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Joint optimization of pricing and capacity allocation for two competitive airlines under demand uncertainty

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Nowadays, airline industries should overcome different barriers regarding the fierce competition and changing consumer behavior. Thus, they attempt to focus on joint decision making which enables them to set pricing and capacity allocation to maximize their profits. In this research, we develop a model to optimize pricing and capacity allocation in a duopoly of single-flight leg for two competitive airlines. The problem considers actual assumptions about flexible partitions in flight's cabins and additionally demand uncertainty. There is a flexible partitioning of business and economy cabins and demand is assumed price-dependent with additive uncertainty. The capacity and pricing decisions are simultaneously determined through indirect channels. Moreover, a numerical study is developed to investigate how market components and competition conditions change pricing, capacity, and profit levels. The results show that increasing market volume like decreasing price sensitivity provides higher levels of price and profits. Moreover, intensified competition never leads to higher prices. Thus, a competitive network of airlines provides better impact on market mechanism to achieve competitive prices for both economy and business classes.

***Keywords**: Revenue management, Pricing, Capacity allocation, Competition, Airline Networks*.

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1. Introduction

In 1978, Revenue management (RM) started as an independent scholar branch by establishment of Airline Deregulation Act. This transformation originally developed when Civil Aviation Board (CAB) of United States considered the voluntary setting of pricing decisions for airlines. These new regulations, prompted rapid change and innovation in airline industry Talluri and van Ryzin [22]. After that, academic and business investigations grew dramatically through publishing research reports and papers in domain of revenue management and pricing. Over the years, RM developed for airline industry, hotel industry, car rental, and other industries which encounter with deteriorating items and demand uncertainty.

The aviation industry plays a critical role in creating wealth and employment for the economy and society. Air transportation not only helps passenger's health and logistics services but also provide basic inputs for economic activities in other sectors, such as tourism, business, investment, and supply chain management. Therefore, it is very important for the economy to reach high-quality air services at competitive expense levels Donehue and Baker [7].

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In airline industries, the first improvements for RM focused on basic models that attempt to optimize selling capacity of a flight for a specific airline. However, currently reserving of airlines for multiple flight legs and setting prices for different classes provide more complex problems. Moreover, such optimization models will become more actual if they can consider the real competitive situations of the industry. For this reason, airlines need new policies for determining joint capacity and pricing regarding the opponents' decisions in a competitive market.

Airlines have a vested interest in providing the best possible service for passengers. The biggest airlines, as they have been, compete each day with the appearance and popularity of smaller companies which have low cost travels and low-cost carriers. The analysis conducted by economists suggests that the state of national airline competition has been tightened in recent years, with fewer choices than when they travel from one city to another. This positive trend is alive as well as a wide range of airlines increase competition and even enter novel markets which help to offer flexible fare prices with more options. Thus, the industry is more competitive compared to first practices of RM.

Many managers identify competition as an effective factor on their pricing. Thus, a critical issue is that "How should we place competition in calculating price responses?". One of the main tools in order to tackle with competitive-based problems in RM is Network Management. Network management is important in any major RM industry that sells products containing more than one source. Network management can result in substantial increases in profit over managing revenue by fare class alone. Network management is also important for airlines offering communication services. The purpose of network management is to improve revenues by managing a combination of products, as well as a combination of fare classes that are sold for each product Phillips [17].

In this article, we study the problem of pricing and capacity allocation for two competitive airlines whit different fare classes on a single flight leg. Each airline has a number of fixed seats that are sold in two different fare classes (economy and business). The model of both airlines developed based on the model of Kyparisis and Koulamas [10] where they focus on joint optimization of capacity and pricing under demand uncertainty. According to the random demand and the competition between the two airlines, the key decision for airlines is determining the optimal price and the number seats to reserve for tow fare classes.

Although the roots of profit management are highly related to the airline industry, the recent developments regarding airlines reservation systems as well as the application of earnings management techniques in many other industries, provides suitable platform for capacity control. The new systems do not necessarily depend on the constraints of reservation systems and directly control the prices of products in a dynamic fashion Straussa et al. [21]. Thus, improvement in new platforms of e-commerce reveals real-time decision-making abilities for capacity allocation and pricing. From distribution channel point of view, the decisions of capacity allocation and pricing are provided through indirect channels (Global Distribution Systems (GDSs) and travel agents) or direct channels (Airline websites) Wang et al. [23]. Herein, the problem of two competitive airlines is modeled only by considering direct channels. In this paper, we assume a fixed partition to separate business and economy cabins. In the sequel, we call this problem as Two Competitive Airlines' Revenue Management Problem (TCARMP).

The objective of TCARMP is the allocation of optimal seating capacity and pricing decision for an airline through different channels in two classes of economic and business fares where both airlines are in a competition and concentrate on profit maximization. At this problem, we follow the improvement of airlines' decisions to sell at the right price to the selected groups of customers. In

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addition, we analyze the impact of market size, price sensitivity, and competition coefficient of demand as well as changes in cost structure on optimal levels of pricing and capacity.

A key aspect of demand in operations/revenue management models, unlike traditional models in the economy, is the presence of uncertainties. Random demand functions are sometimes generated from a major random consumer model. The importance and difficulty of understanding the interactions of pricing decisions and inventory in uncertain demand environment is well-defined. Thus, we model demand as a general stochastic function of price, which encompasses additive-multiplicative models typically used in the literature of newsvendor models with pricing Petruzzi and Dada [16].

In most of allocation models, the market share capacity is considered as a discounted price for economy class. As a result, the economic class capacity is filled and then a business class demand is returned, but in recent research this assumption is done at the same time.

The rest of the paper is structured as follows. After a brief literature review in Section 2, Section 3 defines the problem mentioned above and proposes the mathematical formulation with optimization procedure. Section 4 deals with the numerical study for the case of uniform demand uncertainty and shows the results of optimal decisions for a numerical case. Then, in Section 5 the results of sensitivity analysis based on the main parameters of the model are demonstrated. Finally, Section 6 concludes the paper with proposed future research.

