

A Robust Multi-Objective Optimization Model for Integrated Truck–Quadrotor Patrol Network Design

M. Alinasab Rostam¹, M. Fallah^{2,*}, I. Mahdavi³, G. Tohidi⁴

This study develops a multi-objective mathematical model for planning and optimizing patrol operations in surveillance networks using an integrated truck–quadrotor system. The proposed model simultaneously considers five conflicting objectives: maximizing network coverage, minimizing operational costs, operational risks, mission duration and environmental emission. To balance these objectives, the LP-metric distance-to-ideal-point approach is employed to transform the multi-objective formulation into an equivalent single-objective model. Benders decomposition is subsequently applied to efficiently solve larger-scale instances and reduce computational complexity. The performance of the proposed approach is evaluated through numerical experiments and sensitivity analyses. Results show that network coverage reaches approximately 89% in the baseline scenario and increases to 96% under a coverage-oriented strategy. Meanwhile, total operational cost decreases from approximately 47800 to 39900 cost units under a cost-focused strategy, while operation time varies between 590 and 710 minutes across different scenarios. Sensitivity analysis indicates that a 20% increase in truck deployment cost reduces network coverage by 22% and increases operational cost by 10%, whereas a 30% increase in quadrotor energy consumption increases operation time by 18% and cost by 7%. The findings demonstrate that the proposed integrated framework provides an effective decision-support tool for patrol planning and resource allocation.

Keywords: Optimal patrol, mathematical modeling, quadrotor, truck, multi-objective optimization, operations planning.

1. Introduction

The expanding use of quadrotors in transportation, surveillance, emergency response, monitoring, and field services has led to the emergence of new problems in the fields of operations research and network optimization. One of the most important of these problems is the coordination of the movements of ground vehicles and quadrotors. In this structure, the ground vehicle can serve as a carrier, launch and recovery platform, support center, or mobile hub, while the quadrotor performs tasks at locations where ground access is difficult, costly, or time-consuming. Murray and Chu [15]. Early studies in this field primarily focused on last-mile delivery in logistics; however, their conceptual framework can be extended to patrol

* Corresponding Author.

¹Department of Industrial Engineering.CT.C.Islamic Azad University, Teharan,Iran, Email: maedeh.alinasab@iau.ir.

²Department of Industrial Engineering.CT.C.Islamic Azad University, Teharan,Iran, Email: mohammad.fallah43@yahoo.com.

³Department of Industrial Engineering,Mazandaran University of Science and Technology, babol, iran, Email: irajustmb@gmail.com.

⁴Department of Mathematics.CT.C.Islamic Azad University, Teharan,Iran, Email: gh_tohidi@iauctb.ac.ir.

and monitoring missions. In both applications, the main problem is to determine how missions are allocated between the ground vehicle and the quadrotor, determine the travel route, the quadrotor launch and recovery times, ensure temporal coordination between the two vehicles, and satisfy range and energy constraints. Agatz et al [1]. The importance of this problem in the present context arises from the fact that patrol operations in large, inaccessible, or ground-risk areas are constrained when relying solely on ground vehicles or solely on quadrotors.

The scientific literature indicates that integrated ground–air systems can simultaneously exploit the advantages of ground mobility and aerial accessibility. In early models, this cooperation was primarily considered for reducing time or cost; however, other objectives, such as reliability, energy consumption, emissions, resilience, and uncertainty management, have gradually been incorporated into the models. Liang and Luo [11].

2. Literature Review

Over the past fifteen years, the literature on the design and optimization of patrol networks has evolved from relatively static models for route determination and coverage maximization toward more integrated models incorporating facility location, routing, scheduling, multi-vehicle coordination, continuous coverage, uncertainty, and multi-objective optimization. This evolution can be observed along several main directions: first, the development of exact mathematical models for maximizing the coverage of critical points; second, the simultaneous incorporation of facility location and routing decisions; third, the development of multi-vehicle and coordinated models involving vehicles, quadrotors, vessels, and robots; fourth, the transition from deterministic models to dynamic, stochastic, interval-based, and robust models; and fifth, the increasing use of metaheuristic algorithms, reinforcement learning, and hybrid methods for solving large-scale instances.

Keskin et al [8] investigated the routing of police patrol vehicles with the objective of maximizing the coverage of critical highway locations and developed a mixed-integer linear programming model subject to time and budget constraints. Local search and tabu search-based heuristics were employed to solve larger instances. Computational experiments using both randomly generated and real-world data demonstrated the applicability of the proposed approaches for analyzing coverage levels and designing patrol routes. Subsequently, Lin et al [12] investigated a graph patrolling problem with a focus on detecting potential attacks at network nodes. By considering the temporal distribution of attacks and the heterogeneous costs associated with successful attacks at different nodes, linear programming models were developed for both random and strategic attackers, together with an index-based heuristic policy to reduce computational requirements. Lin et al [13] extended this framework by incorporating imperfect attack detection and formulated the problem with the objective of minimizing the expected cost resulting from successful attacks. Li and Keskin [9] further extended police patrol planning from a pure routing problem to an integrated multi-period location–routing problem, in which the number of patrol units, patrol routes, and temporary starting and ending locations were determined simultaneously. Their multi-objective formulation considered both the coverage of critical locations and the costs associated with facility location, vehicle utilization, and routing. Dewil et al [5] reformulated the patrol coverage and routing problem as a minimum-cost network flow problem and demonstrated that exploiting the underlying network structure can substantially reduce the computational effort required to solve benchmark instances. With the development of the literature, attention gradually shifted from static coverage toward operational and behavioral characteristics of patrol activities.

