on the convexity property. Capacity utilization plays a critical role in operations management, focusing on how effectively an organization utilizes its resources to produce goods or services. This research addressed the performance and CU of non-homogeneous incorporating non-convexity property. Specifically, FDH models under the assumption of variable returns to scale were developed to quantify the CU of non-homogeneous processes based on both meta and group frontiers.

A case study from the Iranian hospital sector illustrates the efficacy of the proposed methodology. The results for CU measures and technology gap ratios are presented, demonstrating that this technique is effective for analysing performance and CU in heterogeneous under the nonconvexity property. Desirable factors are incorporated within the framework established.

Future research could explore the extension of these models to scenarios involving undesirable factors. Additionally, investigating the capacity utilization of production processes with different network structures represents a compelling area for further study.

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