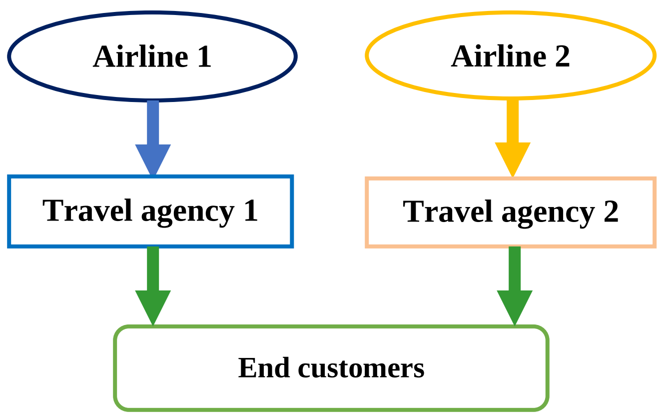

Figure 1. Two competitive airlines' network with direct distribution channels

2. Literature Review

There is a large body of literature on RM problems and airline yield management. RM problems have been reviewed by Weatherford and Bodily [24] and McGill and Van Ryzin [13] while Bitran and Caldentey [3] reviewed dynamic pricing policies and their relation to RM. Capacity control as a traditional and significant aspect of RM has been reported in many studies. Littlewood [12] first considered the two class, single leg, capacity allocation problem in this field. Belobaba [2] examined the multiple fare product problem and proposed the expected marginal seat revenue

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heuristics (ESMRa and EMSRb). After that, some assumptions in static single-leg models were relaxed by Curry [6]; Wollmer [25]; Brumelle and McGill [4] and Robinson [18].

Considering pricing decisions jointly to capacity control, Weatherford [24] first highlighted the importance of joint pricing and seat allocation. Netessine and Shumsky [15] considered the airline yield management problem of optimal allocation of seat inventory among fare classes. Cizaire and Belobaba [5] investigated joint optimization of airline pricing and class seats where two optimization problems analyzed separately.

Another group of research focuses on planning decisions of airlines under competition using different types of space models developed by Harold [8]. Li and Oum [11] describe a seat allocation problem for two airlines in competition. They used a relatively limited assumption about demand rates between airlines and identify a symmetric equilibrium. Zhao and Atkins [26] described a model with two airlines competing for passengers in a demand class. Zhao et al. [27] developed a joint pricing and capacity allocation for a duopoly with linear additive price-demand functions and bivariate normally distributed joint demand. They analyzed the competitive games between the airlines.

The stream of analyzing pricing strategies under competitive situation has been widely used in traditional supply chain management and product marketing literature since the rapid development of e-commerce. Simchi-Levi et al. [20] reviewed the main streams of joint optimization of pricing and inventory control. Netessine and Rudi [15] study newsvendors selling a product in a single sales season and investigate how consumer switching upon a stock-out affects inventory decisions. In the world of airlines, multi-channel distribution has been run for several years, but only a few studies focus on the influence of multi-channel distribution on airline revenues. Jarach [9] indicated that airfare tickets will bring changes to airline competitions through analysis of the impact of different airlines. Shon et al. [19] suggested that online channels dominate the ticket market compared to traditional channels.

In this article, we study the problem of pricing and capacity allocation competition for two airlines with different fare classes on a single flight leg. Each airline has a number of fixed seats that are sold in two different fare classes (economy and business). Obviously, the ticket price for the business class is higher than the economic class. We assume additive demand uncertainty for modeling the demand structure of different classes and airlines. According to the random demand and the competition between the two airlines, the key decision for airlines is determining the optimal price and the number of seats to reserve for two fare classes. The decisions of capacity and price are provided through direct channels for instance airline websites, as shown in Figure 1.

In this paper, we consider a different airline with a movable partition to separate their business and economy cabins which have the same seat pitch. Kyparisis and Koulamas [10] solved the single-flight leg two-cabin airline revenue management problem for an airline in which there is a flexible partition of cabins and determine the optimal cabin partition and the optimal fares for both cabins with both linear and iso-elastic multiplicative price-demand function. Thus, we model TCARMP as a problem for two competing airlines with indirect channels under demand uncertainty and flexible partitioning of classes based on Kyparisis and Koulamas [10] assumptions.

Moreover, the majority of literature (Littlewood, [12]; Netessine and Shumsky [14] and Zhao et al. [27]) assume the economy fare problems with sequential timing of demand fulfilment in a single-cabin of multiple economy fare classes. They also consider preceding of discount customers or last-minute customers. The fundamental distinction between TCARMP and above research is assuming coincident streams of business and economy class demands. In addition, we analyze the impact of market size, price sensitivity as well as changes in competition coefficient on airline optimal levels of pricing, capacity, and profits.

3. The Model

We develop a model of a pricing and allocation problem for two airlines sharing a fixed inventory of seats. Two fare classes, economy and business, are offered by any airline. The two fare classes are in a same assigned flight by any airline where any airline provides differentiated services for its customers based on airline's brand and standards. Nevertheless, these fare classes are associated with two distinct sets, and are therefore priced differentiated.

3.1 Notations and assumptions

In our notation, business class represents the more expensive prices regarding the higher service and standard levels of any airline. In TCARMP model, two airlines sale through their direct channels which are competing on selling prices based on the preference of customers. Customers can purchase one of the economy or business classes $k = 1, 2$ through channels. Airlines are showed by $i = 1, 2$. With loss of generality, we omit the indices of channels because we assume that all the tickets will be sold through direct channels for any airlines. These assumptions can be released for network models that assume part of demands will be fulfilled by indirect channels like GDSs.