Frohlich et al [7] formulated a security patrolling problem as a multi-objective periodic routing problem in which cost minimization and route unpredictability were simultaneously considered to reduce the predictability of patrol patterns. Chircop et al [3] investigated a continuous maritime patrolling problem

with the objective of determining the minimum fleet size required to maintain continuous area coverage. In their formulation, routing and scheduling decisions were integrated while accounting for operational constraints, fuel consumption, and resource replenishment, and a column-generation approach based on a space–time–resource network was developed. These studies indicate that patrol planning extends beyond route determination and requires the simultaneous consideration of coverage, resource allocation, and scheduling decisions. a major direction in the development of this literature has been the introduction of unmanned vehicles and the emergence of integrated ground–air cooperation. Luo et al [14] investigated the cooperation between a ground vehicle and an accompanying drone, where the ground vehicle traveled along a road network while the drone covered targets that were inaccessible to the ground vehicle. The model also incorporated the spatial and temporal coordination required for drone launch, operation, and rendezvous with the ground vehicle. Zhu et al [21] investigated the allocation of patrol tasks to multiple unmanned surface vehicles while considering endurance and mission-duration constraints and developed a hybrid solution approach based on a genetic algorithm and an immune algorithm. Xu et al [19] further investigated the cooperation of a ground vehicle with multiple drones in urban traffic patrolling. in their formulation, the ground vehicle served not only as a carrier but also as a launch, recovery, and recharging platform for the drones. A hybrid algorithm based on randomized variable neighborhood descent and simulated annealing was developed for larger instances.

More recent studies have addressed the dynamic and multi-objective nature of patrol planning in addition to multi-vehicle coordination. Dewinter et al [6] simultaneously considered reactive and proactive police patrol tasks and used discrete-event simulation to evaluate the effects of different deployment and redeployment strategies on area coverage and response time. Tschernutter and Feuerriegel [18] incorporated fine-grained routing decisions directly into a dynamic police patrolling problem using real-world crime data, with the objective of maximizing the expected number of prevented crimes. Sui et al [17] formulated police patrol planning as a constrained multi-objective optimization problem and developed an evolutionary multi-objective algorithm to generate a set of compromise solutions for different operational objectives. another stream of research has focused on the operational performance of patrol systems under varying operating conditions. Li et al [10] investigated a multi-quadrotor patrol routing problem while accounting for variability in flight duration and developed an optimization model incorporating uncertainty constraints. Asghari et al [2] considered patrol coverage, the distance of uncovered locations, environmental conditions, and pollutant emissions simultaneously in a multi-objective maritime patrolling model. Hou et al [4] investigated long-term mission scheduling for multiple fixed-wing UAVs and formulated repeated patrol missions over an extended planning horizon. Their model focused on the coordinated scheduling of a set of recurring patrol missions over multiple time periods.

Recent studies have also introduced data-driven and learning-based approaches to patrol planning. Palma-Borda et al [16] investigated coordinated urban patrolling using multi-agent reinforcement learning and considered the maximization of coverage in crime-prone areas without restricting patrol routes to predefined nodes. Zhang et al [20] investigated patrol robot routing in a chemical-plant environment by simultaneously considering route length and safety risk and developed a multi-objective evolutionary algorithm to obtain trade-offs between these criteria.

3. Problem Statement

In this research, the problem under investigation focuses on the design and planning of an integrated patrol network based on the cooperation of ground trucks and aerial quadrotors, such that the mobility, operational, and coverage advantages of both vehicles are utilized in a complementary manner.

A set of target areas or points for patrol, monitoring, support, or delivery is located within a specified geographical area and must be served using multiple trucks and multiple quadrotors. In this structure, trucks move on the ground and, while traversing accessible routes, undertake the role of operational support for the system. In contrast, quadrotors benefit from rapid accessibility, the ability to overcome natural obstacles, and the capability to cover locations where ground access is difficult, time-consuming, or costly. Therefore, the main problem of this research is not merely to determine the movement routes of the vehicles, but rather to design a coordinated mechanism for mission allocation, operation scheduling, area coverage, and synergy between the ground and aerial components, so that the multiple objectives of the research can be achieved simultaneously.

3.1. Mathematical Formulation

Sets

$$N = \{0\} \cup P \cup L$$

$$P = \text{set of patrol areas}$$

$$L = \text{set of launch /landing nodes}$$

$$A_g \subseteq N \times N = \text{set of feasible ground arcs}$$

$$A_a \subseteq L \times P \times L = \text{set of feasible aerial sorties}$$

$$K = \text{set of ground vehicles (trucks)}$$

$$D = \text{set of quadrotors}$$

Parameters

$$W_p \geq 0 \quad \forall p \in P$$

$$C_{ij}^T \geq 0 \quad \forall (i, j) \in A_g$$

$$C_{lpm}^Q \geq 0 \quad \forall (l, p, m) \in A_a$$

$$R_{ij}^T \geq 0 \quad \forall (i, j) \in A_g$$

$$R_{lpm}^Q \geq 0 \quad \forall (l, p, m) \in A_a$$

$$T_{ij}^T \geq 0 \quad \forall (i, j) \in A_g$$

$$T_{lpm}^Q \geq 0 \quad \forall (l, p, m) \in A_a$$

$$E_{lpm}^Q \geq 0 \quad \forall (l, p, m) \in A_a$$

$$Rel_{ij}^T \geq 0 \quad \forall (i,j) \in A_g$$

$$Rel_{lpm}^Q \geq 0 \quad \forall (l,p,m) \in A_a$$

$$F_k \geq 0 \quad \forall k \in K$$

$$Q_d \in \mathbb{Z}_+ \quad \forall d \in D$$

$$B_d \geq 0 \quad \forall d \in D$$

$$M \gg 0$$

Decision Variables

$$x_{ijk} = \begin{cases} 1 & \text{if truck } k \text{ travels on ground arc } (i,j), \\ 0 & \text{otherwise,} \end{cases} \quad \forall (i,j) \in A_g, k \in K$$

$$y_{lpmd} = \begin{cases} 1 & \text{if quadrotor } d \text{ patrols area } p \text{ by a sortie from } l \text{ to } m, \\ 0 & \text{otherwise,} \end{cases} \quad \forall (l,p,m) \in A_a, d \in D$$

$$z_p = \begin{cases} 1 & \text{if patrol area } p \text{ is covered,} \\ 0 & \text{otherwise,} \end{cases} \quad \forall p \in P$$

$$u_{i,k} \geq 0 \quad \forall i \in N, k \in K$$

Objective Functions

Objective 1: Maximize patrol coverage

$$\max Z_1 = \sum_{p \in P} W_p z_p \quad (1)$$

Objective 2: Minimize total operational cost

$$\min Z_2 = \sum_{(i,j) \in A_g} \sum_{k \in K} C_{ij}^T x_{ijk} + \sum_{(l,p,m) \in A_a} \sum_{d \in D} C_{lpm}^Q y_{lpmd} + \sum_{k \in K} F_k \left(\sum_{j: (0,j) \in A_g} x_{0jk} \right) \quad (2)$$

Objective 3: Minimize total operational risk

$$\min Z_3 = \sum_{(i,j) \in A_g} \sum_{k \in K} R_{ij}^T x_{ijk} + \sum_{(l,p,m) \in A_a} \sum_{d \in D} R_{lpm}^Q y_{lpmd} \quad (3)$$

Objective 4: Minimize total operation time

$$\min Z_4 = \sum_{(i,j) \in A_g} \sum_{k \in K} T_{ij}^T x_{ijk} + \sum_{(l,p,m) \in A_a} \sum_{d \in D} T_{lpm}^Q y_{lpmd} \quad (4)$$

Objective 5: Minimize environmental emissions

$$\min Z_5 = \sum_{(i,j) \in A_g} \sum_{k \in K} E_{ij}^T x_{ijk} + \sum_{(l,p,m) \in A_a} \sum_{d \in D} E_{lpm}^Q y_{lpmd} \quad (5)$$

Constraints**Patrol area coverage**

$$z_p = \sum_{(l,p,m) \in A_a} \sum_{d \in D} y_{lpmd} \quad \forall p \in P \quad (6)$$

$$\sum_{(l,p,m) \in A_a} \sum_{d \in D} y_{lpmd} \leq 1 \quad \forall p \in P \quad (7)$$

Quadrotor capacity

$$\sum_{(l,p,m) \in A_a} y_{lpmd} \leq Q_d \quad \forall d \in D \quad (8)$$

Quadrotor flight endurance

$$\sum_{(l,p,m) \in A_a} T_{lpm}^Q y_{lpmd} \leq B_d \quad \forall d \in D \quad (9)$$

Launch synchronization with truck presence

$$y_{lpmd} \leq \sum_{j:(l,j) \in A_g} \sum_{k \in K} x_{ljk} \quad \forall (l,p,m) \in A_a, d \in D \quad (10)$$

Recovery synchronization with truck presence

$$y_{lpmd} \leq \sum_{i:(l,m) \in A_g} \sum_{k \in K} x_{lmk} \quad \forall (l,p,m) \in A_a, d \in D \quad (11)$$

Truck flow conservation

$$\sum_{j:(i,j) \in A_g} x_{ijk} - \sum_{j:(j,i) \in A_g} x_{jik} = 0 \quad \forall i \in N \setminus \{0\}, k \in K \quad (12)$$

Truck departure from depot

$$\sum_{j:(0,j) \in A_g} x_{0jk} \leq 1 \quad \forall k \in K \quad (13)$$

Truck return to depot

$$\sum_{i:(i,0) \in A_g} x_{i0k} \leq 1 \quad \forall k \in K \quad (14)$$

Route continuity for each truck

$$\sum_{j:(0,j) \in A_g} x_{0jk} = \sum_{i:(i,0) \in A_g} x_{i0k} = 0 \quad \forall k \in K \quad (15)$$

Subtour elimination

$$u_{ik} - u_{jk} + |N|x_{ijk} \leq |N| - 1 \quad \forall i, j \in N \setminus \{0\}, i \neq j, k \in K \quad (16)$$

$$1 \leq u_{ik} \leq |N| - 1 \quad \forall i \in N \setminus \{0\}, k \in K \quad (17)$$

Binary and domain restrictions

$$x_{ijk} \in \{0,1\} \quad \forall (i,j) \in A_g, k \in K$$

$$y_{lpmd} \in \{0,1\} \quad \forall (l,p,m) \in A_a, d \in D$$

$$z_p \in \{0,1\} \quad \forall p \in P$$

$$u_{ik} \geq 0 \quad \forall i \in N, k \in K$$

To model the integrated ground–air patrol problem, a set of assumptions is considered to maintain the realism of the model while also enabling its mathematical formulation and solution. These assumptions define the operational framework of the system and determine how the different components of the system, including trucks, quadrotors, and patrol areas, interact with one another. Several patrol trucks are available in the system, each of which is capable of moving through the ground transportation network. These trucks can stop at different points in the network and support the flight operations of the quadrotors. The presence of multiple ground vehicles in the system enables patrol operations to be conducted simultaneously across different parts of the study area, thereby improving the efficiency of area coverage. Quadrotors with different capacities are available. This difference in capacity may be related to factors such as equipment-carrying capability, flight duration, and the ability to perform different types of missions. Consequently, each quadrotor can be assigned to specific missions according to its operational characteristics. In the proposed model, missions are allocated to quadrotors while accounting for these capacity differences in order to improve the efficiency of the system.