Table 1. Summary of notations

Notations	**Description**
Indices	
$i = 1, 2$	Airline indices
$k = 1, 2$	Indices of fare classes in any airline (economy and business)
Parameters	
a_{ik}	Market volume for demand of class k and airline $\boldsymbol{i}$
b_{ik}	Price sensitivity for demand of class k and airline $\boldsymbol{i}$
γ_{ik}	Price competition coefficient between the same fare classes k of different airlines
Decision variable	
p_{ik}	Ticket selling price of class k and airline $\boldsymbol{i}$
Z_{ik}	Stocking decision of class k and airline $\boldsymbol{i}$
V_{ik}	Capacity decision of class k and airline $\boldsymbol{i}$
C_{ik}	Total cost per ticket of class k and airline $\boldsymbol{i}$
D_{ik}	Demand function of class k and airline $\boldsymbol{i}$
$y_{ik}(p_{ik})$	Price sensitive linear demand of class k and airline $\boldsymbol{i}$

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ε_{ik}	Demand Random variable of class k and airline i

3.2 Demand Function

The price-dependent demand with additive uncertainty for two competing airlines can be formulated as follows:

$$D_{ik}(p_{ik}, p_{3-i,k}, \varepsilon_{ik}) = y_{ik}(p_{ik}) + \varepsilon_{ik}, a_{ik}, b_{ik} > 0 \quad (1)$$

In the function, $y_{ik}(p_{ik}) = a_{ik} - b_{ik}\ p_{ik} + \gamma_{ik}\ p_{3-i,k}$ is the deterministic linear price-sensitive part where $\gamma_{ik}\ p_{3-i,k}$ is the effect of competition between the same classes in two competing airlines. In addition, ε_{ik} is the random demand part with CDF, $F(.)_{ik}$ and density function $f\ (.)_{ik}$. For the sake of simplicity, we neglect the possible competition between different classes for an airline (Internal competition) and additionally for two competing airlines (External competition). These assumptions are applicable because optimal partitioning and pricing is not so sensitive to random demand distribution of different classes in a similar flight Kyparisis and Koulamas [10] . Moreover, if the two assumed airlines have differentiated brands, the comparison between the prices of lower brand business class and the higher brand economy class can be analyzed where herein we leave it for future research.

3.3 The model of TCARMP

Accordingly, the profit function of airline i for class k, $\Pi_{ik}(p_{ik}, V_{ik})$, is as follows:

$$\Pi_{ik} = \begin{cases} p_{ik} D_{ik}(p_{ik}, p_{3-i,k}, \varepsilon_{ik}) - C_{ik} V_{ik} & , D_{ik} < V_{ik} \\ (p_{ik} - C_{ik}) V_{ik} & , D_{ik} > V_{ik} \end{cases} \quad (2)$$

According to Petruzzi and Dada [16], the transformation, $Z_{ik} = V_{ik} - y_{ik}(p_{ik})$, is considered to simplify the problem as follows:

$$\Pi_{ik} = \begin{cases} p_{ik}\left(y_{ik}(p_{ik}) + \varepsilon_{ik}\right) - C_{ik}\left(Z_{ik} + y_{ik}(p_{ik})\right) & \varepsilon_{ik} < Z_{ik} \\ \left(p_{ik} - C_{ik}\right)\left(Z_{ik} + y_{ik}(p_{ik})\right) & \varepsilon_{ik} > Z_{ik} \end{cases} \quad (3)$$

Where Z_{ik} is called as stocking decision. Defining $\Lambda(Z_{ik}) = \int_{A_{ik}}^{Z_{ik}} (Z_{ik} - u_{ik}) f\ (u_{ik}) du$ as expected leftover, and $\Theta(Z_{ik}) = \int_{Z_{ik}}^{B_{ik}} (u_{ik} - Z_{ik}) f\ (u_{ik}) du_{ik}$, as the expected shortage, the expected profit function can be written as follows:

$$E(\Pi_{ik}(Z_{ik}, p_{ik})) = \psi_{ij}(p_{ik}) - L_{ij}(Z_{ik}, p_{ik}) \quad (4)$$

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where

$$\psi_{ij}(p_{ik}) = (p_{ik} - C_{ik})(y_{ik}(p_{ik}) + \mu_{ik}) \tag{5}$$

and

$$L_{ij}(Z_{ik}, p_{ik}) = C_{ik}\Lambda(Z_{ik}) + (p_{ik} - C_{ik})\Theta(Z_{ik}). \tag{6}$$

Equation (5) represents the riskless profit function and Equation (6) is the loss function. Thus, the objective is to maximize the expected profit function with respect to joint optimal pair of stocking and pricing decisions, (Z_{ik}, p_{ik}) :

$$\underset{Z_{ik},\, p_{ik}}{Max} E(\Pi_{ik}(Z_{ik}, p_{ik})). \tag{7}$$

Moreover, after finding the optimal pair of stocking and pricing, (Z_{ik}^*, p_{ik}^*) the optimal capacity for particular class of any airline is determined by $V_{ik}^* = y_{ik}(p_{ik}^*) + Z_{ik}^*$.

Theorem 3.1. The optimal stocking and pricing decisions for any competitive airline i and class k under additive demand uncertainty are developed as follows:

$$p_{ik}^* = p_{i,k}^0 - \frac{\Theta(Z_{ik})}{2b_{ik}} \tag{8}$$

$$Z_{ik}^* = F^{-1}\left(\frac{p_{ik} - C_{ik}}{p_{ik}}\right) \tag{9}$$

where

$$p_{i,k}^0 = \frac{a_{ik} + b_{ik}C_{ik} + \mu_{ik} + \gamma_{ik}\, p_{3-i,k}}{2b_{ik}} \tag{10}$$

The term $p_{i,k}^0$ denotes the optimal riskless price of class k for airline i, which is the price that maximizes $\psi_{ij}(p_{ik})$. Moreover, the optimal capacity of airline i for class k is calculated by $V_{ik}^* = y_{ik}(p_{ik}^*) + Z_{ik}^*$.