3.2. Single-Objective LP-Metric Form

Let Z_r^* and Z_r^{max} denote the ideal and nadir values of objective r , respectively. The normalized deviation of each objective is defined as:

$$\delta_r = \frac{Z_r - Z_r^*}{Z_r^{max} - Z_r^*} \quad r = 2,3,4,5 \quad (18)$$

For maximization objectives:

$$\delta_1 = \frac{Z_1^* - Z_1}{Z_1^* - Z_1^{min}} \quad (19)$$

The aggregated LP-metric objective is:

$$\min Z = \left(\sum_{r=1}^5 \alpha_r |\delta_r|^p \right)^{1/p} \quad (20)$$

Where

$$\sum_{r=1}^5 \alpha_r = 1, \quad \alpha_r \geq 0, \quad p \geq 1 \quad (21)$$

3.3. solution method

In the Benders decomposition algorithm, the master problem includes the main binary decision variables associated with patrol assignment and ground-route decisions. By considering the main constraints and the cuts generated in previous iterations, the master problem provides candidate values for the binary variables and a lower bound for the objective function of the overall problem. Given these decisions, the corresponding subproblem is solved to evaluate the feasibility and objective value of the resulting solution and to generate the information required for Benders cuts. If the subproblem is infeasible or the resulting solution does not satisfy the required conditions, appropriate feasibility or optimality cuts are added to the master problem. This iterative process continues until the prescribed optimality tolerance is satisfied.

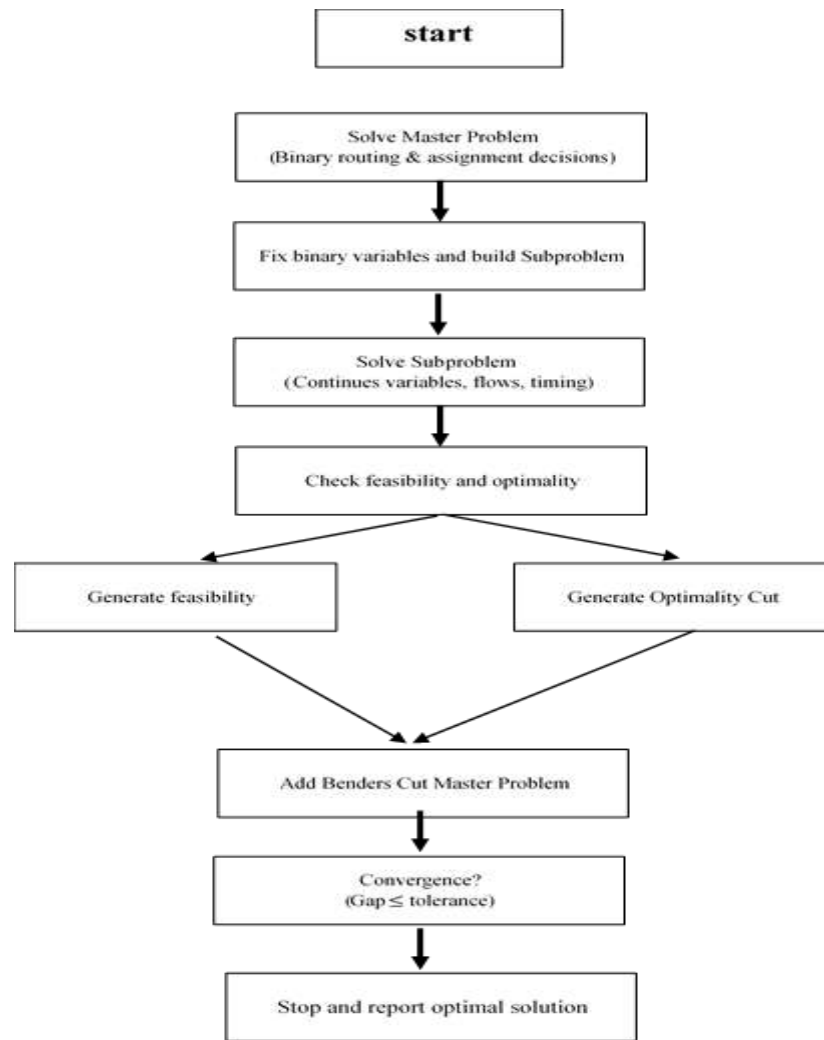


Figure 1. Benders decomposition algorithm

4. Computational Results and Numerical Experiments

In this section, the performance of the proposed robust multi-objective mathematical model for integrated truck–quadrotor patrol network design is evaluated. Numerical experiments are conducted to examine the validity of the model, analyze its behavior in the presence of conflicting objectives, evaluate the effects of different operational scenarios and key parameters, and assess the performance of the Benders decomposition method. Since the problem under investigation is multi-objective and combinatorial in nature and simultaneously incorporates criteria such as coverage, cost, risk, operational time, and pollutant emissions, the results are analyzed not only in terms of the objective function value but also from the perspective of trade-offs among the decision-making criteria.

4.1. Computational Settings and Experimental Design

The proposed model was implemented in GAMS version 42.3, and the CPLEX solver was used to solve small-sized instances. For medium- and large-sized instances, the Benders decomposition algorithm was employed to reduce computational complexity. To investigate the behavior of the model at different scales, test instances were designed with 10, 15, 20, 25, and 30 patrol nodes and different numbers of trucks. In addition, the uncertainty in route length and travel time was considered within a range of 10% to 30%. The carrying capacity of the quadrotors assigned to each truck was also considered in the range of 2 to 4 units. Operating costs were determined based on randomly generated parameters from the distributions used in the model.

Table 1. Computational environment

Parameter	Quantity
Processor Frequency	3.5 GHz
System Memory	32 GB
Modeling Software	GAMS 42.3
Solver	CPLEX
Time Limit	3600

4.2. Model Validation and Evaluation of Small Instances

In the first stage, small-sized instances were solved directly using the CPLEX solver to evaluate the correctness of the model's performance. The instances were defined based on the spatial distribution pattern of the patrol points and were classified into three groups: concentrated, semi-concentrated, and dispersed. The results showed that all small-sized instances were solved in less than 45 seconds, with an optimality gap of zero. In the concentrated instance, complete coverage of the patrol points was achieved, with the coverage reaching 100%. As the spatial dispersion of the points increased, network coverage decreased, while the cost, risk, and operational time increased simultaneously. This result indicates that the spatial structure of the network is one of the determining factors in the configuration of patrol operations.

Table 2. Exact solution results for small instances

Instances	Coverage (%)	Total Cost	Risk	Operational Time (min)	Emissions (kg)	LP-Metric Value
Concentrated	100	14750	0.18	240	1120	0.214
Semi-concentrated	86	12300	0.25	300	940	0.381
Dispersed	62	19600	0.31	380	1560	0.547

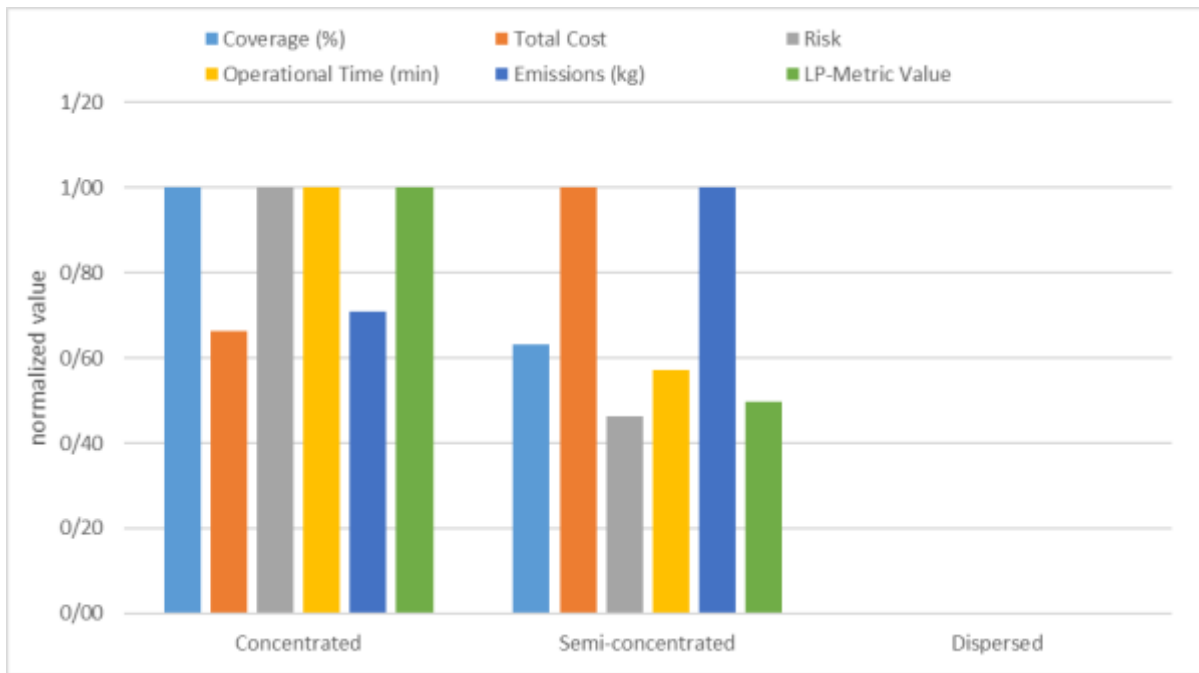


Figure 2. Comparison of base model results for small instances

The results presented in Figure 2 indicate that as the patrol points become more dispersed, achieving high coverage and low operational cost simultaneously becomes more difficult. Specifically, coverage decreased from 100% in the concentrated case to 62% in the dispersed case; in contrast, risk increased from 0.18 to 0.31, and operational time increased from 240 to 380 minutes. Therefore, this initial experiment, in addition to confirming the solvability of the model, also demonstrates the existence of trade-offs among the main objectives of the problem.

4.3. Multi-Objective Analysis and the Effect of Objective Weighting

In the next stage, the performance of the multi-objective model was evaluated under three different weighting scenarios, including the balanced, coverage-oriented, and cost-oriented scenarios. The multi-objective problem was transformed into a single objective function using the LP-metric; therefore, changes in the objective weights directly affect the structure of the final solution and the manner in which ground and aerial resources are allocated. In the balanced scenario, equal weights of 0.2 were assigned to all objectives. In the coverage-oriented scenario, the weight of the coverage criterion was increased to 0.4, while the weights of cost, time, risk and emission were each reduced to 0.15. In the cost-oriented scenario, the weight of operational cost was increased to 0.4, while the weight of coverage, time, risk and emission were each reduced to 0.15.