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Proof. The first and second optimality conditions for maximizing $E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)$ with respect to Z_{ik} and p_{ik} , are as follows:

a) First-order optimality conditions

$$\frac{\partial E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)}{\partial Z_{ik}} = -C_{ik} + (p_{ik})\left[1 - F(Z_{ik})\right] = 0 \tag{11}$$

$$\frac{\partial E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)}{\partial p_{ik}} = a_{ik} - 2b_{ik}\, p_{ik} + \gamma_{ik}\, p_{3-i,k} + \mu_{ik} + b_{ik}C_{ik} - \Theta(Z_{ik}) = 0 \tag{12}$$

b) Second-order optimality conditions

$$\frac{\partial^2 E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)}{\partial Z_{ik}^2} = -P_{ik} f\,(Z_{ik}) < 0 \tag{13}$$

$$\frac{\partial^2 E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)}{\partial^2 p_{ik}} = -2b_{ik} < 0 \tag{14}$$

$$\frac{\partial^2 E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)}{\partial p_{ik}\partial Z_{ik}} = \frac{\partial^2 E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)}{\partial Z_{ik}\partial p_{ik}} = 1 - F(Z_{ik}) > 0 \tag{15}$$

$$\Delta_{ik} = \left(\frac{\partial^2 E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)}{\partial^2 p_{ik}}\right)\left(\frac{\partial^2 E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)}{\partial^2 Z_{ik}}\right) - \left(\frac{\partial^2 E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)}{\partial Z_{ik}\partial p_{ik}}\right)^2 = (-2b_{ik}\, p_{ik} f\,(Z_{ik}) - \left[1 - F(Z_{ik})\right]^2 > 0 \tag{16}$$

It can be seen that by checking relation (16), $E\left(\Pi_{ik}(Z_{ik}, p_{ik})\right)$ is concave in Z_{ik} for a given p_{ik} . Thus, the optimal decisions that are developed by relations (11) and (12) are the joint optimal decisions for airline $\boldsymbol{i}$ and class k . These optimal decisions are demonstrated in (7) and (8).■

Regarding Theorem 3.1 the optimal pair of capacity and pricing for any airline can be determined sequentially because any optimal pricing decision of any airline relates on the proper price for the opponent airline. Herein, we develop a simple procedure in order to find the optimal

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decisions for both airlines. This procedure is depicted in Figure 2. In the next section, a numerical study for two competing airlines is developed.

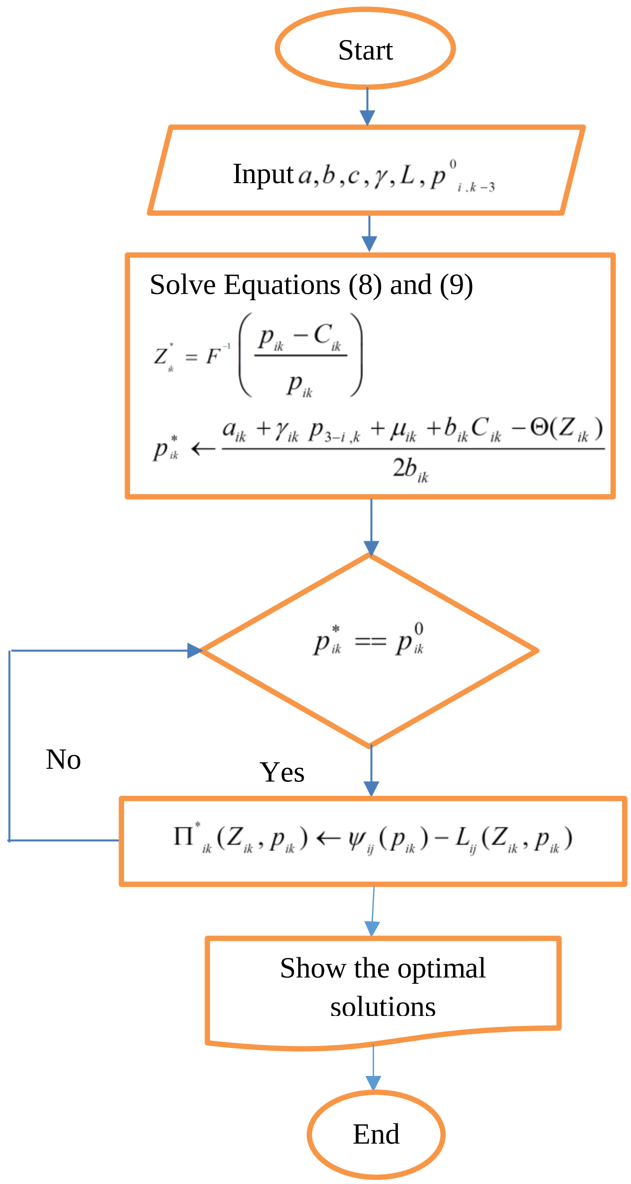

Figure 2. Procedure of finding the optimal solutions of TCARMP

4. Numerical Study

In this section, we illustrate our joint pricing and seat allocation approach with a numerical example.

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4.1 The case of uniform demand uncertainty

According to Kyparisis and Koulamas [10] , we consider a single-aisle aircraft comparable to B737 or A320 family for airline 1 and 2. In addition, we do not have any limited for capacity. We consider uniform distribution $U\left[0, L_{ik}\right]$ with probability distribution $f(u) = \begin{cases} \frac{1}{B-A} & if\ 0 \le u \le L \\ 0 & otherwise \end{cases}$ and cumulative distribution function, $F(z) = \begin{cases} 0 & z < 0 \\ \frac{z}{L}\ if & 0 \le z < L \\ 1 & L \le z \end{cases}$.

Due to the distribution of random demand, the expected leftover and shortage can be developed as follows

$$\Lambda(Z_{ik}) = \int_{A_{ik}}^{Z_{ik}} (Z_{ik} - u_{ik}) f(u_{ik}) du = \frac{(Z_{ik})^2}{2L_{ik}} \tag{17}$$

$$\Theta(Z_{ik}) = \int_{Z_{ik}}^{B_{ik}} (u_{ik} - Z_{ik}) f(u_{ik}) du_{ik} = \frac{(Z_{ik} - L_{ik})^2}{2L_{ik}} \tag{18}$$

It is assumed that the price of the same class from the other competing airline, $p_{3-i,k}$, is set initially based on the procedure of optimization (Fig. 1). Regarding the relation of p_{ik}^* and Z_{ik}^* from (10), it can be seen that:

$$p_{ik}^* = C_{ik} \left(\frac{L_{ik}}{L_{ik} - Z_{ik}}\right) \tag{19}$$

Therefore, concerning to (11) and (18), the integrated equation to find Z_{ik}^* is as follows:

$$G_{ik}(\bar{Z}_{ik}) = (\bar{Z}_{ik})^3 - 4L_{ik}\left(b_{ik} - \gamma_{ik}\right) p_{i,k}^0 (\bar{Z}_{ik}) - 4C_{ik} L_{ik}{}^2 \left(b_{ik} - \gamma_{ik}\right) = 0 \tag{20}$$

Where $\bar{Z}_{ik} = Z_{ik} - L_{ik}$.