Table 3. Model results under different weighting scenarios

Scenario	Objective Function Value	Coverage (%)	Total Cost	Operational Risk	Total Time (min)	Pollutant Emissions (kg)
Balanced Weights	0.312	89	47800	0.26	645	3920
Coverage-Oriented	0.279	96	53600	0.29	710	4350
Cost-Oriented	0.357	78	39900	0.22	590	3150

The results show that increasing the importance of the coverage objective increased the coverage level from 89% in the balanced scenario to 96%; however, this increase in coverage was accompanied by increases in cost, operational time, risk, and pollutant emissions. In contrast, in the cost-oriented scenario, the operational cost decreased to 39,900, but the coverage level also decreased to 78%.

4.4. Effect of Network Spatial Structure on Coverage

To investigate the effect of the spatial distribution of points on system performance, three patterns, namely concentrated, semi-concentrated, and dispersed, were compared. The results showed that the spatial structure of the points has a direct effect on the ability to achieve high coverage.

In the concentrated pattern, complete coverage of 100% was achieved, whereas in the semi-concentrated and dispersed patterns, this value decreased to 86% and 62%, respectively. Therefore, the difference between the best and worst conditions in terms of coverage was 38 percentage points.

Table 4. Coverage results for patrol areas

Instance / Scenario	Area Coverage (%)	Brief Description of Model Performance
Concentrated Instance	100	Complete coverage of all areas in the concentrated structure with optimal flight
allocationSemi-concentrated Instance	86	Partial coverage due to increased spatial distance between nodes and flight range limitations
Dispersed Instance	62	Significant reduction in coverage due to high dispersion and fleet capacity limitations
Balanced Scenario	89	Appropriate balance among coverage, cost, and operational time in the base
caseCoverage-Oriented Scenario	96	Increased coverage at the expense of higher cost and flight time across
missionsCost-Oriented Scenario	78	Reduced coverage to achieve minimum cost and resource savings

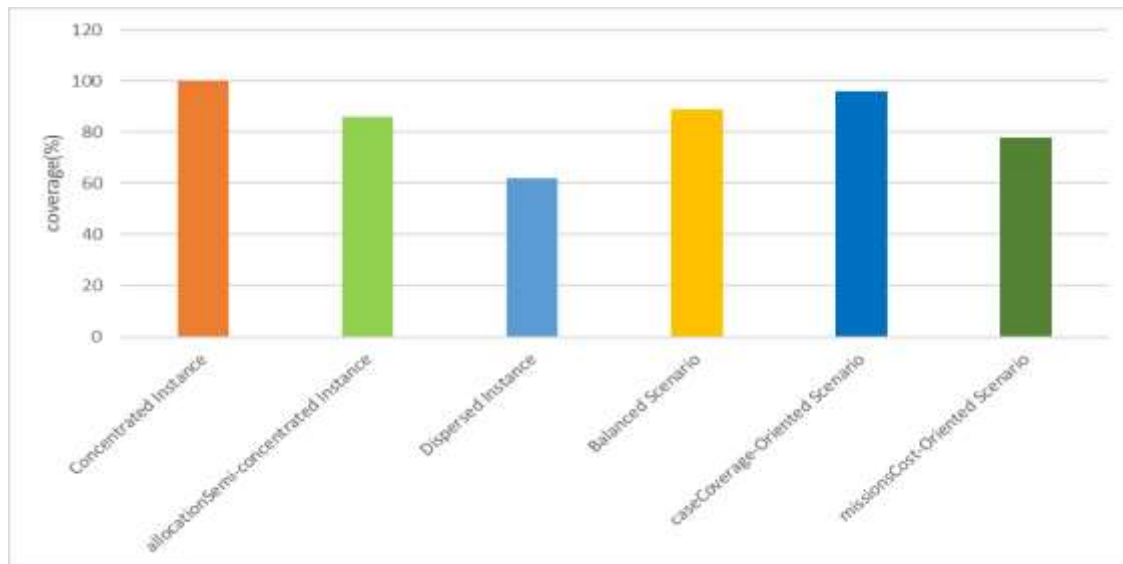


Figure 3. Comparison of the percentage of patrol area coverage under different scenarios

This result is important from the perspective of network design, as it shows that increasing the number of patrol points alone is not an appropriate criterion for evaluating network performance. The spatial dispersion of the points, the range limitations of the quadrotors, truck routes, and the coordination requirements between the two vehicles should also be considered simultaneously in network design.

4.5. Analysis of Operational Scenarios

To evaluate the robustness of the model under different operational conditions, network performance was examined under several scenarios, including the base case, increased patrol demand, increased operational constraints, and a focus on high-importance areas.

In the base case, network coverage was considered to be 89%. An increase in patrol demand resulted in a reduction in coverage to 83% and an increase in the cost and operational time indices. Under increased operational constraints, coverage also decreased to 76%. In contrast, focusing on high-importance areas resulted in an overall coverage of 87%, while approximately 95% of the high-priority points were successfully covered.