4.2 Numerical case

For a sample case, the parameters of the problem are set as shown in Table 2. Thus, the numerical results are summarized in Table 3.

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Table 2. Summary of parameters for numerical case

$a_{11} = a_{21} = 500$	$\gamma_{11} = \gamma_{21} = 0.5$	$C_{11} = 20$
$a_{12} = a_{22} = 150$	$\gamma_{12} = \gamma_{22} = 0.2$	$C_{12} = 40$
$b_{11} = b_{21} = 4$	$\varepsilon_{11}, \varepsilon_{21} \sim U[0,5]$	$C_{21} = 20$
$b_{12} = b_{22} = 1.2$	$\varepsilon_{12}, \varepsilon_{22} \sim U[0,10]$	$C_{22} = 40$

Table 3. Optimal values for economy/business classes

Optimal values	**Economy class**	
	Airline1	Airline 2
Optimal stocking	3	3
Optimal ticket price	78	78
Price-dependent demand	228	228
Capacity	231	231
Optimal profit	13239	13239
	Business class	
Optimal stocking	5	5
Optimal ticket price	92	92
Price-dependent demand	58	58
Capacity	63	63
Optimal profit	3143	3143

5. Sensitivity Analysis and Results

In this section, we present our sensitivity analysis and results. The purpose of these analysis is to examine the performance of the proposed model, to determine the range of variables and additionally the effect of parameters on optimal decisions under demand uncertainty and competition space.

5.1 The impact of market volume

In this section, it is assumed that the potential demand for the business class is changing with respect to economy tickets' market by a constant ratio such as η . Therefore, the sensitivity analysis is developed considering $a_{i2} = \eta a_{i1}$. The market volume for the airline 1 is considered 500 for the economic class and 150 for the business class. Then, a_{21} will be changed in domain (500, 2000) for the market of airline 2 by $a_{i2} = \eta a_{i1}$ when η is 0.3. The results of optimized decisions and profit levels for both airlines are summarized in Table 4 . Figure. (3), shows that the increase of market volumes for the classes of airline 2 results in increase for prices of both business and economy classes. Meanwhile, the price of the business class is at a higher level than the economy class. Furthermore, the increase of price levels is more for airline 2. Figure (4), demonstrates that there is a slight increase in the capacity of airline 1, unlike that, airline 2 increases its capacity and at each

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stage the capacity allocation rate increases for the business class. Figure (5) shows that the profit of airline 2 increases dramatically in comparison to airline 1 where such increase is more for business tickets.

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Table 4. The impact of market volume on optimal decisions and optimal profit of airlines

Market volume		Airline1								Airline2							
a_{21}	a_{22}	V_{11}	V_{12}	Total capacity	p_{11}	p_{12}	Π_{11}	Π_{12}	Total profit	V_{21}	V_{22}	Total capacity	p_{11}	p_{11}	Π_{11}	Π_{12}	Total profit
500	150	231	63	294	78	92	13239	3143	16382	231	63	294	78	92	13239	3143	16382
600	180	234	64	298	78	93	13594	3261	16855	281	79	360	90	104	19634	4906	24540
700	210	237	65	302	79	94	13955	3381	17336	332	94	426	103	117	27308	7034	34342
800	240	240	67	307	80	95	14320	3558	17878	382	109	491	115	130	36219	9539	45758
900	270	243	68	311	81	96	14690	3683	18373	432	125	557	128	142	46385	12450	58835
1000	300	247	69	316	82	97	15126	3810	18936	483	140	623	140	155	57927	15717	73644
1100	330	250	70	320	82	98	15506	3939	19445	533	155	688	153	168	70615	19362	89977
1200	360	253	72	325	83	99	15891	4130	20021	583	170	753	165	180	84558	23384	107942
1300	390	256	74	330	84	100	16280	4283	20563	633	185	818	178	193	99756	27787	127543
1400	420	259	75	334	85	101	16674	4420	21094	683	201	884	191	205	116209	32598	148807
1500	450	262	76	338	85	102	17073	4560	21633	734	216	950	203	218	134101	37758	171859
1600	480	265	77	342	86	103	17477	4701	22178	784	231	1015	216	231	153076	43296	196372
1700	510	269	79	348	87	105	17952	4909	22861	834	247	1081	228	243	173306	49415	222721
1800	540	272	80	352	88	106	18366	5055	23421	884	262	1146	241	256	194792	55720	250512
1900	570	275	81	356	89	107	18784	5204	23988	934	277	1211	253	268	217532	62404	279936
2000	600	278	82	360	89	108	19207	5355	24562	985	292	1277	266	281	241773	69465	311238

Table 5. The impact of price sensitivity on optimal decisions and optimal profit of airlines

Price sensitivity		Airline1								Airline2							
b_{21}	b_{22}	V_{11}	V_{12}	Total capacity	p_{11}	p_{12}	Π_{11}	Π_{12}	Total profit	V_{21}	V_{22}	Total capacity	p_{11}	p_{11}	Π_{11}	Π_{12}	Total profit
2.66	0/8	239	66	305	80	95	14207	3495	17702	4	6	246	72	318	112	128	22374
3.00	0/9	237	65	302	79	94	13920	3376	17296	4	6	242	70	312	100	116	19277
3.33	1	234	64	298	78	93	13610	3271	16881	3	6	238	68	306	91	106	16863
3.66	1/1	232	63	295	78	92	13386	3177	16563	3	5	234	65	299	84	98	14851
4.00	1/2	231	63	294	78	92	13239	3143	16382	3	5	231	63	294	78	92	13239
4.33	1/3	230	62	292	77	91	13106	3063	16169	3	5	227	61	288	72	86	11828
4.66	1/4	228	62	290	77	91	12928	3039	15967	3	5	224	59	283	68	81	10673