Table 5. Model results under different operational scenarios

Scenario	Area Coverage (%)	Operational Cost (Relative Index)	Operational Time (Relative Index)
Base Scenario	89	100	100
Increased Patrol Demand	83	115	112
Increased Operational Constraints	76	111	109
Focus on High-Importance Areas	87	110	100

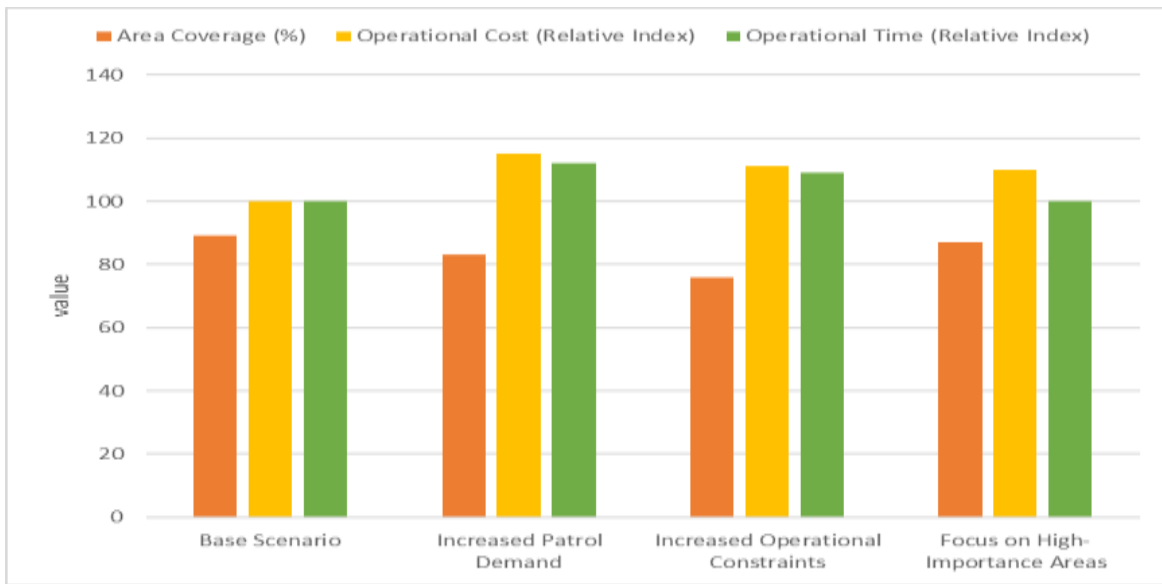


Figure 4. Comparison of system performance under different operational scenarios

The results show that the model responds appropriately to changes in operational conditions. Increased demand or operational constraints leads to higher resource consumption and a reduction in coverage, whereas defining priorities for important areas enables more targeted resource allocation. This feature is particularly important for real-world patrol applications, where not all areas have the same level of importance.

4.6. Sensitivity Analysis of Key Parameters

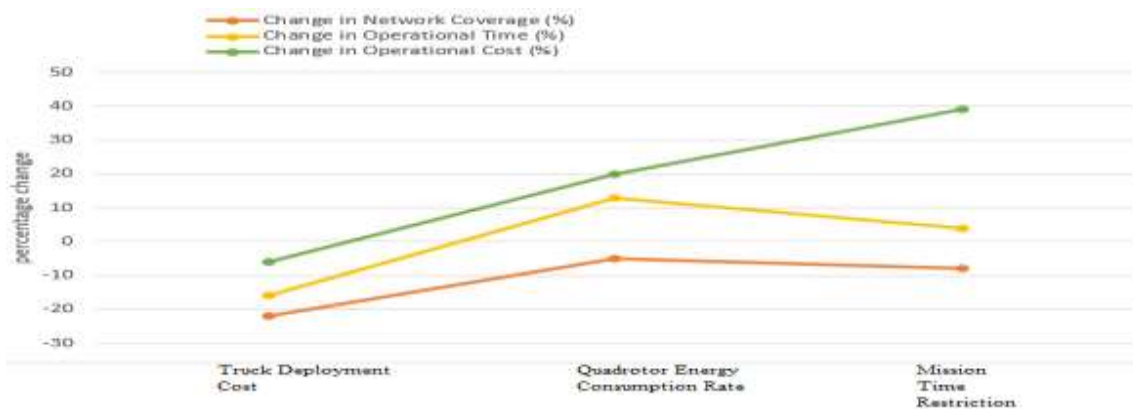
To investigate the sensitivity of the model solution to changes in operational parameters, three key parameters, including truck deployment cost, quadrotor energy consumption rate, and mission time restriction, were examined.

A 20% increase in truck deployment cost resulted in a 22% reduction in network coverage. Under the same conditions, operational time increased by 6% and the overall cost increased by approximately 10%.

Furthermore, a 30% increase in the quadrotor energy consumption rate led to a 5% reduction in coverage, an 18% increase in operational time, and a 7% increase in cost. Under the scenario of a 20% reduction in the mission time restriction, operational cost increased by 35% and coverage decreased by 8%.

Table 6. Sensitivity analysis results for key parameters

Model Parameter	Parameter Change (%)	Change in Network Coverage (%)	Change in Operational Time (%)	Change in Operational Cost (%)
Truck Deployment Cost	+20%	-22	6	10
Quadrotor Energy Consumption Rate	+30%	-5	18	7
Mission Time Restriction	-20%	-8	12	35

**Figure 5.** Comparison of the effects of changes in key model parameters on network coverage, operational time, and operational cost

The sensitivity analysis results indicate that network performance is sensitive to economic and operational parameters. Among the parameters examined, an increase in truck deployment cost has a significant effect on the network structure and coverage level, while an increase in quadrotor energy consumption also directly affects operational time and coverage level. These findings highlight the importance of accurately calibrating cost and energy parameters in real-world applications of the model.