5.00	1/5	227	61	288	77	90	12817	2967	15784	3	4	221	56	277	64	77	9677

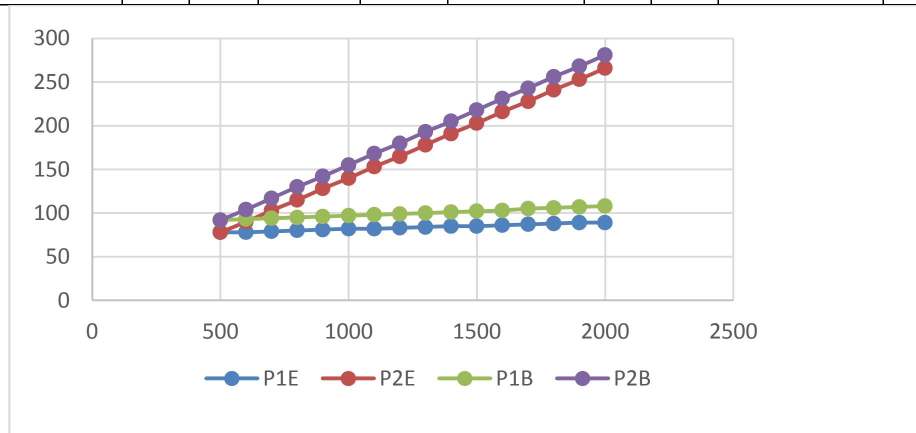

Figure 3. Impact of market volume parameters on pricing for different classes in two airlines

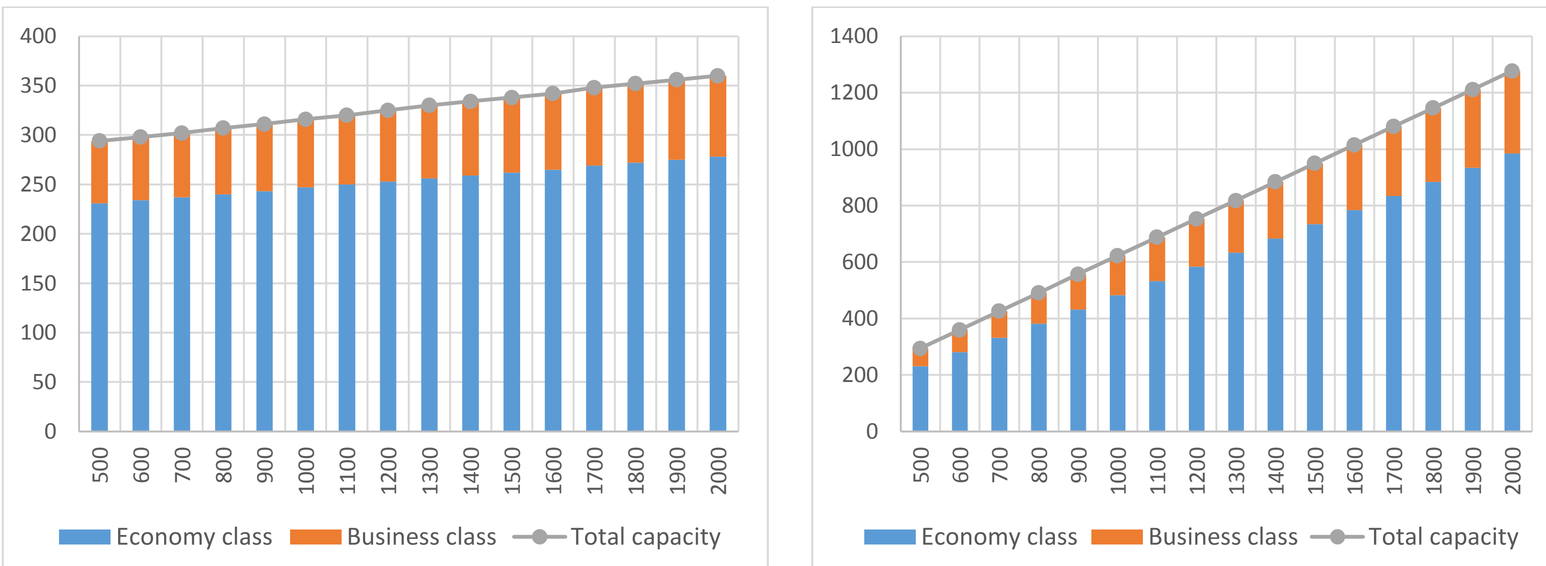

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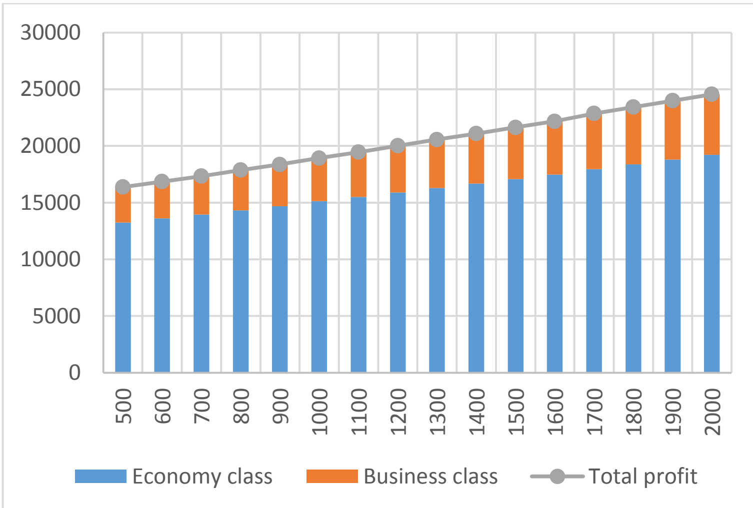
Figure 4. Impact of market volume on capacity of two competing airlines 1 (left) and 2 (right)