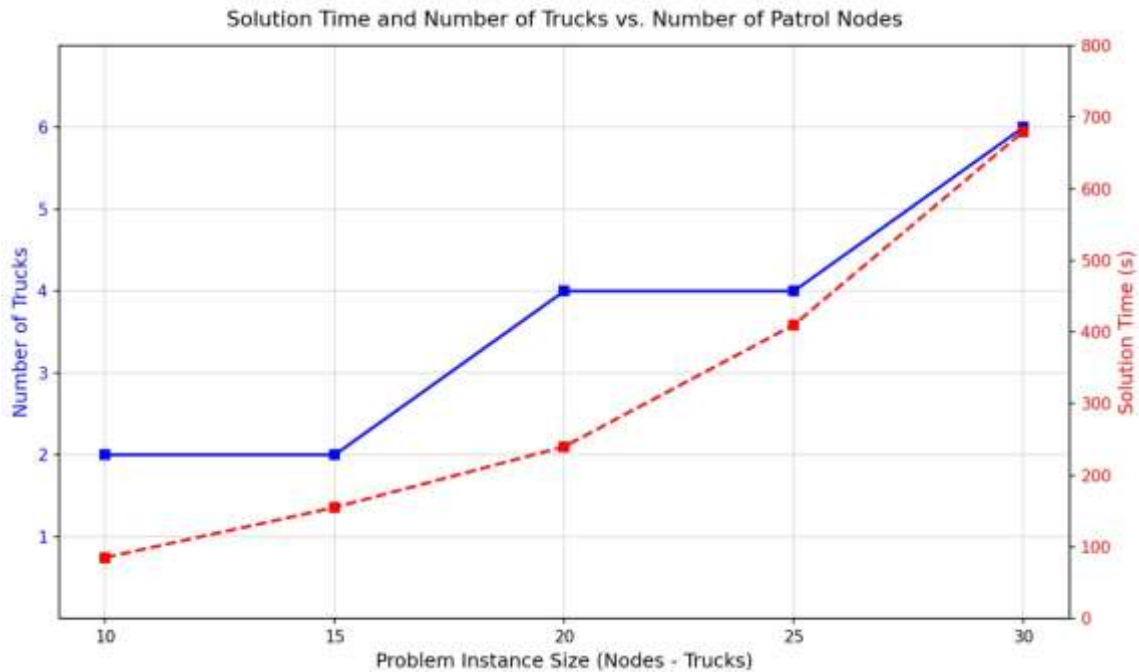
4.7. Evaluation of Benders Decomposition Algorithm Performance

As the problem size increases, directly solving the model faces greater computational limitations. Therefore, the Benders decomposition algorithm was employed to solve medium- and large-sized instances.

The results showed that the algorithm converged to optimal solutions for instances with zero optimality gap and to near-optimal solutions with an optimality gap of less than 0.5% for the remaining examined instances. The solution time increased from 85 seconds for an instance with 10 nodes and 2 trucks to 680 seconds for an instance with 30 nodes and 6 trucks. In addition, the number of algorithm iterations increased from 12 to 55.

Table 7. Evaluation of benders decomposition algorithm performance

Number of Patrol Nodes	Number of Trucks	Solution Time (s)	Number of Algorithm Iterations	Final Optimality Gap (%)	Convergence Status
10	2	85	12	0.12	Near-optimal
15	2	155	18	0.18	Near-optimal
20	4	240	25	0.25	Near-optimal
25	4	410	38	0.35	Near-optimal
30	6	680	55	0.45	Near-optimal

**Figure 6.** Changes in solution time and number of benders decomposition algorithm iterations with respect to problem size

According to the results, increasing the problem size led to an increase in computational time and the number of algorithm iterations. However, even for the largest instance examined, the final optimality gap remained below 0.5%. The increase in computational time with problem size indicates that the growing network complexity and the associated computational burden become more significant as the problem scale increases.

5. Conclusion

In this study, a robust multi-objective mathematical model was developed for the design and planning of an integrated truck–quadrotor patrol network. By integrating allocation, routing, scheduling, and ground–air operational decisions, the proposed model enables the simultaneous management of conflicting operational criteria. By considering operational uncertainties and constraints such as flight range, capacity, mission time, and coordination between vehicles, the proposed structure provides a flexible framework for designing patrol networks in large-scale and dynamic environments.

The computational results showed that the spatial distribution of patrol areas has a significant effect on network performance, with coverage levels of 100%, 86%, and 62% obtained for concentrated, semi-concentrated, and dispersed configurations, respectively. Furthermore, changes in the decision-maker's preferences through objective weighting systematically affected the model's performance. In the balanced scenario, 89% coverage was achieved with an operational cost of 47,800 units, whereas increasing the priority assigned to coverage increased coverage to 96%, while simultaneously increasing operational cost and mission time. In contrast, focusing on cost control reduced the operational cost to 39,900 units, but coverage decreased to 78%. These results confirm the existence of a clear trade-off among coverage level, operational cost, and other operational performance indicators.

The sensitivity analysis also showed that truck deployment cost and mission time restriction are among the parameters affecting the stability of network performance. A 20% increase in truck deployment cost resulted in a 22% reduction in coverage, whereas tightening the mission time restriction by 20% led to a 35% increase in operational cost. In addition, the results highlighted the importance of coordinating ground and aerial operations in improving the operational efficiency and response capacity of the network.

From a computational perspective, the application of Benders decomposition enabled instances of different sizes to be solved to optimality or near-optimality, with a final optimality gap of less than 0.5%. The gradual increase in solution time as the problem size increased demonstrates the applicability of this approach to patrol problems with a hybrid structure. Overall, the findings indicate that the proposed model can be used as a decision-support tool for the design and operation of integrated patrol networks and, by adjusting the priorities assigned to the objectives, can adapt the coverage level and resource allocation to different operational conditions and existing constraints.

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