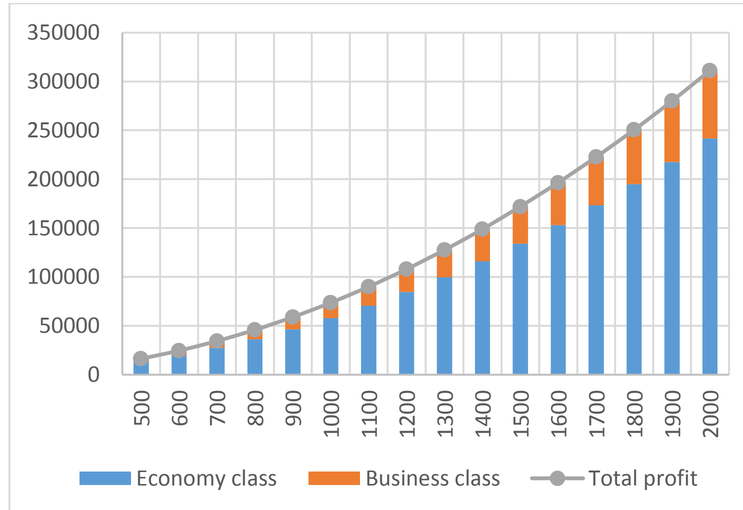
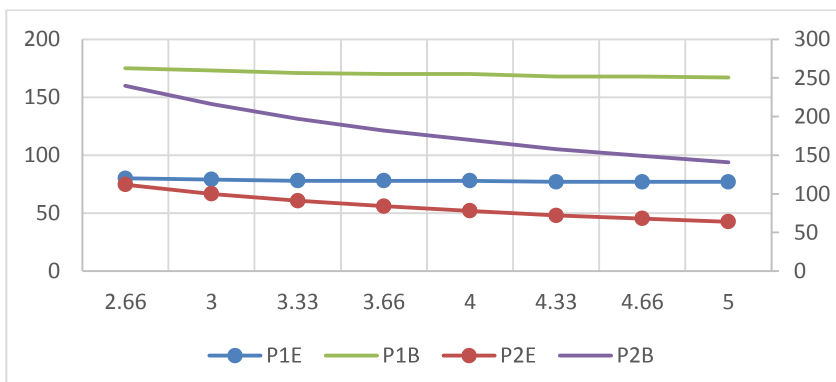


Figure 5. Impact of market volume on capacity of two competing airlines 1 (left) and 2 (right)



[DOI: 10.29252/iors.10.1.43]

Figure 6. Impact of price sensitivity on pricing of two competing airlines

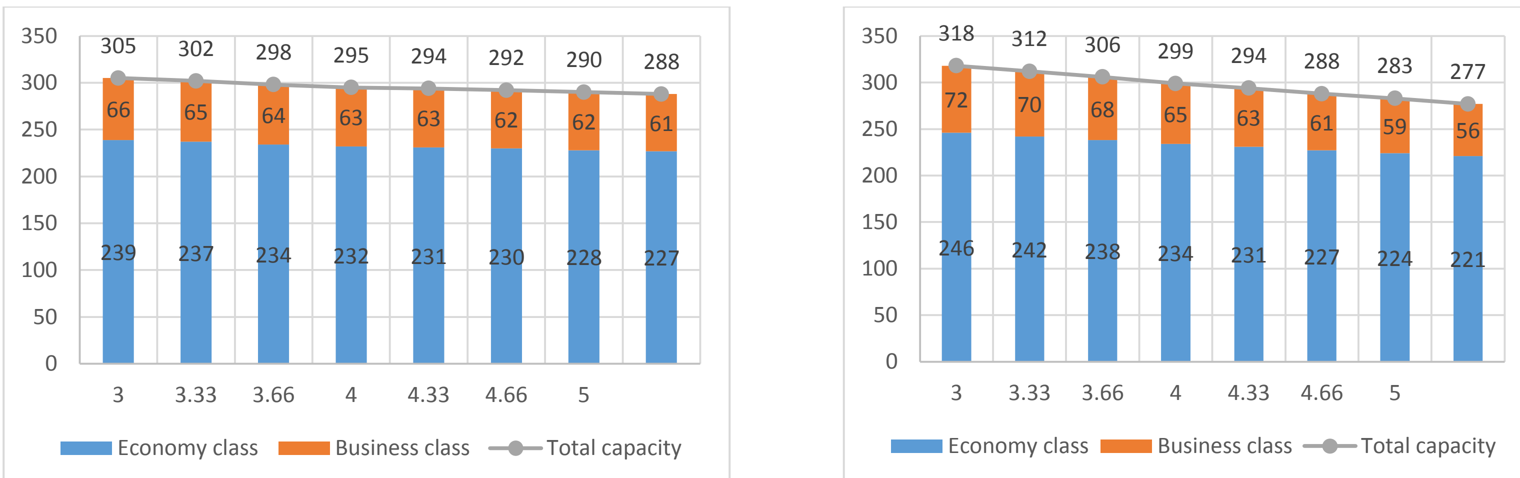


Figure 7. Impact of price sensitivity on capacity of two competing airlines 1 (left) and 2 (right)

[DOI: 10.29252/iors.10.1.43]

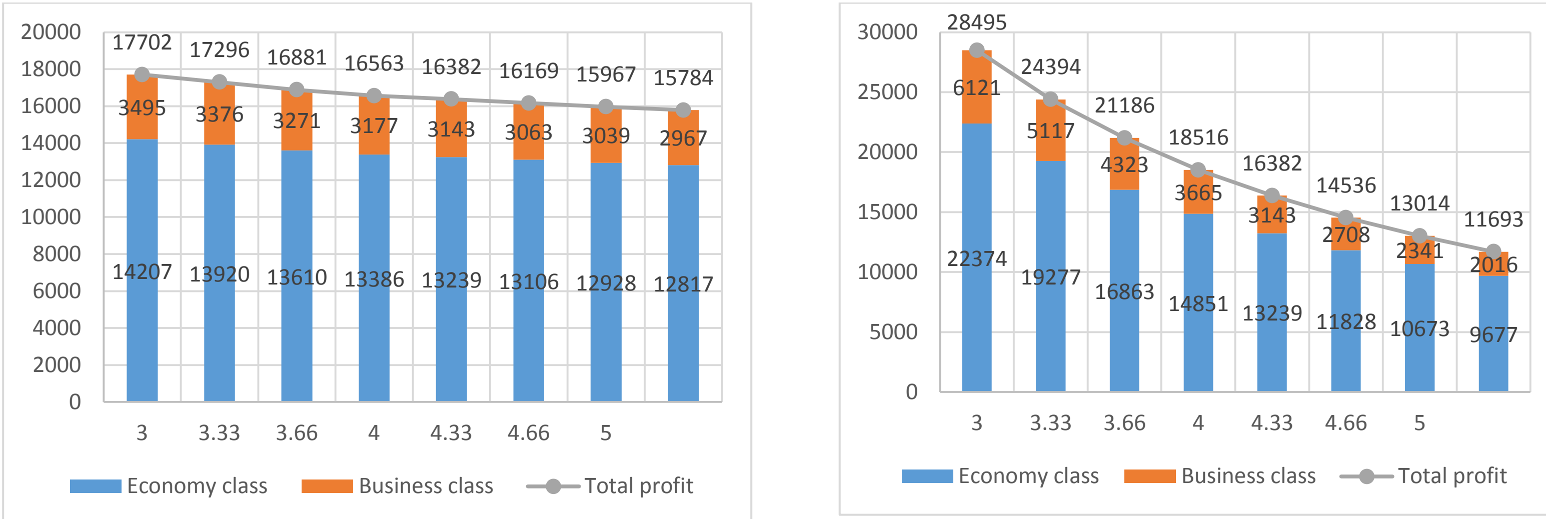


Figure 8. Impact of price sensitivity on profit level of two competing airlines 1 (left) and 2 (right)

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5.2 The impact of price sensitivity

For this section, it is considered that the price sensitivity for economy customers is changing with a constant ratio with respect to the business market. Moreover, regarding the literature it is reasonable to assume that the price sensitivity of economy class is greater than business class. Thus, we set the parameters by $b_{22} = \eta b_{21}$ for airline 2 where b_{21} is changing in the period (0.8, 1.5) and the value of η is 0.3. However, for the airline 1, the price sensitivity for the economic and business classes are set respectively 4 and 1.5. The impact of changing price sensitivity of classes is shows in Table 5. Figure (6) shows that increasing the price sensitivity results in declining the price of the business class relative to the economy class. It can be seen that the ticket price for airline 2 has more reduction than the other airline. Figure (7) presents that the capacity of both airlines is reduced by increasing price sensitivity of classes. In addition, such reduction is more for airline 2 compared to airline 1. In overall view, the change of airlines' profits is similar to the variations of capacity. The profit of airline 2 is greater than airline 1 where the levels of profit for airline 2 is significantly decrease compared to profit level of airline 1 (Figure (8)).

5.3 The impact of coefficient of competition

The airline's coefficient of competition is assumed to be changed according to $\gamma_{1k} = 0.4 + \eta * 0.1$ and $\gamma_{1k} = \gamma_{2k}$. The domain of η is assumed between (0, 3). Table (6) summarizes the results of optimal decisions and profit allocation for both airlines with respect to different coefficients of competition.

Table 6. Model sensitivity analysis to coefficient of competition variation

Coefficient of competition		**Airlines**							
$\gamma_{11} = \gamma_{21}$	$\gamma_{21} = \gamma_{22}$	$V_{11} = V_{21}$	$V_{12} = V_{22}$	Total capacity	$p_{11} = p_{21}$	$p_{12} = p_{22}$	$\Pi_{11} = \Pi_{21}$	$\Pi_{12} = \Pi_{22}$	Total profit
0.4	0.1	227	58	285	77	88	12777	2658	15435
0.5	0.2	231	63	294	78	92	13239	3143	16382
0.6	0.3	235	68	303	79	96	13715	3693	17408
0.7	0.4	239	75	314	80	101	14206	4401	18607

The results of pricing, capacity allocation and optimal profit are similar for both airlines because the changes of coefficient of competition have the same effect on both airlines' decisions. According to Fig (9-a) with increasing coefficient of competition, pricing levels increase on both classes where such increase is more significant in economy class. The similar increase can be seen for capacity control in both classes of airlines concerning to Fig (9-b). Fig. (9-c) shows the increase in airline profitability towards increasing coefficient of competition. In general, the profits from the economic class is higher compared to business class for both airlines.

a) Pricing

b) Capacity

c) Profit

Figure 9. Impact of coefficient of competition on pricing, capacity, and profit of airlines

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6. Conclusion and Future Research

One of the main aspects of revenue management problems for airline industries is analyzing the joint decisions of pricing and capacity allocation under competitive situations. This actual problem is more vital when the competing airlines encounter with demand uncertainty for different segments of their markets. This paper investigates capacity allocation and pricing decisions for two competing airlines that should optimally allocate their seats to economy and bossiness classes.

We consider this problem as Two Competitive Airlines' Revenue Management Problem (TCARMP) where a flexible partition of business and economy cabins is existed and the capacity decision is determined simultaneously by optimal pricing for both cabins. We considered a general linear price-dependent demand function with additive uncertainty and then obtain joint optimal decisions of pricing and capacity level, and optimal profit for each airline.

After providing necessary proofs and developing the procedure of optimization, a numerical study was developed to understand how market situations and competitive conditions affect the price and capacity decisions and additionally profit levels of airlines. We found that increasing market volumes of airline demands results in higher prices, capacities and profits where this impact is different by increasing pricing sensitivities. In both cases, changing the market volume and price sensitivities have more strong effects on increase and decrease of optimized levels of economy classes. Moreover, intensified competition leads to increased prices, and profits and results in lower level of capacities.

Future research should focus on expanding this model with actual constraints about fixed or variable cabin size. Moreover, the structure of TCARMP provide an appropriate basic model to analyze real networks with more direct/indirect and physical/electronic competing distribution channels in airline industries.

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